Ph.D. Qualify Exam: **Classical Electrodynamics**

Feb. 2012

1. (30%) Explain the following terms qualitatively and quantitatively.

(a) Lorentz gauge and Coulomb gauge (5%)

(b) Poynting theorem (5%)

(c) Skin depth (5%)

(d) Group velocity and phase velocity (5%)

(e) Retarded Green function (5%)

(f) Synchrotron radiation (5%)

2. (10%, 10%) Green function

(a) What are Green's first identity and Green's theorem?

(b) For a point charge outside a grounded conducting sphere, find the Green function $G(\mathbf{x},\mathbf{x}')$ that satisfies Dirichlet boundary condition. [Hint: the method of images.]

3. (10%, 10%) Two concentric conducting shells of inner and outer radii a and b ($b>a$), respectively. The outer shell is connected to a given potential V, while the inner shell is grounded $V(a,\theta)=0$.

(a) If $V(b,\theta)=V_0$ (constant), find the potential at $r<a$, $a<r<b$, and $r>b$.

(b) If $V(b,\theta)=V_0\cos^2\theta$, find the potential everywhere between the shells ($a<r<b$). [Hint: use Legendre polynomials $P_0(x)=1,\ P_1(x)=x,$ and $P_2(x)=(3x^2-1)/2$].

4. Find the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ generated by a charged particle q, if

(a) the charged particle is moving with a constant velocity $\mathbf{v}=v_0\hat{\mathbf{x}}$. (10%)

(b) the charged particle is moving with a constant acceleration $\mathbf{a}=a_0\hat{\mathbf{x}}$. (10%)

[Hint: Liénard-Wiechert potentials and fields for a point charge.]

5. If $\mathbf{E}$ and $\mathbf{B}$ are perpendicular in the laboratory and $|E|=2|B|$, find a reference frame such that the field is pure electric or magnetic? If yes, what is the velocity of the reference frame relative to the laboratory? (10%)

[Hint: Gaussian unit $\mathbf{E}'_{\|}=\mathbf{E}_{\|},\ \mathbf{E}'_{\perp}=\gamma_0\left(\mathbf{E}_{\perp}+\frac{\mathbf{v}_0}{c}\times\mathbf{B}_{\perp}\right)$ and $\mathbf{B}'_{\|}=\mathbf{B}_{\|},\ \mathbf{B}'_{\perp}=\gamma_0\left(\mathbf{B}_{\perp}-\frac{\mathbf{v}_0}{c}\times\mathbf{E}_{\perp}\right)$]