2010 Fall

2010 Qualify Examination: Classical Mechanics

1. A particle of mass m can slide freely along a wire AB whose perpendicular distance to the origin O is h (see Fig.1). The line OC rotates about the origin at a constant angular velocity $\dot{\theta} = \omega$. The position of the particle can be described by θ and the distance q to the point C. If the particle is subject to a gravitational force, and if the initial conditions are $\theta(0) = 0$, $q(0) = 0$, and $\dot{q}(0) = 0$, answer the following questions.

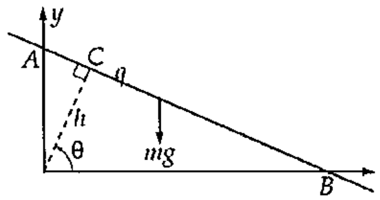

FIG. 1: Setup of problem 1

(a) 10% Find the time dependence of q and sketch $q(t)$ versus t.
(b) 5% If we take $y = 0$ as the zero point for the potential energy, find the total energy at time t.
(c) 5% Find the Hamiltonian for describing the particle at time t.

2. **15%** Apply the Hamilton-Jacobi theory to find $x(t)$ for the simply harmonic oscillator with $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ with the initial conditions: $x(0) = x_0$ and $p(0) = p_0$.

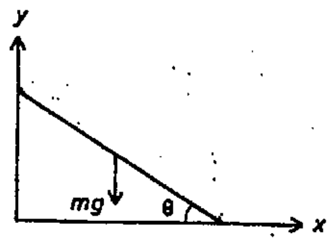

FIG. 2: Setup of problem 3

3. A uniform stick of length $2a$ is held temporarily so that one end leans against a frictionless vertical wall and the other end rests on a frictionless floor making an angle $\theta = \theta_0$ with the floor. When the stick is released, it will slide down under the influence of gravity (see Fig. ??).
(i) 7% Find the expression (as an integral) for the time that it will take for the stick to reach a new angle θ.
(ii) 8% Find the angle when the upper end of the stick leaves the wall.

4. The dynamics for a particle of mass m can be described the Lagrangian

$$L = \frac{1}{2}mv^2 - q\phi(\vec{r}) + q\vec{A}\cdot\vec{v}, \tag{1}$$

where $\vec{r}$ is the displacement vector of the particle, $\vec{v} = \dot{\vec{r}}$, q is a constant, and $\vec{A} = \vec{B}\times\vec{r}/2$ with $\vec{B}$ being a constant vector.
(a) 8% Find the force that acts on the particle. From the form of force, identify a physical system that the Lagrangian may describe.
(b) 10% Find the generalized momentum $\vec{p}$, the Hamiltonian and the total energy for the corresponding physical system.
(c) 5% Suppose that $\vec{B} = B\hat{z}$, $\phi(\vec{r}) = \phi(r)$ (i.e., $q\phi$ is a central-potential), and the motion of the particle is confined to the $x - y$ plane. Find the constant(s) of the motion in the polar coordinates, r and θ, in addition to the total energy.

5. Consider two oscillators with the same mass m and spring constant k. If they are coupled by the potential $V = \epsilon x_1 x_2$, where ϵ is a constant and x_i are coordinates of the oscillators.
(a) 5% Find the condition that two oscillators may not be oscillating.
(b) 4% By choose appropriate unit, we may set $m = 1$ and $k = 3/2$. Suppose $\epsilon = 1/2$. Find the angular frequencies of oscillation.
(c) 10% Following (b), if at $t = 0$, we have $x_1 = 1, x_2 = 0, \dot{x}_1 = 0$, and $\dot{x}_2 = 0$. Find $x_1(t)$ and $x_2(t)$.

6. 8% Consider the pion photoproduction reaction

$$\gamma + p \rightarrow p + \pi_0,$$

where p is the proton, π_0 is the neutral pion, the rest energy is 938 MeV for the proton and 135 MeV for the neutral pion. If the initial proton is at rest in the laboratory, find the minimum energy of the gamma-ray γ such that the reaction can happen in the laboratory.