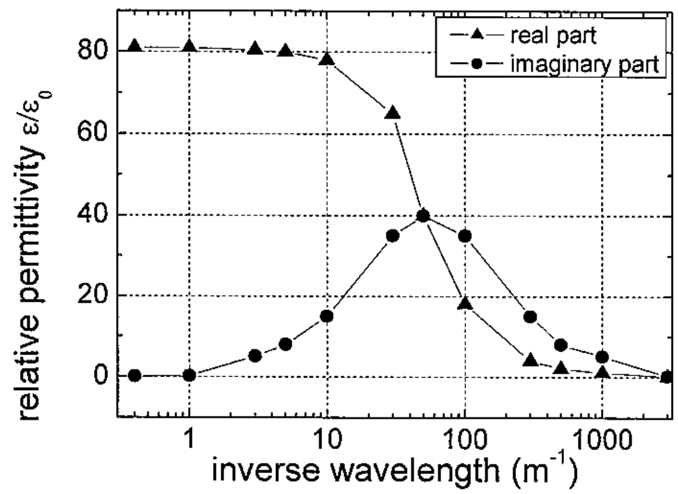
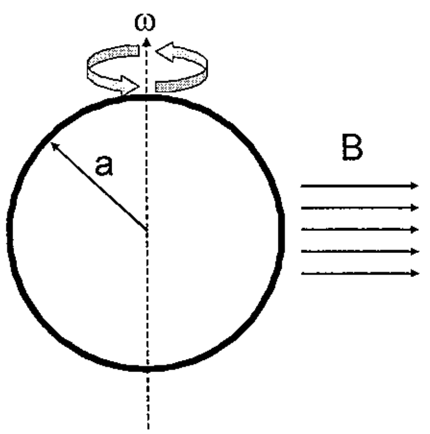
Classical Electrodynamics

1. Maxwell equations:
   (a) Write down the differential form of Maxwell equations. (10%)
   (b) Derive the wave equations in free space for both electric and magnetic fields from Maxwell equations. You may need this identity $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$. (5%)
   (c) Show that the electric field and magnetic field are perpendicular to each other in an EM wave. (5%)
2. Pure water behaves as a dielectric where the electric permittivity and magnetic permeability are ε and μ. Suppose $\mu = \mu_0 =$ constant and $\varepsilon = \varepsilon_{real} + i \cdot \varepsilon_{imag}$ is a complex number. The relative electric permittivity $\varepsilon_{real}/\varepsilon_0$ and $\varepsilon_{imag}/\varepsilon_0$ are plotted in the figure as a function of inverse wavelength, λ^{-1}. As a convention, the wavelength displayed is the wavelength in vacuum, so you can simply get the frequency by $f = c \times (\lambda^{-1})$. A transverse electromagnetic wave in water with angular frequency ω is propagating in the z direction. Its electric field is proportional to $e^{i(kz-\omega t)}$, and the time-independent part of the electric field satisfies $(\nabla^2 + \mu_0 \varepsilon \omega^2)E = 0$. The wave number k is also a complex number, $k = k_{real} + i \cdot k_{imag}$.



   (a) Explain the physical meaning of an imaginary ε. (5%)
   (b) At low frequency, $\varepsilon_{real}/\varepsilon_0 = 81$ and $\varepsilon_{imag}/\varepsilon_0 = 0$, find the numerical value of the phase velocity of the EM wave. (5%)
   (c) At $\lambda^{-1} = 50\ m^{-1}$, $\varepsilon_{real}/\varepsilon_0 = \varepsilon_{imag}/\varepsilon_0 = 40$, find the numerical value of the attenuation length. (the distance over which the electric field drops to 1/e of its initial value). (5%)
3. A thin metallic ring with mass m, radius a, and resistance R is rotating about an axis perpendicular to a uniform magnetic field B as shown in the figure. Initially the rotation angular velocity is ω_0. Assume the fractional change of the rotation speed per cycle is small.
   (a) Find the energy loss per cycle due to joule heating in the beginning of the rotation. (5%)
   (b) Find the time it takes for the rotation frequency dropping to 1/e of its initial value. (10%)



4. Explain the following items qualitatively.
   (a) Synchrotron radiation (5%)
   (b) Gauge invariance (5%)
   (c) Phase velocity and group velocity (5%)
   (d) Rayleigh scattering (5%)
   (e) Liénard-Wiechert potentials (5%)
   (f) Kramers-Kronig relations (5%)
5. A conducting solid sphere has radius R and total charge Q. Half of the sphere is inside a liquid with electric permittivity ε and half of the sphere is in the air (electric permittivity = ε_0).
   (a) Find the electric field at point P outside the sphere in the air. The distance from point P to the center of the sphere is r and the angle to the axis of the sphere is θ. (10%)
   (b) Find the charge density on the surface of both the upper and lower half spheres. (10%)
   (The general solution of separation of variables for the potential with azimuthal symmetry is $V(r,\theta)=\sum_{l=0}^{\infty}\left(A_l r^l+\frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$, where $P_0(x)=1, P_1(x)=x, P_2(x)=\frac{1}{2}(3x^2-1)$ are the Legendre polynomials.