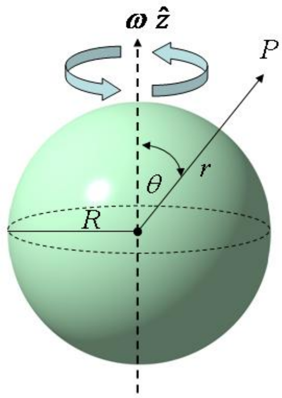
1. (20 %) Explain the following items:
   (a) Linear and circular polarization of light
   (b) Kramers-Kronig relations
   (c) Cherenkov radiation
   (d) Synchrotron radiation
   (e) Skin depth
   (f) Pseudovector (axial vector)

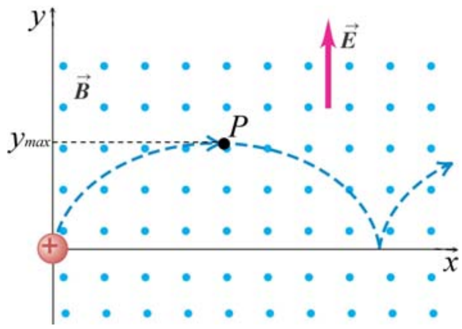
2. (20 %) [Maxwell equations and EM waves]
   (a) Write down the differential form of the four Maxwell equations.
   (b) Derive the wave equations in free space for the electric field from Maxwell equations. You may need this identity $\nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$.
   (c) Show that the electric field and magnetic field are perpendicular to each other in an electromagnetic wave.
   (d) A laser beam at 532 nm has an intensity of 1 mW/mm^2. Find the root-mean-square value of its electric field and magnetic field. ($\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N·m^2, $\mu_0 = 4\pi \times 10^{-7}$ N/A^2)

3. (20 %) [Magnetostatics] A hollow spherical shell of radius R, carrying uniform surface charge density σ is spinning at constant angular velocity ω as shown in the figure. (a) What is the surface current density $\vec{K}$ (current per unit length) at a point on the shell with coordinate (R, θ, ϕ)? (Notice: $\vec{K}$ is a vector, and your answer must include the direction) (b) Find the magnetic field $\vec{B}$ at point P whose coordinate is (r, θ, ϕ). Express your answer for both P inside and outside the shell. (hint: you may find the vector potential $\vec{A}$ or directly use Biot-Savart law as $B(\vec{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{\vec{K}(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}da'$.



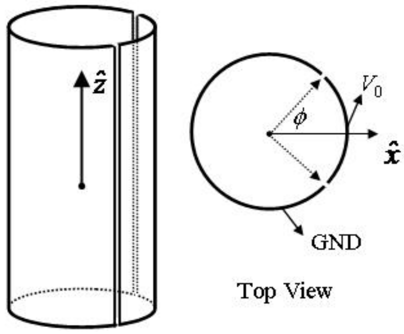
However, since $\nabla\times\vec{B} = \mu_0\vec{J} = 0$ at point P where no current is present, we can define a scalar potential Ψ such that $\vec{B} = \nabla\Psi$, and $\nabla^2\Psi = 0$. Due to ϕ symmetry, you can take the result from separation of variables and the solution of the Laplace equation in this case is $\Psi = \left[A_1 r + \dfrac{B_1}{r^2}\right]\cos\theta$. Then apply appropriate boundary conditions. You may need the gradient operator in spherical coordinate: $\nabla f = \dfrac{\partial f}{\partial r}\hat{r} + \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}\hat{\theta} + \dfrac{1}{r\sin\theta}\dfrac{\partial f}{\partial \phi}\hat{\phi}$.

4. (15 %) [Force on a charged particle] A particle with mass m and charge $+q$ starts from rest at the origin under a uniform electric field $\vec{E} = E\,\hat{j}$ and a uniform magnetic field $\vec{B} = B\,\hat{k}$. Neglecting earth's gravity, the particle is accelerated to the $+y$-direction by the E field, but the magnetic force bends it to the right and the resulting trajectory is shown in the figure. At point P, the particle reaches its maximum height called $y_{\max}$. Determine the curve of the trajectory and express $y_{\max}$ in terms of m, q, E, B.



5. (25 %) [Electrostatics]

A very long hollow conducting tube with radius R is cut into two parts. The right part, which is one-fourth of the whole tube ($\phi = -\frac{\pi}{4}$ to $\frac{\pi}{4}$), is kept at potential V_0 and the left part is kept at ground ($V = 0$). (a) Find the electric potential $V(r,\phi)$ inside the tube. (b) Calculate the surface charge density $\sigma(R,\phi)$ on both parts of the tube and hence determine the capacitance per unit length. The solution of Laplace equation in cylindrical coordinate with z-symmetry is:



$V(r,\phi) = \sum_{n=1}^{\infty} (A_n r^n + B_n r^{-n})(C_n \cos n\phi + D_n \sin n\phi)$. You may also need the orthonormal condition for Fourier series:

$$\int_0^{\pi} \sin n\phi \cdot \sin m\phi \cdot d\phi = \int_0^{\pi} \cos n\phi \cdot \cos m\phi \cdot d\phi = \begin{cases} 0, & if\ n \neq m \\ \frac{\pi}{2}, & if\ n = m \end{cases}.$$