Qualifying Examination – Statistical Mechanics

Fall, 2013

1. (some fundamental concepts in statistical mechanics, 30 points)
   Please explain in brief the following terminologies in Statistical Mechanics.
   (a) Definition of temperature in statistical mechanics
   (b) Debye model versus Einstein model
   (c) Fluctuation-dissipation theorem
   (d) Liouville's theorem
   (e) Pauli paramagnetism

2. (ideal Fermi gas in 3-D, 30 points)
   (a) Considering an ideal Fermi gas of spin S=5/2 at zero temperature, calculate the total internal energy U by summing the electron energy all the way up to the Fermi energy ε_{F}
   (b) Calculate the Fermi Pressure, *P*
   (c) Derive the Pauli magnetic susceptibility in the limit of zero magnetic field.

3. (classical gas and negative temperature, 20 points)
   Consider a collection of N particles sitting on some crystal lattice in which the energy of each particle can assume only two distinct values, 0 and E $(E>0)$. n_0, n_1 are the occupation numbers of these two energy levels, respectively.
   (a) Write down the partition function at a given temperature, *T*.
   (b) Find the values of n_0, n_1 when in equilibrium, and calculate their mean fluctuations.
   (c) Calculate the total energy, *U* , and the entropy, *S*.
   (d) Define the absolute temperature T as $\frac{1}{k_{\mathrm{B}}T} \equiv \frac{1}{k_{\mathrm{B}}}\frac{\partial S}{\partial U}$. Show that T can be negative, and explain when it may happen.

4. (Gibbs paradox and entropy, 20 points)
   (a) Calculate the free energy for an ideal classical gas of N identical particles of individual mass m, confined in a container of volume V and temperature T.
   (Hint: the Gaussian integral $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$)
   (b) Take two such containers of DIFFERENT gas and open the partition to allow them to mix. What is the entropy change after the mixing?
   (c) Had the two gases in (b) been the same, what is the entropy change due to the mixing? Explain what problem happens and how to resolve it.