

## Suppression of phase decoherence in a single atomic qubit

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We study the suppression of noise-induced phase decoherence in a single atomic qubit by employing pulse sequences. The atomic qubit is composed of a single neutral atom in a far-detuned optical dipole trap and the phase decoherence may originate from the laser intensity and beam pointing fluctuations, as well as magnetic field fluctuations. We show that suitable pulse sequences may prolong the qubit coherence time substantially as compared with the conventional spin-echo pulse.

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Suppressing decoherence in a quantum system is of great importance for quantum information processing as well as high-precision spectroscopy. The fault-tolerance quantum computation requires the decoherence rate to be below a threshold level [1]. Low decoherence is also demanded to store quantum information in a quantum memory [2–7]. For high-precision spectroscopy, suppressing decoherence prolongs the measurement time and thus increases the precision of the measurement. In view of achieving long coherence times, many quantum information processing protocols [8–21] and high-precision measurements [22–28] have thus employed the long-lived internal states of ions or neutral atoms.

However, a quantum system cannot be completely isolated from the environment, leading to unavoidable decoherence for quantum states. Therefore a critical question is how to suppress the decoherence to a desired level for various applications. In this paper, we consider the suppression of the phase decoherence in an atomic qubit, which is composed of a single neutral atom confined in a far-detuned optical dipole trap. The qubit is defined by two hyperfine states of the atom. This system is an excellent candidate for quantum computation because it is well isolated from the environment and is also easy to be exploited for storing and processing quantum information. In this system, there are two important types of decoherence mechanisms. The first is the spin relaxation, originating from the inelastic Raman scattering (IRS) of photons from the trapping laser or the spin-exchange collision in hyperfine manifolds [29]. The corresponding decoherence time is known as  $T_1$ . The second type of decoherence mechanism is the fluctuations of laser and magnetic field intensities as well as trap positions, which can modulate the energy splitting between two qubit states and thus lead to phase decoherence of the qubit and loss of quantum information. This type of decoherence is known as dephasing with a decoherence time  $T_2$ . In far-detuned optical traps, the IRS is greatly suppressed because of the large detunings [30]. As a result,  $T_1$  can be very long and  $T_2 \ll T_1$ . The suppression of phase decoherence is hence most relevant to the quantum information processing and quantum measurements in a far-detuned optical trap.

How to suppress the phase decoherence in various quantum systems has attracted much attention both theoretically and experimentally. For many years in the field of nuclear magnetic resonance, applications of external pulse sequences have been investigated in order to refocus the phase diffusion or decouple the qubit from the environment [31]. Some of these techniques have been applied to superconducting qubits where significant enhancement of decoherence time has been observed [32]. Recently, composite pulses have been employed onto an ensemble of atomic qubits [33–35]. Hahn's spin-echo (SE) sequence [36] has also been implemented for an atomic ensemble in an optical dipole trap [37,38] to enhance phase coherence time. Here, we investigate the performance of more elaborate pulse sequences on suppressing the noise-induced phase decoherence of a single atomic qubit. We find that multipulse sequences outperform the conventional SE sequence by orders of magnitude.

Common origins of decoherence for a single atomic qubit in an optical dipole trap are laser intensity fluctuations, beam pointing fluctuations, and magnetic field fluctuations.

(i) *Laser intensity fluctuations.* In a single atomic qubit, magnetic Zeeman sublevels are often exploited as the qubit basis [39–41]. For example, we can define a qubit using  $|\downarrow\rangle = |5S_{1/2}, F_1=1, m_{F_1}=0\rangle$  and  $|\uparrow\rangle = |5S_{1/2}, F_2=2, m_{F_2}=0\rangle$  states of  $^{87}\text{Rb}$  atoms. The energy splitting  $E(r, t)$  of the qubit in an optical dipole trap is related to the intensity of the trapping laser  $I(r, t)$  through

$$E(r, t) = E_H + \frac{\pi c^2 \Gamma}{2 \omega_0^3} \left( \frac{1}{\Delta'_{F_2}} - \frac{1}{\Delta'_{F_1}} \right) I(r, t), \quad (1)$$

where  $E_H$  is the hyperfine splitting between two qubit states without the laser field,  $\Gamma$  is the natural linewidth,  $\omega_0$  is the atomic transition frequency, and  $1/\Delta'_F = (2 + \alpha g_F m_F)/\Delta_{2,F} + (1 - \alpha g_F m_F)/\Delta_{1,F}$ . The quantity  $\alpha = \{1, 0, -1\}$  denotes the polarization of the trapping laser, and  $\Delta_{2,F}$  ( $\Delta_{1,F}$ ) is the detuning with respect to the atomic transition  $\{5S_{1/2}, F\} \rightarrow 5P_{3/2}$  ( $5P_{1/2}$ ). The laser intensity fluctuations,  $I(t) = I_0 [1 + \beta(t)]$ , thus result in temporal fluctuation of the energy splitting  $\delta E(t) = E_L \beta(t)$ , which in turn induces dephasing.

(ii) *Beam pointing fluctuations.* The spatial dependence of  $I(r, t)$  in Eq. (1) for a focused Gaussian beam is given by  $I(r) = I_0 \exp(-r^2/2w_0^2)$ , where  $r$  is the position of the atom

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with respect to the trap center,  $I_0$  is the peak intensity, and  $w_0$  is the beam waist. The beam pointing fluctuations may originate from the air turbulence or mechanical vibration of the mirrors and lenses along the beam path. As a consequence, the position of the trap center  $\gamma(t)$  fluctuates with time, leading to  $r(t) = r_0 - \gamma(t)$ , where  $r_0$  is the actual position of the atom. In experiments, the position fluctuations  $\gamma(t)$  may be suppressed to the order of 10 nm for a typical beam waist of  $\sim 5 \mu\text{m}$ . Since  $r \ll w_0$ , we can approximate the trapping potential by a harmonic trap. For an atom in the ground state of the trap,  $\gamma(t)$  is much smaller than the atom's average position  $\bar{r}_0 \sim \sqrt{\hbar/m\omega} \sim 100 \text{ nm}$  for a typical trapping frequency  $\omega \sim 2\pi \times 10 \text{ kHz}$ . In addition, the beam pointing fluctuations are only significant for frequency  $\bar{\omega}$  below tens of Hz [42]. The atom thus follows the vibration of the trap adiabatically because the moving velocity of the trap  $v_t \sim \gamma(t)\bar{\omega}$  is much smaller than the atom's velocity  $v_a \sim \bar{r}_0\omega$ , leading to the satisfaction of the adiabatic condition

$$\hbar v_t \ll \left| \frac{E_g}{\langle \psi_g | \frac{\partial H}{\partial \gamma(t)} | \psi_e \rangle} \right|. \quad (2)$$

Here  $E_g$  is the energy gap between the ground state  $\psi_g$  and the excited states  $\psi_e$  of the harmonic trap,  $H = p^2/2m + m\omega^2 r^2(t)$  is the Hamiltonian of the system. As a result, the low-frequency beam pointing fluctuations do not induce dephasing in the atom qubit since the atom feels the same trapping potential even if the trap center fluctuates. Moreover, the high-frequency part of the beam pointing fluctuations only leads to negligible dephasing for the atom qubit because of its low magnitude [42]. The dephasing associated with the beam pointing fluctuations is thus not significant. The heating resulted from the beam pointing fluctuations, on the other hand, may induce dephasing but it is negligible within the time scale of the trap lifetime [43].

(iii) *Magnetic field fluctuations.* In the presence of a weak magnetic field  $B_z$ , the energy levels of the atom split linearly according to  $E_B = m_F g_F \mu_B B_z \propto m_F I_B$ , where  $I_B$  is the current of the Helmholtz coil used for generating the magnetic field. Therefore the classical noise of the current source  $\delta I_B(t)$  may give rise to fluctuation of the energy splitting of the qubit, namely,  $\delta E(t) \propto (m_{F_2} - m_{F_1}) \delta I_B(t)$ . In experiments, however, this can be avoided by making use of clock states, such as the superposition state of  $|5S_{1/2}, F_1=1, m_{F_1}=-1\rangle$  and  $|5S_{1/2}, F_1=2, m_{F_1}=1\rangle$  or the  $m_F=0$  Zeeman sublevels in two hyperfine states [37,38,44–47], for the qubit states. For example, the latter has been employed to achieve a coherent time exceeding 15 min for an atomic clock reported in Ref. [48]. As a result, the energy splitting of the qubit is unaffected by the temporal fluctuation of the magnetic field.

To study the dephasing, we consider the following Hamiltonian for a single atomic qubit:

$$\hat{H} = \frac{1}{2} [E_0 + \epsilon(t)] \hat{\sigma}_z, \quad (3)$$

where  $\epsilon(t)$  represents the temporal fluctuation of the energy splitting with respect to the average splitting  $E_0$ . We first

assume that one noise source is dominant. Later on, we will discuss the case in which one needs to take into account multiple noise sources.

In the experiments for studying the decoherence time, one usually prepares the qubit first in the eigenstate of  $\hat{\sigma}_z$ , e.g.,  $|\uparrow\rangle$ , by means of optical pumping. Subsequently, a microwave or two-photon Raman  $\pi/2$ -pulse initializes the qubit in its superposition state  $|\psi(0)\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$  at  $t=0$  with the off-diagonal density-matrix element being  $\rho_{\uparrow\downarrow}(0) = 1/2$ . Then, after a freely evolving time  $t$  in a free-induction decay (FID) experiment, the qubit state becomes

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} (e^{i\phi_{\uparrow}/2} |\uparrow\rangle + e^{i\phi_{\downarrow}/2} |\downarrow\rangle), \quad (4)$$

where  $\phi_{\uparrow} = -\phi_{\downarrow} = -\int_0^t \epsilon(t') dt' / 2$  in a rotating reference frame. The qubit state thus accumulates a phase  $\Delta\phi = \phi_{\uparrow} - \phi_{\downarrow}$  during the free evolution of time  $t$  and the off-diagonal density-matrix element evolves according to

$$\rho_{\uparrow\downarrow}(t) = \rho_{\uparrow\downarrow}(0) \langle e^{-i\Delta\phi(t)} \rangle, \quad (5)$$

where  $\langle \dots \rangle$  denotes averaging over an ensemble of identical systems. For fluctuations whose statistics is stationary, the ensemble average is equivalent to the time average.

To characterize the dephasing for a qubit, we define the decoherence function  $W(t)$  to be

$$W(t) \equiv \frac{|\rho_{\uparrow\downarrow}(t)|}{|\rho_{\uparrow\downarrow}(0)|}. \quad (6)$$

Thus,  $W(t) = 1$  if there is no dephasing and  $W(t) < 1$  if there is dephasing. For a FID experiment, it can then be shown that [49]

$$W^{\text{FID}}(t) = \exp \left( - \int_0^\infty \frac{d\omega}{\pi} S(\omega) \frac{2 \sin^2 \frac{\omega t}{2}}{\omega^2} \right), \quad (7)$$

where  $S(\omega)$  is the power spectrum or the first spectral density of the noise, i.e., the Fourier transform of the correlation function  $S(t) = \langle \epsilon(t) \epsilon(t+\tau) \rangle$  of the noise. The decoherence function is not necessary a Gaussian function, but one can still define the decoherence time  $T_2$  to be  $W(T_2) = 1/e$  for convenience.

Now, we consider simultaneous presence of multiple noise sources  $\beta_i(t)$ . In this case, the correlation function is given by  $S(t_1 - t_2) = \langle \sum_i \epsilon_i(t_1) \sum_j \epsilon_j(t_2) \rangle$ . If the noise sources are uncorrelated, i.e.,  $\langle \epsilon_i(t_1) \epsilon_j(t_2) \rangle = \delta_{ij} S_i(t_1 - t_2)$ , the correlation function can be reduced to  $S(t_1 - t_2) = \sum_i S_i(t_1 - t_2)$ . The power spectrum of the noise is then given by the summation of individual power spectrum,  $S(\omega) = \int_{-\infty}^\infty e^{i\omega t} S(t) dt = \sum_i S_i(\omega)$ , where  $S_i(\omega) = \int_{-\infty}^\infty e^{i\omega t} S_i(t) dt$ . Accordingly, the decoherence function is the product of each decoherence function,  $W(t) = \prod_i W_i(t)$ . We see that the decoherence is dominated by the noise source with shorter dephasing time.

Figure 1 shows the decoherence function for a single atomic qubit in a simulated FID experiment. The decoherence time is found to be  $T_2 \sim 1 \text{ s}$  for the following conditions. The two qubit states are  $|\downarrow\rangle = |5S_{1/2}, F_1=1, m_{F_1}=0\rangle$  and  $|\uparrow\rangle = |5S_{1/2}, F_2=2, m_{F_2}=0\rangle$  states of  $^{87}\text{Rb}$  atom. The atom is

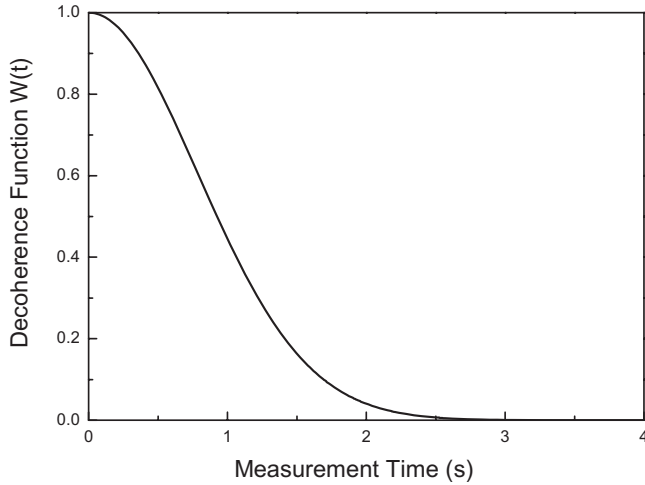


FIG. 1. Decoherence function for a simulated FID experiment in the presence of intensity fluctuation of the trapping laser.

trapped at the bottom of an optical dipole trap that is generated by a YAG laser with a trap depth of  $\sim 500 \mu\text{K}$ . The only relevant classical noise taken into account here is the intensity fluctuation of the trapping laser. The power spectrum is adopted from Ref. [50], which can be approximated by  $S(f)/E_L^2 = 10^{-8.5} f^{-5/3} \text{ Hz}^{-1}$  for frequencies below 1 kHz. As the longest trap lifetime reported thus far is  $\sim 400 \text{ s}$ , we choose the infrared cutoff frequency to be  $\omega_{ir} \sim 0.016 \text{ s}^{-1}$ . We note that the decoherence time depends strongly on the characteristics of the power spectrum. For the power spectrum given in Ref. [51], we obtain  $T_2 \sim 20 \text{ ms}$  for the same trap configuration.

Dephasing in a single atomic qubit may be reversed by applying a sequence of  $\pi$  pulses. The simplest case is a SE sequence in which one applies a microwave or two-photon Raman  $\pi$  pulse at halftime  $\tau$  of the free evolution. By doing this, one can partially cancel the dephasing due to low-frequency ( $< 1/\tau$ ) noise. However, SE becomes less effective when high-frequency noise is present. Furthermore, the imperfection of the  $\pi$  pulse inherently introduces additional phase diffusion onto the qubit state (for example, one applies a “ $\pi + \delta$ ” pulse with  $\delta < \pi$  instead of a  $\pi$  pulse). Accordingly, multipulse sequences may be a better choice for suppressing the dephasing more effectively as well as compensating the phase error of the  $\pi$  pulses.

We consider a general pulse sequence that is composed of  $n$  instantaneous  $\pi$  pulses at time  $t_1, t_2, \dots, t_n \in [0, t]$ . The  $\pi$  pulse rotates the qubit state about the  $x$  axis; therefore the qubit state after the application of the pulse sequence evolves as

$$\begin{aligned}
 |\psi(t)\rangle = & \exp\left[-i \int_{t_n}^t \hat{H}(t') dt'\right] (-i\hat{\sigma}_x) \cdots \\
 & \times \exp\left[-i \int_{t_1}^{t_2} \hat{H}(t') dt'\right] (-i\hat{\sigma}_x) \\
 & \times \exp\left[-i \int_0^{t_1} \hat{H}(t') dt'\right] |\psi(0)\rangle. \quad (8)
 \end{aligned}$$

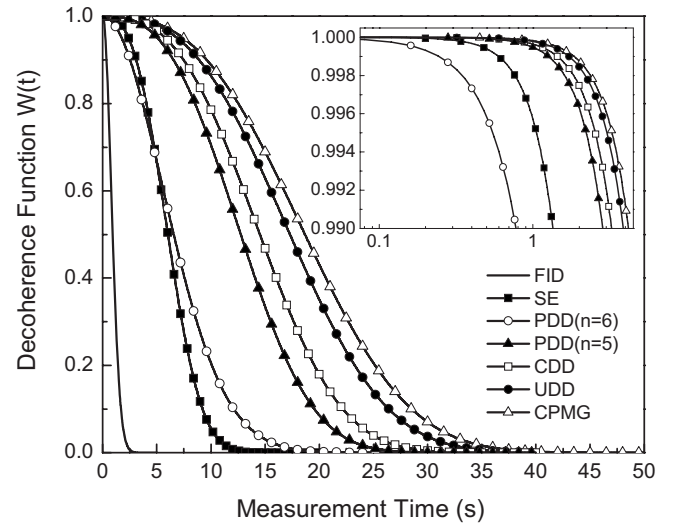


FIG. 2. Decoherence functions with applications of SE, PDD, CDD, UDD, and CPMG pulse sequences. FID is also shown for comparison. The inset shows the short-time performance for various pulse sequences.

The decoherence function defined in Eq. (6) can then be shown to be [49]

$$W(t) = \exp\left(-\int_0^\infty \frac{d\omega}{\pi} S(\omega) \frac{F(\omega t)}{\omega^2}\right), \quad (9)$$

where  $F(\omega t) = \frac{1}{2} |\sum_{k=0}^n (-1)^k (e^{i\omega t_{k+1}} - e^{i\omega t_k})|^2$  corresponds to a certain pulse sequence, which has a specific set of  $t_k$  with  $t_0=0$  and  $t_{n+1}=t$ . In the following, we focus on the performance of various pulse sequences listed below.

(i) *SE pulse sequence.* SE is an efficient technique to reverse the low-frequency dephasing, which exists prior to the application of the  $\pi$  pulse. The pulse sequence comprises a single  $\pi$  pulse at  $t_k = t/2$  ( $n=1$ ) with  $F(\omega t) = 8 \sin^4(\omega t/4)$ .

(ii) *Carr-Purcell-Meiboom-Gill (CPMG) pulse sequence.* CPMG is the  $N$  times repetition of SE sequence [52,53]. For CPMG, we have  $t_k = (k-1/2)t/n$  and  $F(\omega t) = 8 \sin^4(\omega t/4n) G(\omega t) \cos^{-2}(\omega t/2n)$ , where  $G(\omega t) = \sin^2(\omega t/2n)$  for even  $n$  and  $G(\omega t) = \cos^2(\omega t/2n)$  for odd  $n$ .

(iii) *Periodic dynamical decoupling (PDD) pulse sequence.* Dynamical decoupling (DD) sequences are designed to decouple the qubit from the influence of environment. For PDD, the  $n$  pulses are equally distributed over the entire measurement time:  $t_k = kt/(n+1)$  and  $F(\omega t) = 2 \tan^2[\omega t/(2n+2)][1-G(\omega t)]$ . A property of PDD is that only the odd order of the sequence can suppress the low-frequency noise ( $\omega < 2/t$ ) [49].

(iv) *Concatenated dynamical decoupling (CDD) pulse sequence.* CDD is a concatenated DD sequence [54]. The  $l$ th order of the pulse sequence  $\text{CDD}_l(t)$  is defined as  $\text{CDD}_{l-1}(t/2) \rightarrow \Pi \rightarrow \text{CDD}_{l-1}(t/2)$  for odd  $l$  and  $\text{CDD}_{l-1}(t/2) \rightarrow \text{CDD}_{l-1}(t/2)$  for even  $l$ , where  $\Pi$  refers to an instantaneous  $\pi$  pulse and  $\text{CDD}_0(t)$  denotes free evolution for duration  $t$ . As a result,  $F(\omega t) = 2^{2l+1} \sin^2(\omega t/2^{2l+1}) \prod_1^l \sin(\omega t/2^{k+1})$  with  $l \approx \log_2 n$ .

(v) *Uhrig dynamical decoupling (UDD) pulse sequence.*

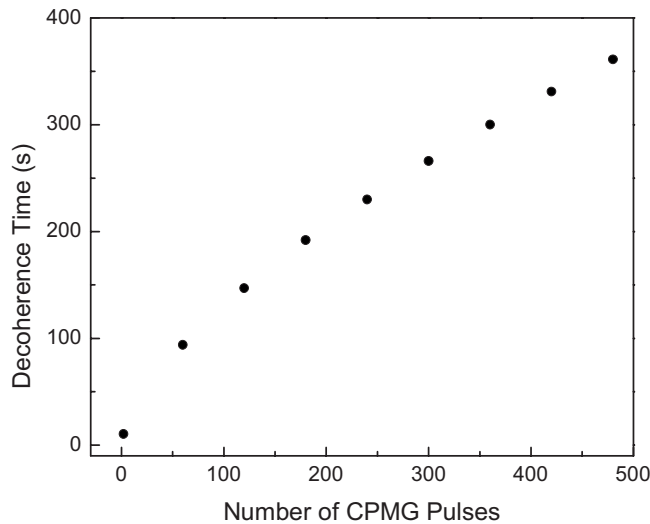


FIG. 3. Decoherence time as a function of number of CPMG pulses applied to a single atomic qubit.

Originally proposed by Uhrig [55], UDD was later shown to be an optimal DD sequence when the delay times between pulses are sufficiently short [56]. For UDD, the sequence is defined as  $t_k = \sin^2[\pi k / (2n+2)]t$  and  $F(\omega t) = \frac{1}{2} |\sum_{n-1}^n (-1)^k \exp\{\cos[\pi k / (n+1)]\omega t / 2\}|^2$ .

The decoherence functions with the applications of various pulse sequences as well as free evolution (FID) are shown in Fig. 2. The number of pulses used during the measurement time is  $n=6$ . For PDD, five-pulse sequence is also shown. One can see that the even order ( $n=6$ ) of PDD sequence is less effective than the odd order ( $n=5$ ) sequence. Due to the presence of a substantial portion of low-frequency

noise in the power spectrum, SE sequence already exhibits a pronounced prolongation of decoherence time. Nonetheless, multipulse sequences (CDD, UDD, CPMG, and odd- $n$  PDD) still outperform SE by prolonging the decoherence time for more than a factor of 20 as compared to FID. Moreover, for short-time performance (inset of Fig. 2), multipulse sequences are clearly more effective than SE. This could be useful when high fidelity but not long coherence time is preferred.

Among different multipulse sequences, CPMG is the most effective sequence in terms of number of pulses. We investigate further prolongation of decoherence time by applying more CPMG pulses. As shown in Fig. 3, the decoherence time increases approximately linearly with the number of pulses. For 50 pulses, the decoherence time is prolonged by a factor of 100; for 500 pulses, the decoherence time is prolonged by a factor of 350. Since the length of a  $\pi$  pulse can be as short as  $\sim 10 \mu\text{s}$ , the decoherence time is eventually limited by the lifetime of the atom in the trap.

In summary, we have examined the performance of variety of external pulse sequences on the suppression of phase decoherence in a single atomic qubit. We find that, at  $n=6$ , pulse sequences ( $n=5$  for PDD) already outperform SE by more than a factor of 2 in terms of decoherence time. Among the pulse sequences considered here, CPMG sequence is optimal for suppressing the phase decoherence induced by the laser intensity fluctuations. We also show that application of large number of CPMG pulses may achieve decoherence time in the regime of minutes.

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- [1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
- [2] C. H. van der Wal, M. D. Eisaman, A. André, R. L. Walsworth, D. F. Phillips, A. S. Zibrov, and M. D. Lukin, *Science* **301**, 196 (2003).
- [3] B. Julsgaard, J. Sherson, J. I. Cirac, J. Fiurášek, and E. S. Polzik, *Nature (London)* **432**, 482 (2004).
- [4] C.-S. Chuu, T. Strassel, B. Zhao, M. Koch, Y.-A. Chen, S. Chen, Z.-S. Yuan, J. Schmiedmayer, and J.-W. Pan, *Phys. Rev. Lett.* **101**, 120501 (2008).
- [5] B. Zhao, Y.-A. Chen, X.-H. Bao, T. Strassel, C.-S. Chuu, X.-M. Jin, J. Schmiedmayer, Z.-S. Yuan, S. Chen, and J.-W. Pan, *Nat. Phys.* **5**, 95 (2009).
- [6] R. Zhao, Y. O. Dudin, S. D. Jenkins, C. J. Campbell, D. N. Matsukevich, T. A. B. Kennedy, and A. Kuzmich, *Nat. Phys.* **5**, 100 (2009).
- [7] P. Maunz, S. Olmschenk, D. Hayes, D. N. Matsukevich, L.-M. Duan, and C. Monroe, *Phys. Rev. Lett.* **102**, 250502 (2009).
- [8] J. I. Cirac and P. Zoller, *Phys. Rev. Lett.* **74**, 4091 (1995).
- [9] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, *Phys. Rev. Lett.* **78**, 3221 (1997).
- [10] S. J. van Enk, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **78**, 4293 (1997).
- [11] G. K. Brennen, C. M. Caves, P. S. Jessen, and I. H. Deutsch, *Phys. Rev. Lett.* **82**, 1060 (1999).
- [12] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **82**, 1975 (1999).
- [13] A. Sørensen and K. Mølmer, *Phys. Rev. Lett.* **82**, 1971 (1999).
- [14] J. Hald, J. L. Sorensen, C. Schori, and E. S. Polzik, *Phys. Rev. Lett.* **83**, 1319 (1999).
- [15] J. I. Cirac and P. Zoller, *Nature (London)* **404**, 579 (2000).
- [16] M. Fleischhauer and M. D. Lukin, *Phys. Rev. Lett.* **84**, 5094 (2000).
- [17] A. Kuzmich, L. Mandel, and N. P. Bigelow, *Phys. Rev. Lett.* **85**, 1594 (2000).
- [18] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, *Nature (London)* **414**, 413 (2001).
- [19] B. B. Blinov, D. Leibfried, C. Monroe, and D. J. Wineland, *Quantum Inf. Process* **3**, 45 (2004).
- [20] D. L. Moehring, M. J. Madsen, K. C. Younge, R. N. Kohn, Jr., P. Maunz, L.-M. Duan, C. Monroe, and B. Blinov, *J. Opt. Soc. Am. B* **24**, 300 (2007).
- [21] L.-M. Duan and C. Monroe, *Adv. At., Mol., Opt. Phys.* **55**,



- 419 (2008).
- [22] M. Kasevich and S. Chu, *Appl. Phys. B: Lasers Opt.* **54**, 321 (1992).
- [23] N. Davidson, H. J. Lee, C. S. Adams, M. Kasevich, and S. Chu, *Phys. Rev. Lett.* **74**, 1311 (1995).
- [24] T. L. Gustavson, P. Bouyer, and M. A. Kasevich, *Phys. Rev. Lett.* **78**, 2046 (1997).
- [25] A. Peters, K. Y. Chung, and S. Chu, *Nature (London)* **400**, 849 (1999).
- [26] M. Takamoto, F.-L. Hong, R. Higashi, and H. Katori, *Nature (London)* **435**, 321 (2005).
- [27] T. Ido, T. H. Loftus, M. M. Boyd, A. D. Ludlow, K. W. Holman, and J. Ye, *Phys. Rev. Lett.* **94**, 153001 (2005).
- [28] M. M. Boyd, A. D. Ludlow, S. Blatt, S. M. Foreman, T. Ido, T. Zelevinsky, and J. Ye, *Phys. Rev. Lett.* **98**, 083002 (2007).
- [29] R. Grimm, M. Weidemüller, and Y. B. Ovchinnikov, *Adv. At., Mol., Opt. Phys.* **42**, 95 (2000).
- [30] R. A. Cline, J. D. Miller, M. R. Matthews, and D. J. Heinzen, *Opt. Lett.* **19**, 207 (1994).
- [31] L. M. K. Vandersypen and I. L. Chuang, *Rev. Mod. Phys.* **76**, 1037 (2005).
- [32] Y. Nakamura, Y. A. Pashkin, T. Yamamoto, and J. S. Tsai, *Phys. Rev. Lett.* **88**, 047901 (2002).
- [33] M. F. Andersen, A. Kaplan, T. Grunzweig, and N. Davidson, *Phys. Rev. A* **70**, 013405 (2004).
- [34] M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, *Nature (London)* **458**, 996 (2009).
- [35] N. Lundblad, J. M. Obrecht, I. B. Spielman, and J. V. Porto, *Nat. Phys.* **5**, 575 (2009).
- [36] E. L. Hahn, *Phys. Rev.* **80**, 580 (1950).
- [37] M. F. Andersen, A. Kaplan, and N. Davidson, *Phys. Rev. Lett.* **90**, 023001 (2003).
- [38] S. Kuhr, W. Alt, D. Schrader, I. Dotsenko, Y. Miroshnychenko, A. Rauschenbeutel, and D. Meschede, *Phys. Rev. A* **72**, 023406 (2005).
- [39] S. Kuhr, W. Alt, D. Schrader, M. Müller, V. Gomer, and D. Meschede, *Science* **293**, 278 (2001).
- [40] N. Schlosser, G. Reymond, I. Protsenko, and P. Grangier, *Nature (London)* **411**, 1024 (2001).
- [41] M. Weber, J. Volz, K. Saucke, C. Kurtsiefer, and H. Weinfurter, *Phys. Rev. A* **73**, 043406 (2006).
- [42] T. A. Savard, K. M. O'Hara, and J. E. Thomas, *Phys. Rev. A* **56**, R1095 (1997).
- [43] K. M. O'Hara, S. R. Granade, M. E. Gehm, T. A. Savard, S. Bali, C. Freed, and J. E. Thomas, *Phys. Rev. Lett.* **82**, 4204 (1999).
- [44] D. M. Harber, H. J. Lewandowski, J. M. McGuirk, and E. A. Cornell, *Phys. Rev. A* **66**, 053616 (2002).
- [45] P. Treutlein, P. Hommelhoff, T. Steinmetz, T. W. Hansch, and J. Reichel, *Phys. Rev. Lett.* **92**, 203005 (2004).
- [46] P. C. Haljan, P. J. Lee, K.-A. Brickman, M. Acton, L. Deslauriers, and C. Monroe, *Phys. Rev. A* **72**, 062316 (2005).
- [47] S. Olmschenk, K. C. Younge, D. L. Moehring, D. N. Matsukevich, P. Maunz, and C. Monroe, *Phys. Rev. A* **76**, 052314 (2007).
- [48] P. T. H. Fisk, M. J. Sellars, M. A. Lawn, and C. Coles, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **44**, 344 (1997).
- [49] L. Cywinski, R. M. Lutchyn, C. P. Nave, and S. DasSarma, *Phys. Rev. B* **77**, 174509 (2008).
- [50] D. Eliyahu, D. Seidel, and L. Maleki, *IEEE Trans. Microwave Theory Tech.* **56**, 449 (2008).
- [51] M. Koch, Diploma thesis, University of Heidelberg, 2007.
- [52] H. Carr and E. M. Purcell, *Phys. Rev.* **94**, 630 (1954).
- [53] S. Meiboom and D. Gill, *Rev. Sci. Instrum.* **29**, 688 (1958).
- [54] W. M. Witzel and S. DasSarma, *Phys. Rev. B* **76**, 241303(R) (2007).
- [55] G. S. Uhrig, *Phys. Rev. Lett.* **98**, 100504 (2007).
- [56] B. Lee, W. M. Witzel, and S. DasSarma, *Phys. Rev. Lett.* **100**, 160505 (2008).