Qualification Exam for PhD Candidates (Spring 2010)

Statistical Mechanics

1. (5%) What is the fundamental assumption of the equilibrium statistical mechanics?

2. (10%) A crystal consists of N identical atoms. At very low temperature, the interaction between atoms can be neglected. If the ground state of each atom is doubly degenerate, what is the entropy of the crystal at zero absolute temperature?

3. (10%) Consider a classical ideal gas at temperature T. The energy of a molecule is

$$\epsilon = ax^2 + bx + (p_x^2 + p_y^2 + p_z^2)/2m$$

where a and b are constants. Determine the average energy of a molecule.

4. (15%) The Hamiltonian of the one-dimensional Ising model is

$$H = -J \sum_{j=1}^{N-1} \sigma_j \sigma_{j+1}$$

where J is a positive constant, and $\sigma = \pm 1$ (two spin states). Calculate the partition function of this model.

5. (15%) Two identical bodies have the same constant heat capacity C. Their initial temperatures are T_1 and T_2 ($T_1 > T_2$). These bodies are connected to a reversible heat engine, which absorbs heat from the hot body and releases heat to the cold body until both bodies approach to the same final temperature T. Calculate T. You should write down the procedure of your derivation.

6. (15%) A surface with N_0 absorbing centers has $N(\leq N_0)$ gas molecules absorbed on it. The partition function of a single molecule is $a(T)$, and

the interaction between the absorbed molecules can be neglected. Find the chemical potential of the absorbed molecules by constructing the grand partition function.

7. (15%) A one-dmensional quantum harmonic oscillator (whose ground state energy is $\hbar\omega/2$ is in equilibrium with a heat reservoir at temperature T. Determine the mean value of the oscillator's energy.

8. (15%)

   Molecular hydrogen is usually found in two forms, orthohydrogen (parallel nuclear spins) and parahydrogen (anti-parallel nuclear spins). Assuming that each kind of hydrogen is mostly in its lowest energy states, what fraction of hydrogen gas is parahydrogen? The result is a function of T/T_c, where T_c is the characteristic temperature of hygrogen molecule ($T_c = h^2/8\pi^2 Ik$, I is the rotational moment of inertia.)