Qualification Exam for PhD Candidates (spring 2013)
Classical Mechanics

You should derive your results step by step.

1. (15) Consider the elastic scattering of two particles with masses M and m ($M > m$). The speed of M is V and the particle m is at rest. Find the maximum scattering angle θ of M after collision.

2. (15) A particle with rest mass M decays at rest into another particle with rest mass m and two photons. Calculate the maximum kinetic energy of the particle with mass m in the theory of relativity.

3. (15) A bucket of water is set spinning about its symmetry axis with constant angular velocity ω. Determine the shape of the water in the bucket.

4. (15) Write down the Hamiltonian for a single particle with potential energy U in three-dimensional Cartesian, cylindrical and spherical coordinates. No derivation is required.

5. (15) Consider a damped oscillator. An external driving force is applied to the oscillator. The equation of motion is

$$d^2x/dt^2 + 2\beta dx/dt + \omega_0^2 x = A\cos(\omega t).$$

Determine the particular solution of this equation.

6. (10) Consider a string with linear mass density ρ (mass per unit length) and tension force τ. (a) Write down the wave equation for $q(x,t)$, where q is the displacement of the string. (b) What is the general solution of this wave equation? No derivation is required.

7. (15) A sphere of radius r is constrained to roll without slipping on the lower half of the inner surface of a hollow cylinder of radius R. Determine the Lagrangian function and the Lagrange's equation of motion. Find the frequency of small oscillation.