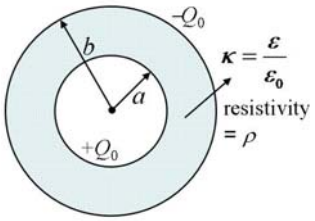
Ph.D. Qualify Exam **Classical Electrodynamics** Fall 2012

1. (25 %) A hollow sphere made of dielectric material with dielectric constant κ and resistivity ρ has inner radius a and outer radius b as shown in the right figure. At time $t = 0$, positive charge $+Q_0$ and negative charge $-Q_0$ are placed uniformly on the inner and outer surfaces.

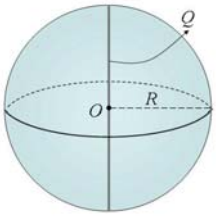


(a) At $t = \infty$, find the charge distribution on the sphere.

(b) Between $t = 0$ and $t = \infty$, how much Joule heat is dissipated due to the charge flow?

(c) Show that the charge on the inner surface decays exponentially and determine the time constant.

2. (20 %) Consider a hollow grounded sphere of radius R in which there is a uniform line charge of total charge Q located on the z-axis between the north and south poles.



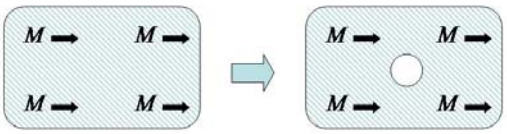
(a) Verify the charge density within the sphere ($r < R$) can be written as: (in spherical polar coordinates)

$\rho(r,\theta,\phi) = \frac{Q}{R}\frac{1}{4\pi r^2}\left[\delta(\cos\theta - 1) + \delta(\cos\theta + 1)\right]$, where δ is the Dirac delta function with normalization condition: $\int_0^{\pi} \delta(\cos\theta \pm 1)\cdot d(\cos\theta) = 1$.

(b) Suppose the potential inside the sphere has been calculated as:

$\Phi(r,\theta,\phi) = \frac{Q}{4\pi\varepsilon_0 R}\left\{\ln(\frac{R}{r}) + \sum_{l=1}^{\infty}\frac{4l+1}{2l(2l+1)}\left[1 - \left(\frac{r}{R}\right)^{2l}\right]P_{2l}(\cos\theta)\right\}$, find the surface charge density on the sphere.

3. (25 %) A large magnet originally has a uniform magnetization M (magnetic moment per unit volume).



(a) Find the magnetic field B inside the magnet. (Hint: you can start from a uniformly magnetized sphere and then make the radius very large.) (b) Now a small hole with radius R is cut inside the magnet. Find the magnetic field inside and outside the hole. You may take the center of the hole as origin.

4. (30 %, 5 % each) Explain the following items qualitatively and quantitatively.
   (a) Lorentz gauge and Coulomb gauge.
   (b) Kramers–Kronig relations.
   (c) Electric polarizability.
   (d) Circular polarization of light.
   (e) Diamagnetic and paramagnetic substances and explain the reasons causing their properties.
   (f) Explain why the sky is blue during the daytime and is red at sunset.