

Qualification Exam [Classical Mechanics] (Feb. 2012)

1. (5%) A car moves on a straight road with a constant speed of $50km/h$. The radius of each tire is $50cm$. The tires roll without slipping. Calculate the acceleration of the point on the top of the tire.
2. (10%) What is the total cross section for the elastic scattering of a beam of particles of radius r from a fixed solid sphere with radius R ?
3. (15%) Consider two coupling oscillators with the same mass m and spring constant k . The potential energy is

$$V = k(x_1^2 + x_2^2)/2 + \epsilon x_1 x_2$$

where ϵ is the coupling constant and x is the coordinate of the oscillator. Find the eigenfrequencies of vibration.

4. (15%) a pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k . Find Lagrange's equations of motion. (Do not solve these equations.)

5. (15%) A photon γ hits an electron at rest, producing an electron-positron pair:

$$\gamma + e^- \rightarrow e^+ + e^- + e^-.$$

Calculate the minimum energy of the incident photon. The electron's rest mass is $0.5 \text{ MeV}/c^2$.

6. (15%) A rain drop falls through a cloud collecting mass as it falls. Assume that the drop is spherical and that the mass increased at a rate proportional to the cross sectional area of the drop. If the drop starts from rest where it is infinitely small, what is the acceleration?
7. (10%) A particle of mass m and initial speed v_0 is scattered by a fixed center of force. The perpendicular distance between the center and the line along the initial velocity is d (impact parameter). The force is repulsive and the magnitude of this force is mc/r^2 where c is a positive constant and r is the distance of the particle from the center. Calculate the minimum value of r .

8. (15%) The potential energy of a particle is $V = kx^2/2 + bx^4/4$ where k and b are constants. Find Hamilton's equations of motion. (Do not solve these equations).