Qualification Examiniation on Physics A
For Ph. D. Candidates

3/5/2011

Classical Mechanics (5 problems, 2 pages)

Problem [1] (15%)

Consider a particle of mass m moves in a circular orbit under the influence of a central potential $V(r) = -km/r^n$, where k is a positive constant. For small perturbation to a circular orbit, what is the range of n for a stable orbit?

Problem [2] (20%)

Consider a double pendulum consists of two particles of mass m_1 and m_2 suspended by masless rods as shown in Fig. 1. l_1, l_2 are the lengthes of the rods.

- [1] Assuming that all motion is in a vertical plane, find the Lagrangian and the equations of motion.
- [2] Find the normal-mode frequencies assuming small angles and $m_1 = m_2$, $l_1 = l_2$.

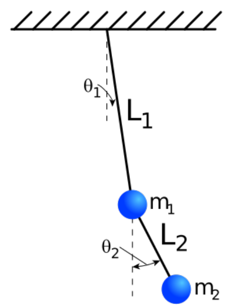

Figure 1:

Problem [3] (20%)

A uniform rod with mass M and radisu ρ rolls without slipping in a bigger cylinder of radisu R as shown in Fig 2.

- [1] Find the Lagrangian, the equation of constraint, and equations of motion.
- [2] Calculate the frequency of small oscillation.

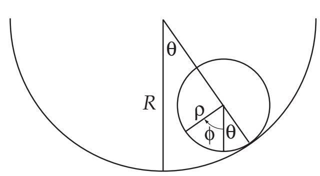

Figure 2:

Problem [4] (15%)

Calculate the minimum total energy of a proton (in the lab system) which hits another proton at rest (in the lab system) to produce an anti-proton in the following process

$$p + p \to p + p + p + \bar{p}.$$

Note: The rest energy of proton and anti-proton are 938 Mev but you may use 1 Gev in your calculation. You must use the relativistic forms of momentum and energy.

Problem [5] (30%)

Consider a simple harmonic oscillator with

$$H = \frac{1}{2m}\left(p^2 + m^2\omega^2 q^2\right).$$

- [1] Apply the canonical transformation with generating function $F = F_1(q, \mathbb{Q}) = \frac{1}{2} m\omega q^2 \cot \mathbb{Q}$ to find the transformed Hamiltonian K. Find the solution for $\mathbb{P}(t)$, $\mathbb{Q}(t)$, and then $q(t)$.
- [2] Apply the Hamiltonian-Jacobi theory to find $q(t)$, assuming the initial conditions $q(0) = q_0$ and $p(0) = p_0$. (Hint: The generating function $F = F_2(q, \mathbb{P}) - \mathbb{Q}\mathbb{P}$ and the transformed Hamiltonian $K = 0$).
- [3] Use action-anle variable to find the frequency of oscillation, starting from defining the constant action variable $J = \oint p dq$.

You might need to use following integral:

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = -\frac{1}{\sqrt{-a}} \sin^{-1}\left(\frac{2ax + b}{\sqrt{b^2 - 4ac}}\right), \; a < 0, b^2 > 4ac.$$