Quantum Mechanics Qualification
Fall, 2014.

1. (5% each)Briefly answer the following questions:

   - What is Hilbert space?
   - What is the value of $\hbar$ (and specify the unit clearly)? And give one example how to experimentally determine $\hbar$.
   - In a spherically symmetrical potential, a state is described by a normalized wave function $\psi(r,\theta,\phi) = R(r)\times\frac{1}{\sqrt{2}}[\frac{3}{5}Y_3^2 - i\frac{4}{5}Y_3^1 + iY_3^{-1}]$. What is $\langle\psi|L_x^2+L_y^2|\psi\rangle$?
   - What is the Hermitian conjugate of the operator $x\frac{\partial}{\partial x} - iy\frac{\partial}{\partial z}$?
   - What are the Clebsch-Gordan coefficients?
   - How the magnetic resonance imaging (MRI) works?
   - What is the Aharonov-Bohm effect?
   - What is Bell's inequality?

2. (15%) Two particles of the same mass m are confined to move in one dimension and they are connected by a spring with spring constant k. Suppose that the total momentum of the system is p, find all possible total energies for the following cases: (i) two particles are different (ii) two particles are identical fermions (iii) two particles are identical bosons.

3. (8%+7%) A point particle of mass m is subject to the following central potential

$$V(r) = \begin{cases} \infty, & r < r_1 \\ -\frac{\hbar^2}{m}\frac{1}{r^2}, & r_1 < r < r_1 + a \\ V_0(>0), & r_1 + a < r < r_1 + a + \triangle \\ 0, & r_1 + a + \triangle < r\,. \end{cases}$$

   (a) In the limit of $V_0 \to \infty$, find the corresponding eigen-energies and the normalized wavefunctions for $l = 1$ bound state. Recall that the Laplacian operator is given by

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

   in the spherical coordinate.
   (b) Now consider the case that V_0 is finite but much larger than the ground state energy obtained in (a). If initially the particle was in the first excited state, what properties (e.g. energy, wave function, angular momentum, and etc) can you say about the particle which tunnels through the V_0 barrier and escapes?

4. (15 %) A particle is initially in its ground state in a one-dimensional harmonic oscillator potential. At $t = 0$, a perturbation $V(x,t) = V_0x^3e^{-t/\tau}$ is turned on. Calculate to first order the probability that, after a sufficiently long time ($t \gg \tau$), the system will have made a transition to a given excited state; consider all final states.

5. (15%) Consider the S-wave neutron-neutron scattering where the interaction potential is approximated by $V(r) = V_0 \vec{S}_1 \cdot \vec{S}_2 e^{-r/a}$, where $\vec{S}_1$ and $\vec{S}_2$ are the spin vector operators of the two neutrons, and $V_0 > 0$. Find the differential cross section in the first Born approximation.