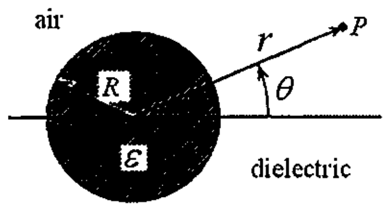
Ph.D. Qualify Exam: **Classical Electrodynamics**

September 2010

1. (10%, 10%) A conducting metal sphere has radius R and charge Q. Half of the sphere is embedded in a dielectric material with permittivity ε and half of the sphere is in the air (ε_0).

(a) Find the electric field at point P outside the sphere in the air. [Hint: use the spherical coordinate with azimuthal symmetry $\mathbf{E}(r,\theta)$.]

(b) Find the charge density on the upper half sphere. [Hint: The Legendre polynomials $P_0(x)=1$, $P_1(x)=x$, and $P_2(x)=(3x^2-1)/2$.]



2. (7%, 7%, 6%)

(a) What are Green's first identity and Green's theorem?

(b) What are the requirements on the Green function for the Dirichlet and Neumann boundary conditions?

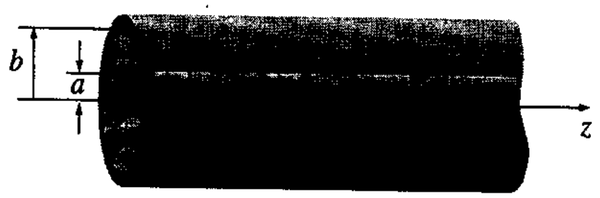
(c) For a point charge outside a grounded conducting sphere, find the Green function $G(\mathbf{x},\mathbf{x}')$ that satisfies Dirichlet boundary conditions. [Hint: the method of images.]

3. (10%, 10%) For an air-filled ($\varepsilon=\varepsilon_0$, $\mu=\mu_0$) coaxial waveguide with inner radius a and outer radius b as shown below, the electric and magnetic fields are:

$$\mathbf{E}(s,\phi,z,t)=\frac{E_0\cos(kz-\omega t)}{s}\hat{\mathbf{s}} \text{ and } \mathbf{B}(s,\phi,z,t)=\frac{E_0\cos(kz-\omega t)}{cs}\hat{\boldsymbol{\phi}}$$

(a) Find the time-averaged energy density u.

(b) Find the energy flow $\mathbf{S}$ (energy per unit area per unit time) along the line.



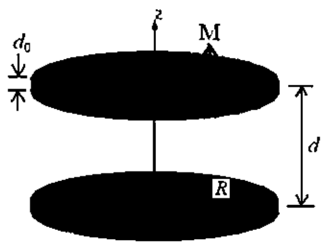
4. (10%, 10%) Consider two thin ferrite disks of radius R and thickness d_0, separated by a distance d ($>>d_0$). Assume the ferrite disks carry a uniform magnetization, $\mathbf{M} = M_\gamma \hat{z}$.

(a) Find the bound surface currents $\mathbf{K}_b$ on the outer surfaces of the ferrite disk and the bound volume current density $\mathbf{J}_b$.

(b) Find the magnetic field at the midpoint of the central axis when $d=R$.

[Hint: The arrangement is similar to a Helmholtz coil.]

[Hint: $\mathbf{A}_M(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\nabla' \times \mathbf{M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' + \frac{\mu_0}{4\pi}\oint_s \frac{\mathbf{M}(\mathbf{x}') \times \hat{\mathbf{n}}'}{|\mathbf{x}-\mathbf{x}'|} da'$].



5. (10%, 10%) The generalized dielectric constant.

Assume there are N molecules per unit volume and Z electrons per molecule. Each molecule contains f_j electrons with binding frequency ω_j and damping factor γ_j, where $Z=\Sigma f_j$. [Hint: Neglect the free electrons.]

(a) Find the polarization $\mathbf{P}$.

(b) Find the form of the complex dielectric constant.