ATOM INTERFEROMETERS WITH BEC

A theoretical analysis based on mean-field approximation

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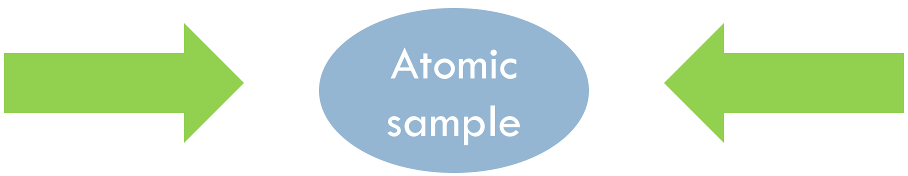
Outline

- What is an atom interferometer?
  - Example: Mach-Zehnder interferometer and the gravity gradiometer at Stanford
- Bose-Einstein condensation of dilute gases
- Atom interferometers with BEC
- Degradation of contrast of interference signals
- Our work
  - Formulation and theoretical tools
  - Origin of contrast degradation
  - Optimization of the contrast of interference signals

Atom interferometers

- Wave nature of atoms
- Atom optics/atom optical elements
  - Use optical pulses as beam splitters and mirror
    - **Very high precision coherent control**
  - Adiabatic manipulation of the trapping potential
- Precision inertial and atomic measurements
  - The interference signal, sensitive to the phases of the wave functions, gives dynamical information.

Bragg diffraction



$$\omega, k \qquad \omega + \delta\omega, k' \qquad k \approx -k'$$

$$V_{\text{opt}}(t) = \hbar\Omega \cos(\delta k z - \delta\omega t)$$

- Resonance condition

$$\hbar\delta k = 2\hbar k$$
$$\hbar\delta\omega = \frac{2\hbar^2 k^2}{m}$$

Coupling between $|2\hbar k\rangle$ and $|0\hbar k\rangle$

- A coherent beam splitter in atom optics

Bragg diffraction

- The effect of the laser pulse

$$U = \begin{bmatrix} \cos\left(\frac{\Omega\tau}{2}\right) & -i\sin\left(\frac{\Omega\tau}{2}\right)e^{i\phi_L} \\ -i\sin\left(\frac{\Omega\tau}{2}\right)e^{-i\phi_L} & \cos\left(\frac{\Omega\tau}{2}\right) \end{bmatrix}$$

- Initial π/2 pulse

consider a 1D case

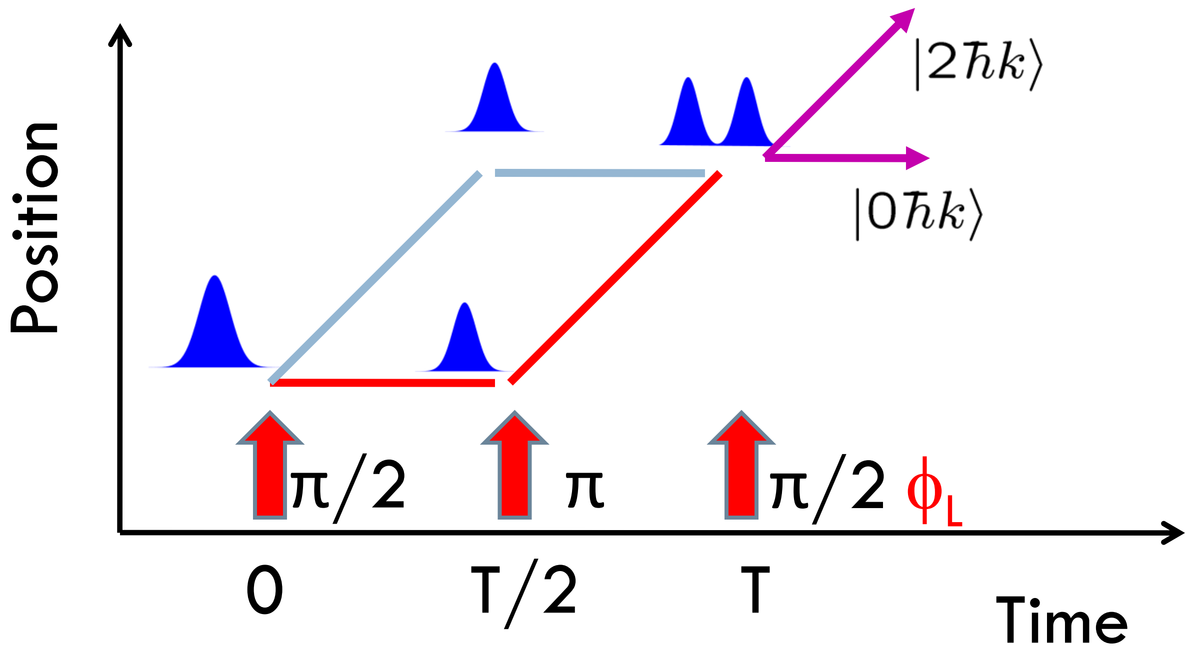
$$\begin{pmatrix} \Psi_{|0\hbar k\rangle} \\ \Psi_{|2\hbar k\rangle} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{i\phi_L} \\ -ie^{-i\phi_L} & 1 \end{pmatrix} \begin{pmatrix} \phi(z) \\ 0 \end{pmatrix}$$

Bragg diffraction

- π pulse – **population inversion**

$$\begin{pmatrix} \Psi_{|0\hbar k\rangle} \\ \Psi_{|2\hbar k\rangle} \end{pmatrix} = \begin{pmatrix} 0 & -ie^{i\phi_L} \\ -ie^{-i\phi_L} & 0 \end{pmatrix} \begin{pmatrix} \phi_{|0\hbar k\rangle}(z) \\ \phi_{|2\hbar k\rangle}(z) \end{pmatrix}$$

Mach-Zehnder interferometer

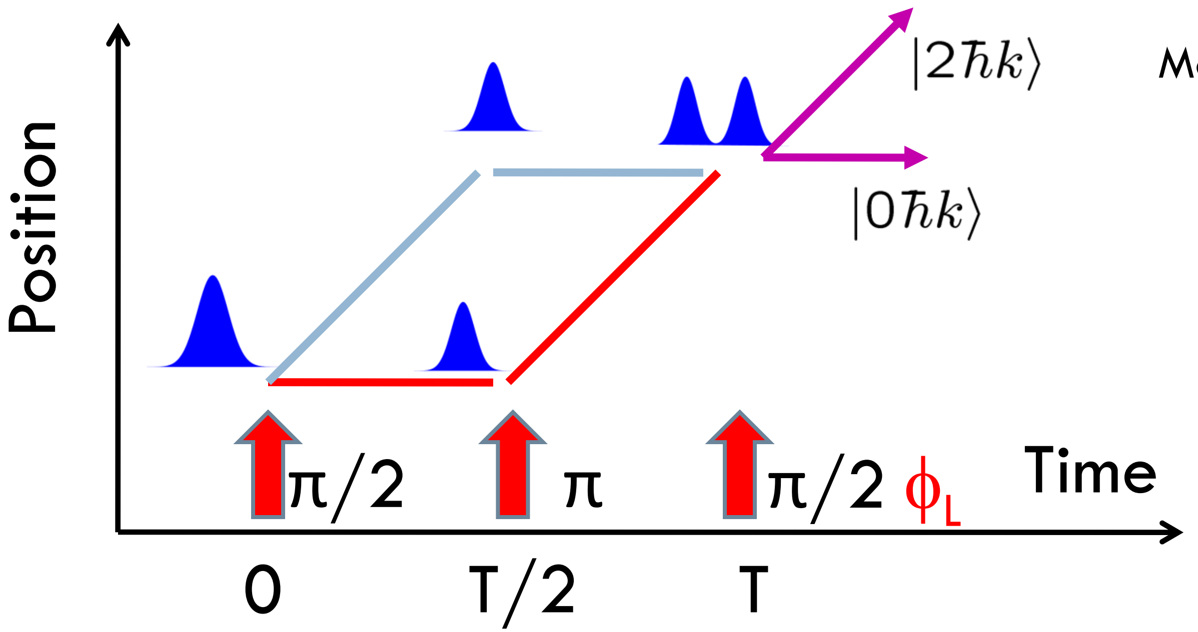


- Beam splitter/mirror → Use optical Bragg pulses
- Interference signals : the population as a function of the laser phase ϕ_L

Bragg diffraction

- Recombining π/2 pulse

$$\begin{pmatrix} \Psi_{|0\hbar k\rangle} \\ \Psi_{|2\hbar k\rangle} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{i\phi_L} \\ -ie^{-i\phi_L} & 1 \end{pmatrix} \begin{pmatrix} f(z) \\ \boxed{f(z-\Delta z)e^{i\phi(z)}} \end{pmatrix}$$



More generally, $g(z)e^{i\phi(z)}$

If the two component are not overlapped in configuration space, the population would NOT depend on ϕ_L

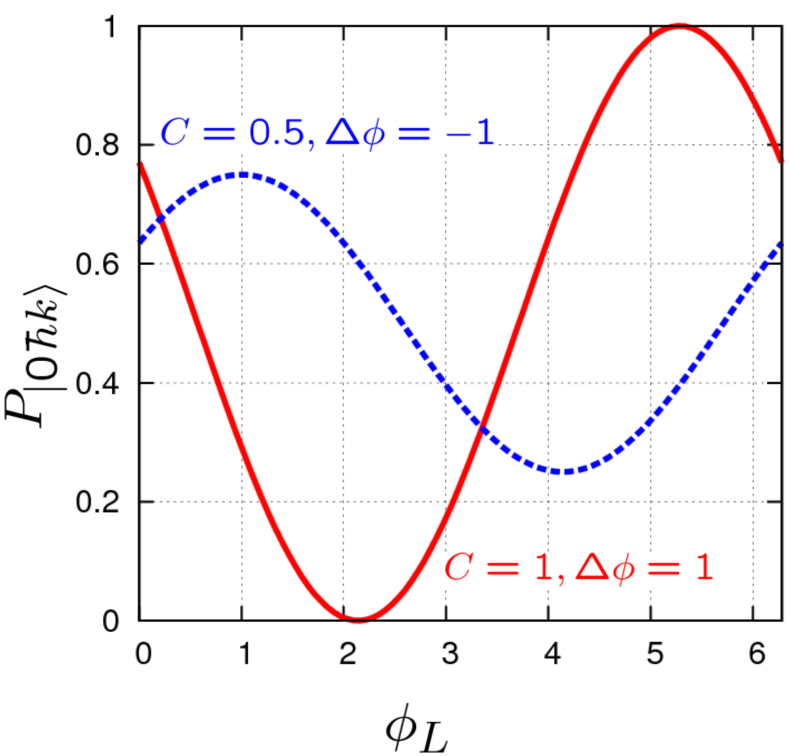
→ zero contrast.

Interference signal and contrast

- Interference signal
- Contrast

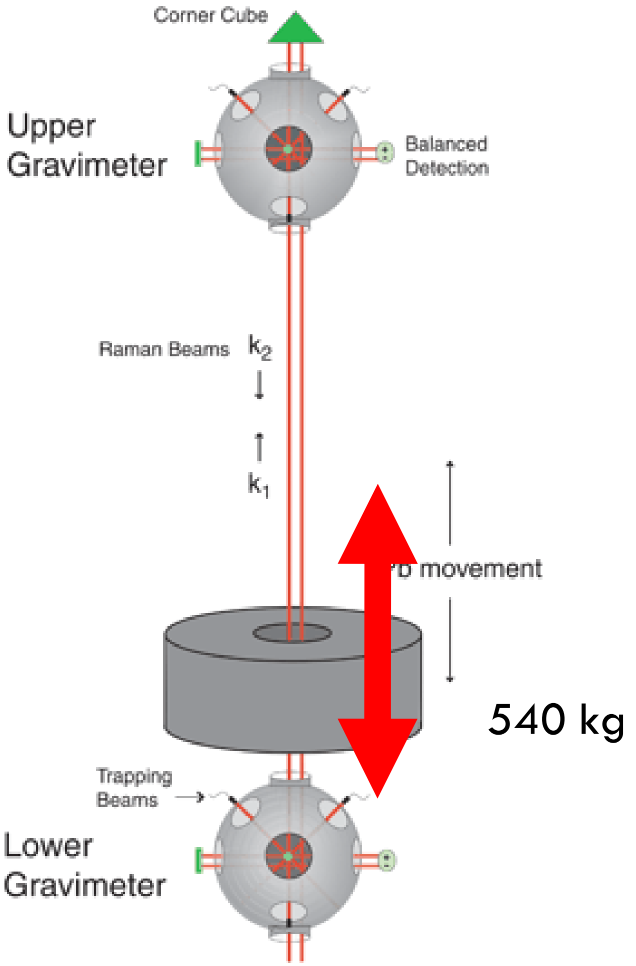
$$P_{|0\hbar k\rangle} = \frac{1}{2}[1 + C\cos(\phi_L + \Delta\phi)]$$

- What are the deciding factors for contrast degradation?
- The phase shift $\Delta\phi$ carries dynamical information useful for applications in inertia measurements.



Gravity graviometer

- Use a gravity gradiometer to measure the differential acceleration of two Cs gases, making a proof-of-principle measurement of the Newtonian constant of gravity.
- The gradiometer is composed of two gravimeters implementing Mach-Zehnder type atom interferometers.



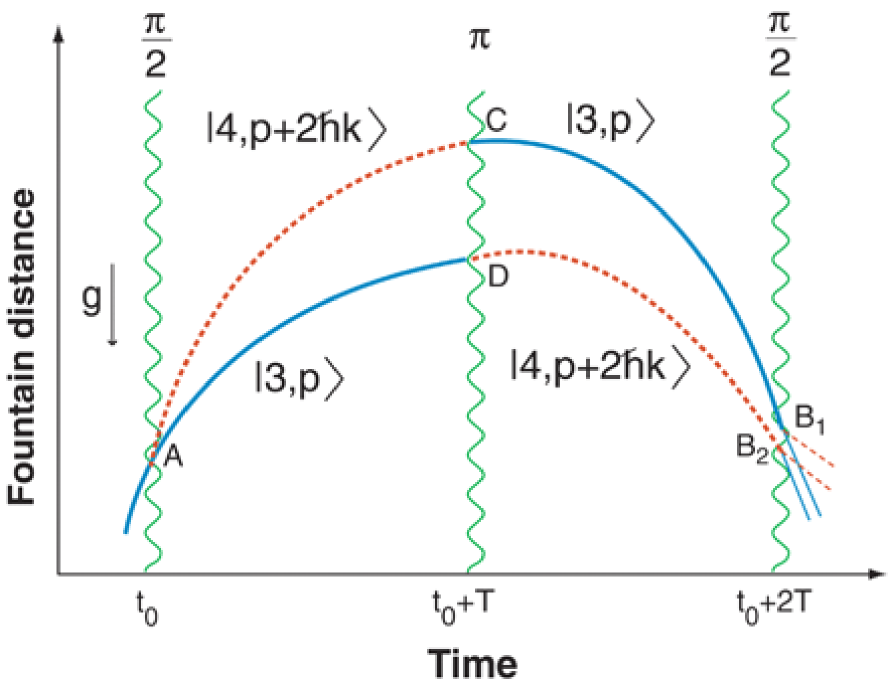
J. B. Fixler, G. T. Foster, J. M. McGuirk, M. A. Kasevich, Science **315**, 74 (2007).

Gravity graviometer

- Quantum interference of atomic Cs is used to directly probe the gravitational scalar potential
  - Momentum recoil creates different trajectories for the wave packets that acquire a relative gravitationally induced atomic phase shift during the interferometer, resulting in a sensitivity to accelerations.
  - With accurate knowledge of the atomic trajectories and the Pb source geometry and composition, they calculated the gravitationally induced phase shift in our atom interferometer and extracted a value for G.

Gravimeter

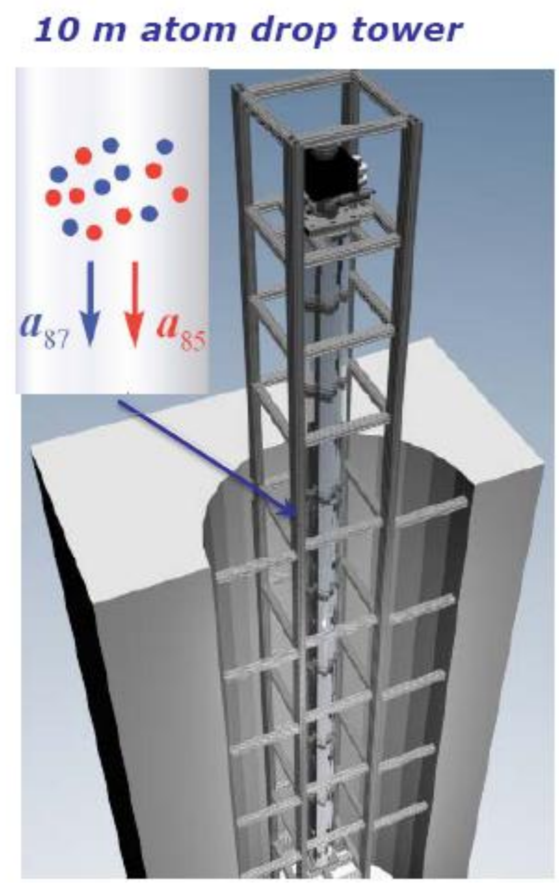
$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{Laser}} + \Delta\phi_{\text{path}} + \Delta\phi_{\text{Separation}} \Rightarrow \textbf{determine G}$$



- Path integral formulation to obtain quantum phases.
- Current status: error $\delta G/G \sim 3$ ppt, limited by the thermal distribution of the sample.
- Next experiment using new sensors aims $\delta G/G \sim 10^{-5}$.

Gravity gravimeter

- Objective: Ground-based precision tests of post-Newtonian gravity
  - Evaporative cooling to < 1 μK to enforce tight control over kinematic degrees of freedom
  - $\delta g \sim 10^{-15} g$ with one month data collection
  - New test of general relativity

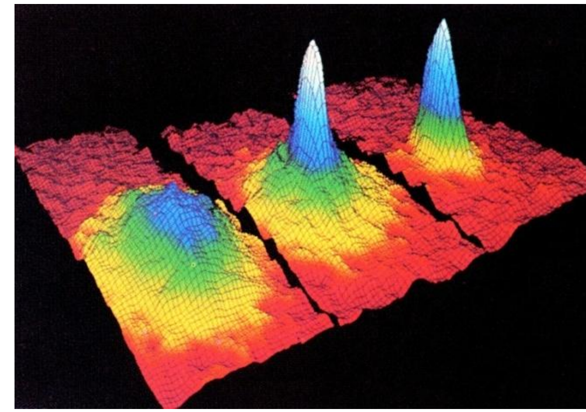


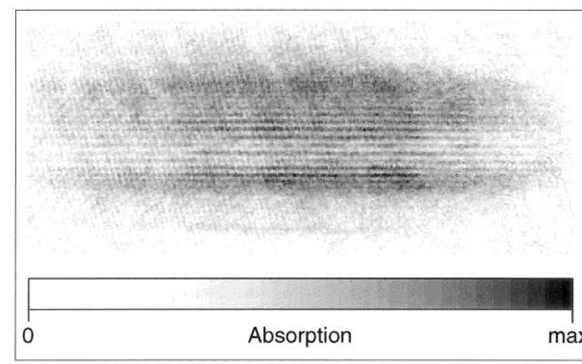
From Kasevich's presentation

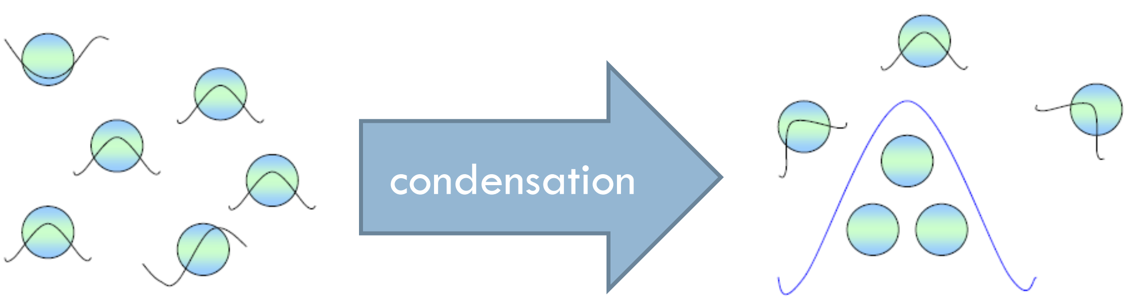
Bose-Einstein Condensation

- Bose-Einstein Condensation
  → almost all of the atoms enter the same state, represented by a common wave function with a common phase.

  ⇨ **Mean-field treatment**







Comparing thermal-atom and condensate interferometers

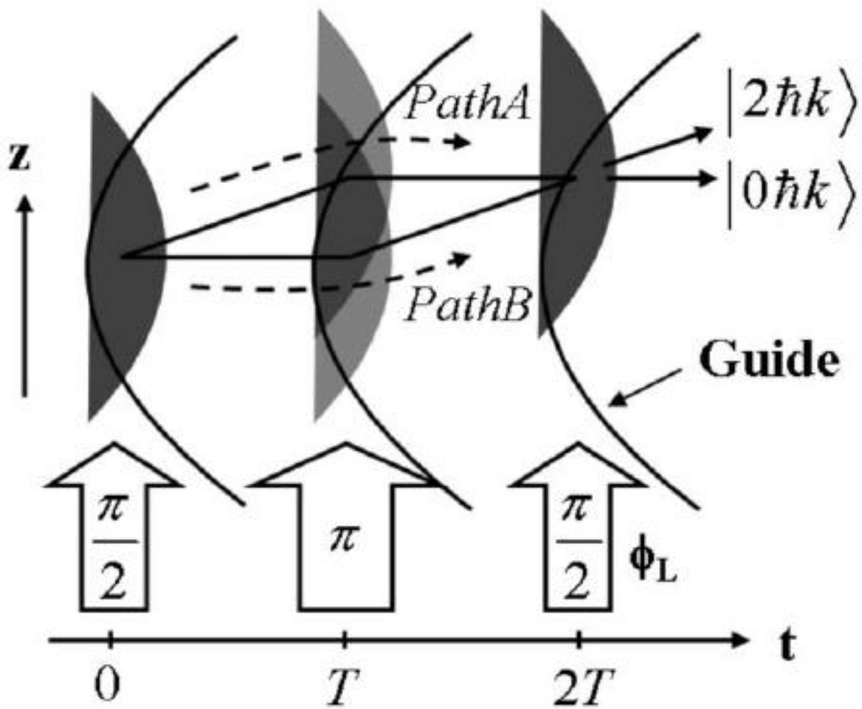
- Atoms in thermal beam move faster
  - difficult to manipulate or to reach large deflection
- Spatial coherence property
- Thermal-atom interferometers, analogous to “whitelight” interferometers, is prone to dispersion.
- Interaction between atoms
  - Dynamics of the BEC phase coherence remains to be studied in detail.
  - Possibility of using non-classical states.

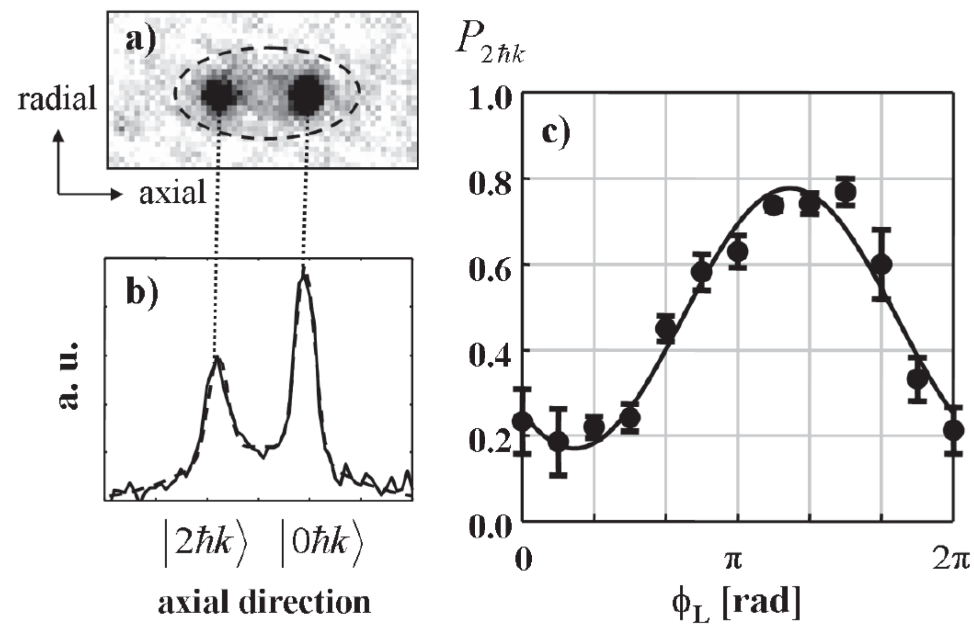
Proof-of-principle measurements

using interferometers with BEC

- Using Bragg pulses as beam splitters/mirrors
  - Various schemes (Mach-Zehnder, Michelson, ...)
  - MOT and Atom chip (JILA, NIST, Harvard, MIT, Virginia, UEC)
- Adiabatic splitting
  - By deforming the trap into a double well
- General goal: To achieve longer interrogation time, larger separation and larger enclosed area.

Mach-Zehnder interferometer with BEC





$$P_{|2\hbar k\rangle} = \frac{1}{2}[1 + C\cos(\phi_L + \Delta\phi)]$$

interrogation time = 2 ms and condensate not fully separated.

Contrast of interference signal ≈ 0.6

Munekazu Horikoshi and Ken'ichi Nakagawa,
PRA **74**, 031602(R) (2006).

Some important issues

$$\begin{pmatrix} \Psi_{|0\hbar k\rangle} \\ \Psi_{|2\hbar k\rangle} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{i\phi_L} \\ -ie^{-i\phi_L} & 1 \end{pmatrix} \begin{pmatrix} f(z) \\ \boxed{g(z)e^{i\phi(z)}} \end{pmatrix}$$

$f(z)e^{ia}$

$$P_{|0\hbar k\rangle} = \frac{1}{2}[1 + C\cos(\phi_L + \Delta\phi)]$$

- Perfect contrast conditions
  - **Perfect spatial overlap and uniform relative phase**

→ wave function profiles of the two recombining components in momentum space are related by a translation of **2ħk**

→ **perfect overlap in momentum space**

Some important issues

$$\begin{pmatrix} \Psi_{|0\hbar k\rangle} \\ \Psi_{|2\hbar k\rangle} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -ie^{i\phi_L} \\ -ie^{-i\phi_L} & 1 \end{pmatrix} \begin{pmatrix} f(z) \\ \boxed{g(z)e^{i\phi(z)}} \end{pmatrix}$$

$$f(z)e^{ia}$$

$$P_{|0\hbar k\rangle} = \frac{1}{2}[1 + C\cos(\phi_L + \Delta\phi)]$$

- Origins of contrast degradation
  - Dephasing -- relative position-dependent phase shift
  - Wave function profiles of the two components, and spatial overlap between them
  - Decoherence (not discussed in this talk)

Contrast of interference signals

- Origins of position-dependent relative phase:
  - Trapping potential
  - Atomic interactions
- How do we get phase information?
  - Go to **momentum space** in theoretical study!!
  - Classical picture for momentum changes due to forces
  - Monitor the dynamics in phase space

Our approach and goals

- A mean-field description using a single wave function representing the condensate.
  - Time-dependent Gross-Pitaevskii equation

$$i\hbar\frac{\partial\Psi(z,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V_{\text{tot}}(z,t) + g_{1\text{d}}N|\Psi|^2\right)\Psi$$

$$V_{\text{opt}}(t) = \hbar\Omega\cos(\delta k z - \delta\omega t)$$

Optical potential describes the action of the Bragg pulses.

$$I_{|2\hbar k\rangle}(\phi_L) = \frac{P_{|2\hbar k\rangle}}{P_{|0\hbar k\rangle} + P_{|2\hbar k\rangle}}$$

Interference signal.

$$C_{|2\hbar k\rangle} = \frac{\max(I_{|2\hbar k\rangle}) - \min(\mathrm{I}_{|2\hbar k\rangle})}{\max(I_{|2\hbar k\rangle}) + \min(I_{|2\hbar k\rangle})}$$

Contrast

Husimi distribution function

- Wigner distribution with a finite resolution (~ħ)
- Positive definite
- Projection on coherent states

$$P_\alpha(z,p) = \frac{1}{2\pi}\frac{1}{\sqrt{\pi\alpha}}\left|\int_\infty^\infty e^{-\frac{(q-z)^2}{2\alpha}-ipq}\Psi(q)dq\right|^2$$

- A useful tool to monitor the phase space dynamics
- **Perfect contrast condition – perfect overlap in phase space!**

Our approach and goals

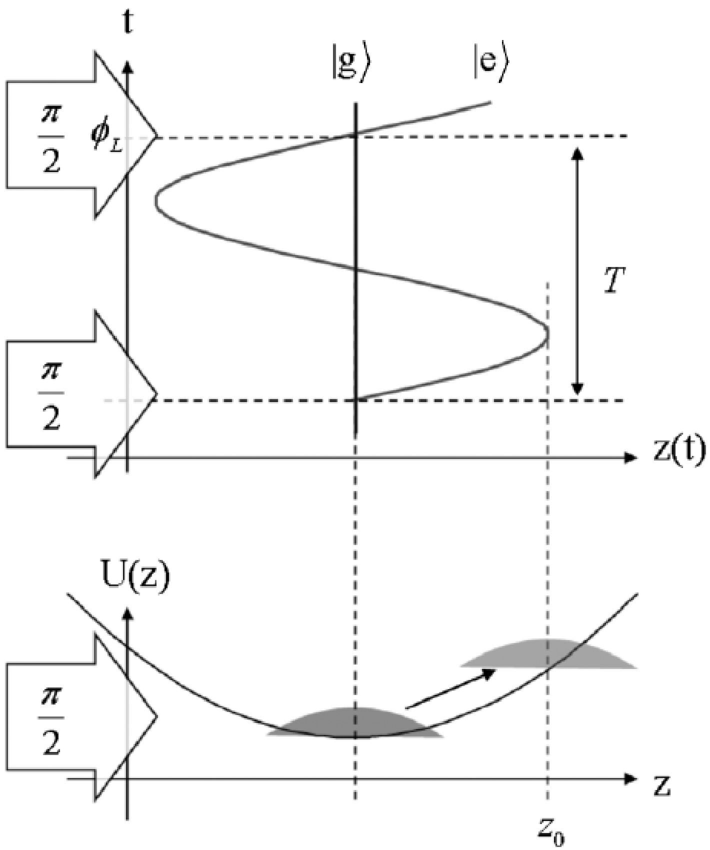
- Use the momentum space wave function to provide detailed complementary dynamical information.
- Husimi distribution function provides a useful tool to visualize and to monitor dynamics in phase space.
- **Goals** : understand the dephasing factors and control them if possible!
  - Extend the applicability of BEC interferometer

Strategies for better contrast

- Better contrast can be achieved by maximizing the overlap in phase space at recombination.
  - Shift the recombination time – referred to as ΔT scheme
  - Change the frequency of the recombination pulse – referred to as Δk scheme
  - Pros and cons

- There have been some experimental efforts.

Dephasing-free interferometer

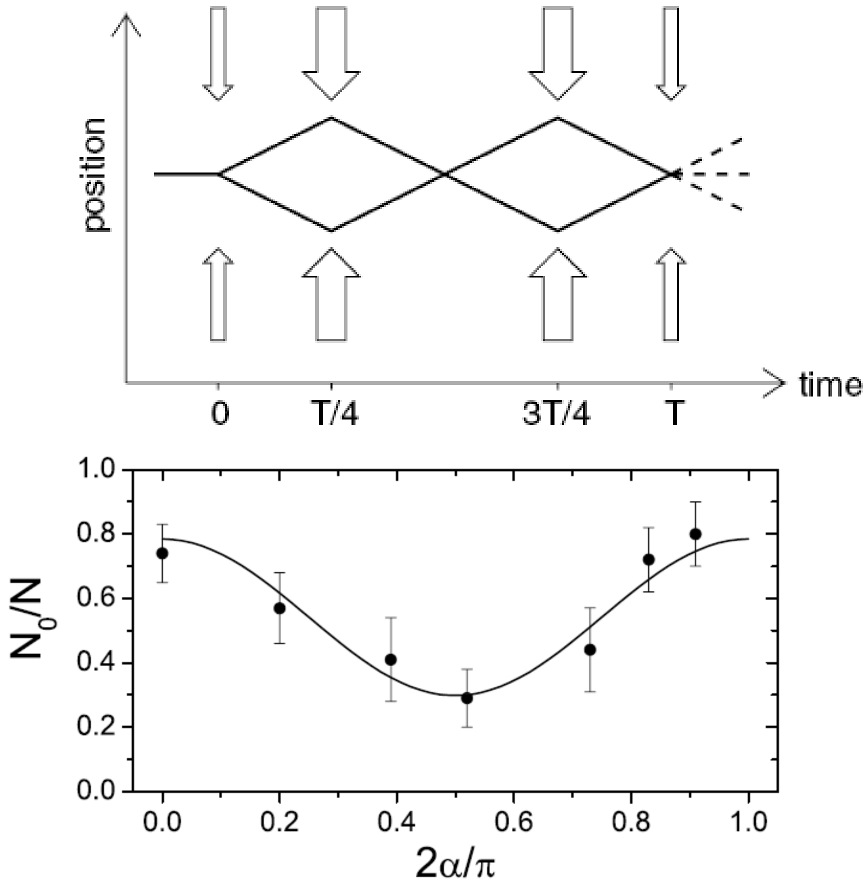


interrogation time = 58 ms and oscillation amplitude= 110 μm.

Munekazu Horikoshi and Ken'ichi Nakagawa,
PRL **99**, 180401 (2007).

- Cancellation of the dephasing due to the potential.
- Interrogation time equal to the time period of the trap.

Double-reflection interferometer

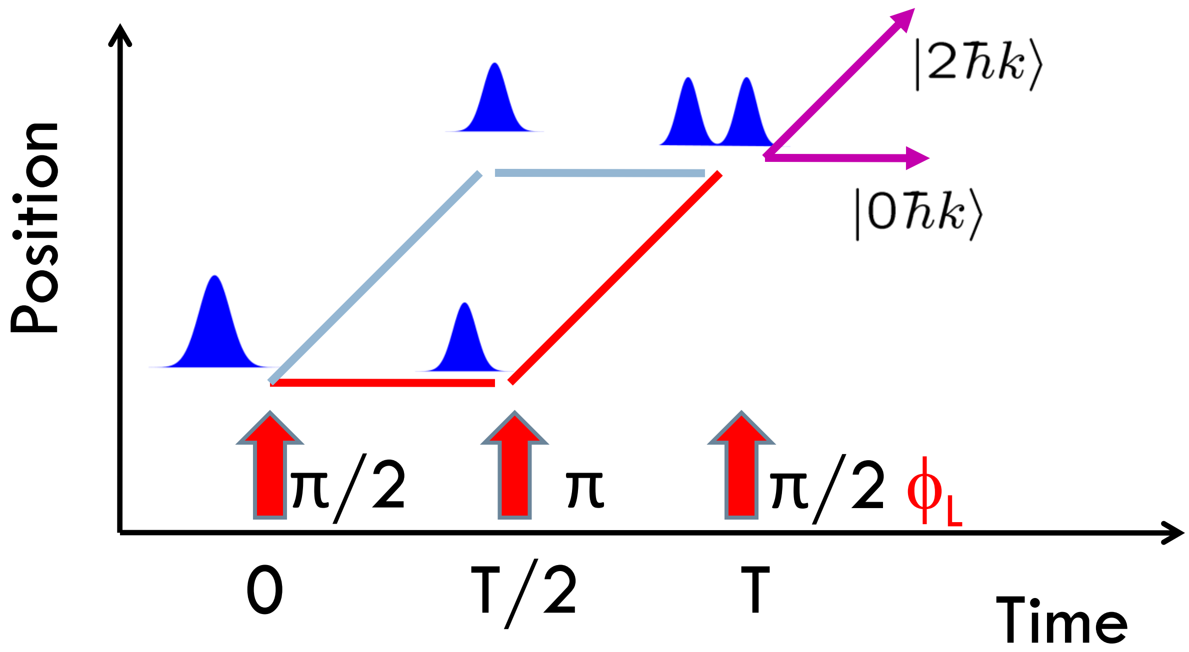


interrogation time = 44 ms and arm separation = 260 μm.

B. Deissler, K. J. Hughes, J. M. Reeves, J.H.T Burke, and C. A. Sackett PRA **77**, 031604(R) (2008).

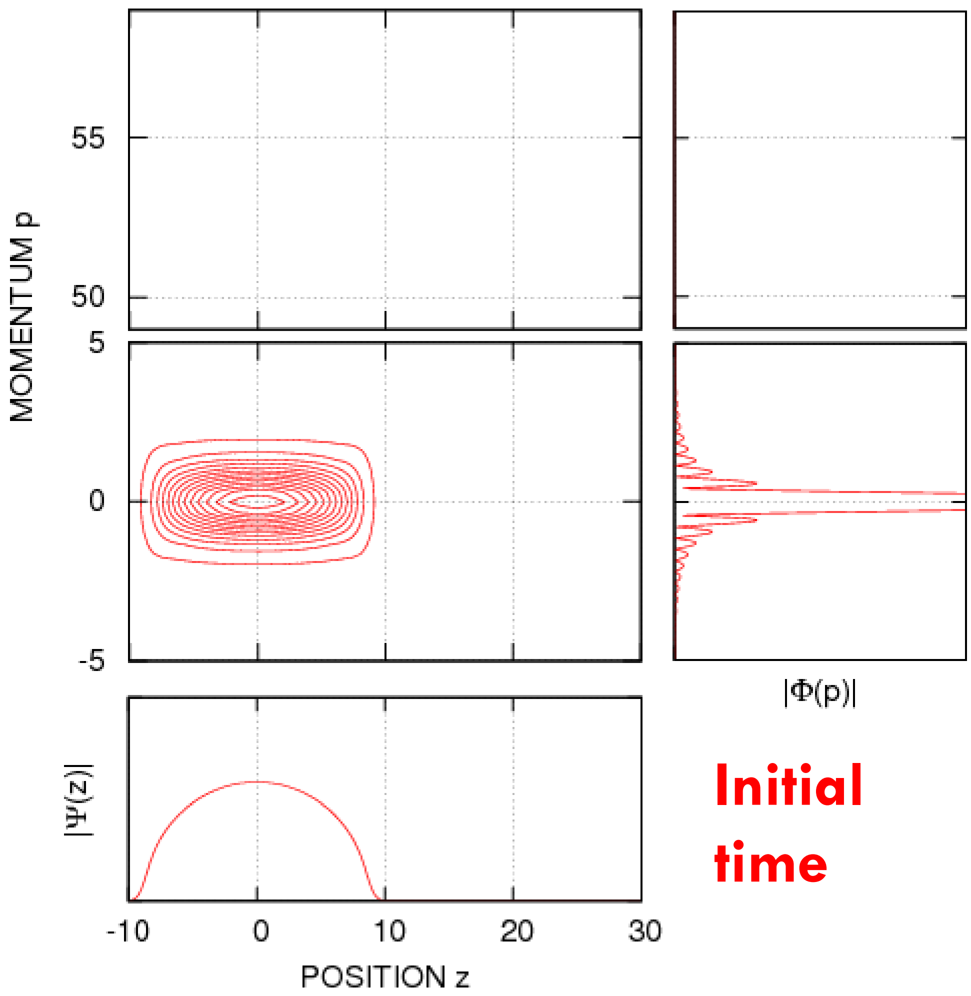
- More symmetric trajectories lead to cancellation of dephasing due to trapping potential.
- The effective interrogation time can be increased by shifting the recombination time.
- Cancellation of first order effects.

Mach-Zehnder interferometer



Time evolution of the condensate

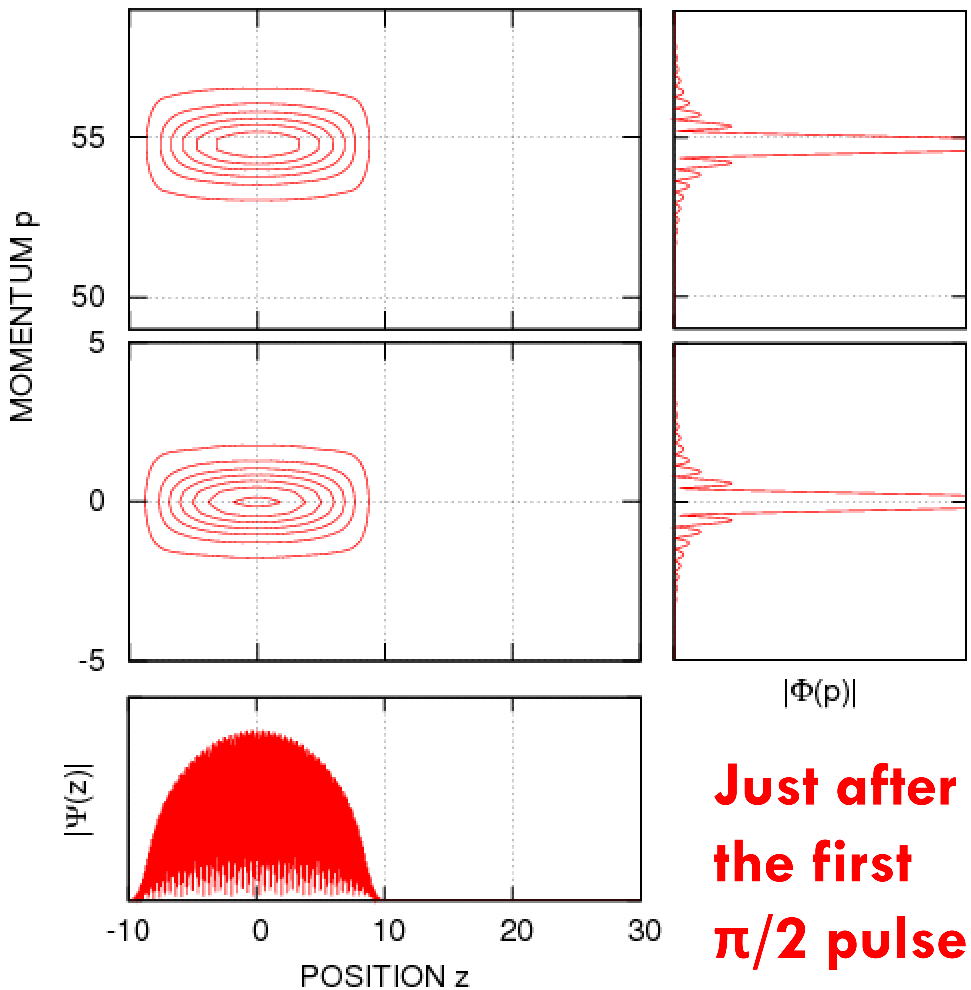
Mach-Zehnder interferometer in the presence of trapping



Initial time

Time evolution of the condensate

Mach-Zehnder interferometer in the presence of trapping

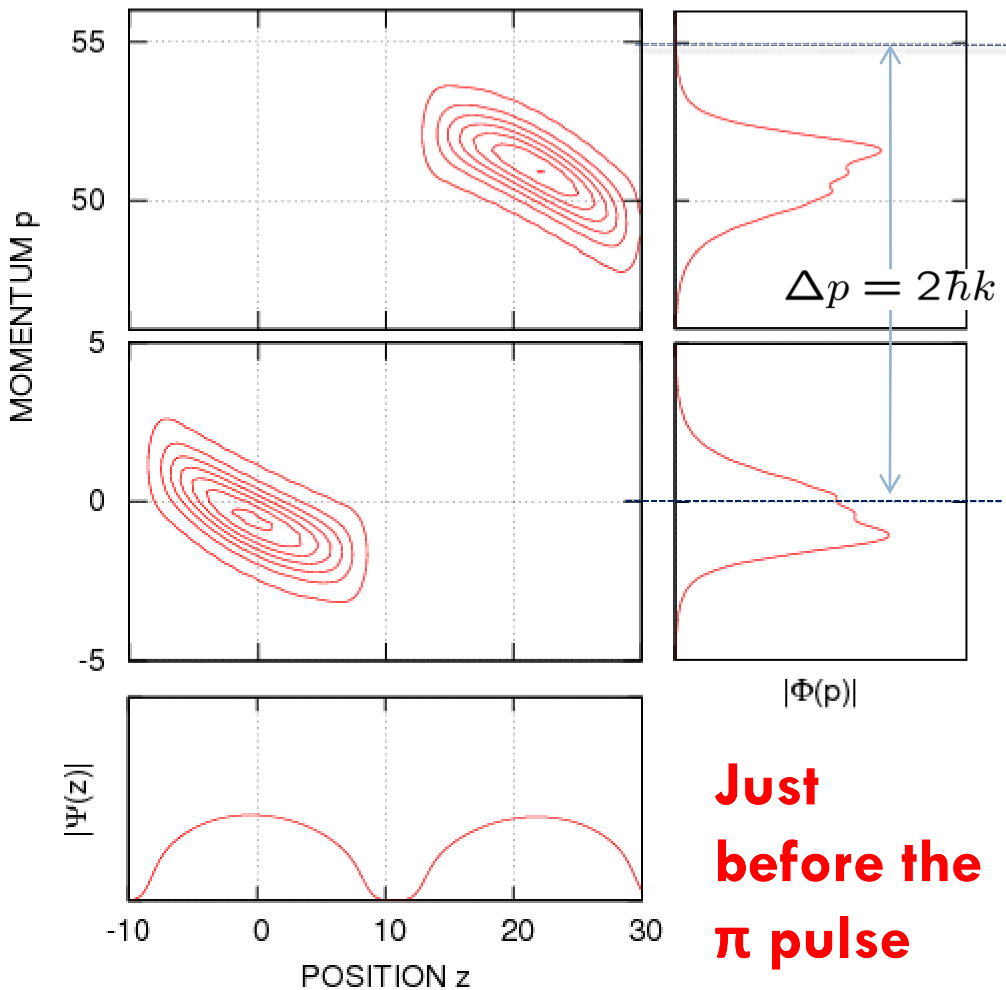


Just after the first $\pi/2$ pulse

- $\pi/2$ pulse results in a 50/50 split
- Split and **translation** by $2\hbar k$ in momentum in phase space.

Time evolution of the condensate

Mach-Zehnder interferometer in the presence of trapping

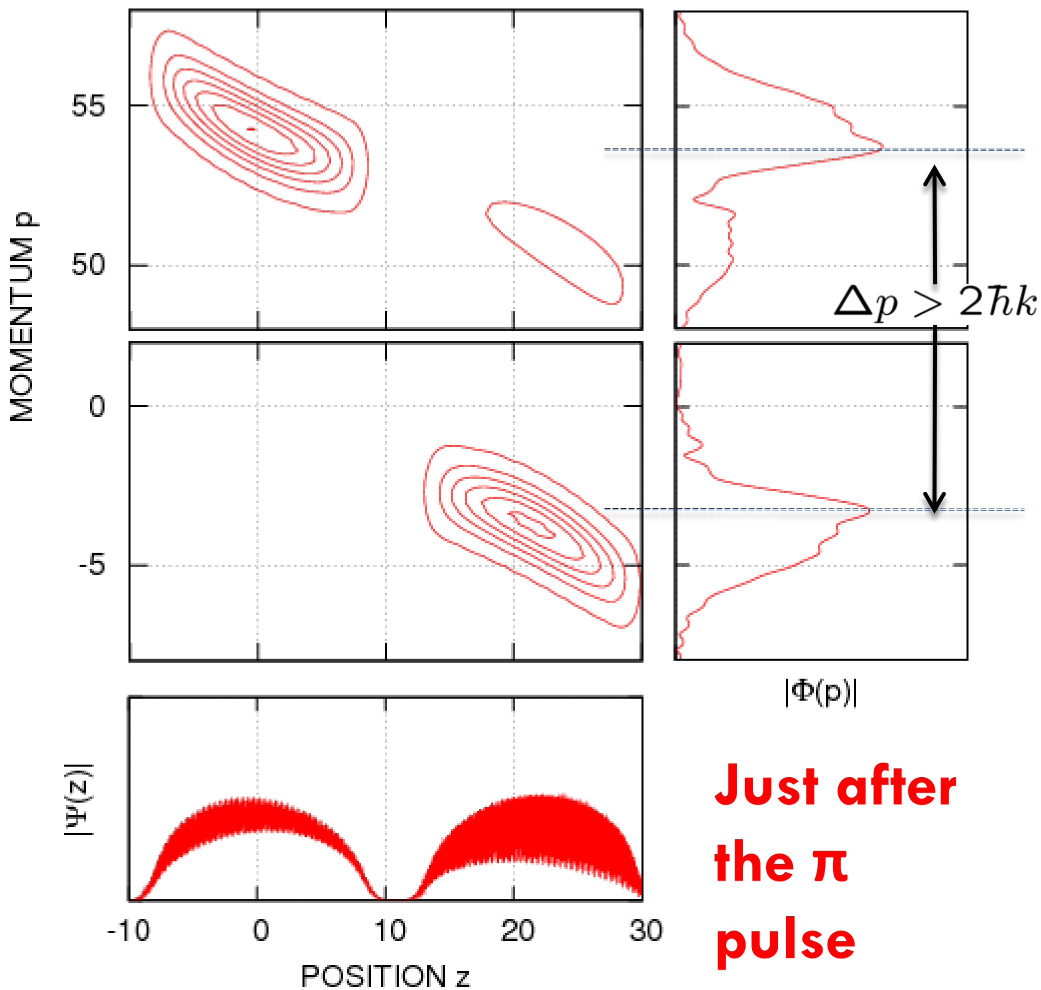


Just before the π pulse

- Asymmetric and broadened momentum profile due to nonlinear interaction.
- Different shifts of the momentum peak positions
- **Tilting of the Husimi representation results from forces**
- Positive tilting – trapping force dominates

Time evolution of the condensate

Mach-Zehnder interferometer in the presence of trapping

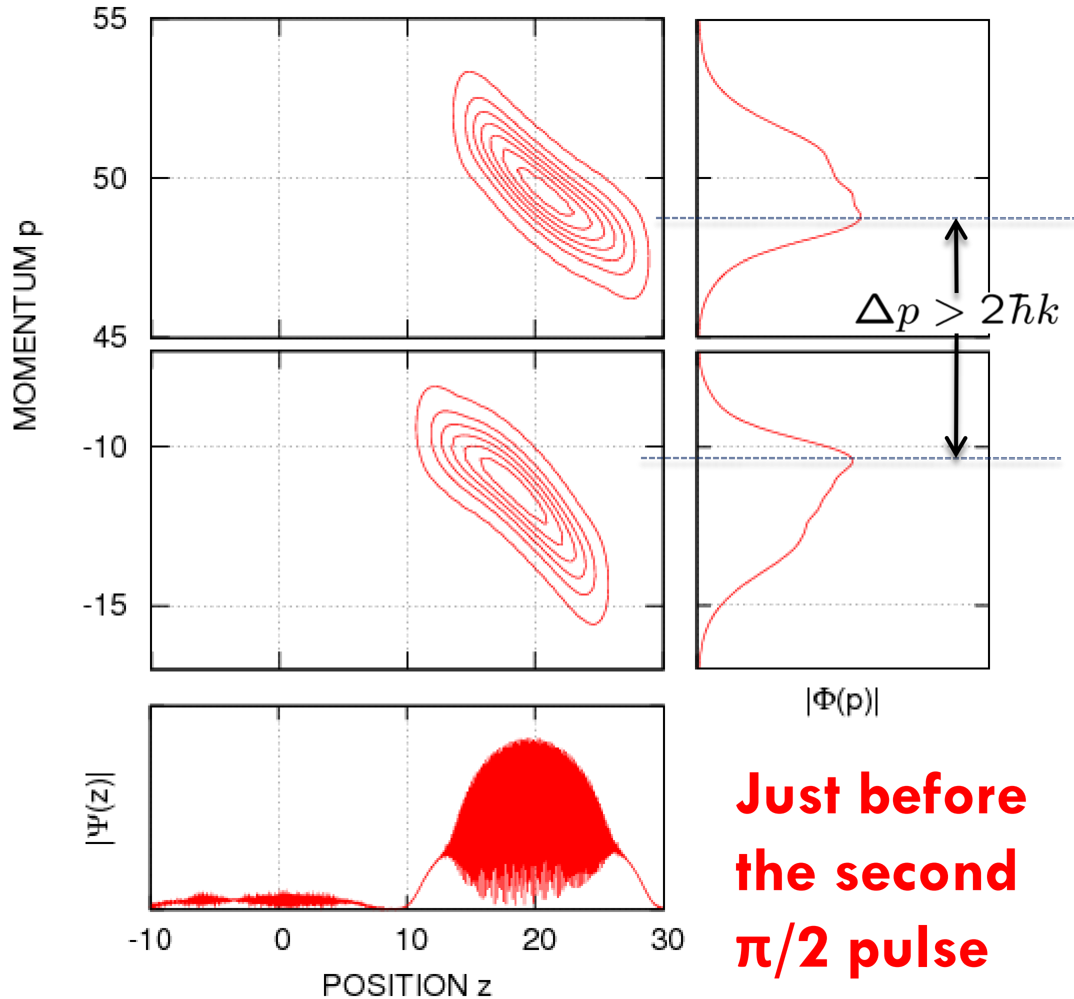


Just after the π pulse

- Not a perfect reflection since the two components are shifted away from |0ħk> and |2ħk>, respectively.

Time evolution of the condensate

Mach-Zehnder interferometer in the presence of trapping

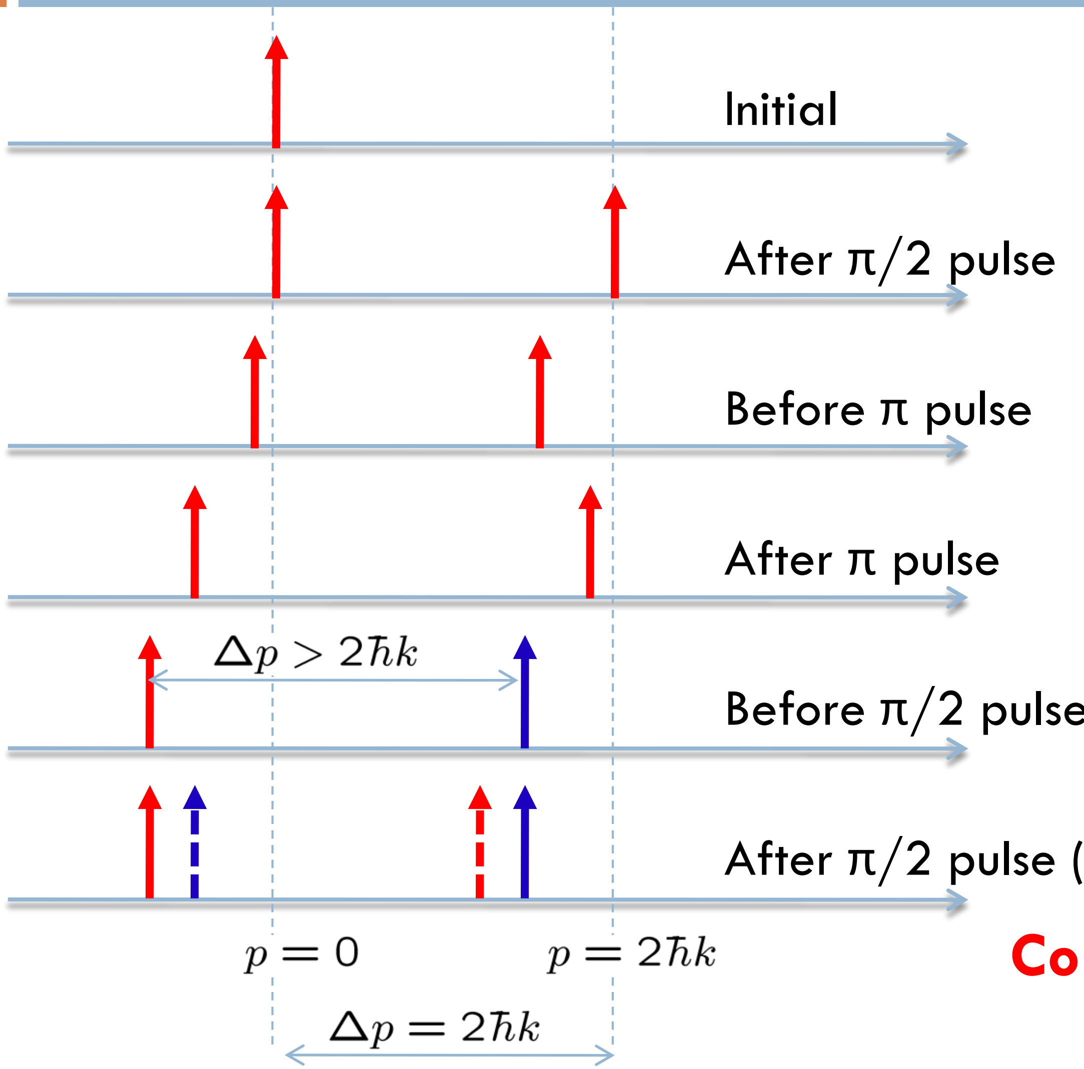


Just before the second π/2 pulse

- The same Bragg pulse would not bring these two components into overlap in phase space.
- Zero contrast in this case!!
- Can we improve the contrast?

Simple-minded picture

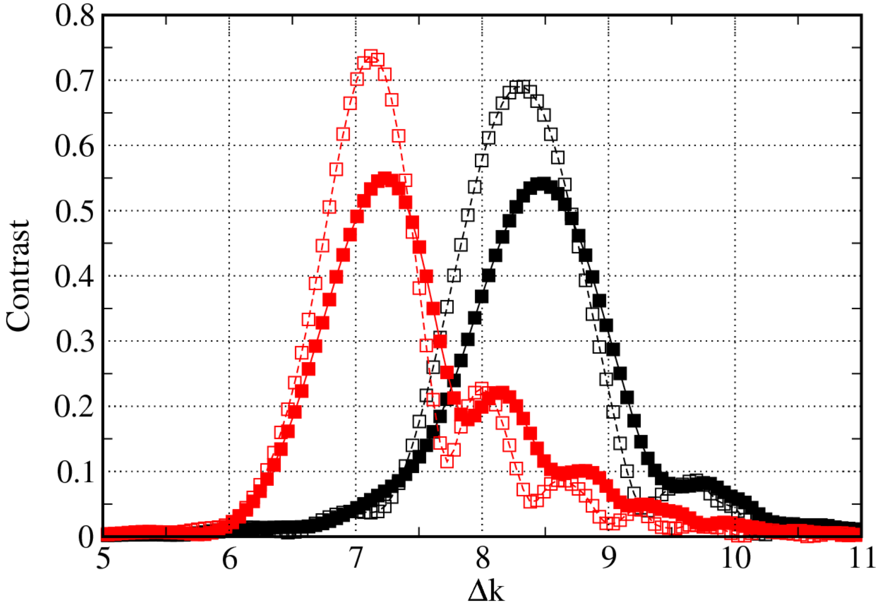
Mach-Zehnder interferometer in the presence of trapping

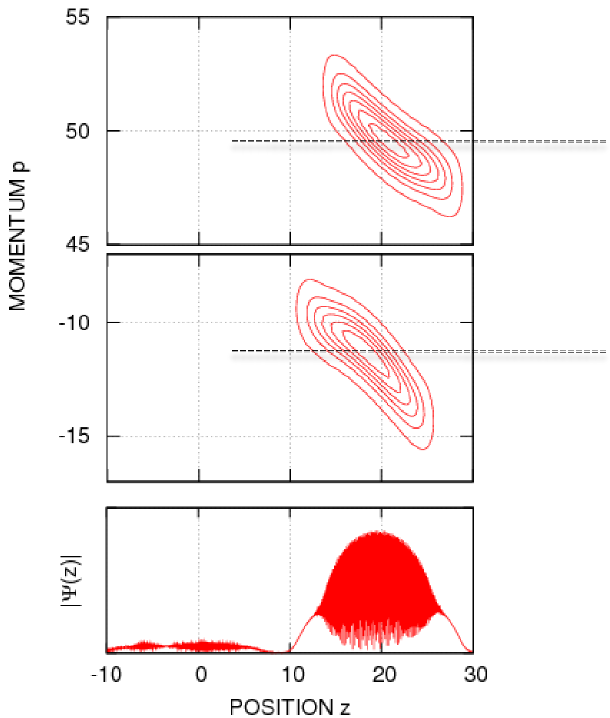


- Consider forces due to trapping and atomic interaction
- The recombination Bragg pulse no longer couples the two components.

Complete lost of contrast

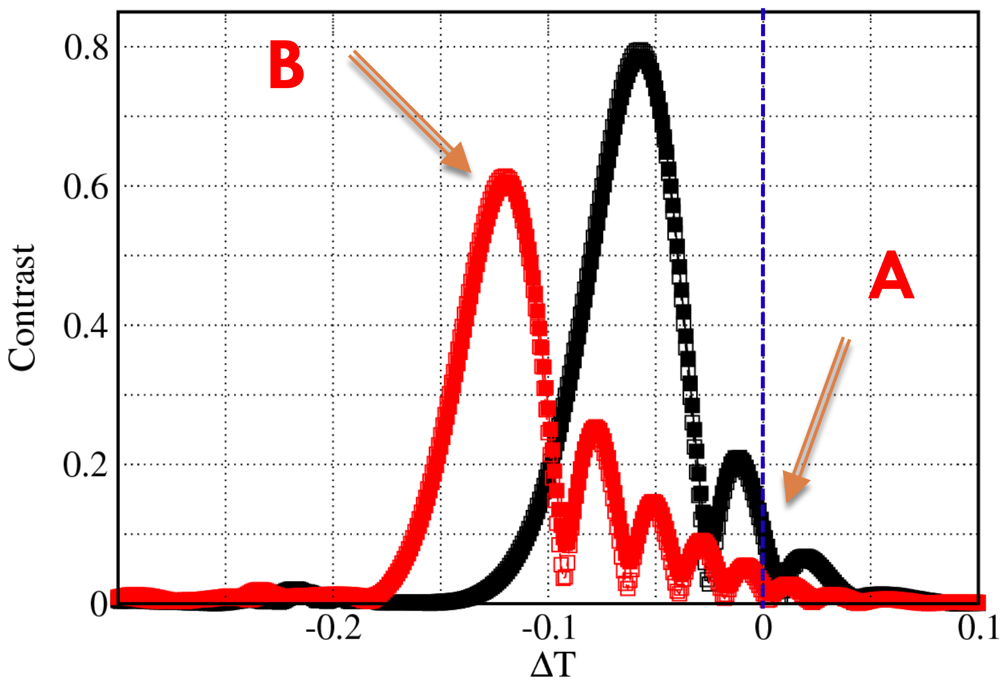
Mach-Zehnder interferometer in the presence of trapping





- Asymmetry of contrast
- Oscillations in contrast
- Repulsive nonlinear interaction is counteracting the trapping. (Red symbol → larger number of atoms)

Mach-Zehnder interferometer in the absence of trapping

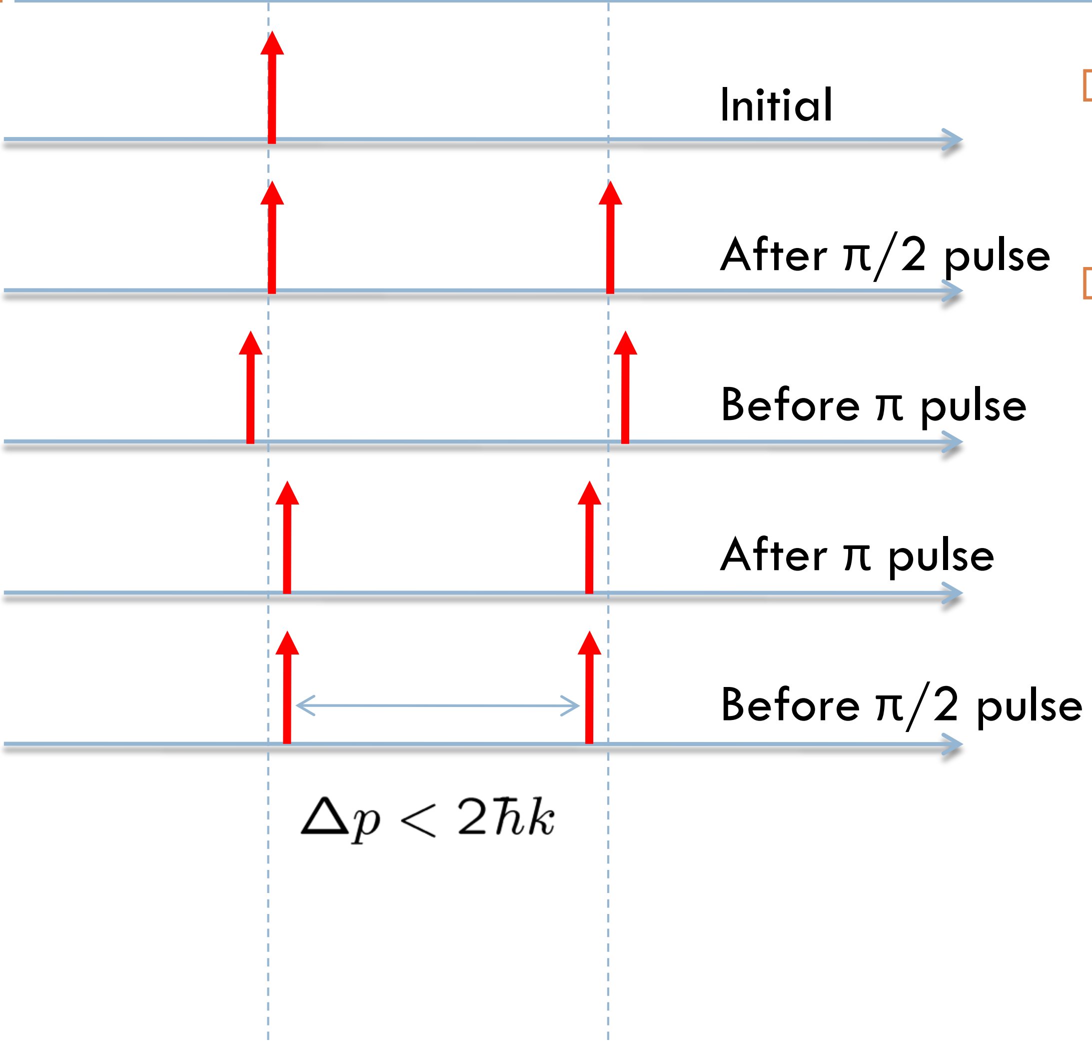


Almost zero contrast at $\Delta T=0$.

- Asymmetry
- Oscillation in contrast with respect to time shift
- If g=0 or symmetric momentum profile
  - No oscillation
  - Maximum contrast at $\Delta T=0$
- Maximum contrast occurs at a **negative time shift** is counterintuitive.

Simple-minded picture

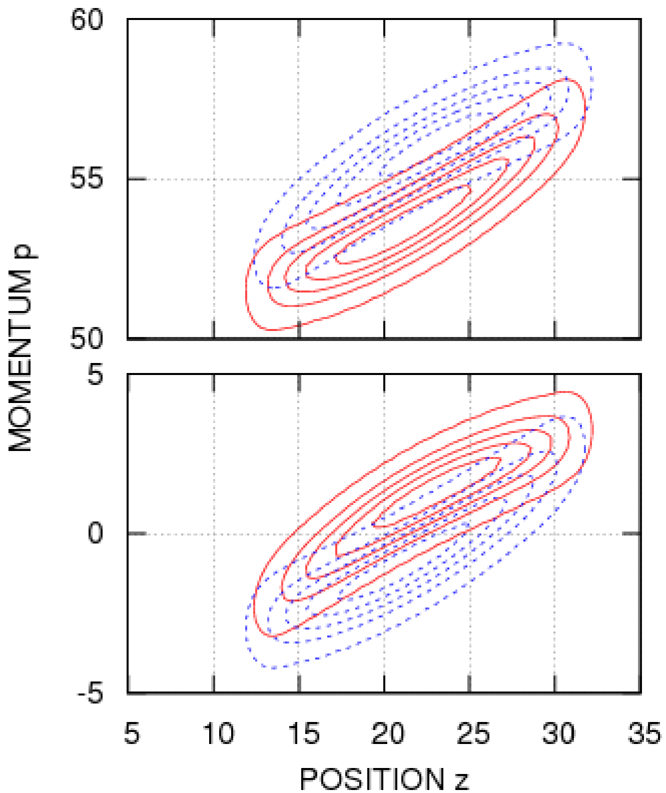
Mach-Zehnder interferometer in the **absence** of trapping



- Atomic interaction is the only force
- Intuitively, perfect spatial overlap requires **positive time** shift, in contrary to the results from full calculations.

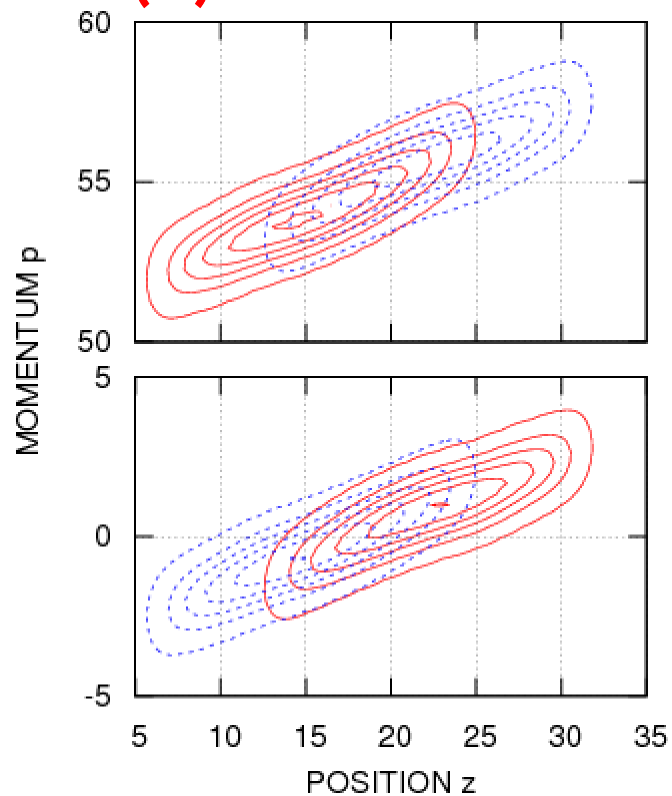
Not-so-surprising reason

(A) ΔT=0



Key ideas: asymmetric momentum profile and overlap in phase space.

(B) ΔT<0

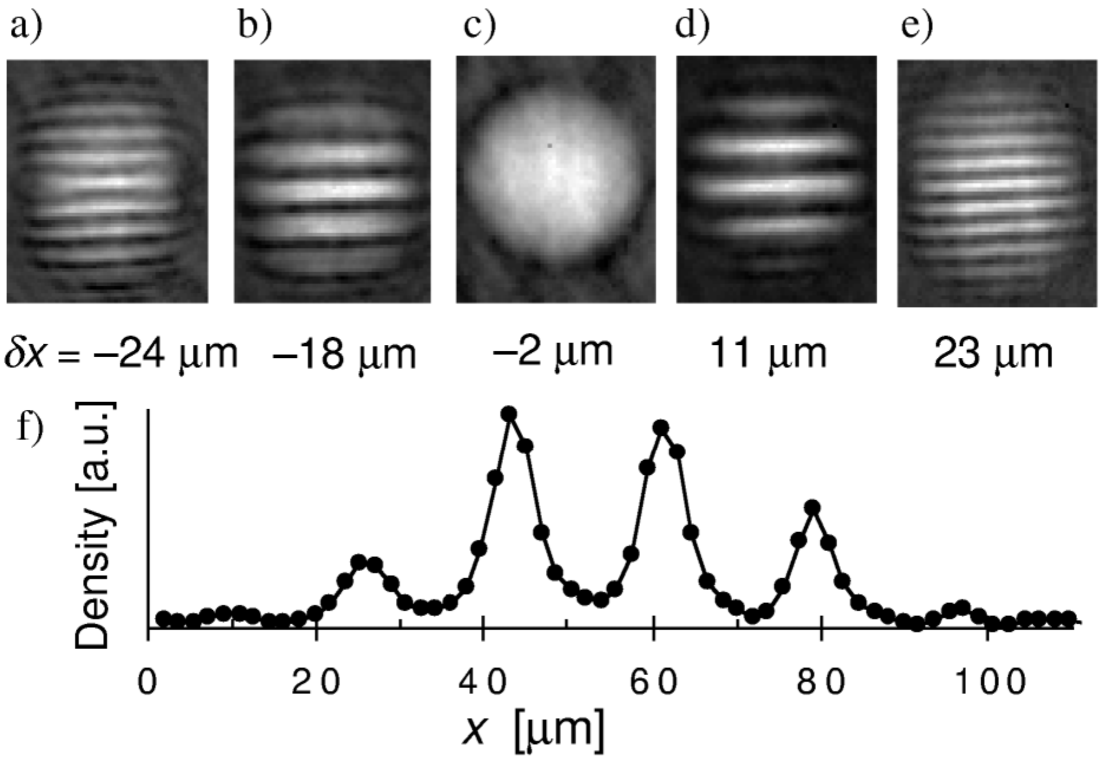


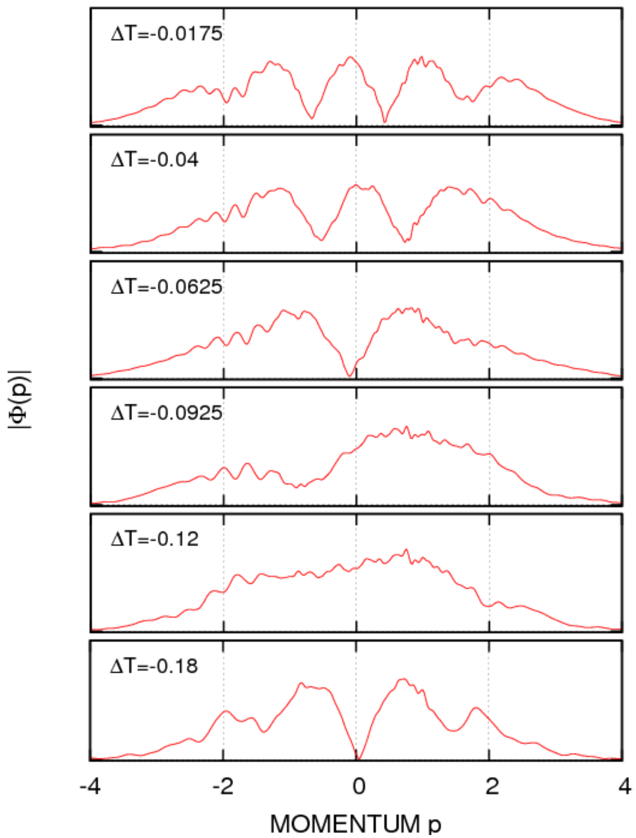
Solid line: Just before the recombination pulse

Dashed line: After the recombination pulse

Absorption images

time-of-flight measurements

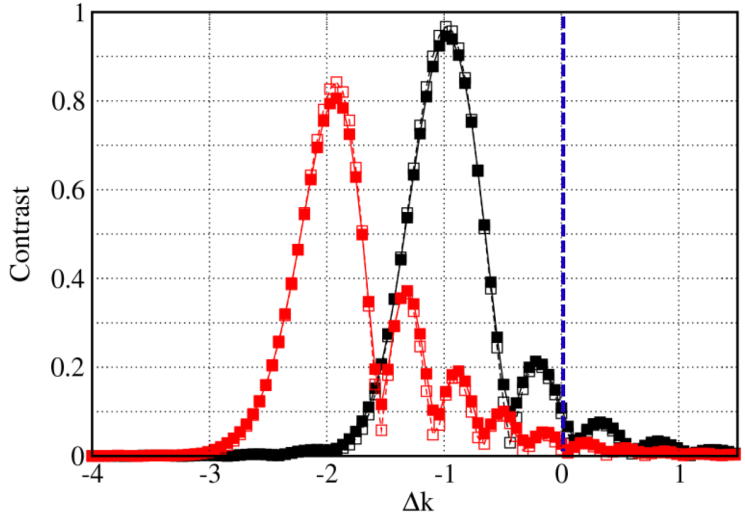




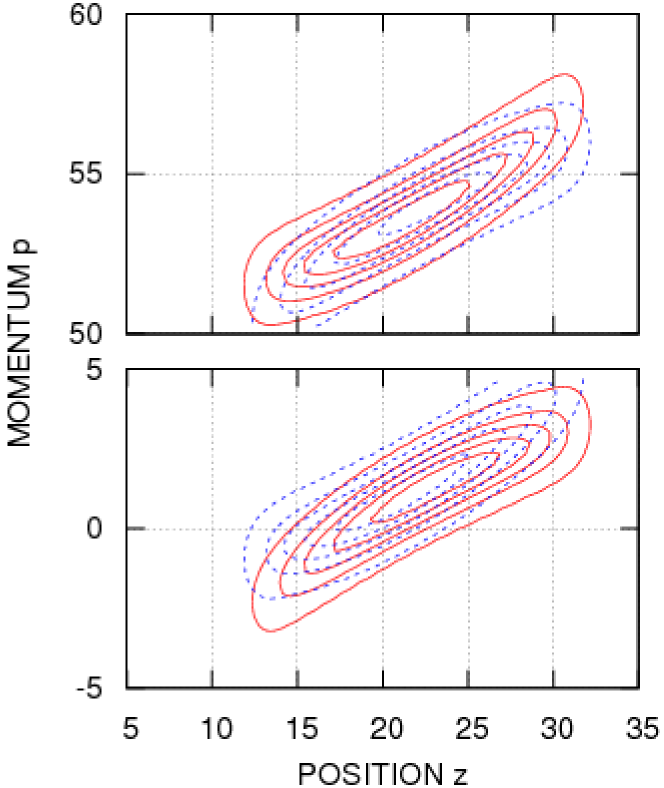
Figures (a–e) show one interferometer output port for different dx *(different T_2) after an expansion time $T_0 = 4$ ms.*

J. E. Simsarian, J. Denschlag, Mark Edwards, Charles W. Clark, L. Deng, E.W. Hagley, K. Helmerson, S. L. Rolston, and W. D. Phillips, PRL **85**, 2040 (2000).

Mach-Zehnder interferometer in the absence of trapping

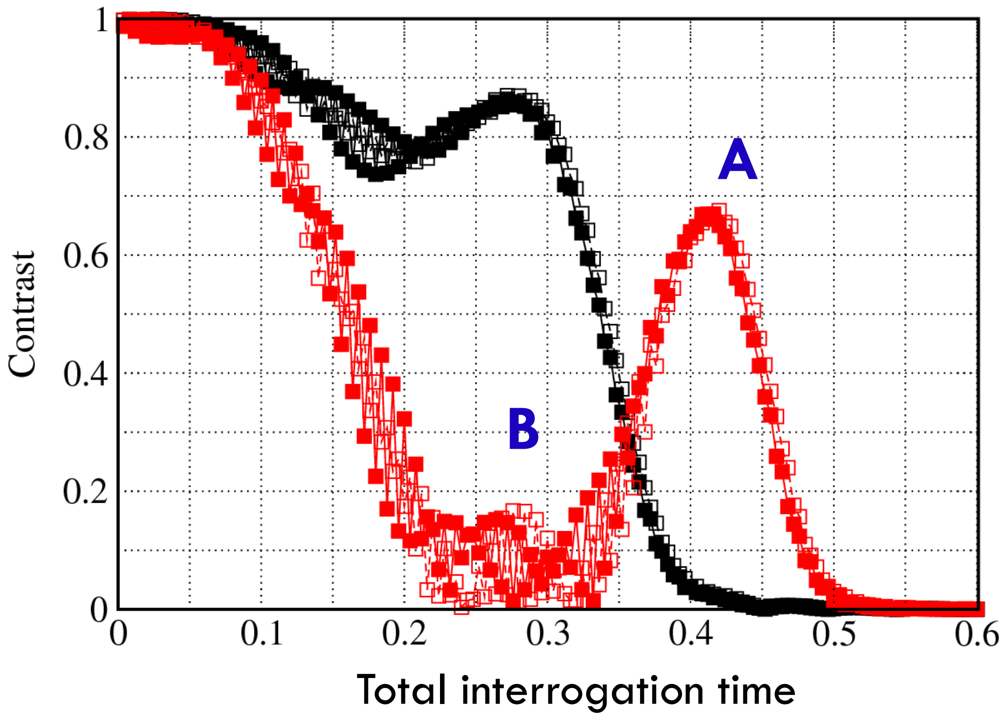


Almost zero contrast at Δk=0.



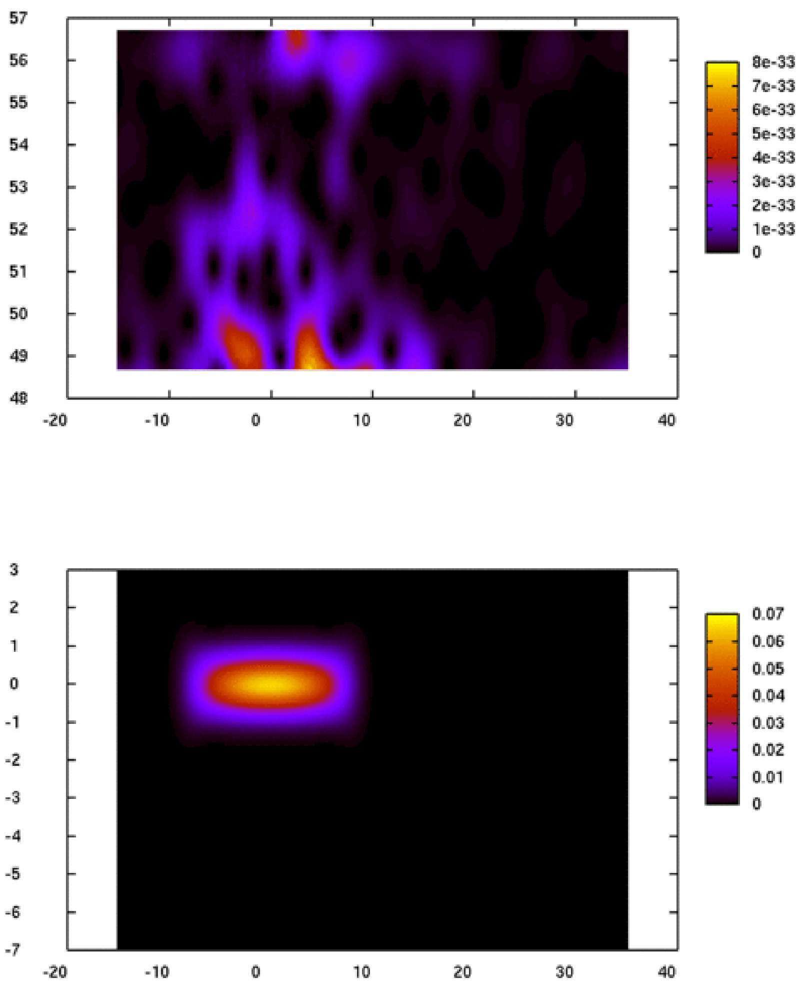
- Again, Δk scheme is intuitive.

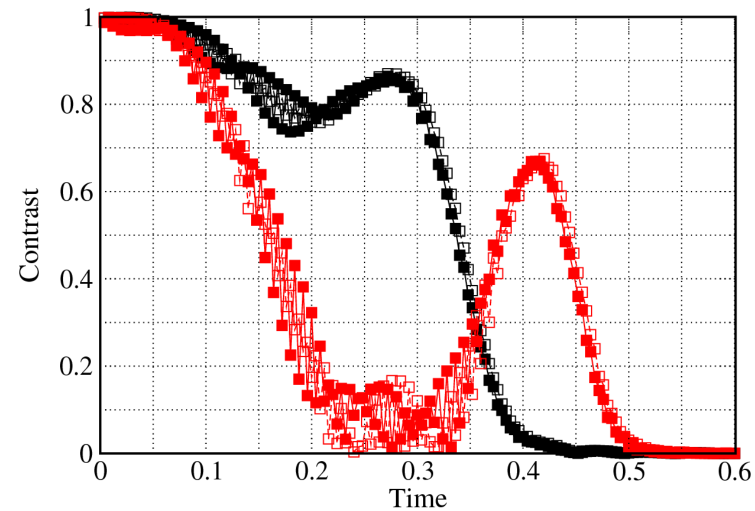
Effects of nonlinear atomic interaction



- Revival of contrast due to the interplay between the atom-atom interaction and trapping force

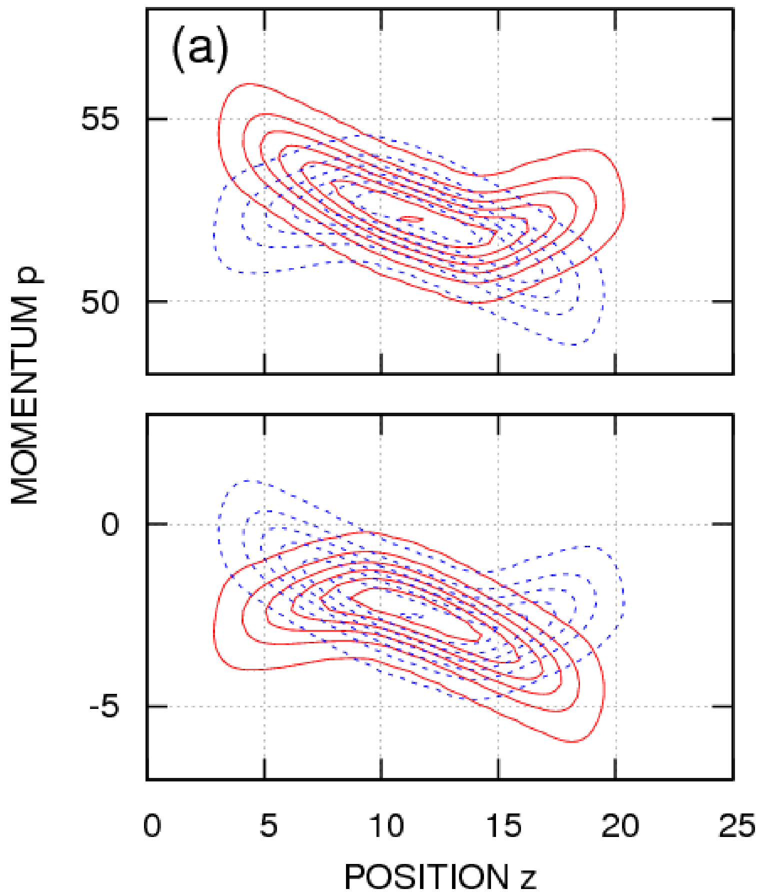
Time evolution of Husimi distribution functions



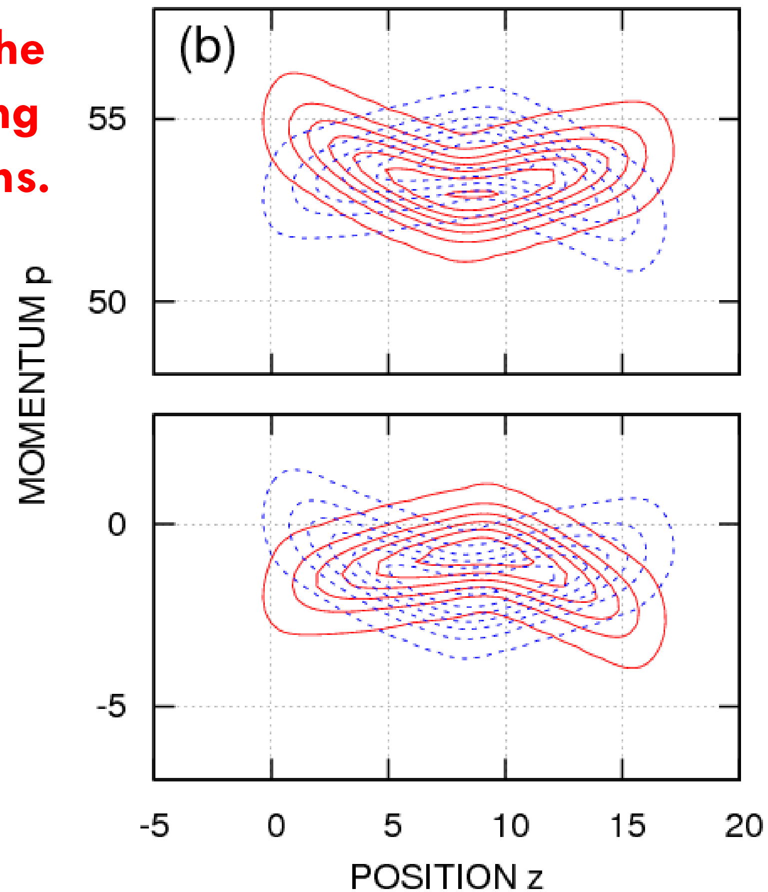


The time evolution corresponding to T=0.42 when a revived maximum contrast is observed.

Effects of nonlinear interaction



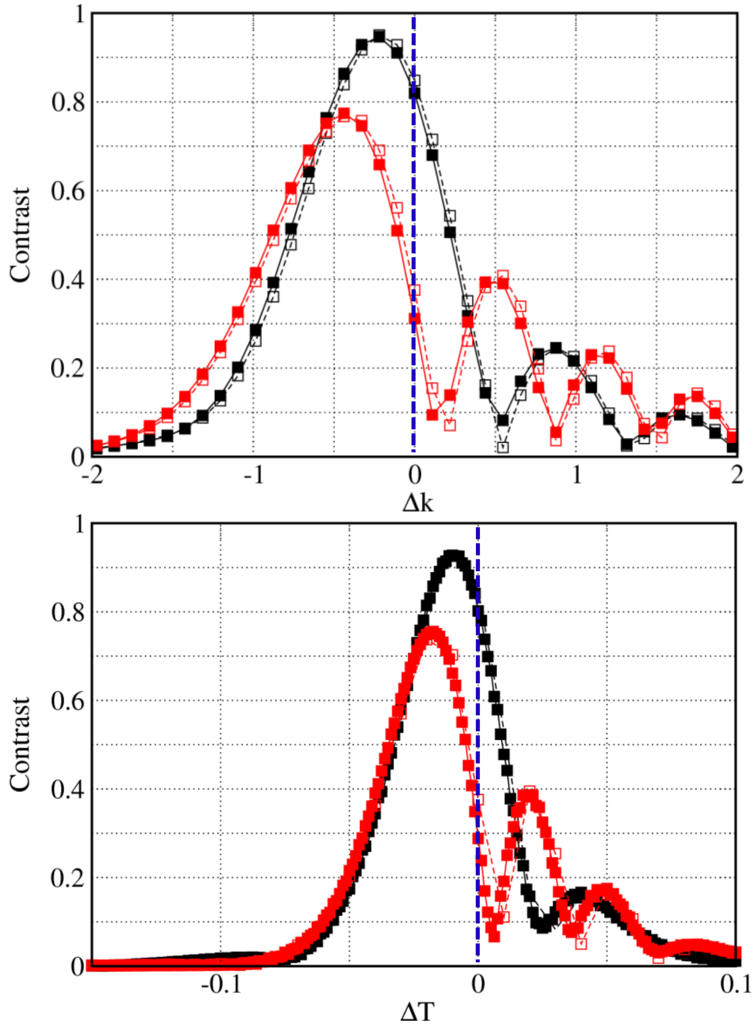
Note the bending patterns.



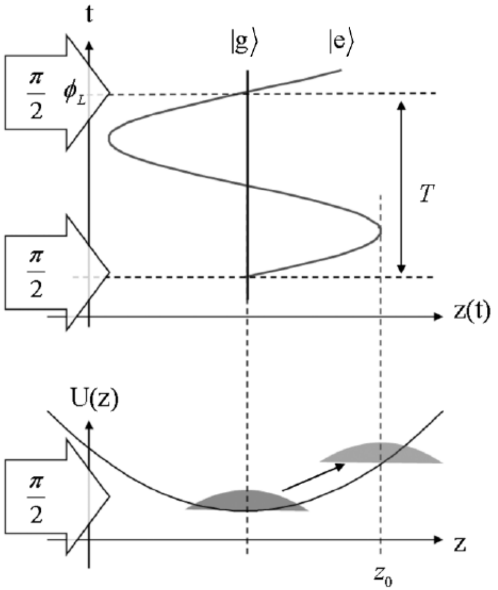
T=0.42 : optimum contrast

T=0.3 : vanishing contrast

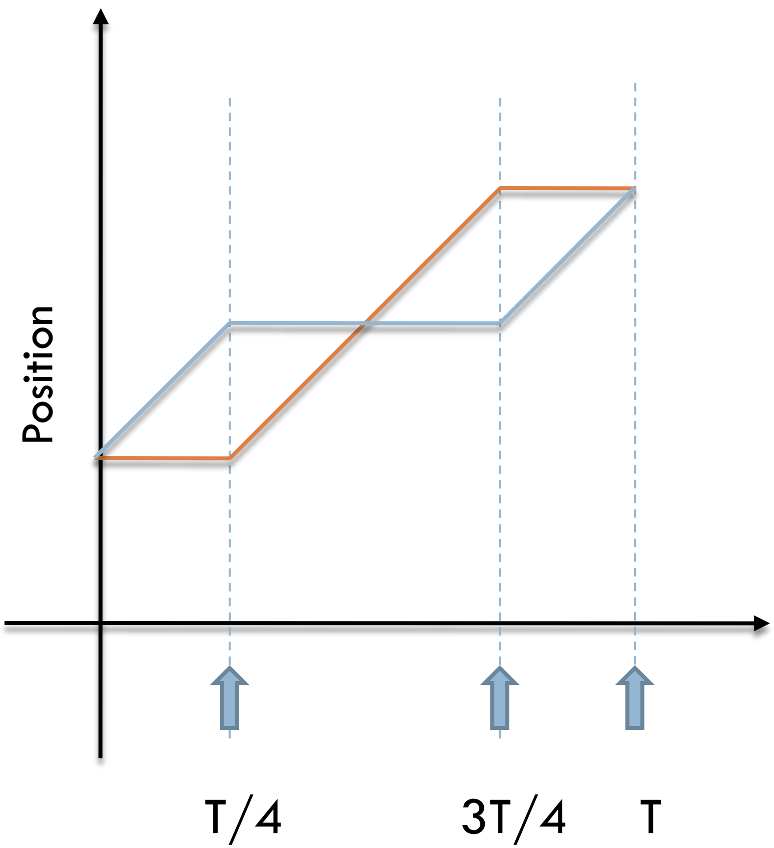
Dephasing-free interferometer



- In the presence of trapping.
- Good contrast at zero shift in Δk or ΔT.



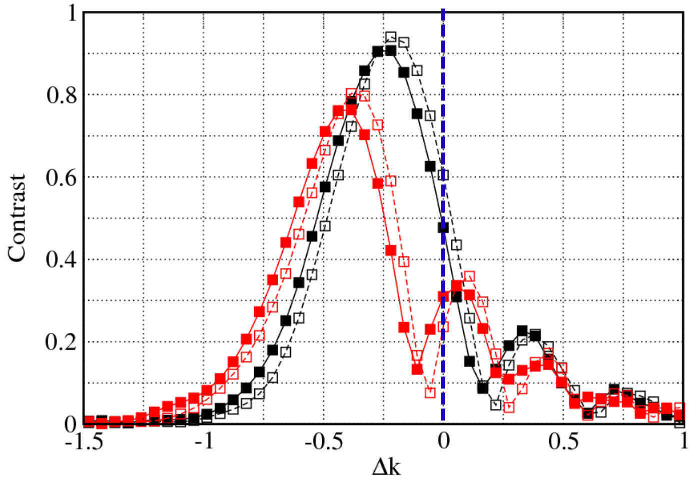
Double-reflection interferometer

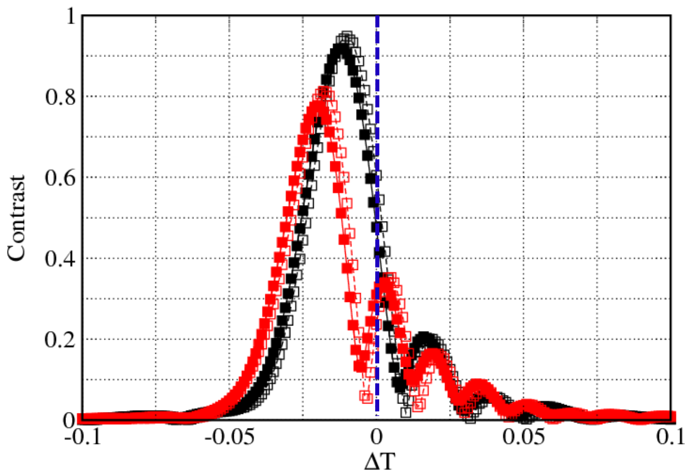


- Slight modification from the original proposal.
- The scheme is of value for second order inertia measurement.
- The principle works in the absence of trapping.

Double-reflection interferometer

in the absence of trapping





- The modifies DR interferometer in the presence of trap cannot be improved by either scheme.
- Gives better optimal contrasts, compared with MZ.

Conclusions

- Husimi representation is useful in illustrating and monitoring the quantum dynamics in phase space as well as the wave function overlap.
- The contrast of interference signal depends on the degree of overlap in phase space.
- We have demonstrated that both Δk and ΔT schemes are effective in improving the contrast of interference signal.
  - Asymmetric oscillations of contrast with respect to Δk or ΔT reflect the asymmetry in the wave functions.

Future work

- How do the optimization strategies improve precision in “real” measurement?
- Explore the dephasing limits on interrogation time.
  - Atomic interaction and trapping potentials are counteracting
- Explore different interferometer schemes.
- Non-classical states/beyond mean-field approximation.
- Simulate real experiments.

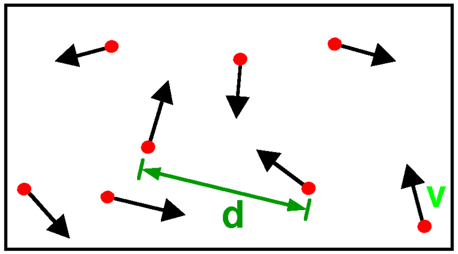
Other issues

- Decoherence
  - Phase fluctuation
  - Finite temperature phase diffusion

Inherent advantages of atom interferometry

1. Laser cooling and manipulation techniques extend the interferometer measurement time, defined as the drift time of an atom through the interferometer, by orders of magnitude over interferometers based on photons, electrons or neutrons.
2. Wavelength of matter waves is much shorter, leading to very compact interferometers.
3. The effects of beam splitters and mirrors based on optical pulses can be calculated to high precision since the interactions of light with matter are well understood.
4. The internal degrees of freedom of an atom offer the possibility of designing better interferometer components.

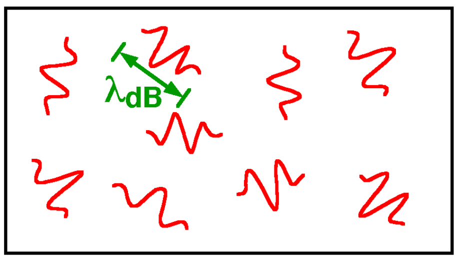
What is Bose-Einstein condensation (BEC)?



High Temperature T:

thermal velocity v
density d^{-3}

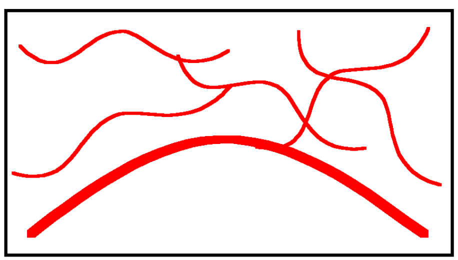
"Billiard balls"



Low Temperature T:

De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$

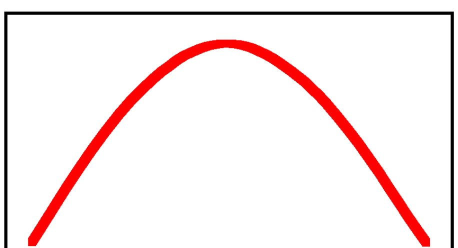
"Wave packets"



$T=T_{crit}$: Bose-Einstein Condensation

$\lambda_{dB} \approx d$

"Matter wave overlap"

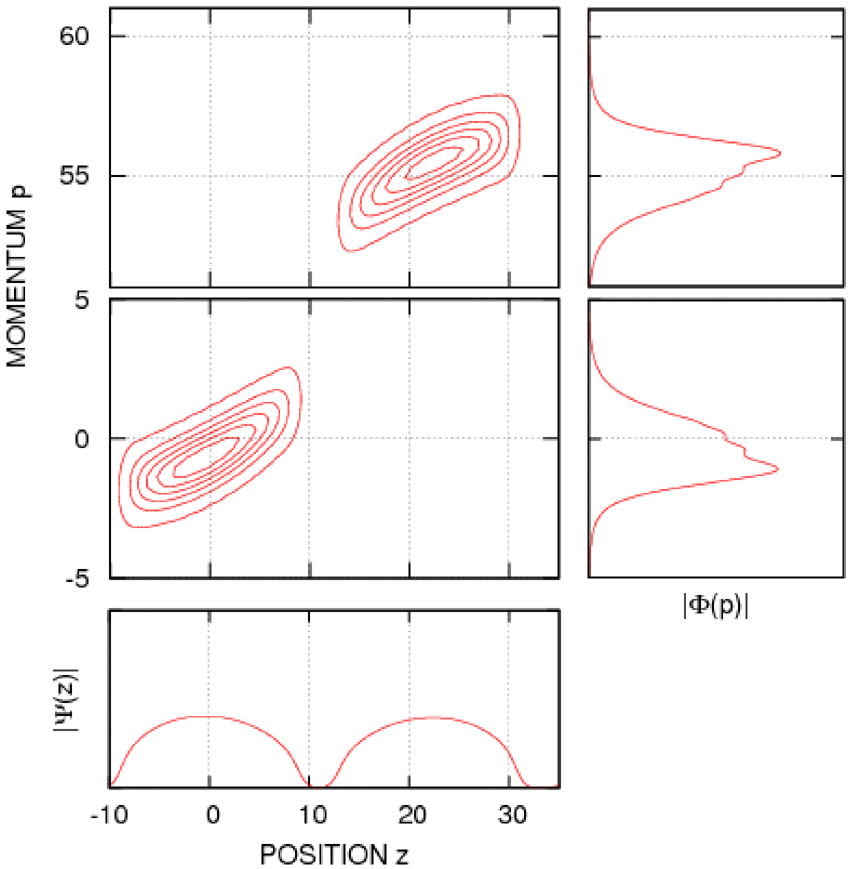


T=0: Pure Bose condensate

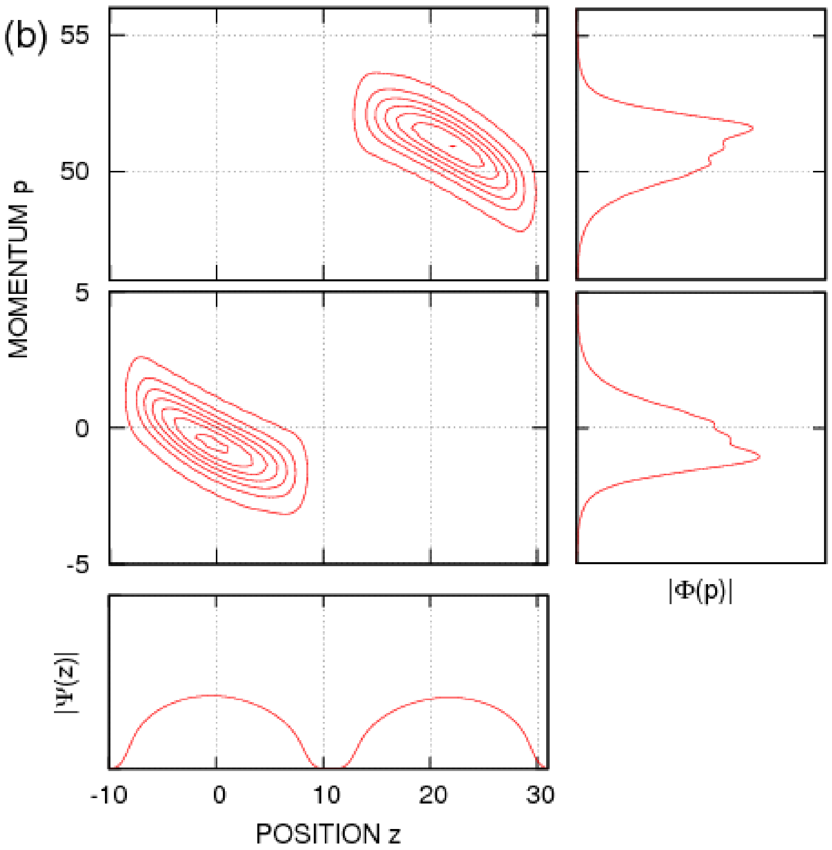
"Giant matter wave"

Positive and negative tilting of the Husimi distribution function

MZ interferometer, just before the π pulse.



In the absence of trapping



In the presence of trapping

Gravimeter

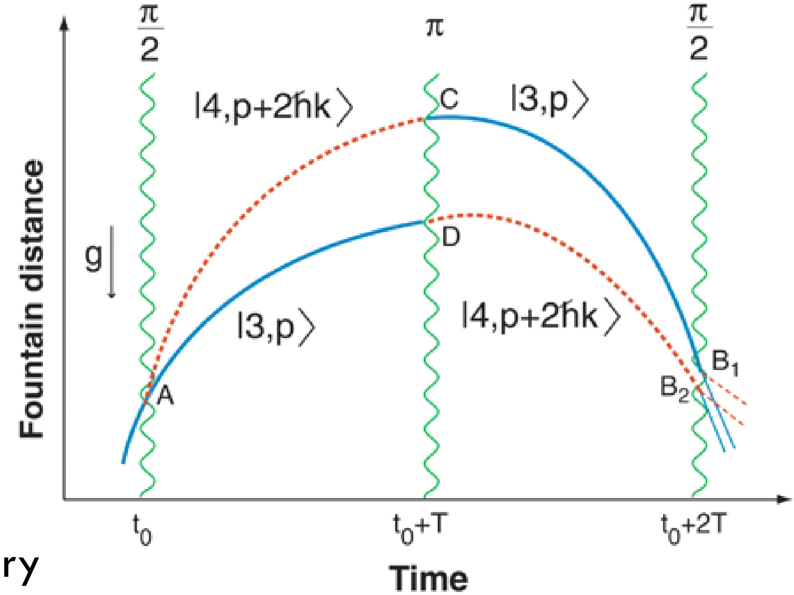
Atomic trajectories are determined using the exact potential of the source mass distribution and the Earth potential.

The quantum propagation phase accrued by each wave packet over its trajectory involves an integral over the classical action for the calculated atomic trajectories.

Separation phase arises from the spatial separation of the two interfering wave packets following the final p/2 pulse.

$$\Delta\phi_{\text{tot}} = \Delta\phi_{\text{Laser}} + \Delta\phi_{\text{path}} + \Delta\phi_{\text{Separation}}$$

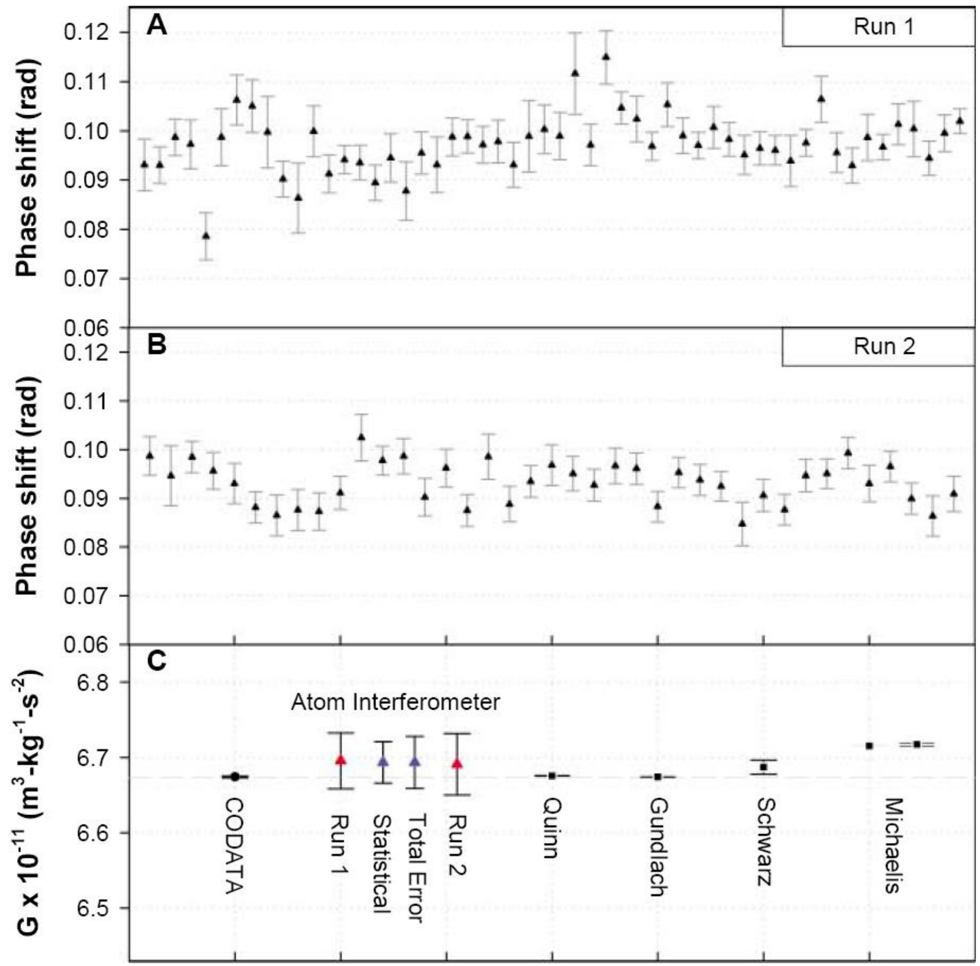
Results from the light-pulse interactions and is determined by the phases of the laser fields evaluated at the semi-classical (mean) positions of the wave packets during each of the laser-atom interactions.



~10 cm ballistic trajectory

Determination of G

A. Analysis of the phase shifts resulted in a value for $G = 6.696\times10^{-11}\pm0.037\times10^{-11}$ $m^3/(kg.s^2)$.
B. A second measurement of G with a different initial vertical position of the source mass and a redistribution of the individual discs comprising the Pb stack. The same analysis as the first measurement gave a value of $G = 6.691\times10^{-11}\pm0.041\times10^{-11} m^3/(kg.s^2)$.
C. Combined results of our measurements agree within statistical uncertainties of each other and of the CODATA value, resulting in $G = 6.693\times 10^{-11}\pm0.027\times10^{-11} m^3/(kg.s^2)$ (statistical error) and $6.693\pm0.021\times10^{-11}$ $m^3/(kg.s^2)$ (systematic error). Also shown (left to right) are values reported in other experiments. Error bars show means ± SD.



J. B. Fixler, G. T. Foster, J. M. McGuirk, M. A. Kasevich, Science **315**, 74 (2007).

Previous theoretical work

- Analytical results based on Thomas-Fermi approximation and harmonic decomposition

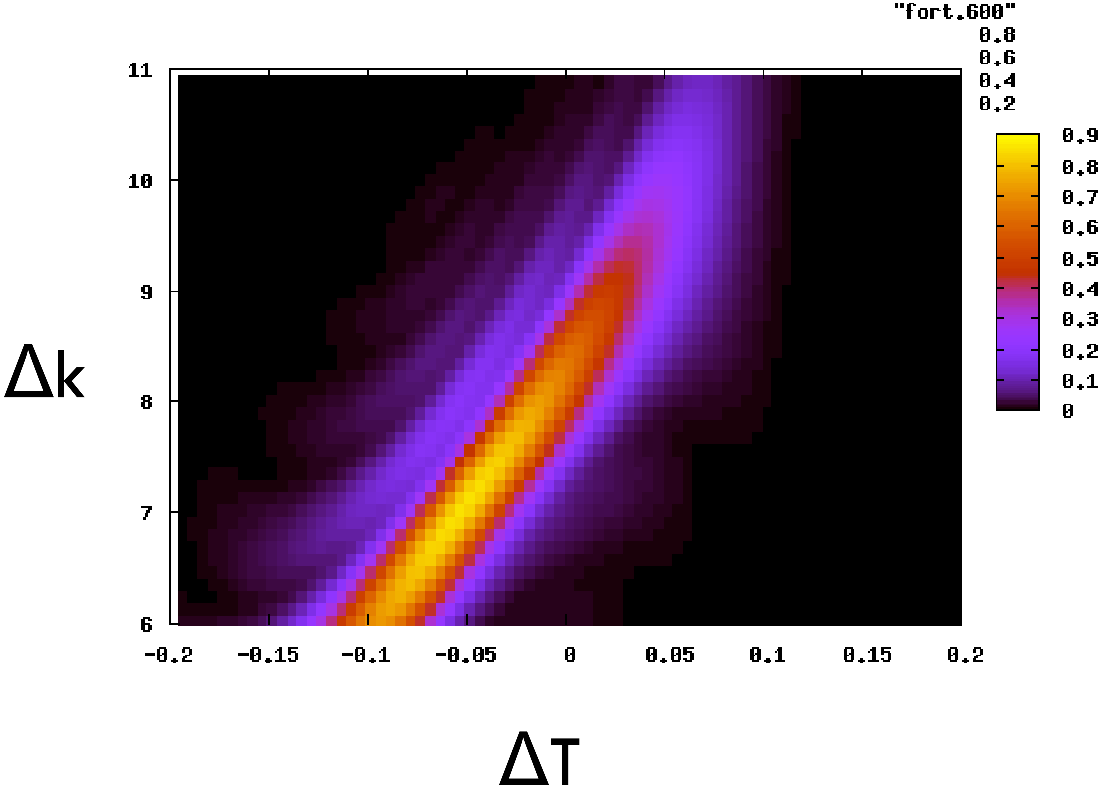
$$\psi_\pm(x,t) = \sqrt{n_\pm(x,t)}\,\mathrm{exp}(i\phi_\pm(x,t))$$

  - Parabolic approximation for density and the phase
  - The time evolution of the density and the phase can be derived.
  - Similar strategies are proposed.

Contrast optimization

Mach-Zehnder interferometer in the presence of trapping

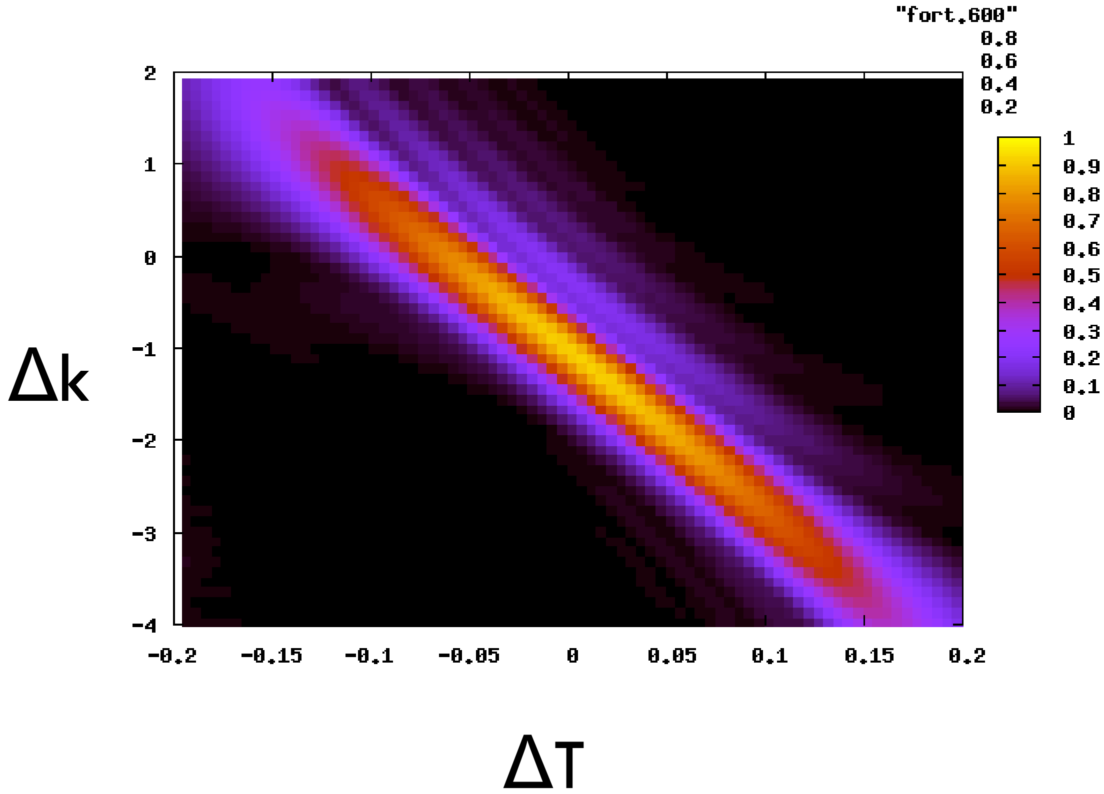
- Contrast as the function of time lag ΔT and the change in the wave vector Δk of recombination Bragg pulse with N=3000 in the presence of trapping.



Contrast optimization

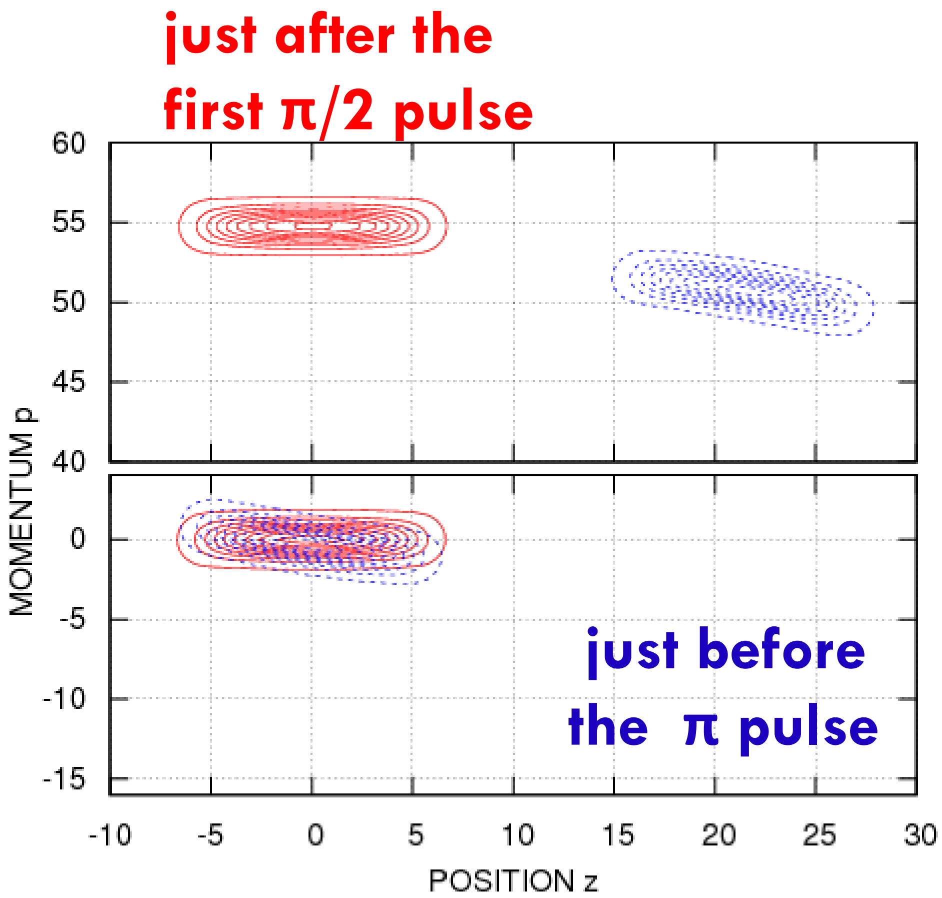
Mach-Zehnder interferometer in the absence of trapping

- Contrast as the function of time lag ΔT and the change in the wave vector Δk of recombination Bragg pulse with N=3000 in the presence of trapping.



Time evolution of Husimi representation

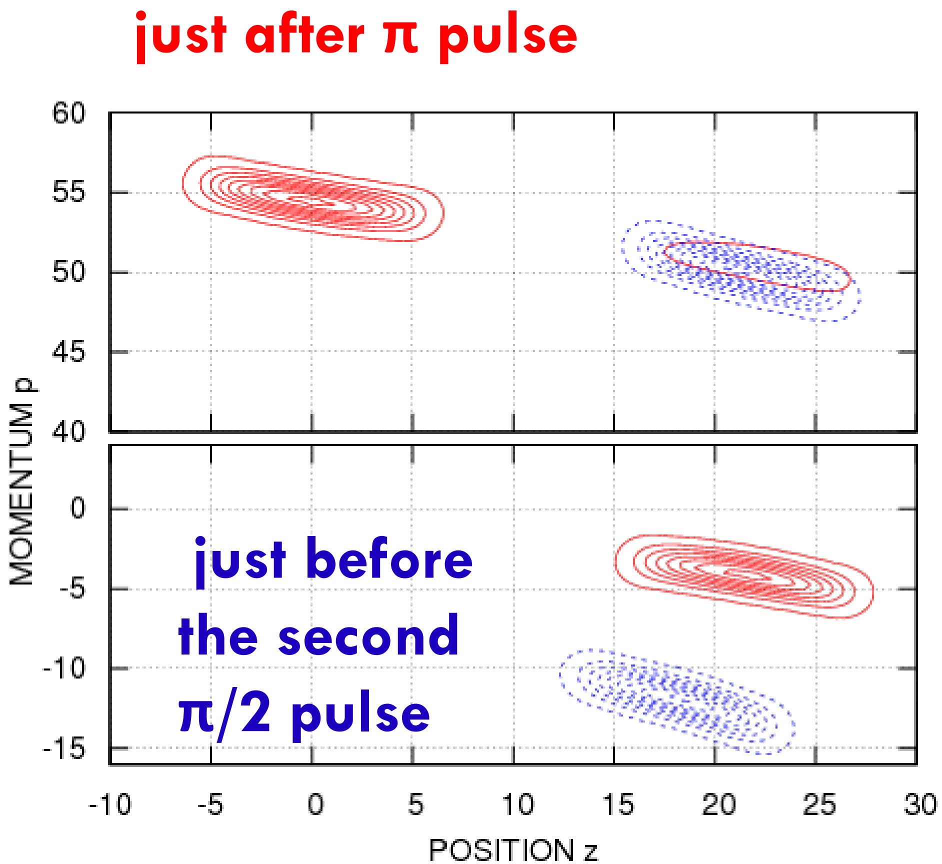
Mach-Zehnder interferometer in the presence of trapping



- Circular trajectory in phase space due to the harmonic trap.

Time evolution of Husimi representation

Mach-Zehnder interferometer in the presence of trapping



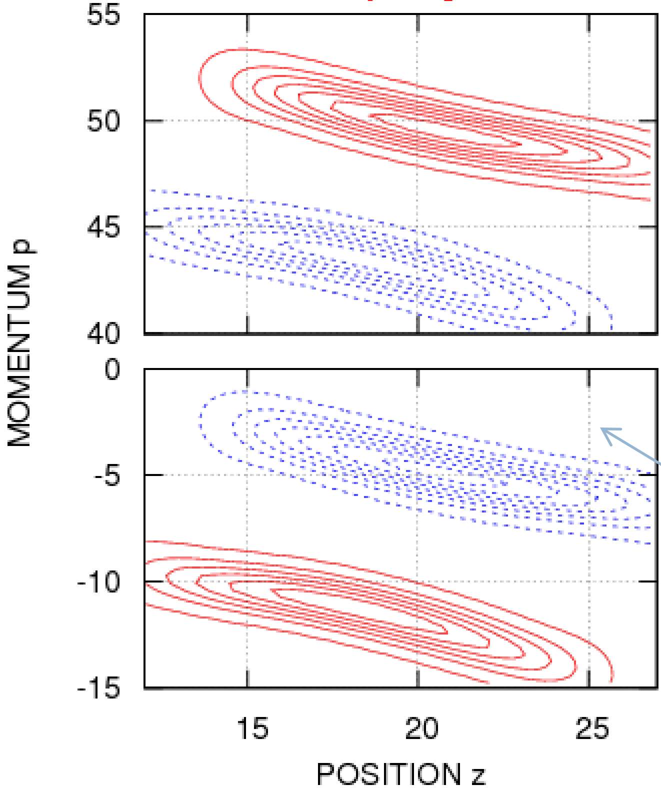
- Condition of a perfect contrast of interference signal:

  The Husimi representation of the two components just before the recombination are related by a **translation** of **2ħk in momentum.**

Time evolution of Husimi representation

Mach-Zehnder interferometer in the presence of trapping

just before the second $\pi/2$ pulse



after the second $\pi/2$ pulse

- Complete loss of contrast!
- Can we fix this?

An experimentalists' comment

- An exact calculation would involve integration of Schödinger's equation for a wavepacket subject to the laser-pulse sequence and the gravitational potential of the source mass and Earth. Such a calculation is computationally intractable.
  - J. B. Fixler, G. T. Foster, J. M. McGuirk, M. A. Kasevich, Science **315**, 74 (2007).