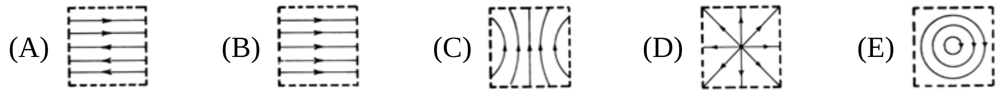
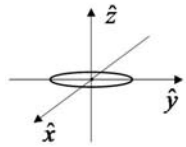
1. (25 %) [Maxwell equations and EM waves]

   (a) Write down the differential form of the four Maxwell equations.

   (b) The figures are static magnetic field lines. Which one(s) violate Maxwell's equation? Explain.



   (c) If magnetic monopoles exist, which equation(s) will be modified? And how?

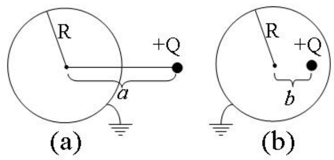


   (d) A small current loop lying on the *x-y* plane is driven by a AC current $I = I_0 \cos \omega t$. Describe the angular distribution of the emitted power of the radiation.

   (e) The average intensity of the sunlight at Earth's surface is 1400 W/m^2. Estimate the magnitude of the magnetic field produced by the sunlight. $\mu_0 = 4\pi \times 10^{-7}$ N/A^2

2. (25 %) [Electrostatics]

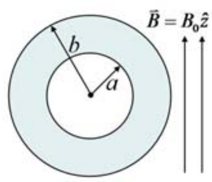
   (a) A point charge Q is placed outside a grounded conducting hollow sphere. Find the net force between the point charge and the sphere. (Hint: you may find an image charge $q' = -(RQ/a)$, placed a distance R^2/a to the right of the sphere center. But first you need to prove this arrangement can satisfy proper boundary conditions.)



   (b) Now the point charge is moved inside the sphere and is at a distance *b* from the center. Find the force between the point charge and the sphere. (Hint: again, find an image charge outside the sphere with negative charge.)

3. (25 %) [Magnetostatics]

   A hollow sphere with magnetic permeability μ has inner radius a and outer radius b. The sphere is placed in a uniform magnetic field B_0. Find the field inside the sphere and conclude this can be used for magnetic field shielding when $\mu >> \mu_0$. (Hint: Since no current is present, we can define a scalar potential ψ_m such that $\vec{B} = \nabla \psi_m$, and $\nabla^2 \psi_m = 0$. Due to ϕ symmetry, the general solution is $\psi_m = \sum_{l=0}^{\infty} \left(A_l r^l + B_l r^{-(l+1)} \right) P_l(\cos\theta)$. Then apply continuity conditions for $\vec{B}$ and $\vec{H}$ at $r = a$ and $r = b$. In this case, only $l = 1$ exists.) In spherical coordinate: $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r \sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$, and the Legendre polynomials are $P_0(x) = 1,\ P_1(x) = x,\ P_2(x) = \frac{1}{2}(3x^2 - 1)...etc$



4. (25 %) Explain the following items:

   (a) Lorentz gauge and Coulomb gauge.
   (b) Kramers-Kronig relations.
   (c) Linear and circular polarization of light.
   (d) Rayleigh scattering.
   (e) Multipole expansion of electric potential.
   (f) Maxwell stress tensor.
   (g) Electric polarizability and magnetic susceptibility.
   (h) Synchrotron radiation.