

Possible origin of convection flow in granular systems

D.-W. WANG, Y.-C. CHOU and T.-M. HONG

Department of Physics, National Tsing-Hua University - Hsin Chu, Taiwan, ROC

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Abstract. – From the observation of a convection flow in a *single* layer of vertically vibrated sand, we conclude that the amplitude gradient of the vibrating bed, which is naturally expected but often neglected, can be crucial for the same phenomenon in containers with more sand. A simple model is proposed to explain the dependence of the drift velocity on the vibration amplitude and frequency.

The convection phenomena of a noncohesive granular material subjected to vertical vibrations have attracted much attention [1]-[9]. A typical experimental setup is shown in fig. 1 *a*). Glass beads in the container are vertically vibrated through a speaker. Beyond a certain frequency and/or amplitude, the pile of glass beads becomes unstable, and a steady convection flow occurs. The threshold of the vibration acceleration is slightly above the acceleration of gravity, g [1]. Such a convecting flow can occur in many granular materials, such as sand, glass (and metal) beads and milk powder; we shall use the word “sand” to represent the granular material in the rest of this paper. The surface of the convecting pile can have two shapes: *a*) an inclined plain [1], [2], and *b*) a mountain-shape heap with its peak near the centre of the container [2], [5], [9]. We shall focus on the case *a*) where the vibration is relatively moderate, in this paper, *i.e.* the vibration acceleration is less than $3g$ and the frequency is less than 40 Hz.

The inclination angle of the surface of the convecting sand pile is known to be smaller than the critical angle of a stable static sand pile. The skew direction of the surface is reproducible in an experiment under the same conditions, and cannot be predetermined [1]. We shall demonstrate in a simple way how to determine the skew direction, and propose a model to explain the relations between the measurables (such as the convection velocity) and the controlling parameters (such as the vibration frequency and amplitude).

Firstly, it is found that one can change the skew direction of the sand pile by putting a small flexible cushion between the bottom edge of the container and the speaker. Such a change occurs very rapidly. This observation suggests that there is a large driving force for the convection flow originated from the bottom of the container. As a result, the distribution of the vibration amplitude at the bottom of the container might be the key factor to determine the skew direction. That is, the sand particles tend to drift toward the place where the vibration amplitude is the smallest. This has been consistently verified by our measurement using accelerometers (fig. 1 *a*)).

We imagine that there are three stages to complete a cycle of convection. 1) The sand particles move upward when their drift motion at the bottom is blocked by the walls of the container or by the opposite stream of drifting particles for the mountain-shape heap.

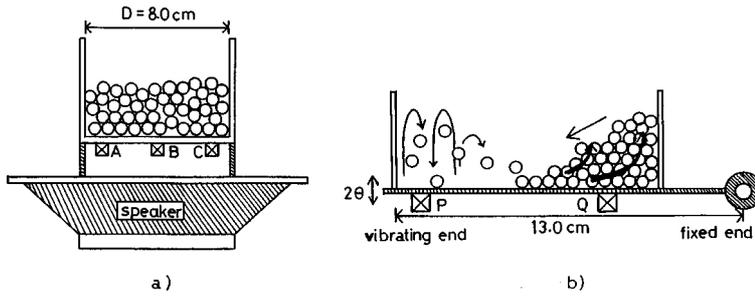


Fig. 1. – a) Schematic diagram of the general vertical vibration apparatus. Accelerometers, A, B, and C are attached to the bed to detect the possible difference of amplitude at different points. b) The few-layer sand system on a modulatory vibrating board. Accelerometers are attached to points P and Q.

2) While on the surface of heap, the vibrated sand particles avalanche when a maximum angle of inclination is reached. 3) At the valley, the sand particles immerse into the pile to fill the vacancies left behind by the drifting particles. These stages are clearly observed in a system consisting of only one layer of sand. If one starts the experiment from a small vibrational acceleration, firstly the sand particles stand still for a vibrational acceleration γ less than acceleration g . For $\gamma = \gamma_{\text{drift}} \geq g$ the drift motion of sand starts until the sand particles are close against the wall. At this stage, the sand still forms a single layer. For $\gamma = \gamma_{\text{heap}} > \gamma_{\text{drift}}$ the sand particles start to pile up against the wall. For an even larger γ_{convect} the sand pile is high enough that the skew angle reaches a critical value, and the convection flow begins.

To further illustrate that the distribution of the vibrational amplitudes is a key factor in producing the convection flow, we designed a gradually vibrated setup (fig. 1 b)). Here, the container of the sand is fixed on a stainless-steel plate with one end fixed by a hinge, which produces an artificial gradient of the vibration amplitude. With no surprise, the sand particles always pile up against the side of the wall near the hinge, and convect at high enough vibrational acceleration. Tilting the stainless-steel plate by 5° from the horizon does not change the above observation. Such a simple design has another advantage, that the drift speed of the sand particles can be measured as a function of the vibrational amplitude (a) and frequency (ω). These relations are shown in fig. 2 a) and b). For sand with an average particle diameter $d = 1$ mm, the drift velocity can be fitted to the form

$$v_{\text{drift}} \propto a^{3.7 \pm 0.3} \omega^{5.4 \pm 0.4}, \quad (1)$$

and it is greater for larger beads [1]-[6]. In many-layer systems, the interaction between layers has to be considered, so a quantitative relation between drift velocity and a (and/or ω) is difficult to derive. However, we did not observe any qualitative difference in drift and convection of the sand particles for many-layer sand piles. The above relation is derived in the following.

One basic difference between the granular system and the continuum system is the way how individual particles dissipate their momentum [2]-[5]. In the granular material, this is through the friction force between the particles themselves and between the particles and the container. For a single-layer system, the latter is important, that is, the energy input from the vibrating board is mainly dissipated by the friction force between the sand and the board. Before going into detailed model estimation, we present in fig. 3 the measured acceleration as a function of time on the bottom of the container. For comparison, fig. 3 a) shows it just before the convection of the sand occurs. Figure 3 b) is for the convecting sand. The bottom of the sand does not touch the vibrating bed during the interval Δt_1 , but hits and perturbs the bed within the interval Δt_0 . The subvibration interval, Δt_2 , is verified to be the damped oscillation of the

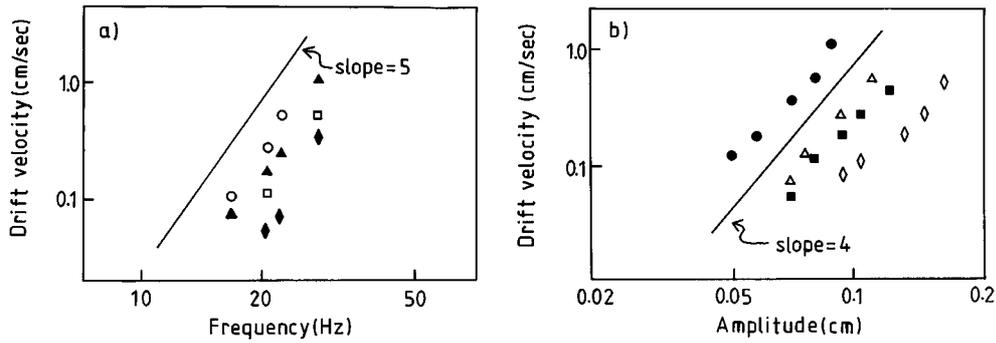


Fig. 2. - *a*) The drift velocity as a function of the vibration frequency, $f = \omega/2\pi$. Data are presented for different amplitudes: $a = 0.07$ cm (full diamonds), 0.08 cm (empty squares), 0.09 cm (full triangles), and 0.10 cm (empty circles). *b*) The drift velocity as a function of the vibration amplitude. Data are presented for different values of frequency: $f = 17.2$ Hz (empty diamonds), 20.4 Hz (full squares), 22.2 Hz (empty triangles), and 27.7 Hz (full circles).

container after colliding with the sand. The frequency of the damped oscillation is independent of the applied frequency, and the damped oscillation vanishes when the acceleration is smaller than g . The same results were observed for experimental configurations shown in fig. 1 *a*) and *b*). What actually happens is that the sand particles are constantly bouncing off the bottom of the container while they drift.

In a steady state, we have

$$dE_k/dt \approx -\mathbf{f} \cdot \mathbf{v}_{\text{drift}}, \tag{2}$$

where \mathbf{f} is the friction force. We can estimate the left-hand side as

$$\frac{dE_k}{dt} \propto \frac{\omega}{2\pi} \frac{m(v\theta)^2}{2} = \frac{1}{4\pi} m\omega(a\omega\theta)^2 = \frac{m}{4\pi L^2} a^4 \omega^3. \tag{3}$$

The velocity v here is the instant velocity of the beads when jumping off the board. Its horizontal component is estimated as the vibration velocity, $a\omega$, multiplied by the vibration angle $\theta = a/L$, where L is the distance from the fixed end to the measurement position. The frequency $\omega/2\pi$ indicates the rate of energy transportation via collision with the board in a unit of time.

The friction force, f , can be represented by $mg\mu_{\text{eff}}$ where the effective friction coefficient μ_{eff} is a function of γ , ω and d because it is affected by the bead ($\gamma \equiv a_{\text{max}}\omega^2$ is the acceleration with a_{max} the maximum amplitude). The most important dimensionless parameter is a_c/d , where $a_c \equiv g/\omega^2$ is the maximum vertical displacement from the equilibrium position of a

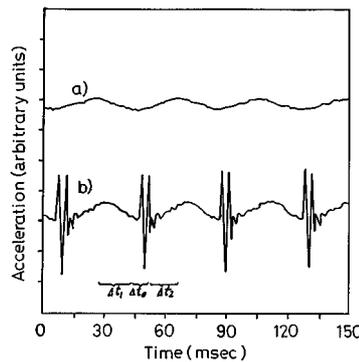


Fig. 3. - Acceleration of the bed as a function of time: *a*) before the convection occurs, *b*) for convecting sand ($f = 25$ Hz, $\gamma = 1.28 g$).

bead before jumping ($a_c < a$, as $a\omega^2 > g$). This is because the less time the beads spend in touch with the board (for a small a_c), and the higher their centre of mass is (for a large d), the more easily for them to roll away from their original positions. We express μ_{eff} as

$$\mu_{\text{eff}}(a_c, d, \mu_0) \approx \mu_0 + \mu_1 \frac{a_c}{d} \approx \mu_0 + \mu_1 \frac{g}{d\omega^2}, \quad (4)$$

in a form linear to a_c/d for simplicity. Here μ_0 and μ_1 depend only on the interface features between the bead and the board. Note that eq. (4) is valid only when the beads drift on the board (when $\omega/2\pi$ ranges from 15 to 80 Hz, a_{max} from 0.01 to 0.45 cm and $\gamma > g$). When the diameter is much larger than a_c , the last term is smaller than μ_0 , and we have $\mu_{\text{eff}} \approx \mu_0$. But in this case the friction effect is unimportant because such large beads can roll on the board freely and be accelerated. Since the small beads (say, d is of the same order as a_c) drift with constant speed in this experiment, we can assume that $\mu_1 a_c/d \approx \mu_{\text{eff}} \gg \mu_0$ in eq. (4). A similar frequency dependence of the mobility coefficient has also been reported by Zik, Stavans, and Rabin [3]. By substituting eqs. (3) and (4) (with μ_0 neglected) into eq. (2), we obtain

$$v_{\text{drift}} \propto da^4 \omega^5, \quad (5)$$

which is consistent with the experimental results shown in fig. 2.

In summary, our model is principally based on the effect of the vibrating force and the size effect in a granular system. In the first part of this letter, we present evidence that the skew direction of the inclined surface of a vibrated and convecting granular system can be determined through the spatial distribution of the vibrational amplitude at the bottom of the container. The sand piles up against the wall where the vibration amplitude is the smallest. Observations in a system containing less than one layer of sand illustrate clearly the initial stage of the convection flow. This result suggests that the origin and the direction of the inclined convecting heap lie in the distribution of the vibration amplitude in the board, instead of the many-particle effect such as thermodynamics [10], [11].

There are threshold vibration accelerations for a bead to drift, heap and convect, respectively, which are measured to obey

$$g \cong \gamma_{\text{drift}} < \gamma_{\text{heap}} < \gamma_{\text{convect}}.$$

The differences between these critical accelerations are about 0.05–0.4 g , which depend strongly on the size of the beads.

When the sand convects, at least for a one-layer system, the motion of the sand particles is governed by the friction force between the particles and between the particles and the wall of the container. The measured functional dependences agree very well with a model estimation based on the energy dissipation through the friction. The thermodynamics and the “effective temperature” concept have little or no role in our approach to explain the convection flow.

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