Qualification Exam for PhD Candidates (Classical Mechanics, March 2015)

1. (10%) Write down (without proof) the general solution of the wave equation
$$\frac{\partial^2 F(x,t)}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 F(x,t)}{\partial t^2} = 0$$

2. (15%) A particle with mass M decays at rest into a photon and another particle with mass m. Use the theory of relativity to calculate the total energy (including the rest energy) of the particle with mass m.

3. (15%) Consider the elastic scattering of two particles with masses M and m where $M > m$. The speed of M is V and the particle m is at rest. Find the maximum scattering angle θ of M after scattering.

4. (15%) Consider two coupled oscillators with the same mass m and spring constant k. The potential energy is
$$V = k(x_1^2 + x_2^2)/2 + \epsilon x_1 x_2$$
where ϵ is the coupling constant and x_i are the coordinates of the oscillators. Find the angular frequencies of vibration.

5. (15%) Consider the motion of a particle with mass m in a central force field and use the spherical polar coordinates (r, θ, ϕ). The potential energy is $V(r)$. (a) Write down the Lagrangian of the system. (b) Derive the Hamiltonian from the Lagrangian. (c) Write down the Hamilton's equations. (Do not solve the equations).

6. (15%) A rocket is fired at 60^0 to the local vertical line with an initial speed $v_0 = \sqrt{GM/R}$ where M is the mass of earth and R is its radius. Find the maximum distance from the center of the earth. The effect of rotation of earth can be neglected.

7. (15%) A ball of radius r and mass m is placed at rest on the top of a hemisphere of radius R. The moment of inertia of the ball about an axis passing through the center is $I = (2/5)mr^2$. If the ball rolls down from the top without slipping, at what height the ball loses contact with the hemisphere?