Qualification Exam [Classical Mechanics]

1. (20 %) Consider a system of two coupled oscillators with the same mass m and spring constant k. The potential energy is

$$V(x_1, x_2) = \frac{k}{2}\left(x_1^2 + x_2^2\right) + \epsilon x_1 x_2$$

where ϵ is the coupling constant and x_1, x_2 are the coordinates of the two oscillators. Find the eigenfrequencies of vibration.

2. (20 %) Find Hamilton's equations of motion for an unharmonic oscillator system for a particle of mass m and the potential energy is given by

$$V(x) = \frac{k}{2}x^2 + \frac{a}{4}x^4$$

where k and a are constants. (You don't have to solve these equations)

3. (20 %) A satellite of mass m was orbiting around the earth in a stable circular orbit with a velocity of v. It was then hit by a space debris vertically with an impulse of Δp. Find the motion of the satellite after the incident.

4. (20 %) A charge particle with mass m enters into a static electro-magnetic field with a velocity v in the x-direction. The electric field is in the y-direction and the magnetic field is in the z-direction: $\mathbf{E} = (0, E, 0), \quad \mathbf{B} = (0, 0, B)$. Find the motion of the charge particle.

5. (20 %) A proton hits a proton at rest, producing antiprotons:

$$p + p \;\rightarrow\; p + p + p + \bar{p}$$

Calculate the minimum energy of th incident proton. The proton's rest mass is 1 GeV/c^2.