Qualifying examination – Statistical Mechanics

Spring, 2013

1. Briefly explain the physical meaning of following items (20 points)
   (a) Basic assumption in Statistical Mechanics
   (b) Canonical ensemble and Grand canonical ensemble
   (c) Bose-Einstein condensation
   (d) Fermi pressure
   (e) Phonons

2. (About partition function, Gibbs paradox, ideal classical gas, and entropy, 20 points)
   (a) Write down the partition function for an ideal classical gas of N identical particles of individual mass m, confined in a container of volume V and temperature T. (5 points)
   (b) Solve for the corresponding free energy and use it to derive the chemical potential μ, pressure P, and entropy S. (9 points, You may need the formula for the Gaussian integral, $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$)
   (c) Take two such containers of the same gas and open the partition to allow them to mix. What is the entropy change after the mixing? (6 points) Explain the results you have)

3. (About quantum statistics, black-body radiation, 20 points)
   (d) It is known that photons obey Bose-Einstein distribution $f_{\mathrm{BE}}(\varepsilon) = \dfrac{1}{e^{\beta\varepsilon} - 1}$ where $\beta \equiv 1/k_B T$ and chemical potential $\mu = 0$. $\varepsilon = pc$ is the photon energy of momentum p (c is speed of light). Write down the total energy stored in the electromagnetic waves inside a cubic black box of length L and temperature T.
   (e) Show that the total energy emitted in all frequencies per second per cm^2 from the surface of this black box is proportional to T^4 (you do not need to do the full calculation).

4. (About phonons, Debye and Einstein models, 20 points)
   (f) Describe Einstein's theory of the heat capacity of a solid, and explain why his theory disagrees with the experimental data for real crystals at very low temperatures. (10 points)

(g) Describe Debye's alternative construction of the same problem, and show his prediction for the heat capacity at very low and very high temperatures, respectively. (10 points)

5. (About Fermions, quantum pressure, 20 points)

(h) Show that the Fermi pressure for an ideal Fermi gas obeys $P = 2U/(3V)$ where U denotes the internal energy and V is the volume.

(i) Derive P as a function of the particle density n at zero temperature.