Qualify Exam for Classical Mechanics

1. (20%) Consider two coupling harmonic oscillators with the same magnitude of instrinsic spring constant k but with different masses $m_2 = 2\ m_1$. The potential energy is given by

$$V(x_1, x_2) = \frac{k}{2}(x_1^2 + x_2^2) + \epsilon x_1 x_2\ ,$$

where x_1 and x_2 are the coordinates of particle 1 and particle 2 respectively. Calculate the frequencies of the eigen modes. Hint: First make a transformation so that the kinetic energy terms are isotropic!

2. (20%) Calculate the minimum energy needed for an incident photon γ to hit an electron at rest, producing an electron-positron pair:

$$\gamma + e^- \rightarrow\ e^-\ +\ e^+\ +\ e^+\ .$$

The energy of the electron may be put at $0.5\text{MeV}/c^2$.

3. (20%) The potential energy of an anharmonic oscillator with mass m is given by

$$V(x) = \frac{k}{2}x^2 + \epsilon x^4$$

where x is the coordinate of the oscillator. (1). What is the Hamilton's equation for the system? (2). Assuming that ϵ is small, calcuate the correction to the frequency for small oscillations up to lowest orders in ϵ.

4. (20%) A stone falls without initial velocity into a 300 m deep mine shaft at a latitude of 60° north. How far does it deviate from the vertical?