Qualifying examination on Classical Electrodynamics (2/14/2011)

1. (Chap. 1, Boundary-value problems in electrostatics，20 分)
   A point charge q is placed at $(0,0,d)$ inside an ungrounded hollow metal sphere centered at the origin and of inner/outer radius of a/b and $d < a$. Use the **image method** to determine the charge distribution on both the inner and outer surfaces.
2. (Chap. 4, Multipoles, electrostatics of macroscopic media, dielectrics，25 分)
   A charge-neutral dielectric sphere of radius a with dielectric constant ε is placed in an initially uniform electric field which at large distances from the sphere is directed along the z axis and has magnitude E_0. Please
   (a) Solve for the potential both inside and outside of the sphere.
   (b) Find the polarization charge density on the surface of the sphere.
3. (Chap. 7, Plane electromagnetic waves and wave propagation，30 分)
   Copper plates were used as mirrors in ancient times tell us that electromagnetic (EM) waves in the optical range are mostly reflected. We also know that x-ray can be utilized to detect possible dislocations in the fuselage of airplanes. So there must be a characteristic frequency ω which separates these two behaviors.
   (a) Within the Drude classical model, the metallic electrons obey the equation of motion: $m\vec{a} = q\left[\vec{E} + \left(\vec{v} \times \vec{B}/c\right)\right] - m\vec{v}/\tau$ where τ is the mean free time. Since the drift velocity v is much smaller than the speed of light c, you can neglect the force from $\vec{B}$ field. Please derive the expression for **AC conductivity** $\sigma(\omega) \equiv j/E$ where j denotes the current density.
   (b) Derive the AC dielectric constant $\varepsilon(\omega)$ for good conductors $\omega\tau >> 1$. Identify the **plasma frequency** ω_P, which renders ε negative when $\omega < \omega_P$.
   (c) Explain why a negative ε implies dissipations for the EM waves. Determine the **skin depth**, at which the amplitude is diminished by $1/e$ into the metal.
   (d) What are the **Kramers-Kronig relations**? Check if the $\sigma(\omega)$ and $\varepsilon(\omega)$ you obtained in (a) and (b) satisfy them. If not, explain why. Be careful not to assume $\omega\tau >> 1$ since the relations require you to integrate over all ω.
4. (Chap.8, Wave guides and resonant cavities，25 分)
   A coaxial transmission line consists of a long straight wire of radius a and a cylindrical conducting sheath of radius b. The electric and magnetic fields are given by: $\vec{E}(\rho,\phi,z,t) = \dfrac{E_0 \cos(kz - \omega t)}{\rho}\hat{e}_\rho$ and $\vec{B}(\rho,\phi,z,t) = \dfrac{B_0 \cos(kz - \omega t)}{\rho}\hat{e}_\phi$.
   (a) What is the relation between E_0 and B_0?
   (b) Find the line charge density and current on the inner conductor.
   (c) Determine the time-averaged power flow along the line..