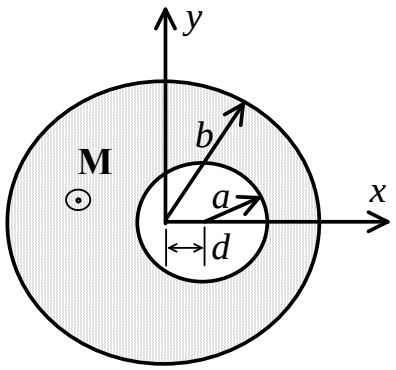
Qualifying Exam: **Electrodynamics** Sep. 20, 2015

1. (20%) Explain the following terms qualitatively and quantitatively.
   (a) Plasma frequency (5%)
   (b) Lorentz gauge and Coulomb gauge (5%)
   (c) Retarded Green function (5%)
   (d) Synchrotron radiation (5%)

2. (10%, 10%) Green theorem and application
   (a) Use the Green's 1st identity to prove the uniqueness theorem for the Poisson's equation with Dirichlet boundary condition.
   (b) For a point charge q **outside** a grounded conducting spherical shell of radius a, find the Green function $G(\mathbf{x}, \mathbf{x}')$ that satisfies Dirichlet boundary condition. [Hint: the method of images.]

3. (10%, 10%) A long cylindrical bar magnet of radius b has a hole of radius a bored parallel to, and centered a distance d from the cylinder axis. The magnetization **M** is uniform throughout the remaining magnet and is parallel to the axis.
   (a) Find the volume current density $\mathbf{J}_M$ inside the magnet and surface current density $\mathbf{K}_M$ in the hole.
   (b) Find the magnitude and the direction of the magnetic-flux density in the hole.
   [Hint: Use Ampere's law and the principle of superposition].



4. (10%, 10%) An ideal circular parallel plate capacitor of radius *a* and plate separation $d \ll a$ is connected to a current source by axial leads. The current in the wire is $I(t) = I_0 \cos \omega t$.
   [Hint: Calculate to the zeroth order in powers of the frequency and $\omega a/c \ll 1$]
   (a) Calculate the electric and magnetic fields between the plate and neglecting the effects of fringing fields.
   (b) Show that the equivalent series circuit has $C \simeq \pi \varepsilon_0 a^2 / d$ and $L \simeq \mu_0 d / 8\pi$.

5. (10%, 10%) A circularly polarized plane wave $\mathbf{E}(\mathbf{x}, t) = E_0(\boldsymbol{\varepsilon}_{||} + i\boldsymbol{\varepsilon}_{\perp})e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}$ is incident on a plane surface between two media of different dielectric properties, where $\boldsymbol{\varepsilon}_{||}$ and $\boldsymbol{\varepsilon}_{\perp}$ are the electric field parallel and perpendicular to the plane of incident, respectively. The media below and above the plane $z = 0$ have the permeabilities and permittivities μ_0, ε and μ_0, ε', respectively.
   (a) When the angle of incident is equal to Brewster's angle, $i_B = \tan^{-1}(n'/n)$, does the reflected radiation polarize linearly, elliptically, or circularly? Explain in details.
   (b) When the wave is incident normally $i = 0$, find the refracted (transmitted) wave $\mathbf{E}'(\mathbf{x}, t)$.