# Qualification Exam for Ph.D Candidacy

## *Statistical Mechanics*

1. Please explain the following concepts clearly (5 points each)

(a) The fundamental assumption of the equilibrium statistical mechanics
(b) Bose-Einstein condensation
(c) Fermi pressure
(d) First and second order phase transition
(e) Ginsburg Landau free energy
(f) Canonical ensemble and grand canonical ensemble
(g) Pauli paramagnetism

2. Consider a collection of noninteracting cold polar molecules. In very low temperature, a single molecule can be treated as a quantum rigid rotor, so that the system Hamiltonian is given by

$$\mathrm{H} = \sum_{\mathrm{i}}^{\mathrm{N}} \left( \frac{\mathbf{p}_{\mathrm{i}}^2}{2\mathrm{m}} + \mathrm{B}\mathbf{J}_{\mathrm{i}}^2 \right)$$

wherem is molecule mass, p is momentum operator, and J is angular momentum operator of the molecule. B is a rotational constant.
(a) (5 pts) Please write down the eigen-energy of a single particle. Explain the definition of your notations, especially those good quantum numbers.
(b) (10 pts) Assuming these polar molecules are classical particle, write down the general expression of the partition function at temperature T.
(c) (5 pts) Using the above expression to calculate the free energy and entropy

3. Consider a system of *N* distinguishable particles, which have two energy levels, $E_0 = -\mu \mathrm{B}$ and $E_1 = \mu B$, for each particles. Here $\mu$ is magnetic moment and *B* is magnetic field. The particles populate the energy levels according to the classical distribution law.
(a) (10 pts) Calculate the average energy of such system at temperature *T*, and
(b) (5 pts) the specific heat of the system.
(c) (5 pts) Calculate the magnetic susceptibility.

4. Consider a 3D system of *N* noninteracting particles, whose energy dispersion is *E* =

$c|\mathbf{p}|$, where $c$ is speed of light. Assuming they are classical particles at an equilibrium temperature *T.* The system volume is *V*. Use the following steps to construct its thermodynamical properties:

(a) (10 pts) Calculate the partition function of such system, *Q*, and then obtain its Helmholtz free energy, *F*.

(b) (5 pts) Use *Q* to show that the average energy, $U = 3N\mathrm{k_B}T$.

(c) (5 pts)Use *F* to show the pressure, $P = Nk_BT/V$. (5%)

(d) (5 pts) Use *F* to calculate entropy *S*. (5%)