Qualifying Examination - Quantum Mechanics

Fall, 2013

1. (important terminologies in quantum mechanics, 30 points)
   Please explain in brief the following terminologies in QM.
   (a) Aharonov-Bohm effect
   (b) Clebsch-Gordan coefficients
   (c) Hyperfine interaction
   (d) Selection rules (for angular momentum, for instance)
   (e) Spontaneous emission
   (f) WKB approximation

2. (Delta-function potential, 20 points)
   (a) Find the ground state energy for attractive delta-function potentials $V(x) = -\alpha \cdot \delta(x)$
   (b) Using above results, present an argument why there must be a bound state for a general 1D short-ranged attractive potential (not delta function).
   (c) Find the ground state energy for $V(x) = -\alpha \cdot \delta(x - L/2) - \alpha \cdot \delta(x + L/2)$.
   (d) Draw a schematic plot to show how the– ground-state energy changes with L, but there is no need to solve for its exact value except at the asymptotic cases of $L = 0$ and $L = \infty$.

3. (Spin, 15 points)
   An electron of spin $\left| S_x = \frac{1}{2} \right\rangle$ is injected with velocity $(v,0,0)$ into $x \geq 0$ regime where there is a magnetic field $(0,0,B)$ (while $B = 0$ elsewhere).
   (a) Assuming the electron follows a classical trajectory, what will be the radius of its trajectory ? What is the time, T, it will need to eventually exist the regime of finite magnetic field ?
   (b) Write down the Hamiltonian to describe the spin degree of freedom (quantum mechanically) in the co-moving frame.
   (c) What is its spin state after time, T ?

4. (time-dependent and –independent perturbation theories, 20 points)
   Consider a perturbation $H' = \alpha x^4$ to the harmonic oscillator problem.
   (a) Show that the first-order correction to the unperturbed eigen-energies are
   $$E_n^1 = \frac{3\hbar^2\alpha}{4m^2\omega^2}\left[1 + 2n + 2n^2\right].$$ (8 points)
   (b) Argue that no matter how small α is, the perturbation expansion will break down for some large enough n. What is the physical reason? (4 points)

(c) If we are careful with how the perturbation is turned on; say, $H' = \alpha x^4 e^{-t^2/\tau^2}$ between $t = -\infty$ and $t = \infty$, what is the probability that the oscillator originally in the ground state ends up in the state $|n\rangle$ at $t = \infty$? (8 points)

5. (Addition of angular momentum, 15 points)
Derive all spin eigenstates for two identical bosons of spin-1.