Qualification Exam for PhD Candidates (Classical Mechanics, September 2015)

1. (15%) Two particles move about each other in circular orbits under the influence of gravitational forces, with a period T_0. Their motion is suddenly stopped and then allowed to fall into each other. They collide after a time T_1. Calculate the ratio T_1/T_0.

2. (10%) What is the total cross section for the elastic scattering of a beam of particles of radius r from a fixed solid sphere with radius R?

3. (15%) Calculate the minimum total energy of a proton which hits another proton at rest to produce an anti-proton in the process

$$p + p \to p + p + p + \bar{p}.$$

   The rest energies of proton and anti-proton are 938 MeV, but you may use 1 GeV in your calculation.

4. (15%) The Hamiltonian of a harmonic oscillator is

$$H = \frac{p^2 + m^2\omega^2 q^2}{2m} = E.$$

   (a) Write down the Hamilton-Jacobi equation (do not solve it). (b) Use the method of action-angle variables to calculate the frequency of oscillation.

5. (15%) Consider a damped oscillator. An external driving force is applied to the oscillator The equation of motion is

$$d^2x/dt^2 + 2\beta dx/dt + \omega_0^2 x = A\cos(\omega t).$$

   Determine the particular solution of this equation.

6. (15%) Three stars with the same mass M form an equilateral triangle that rotates around the triangle's center as the stars move in a common circle around that center. The triangle has edge length L, what is the speed of the stars?

7. (15%) At the origin $x = y = 0$ a machine can shoot a ball with constant speed v_0 at any angle. Find the boundary of the region which can be reached by the ball.