Qualifying Exam: **Electrodynamics** March 8, 2015

1. (30%) Explain the following terms qualitatively and quantitatively.

(a) Group velocity and phase velocity (5%)

(b) Lorentz gauge and Coulomb gauge (5%)

(c) Maxwell stress tensor (5%)

(d) Kramers-Kronig relations (5%)

(e) Retarded Green function (5%)

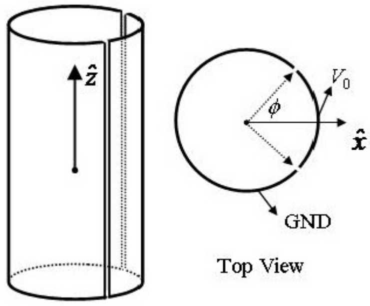
(f) Perfect conductor and super conductor (5%)

2. (10%, 10%) Green function

(a) What are Green's first identity and Green's theorem?

(b) For a point charge q **inside** a grounded conducting spherical shell of radius a, find the Green function $G(\mathbf{x},\mathbf{x}')$ that satisfies Dirichlet boundary condition. [Hint: the method of images.]

3. (10%, 10%) A very long hollow conducting tube with radius R is cut into two parts. The right part, which is one-fourth of the whole tube ($\phi = -\frac{\pi}{4}$ to $\frac{\pi}{4}$), is kept at potential V_0 and the left part is kept at ground ($V = 0$).



(a) Find the electric potential $\phi(r, \phi)$ inside the tube.

(b) Calculate the surface charge density $\sigma(r = R, \phi)$ on both parts of the tube and determine the capacitance per unit length. The solution of Laplace equation in cylindrical coordinate with z-symmetry is:

$$V(r,\ \phi) = \sum_{n=1}^{\infty}\left(A_n r^n + B_n r^{-n}\right)\left(C_n \sin n\phi + D_n \cos n\phi\right).$$

[Hint: $\int_0^{\pi} \sin n\phi \cdot \sin m\phi \, d\phi = \int_0^{\pi} \cos n\phi \cdot \cos m\phi \, d\phi = \begin{cases} 0 & \text{if } n \neq m \\ \frac{\pi}{2} & \text{if } n = m \end{cases}$

4. (10%) If $\mathbf{E}$ and $\mathbf{B}$ are perpendicular in the laboratory and $|E| = 2|B|$, find a reference frame such that the field is pure electric or magnetic? If yes, what is the velocity of the reference frame relative to the laboratory? (10%)

[Hint: Gaussian unit $\mathbf{E}'_{\|} = \mathbf{E}_{\|}$, $\mathbf{E}'_{\perp} = \gamma_0\left(\mathbf{E}_{\perp} + \frac{\mathbf{v}_0}{c}\times\mathbf{B}_{\perp}\right)$ and $\mathbf{B}'_{\|} = \mathbf{B}_{\|}$, $\mathbf{B}'_{\perp} = \gamma_0\left(\mathbf{B}_{\perp} - \frac{\mathbf{v}_0}{c}\times\mathbf{E}_{\perp}\right)$]

5.(10%, 10%)

(a) Starting from the Maxwell equations, derive the dispersion relation (i.e. the relation between the wave frequency ω and the propagation constant k) for a plane electromagnetic wave in an infinite and uniform medium of conductivity σ, electrical permittivity ε, and magnetic permeability μ.

(b) Assume that the medium is a good conductor, derive an expression for its skin depth δ.
[vector formula: $\nabla\times(\nabla\times A) = \nabla(\nabla\cdot A) - \nabla^2 A$]