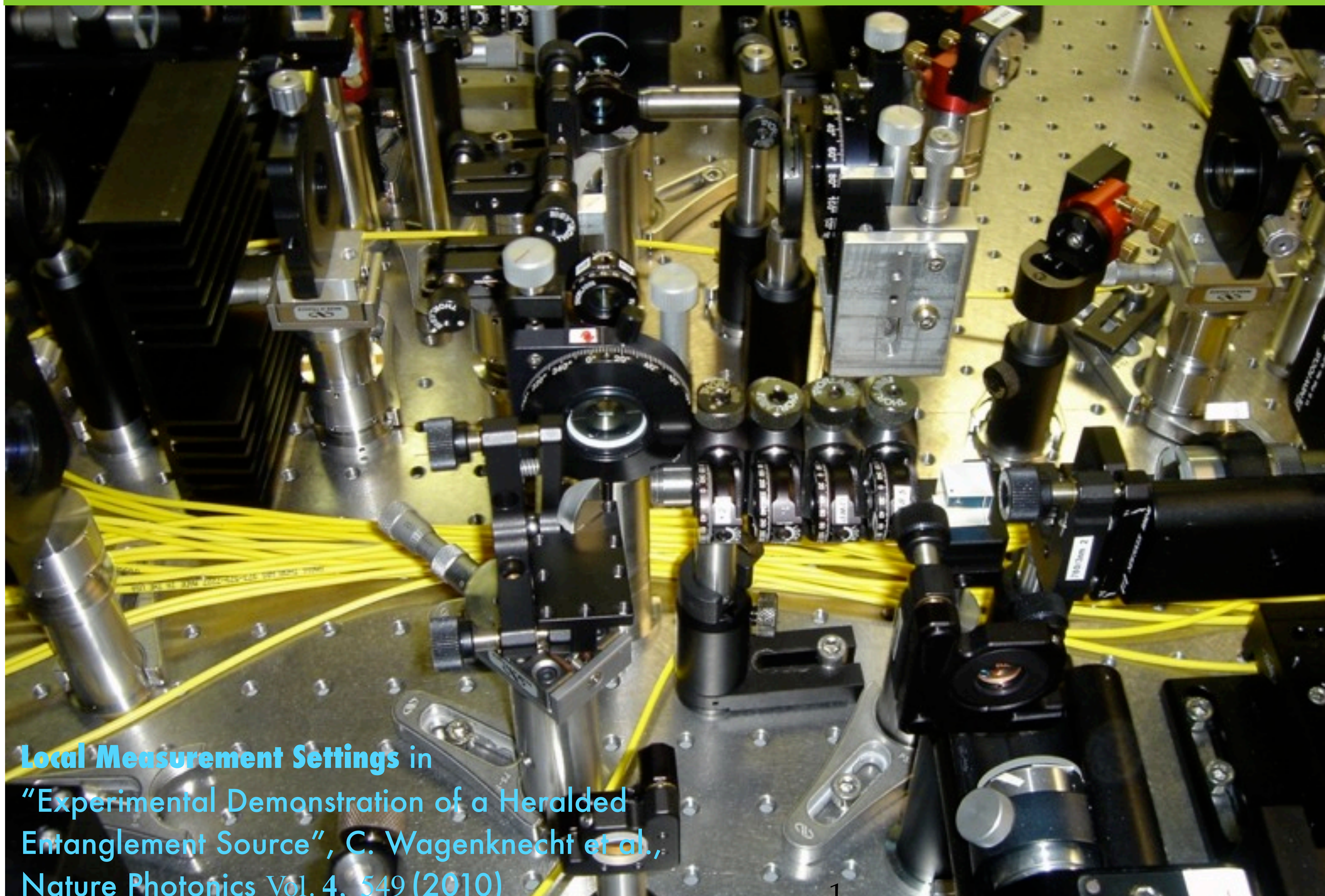
Verification of Genuine High-order Photon Entanglement



Local Measurement Settings in "Experimental Demonstration of a Heralded Entanglement Source", C. Wagenknecht et al., Nature Photonics Vol. 4, 549 (2010)

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General Trend towards Entanglement-based Quantum Information Processing

General Trend towards Entanglement-based Quantum Information Processing

Deterministic Entanglement Source

General Trend towards Entanglement-based Quantum Information Processing

Deterministic Entanglement Source

Single-Degree-of-Freedom Entangled Qubits

General Trend towards Entanglement-based Quantum Information Processing

Deterministic Entanglement Source

Single-Degree-of-Freedom Entangled Qubits

High-order Entanglement

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Detection of Genuine High-order Entanglement

General Protocols for QIP

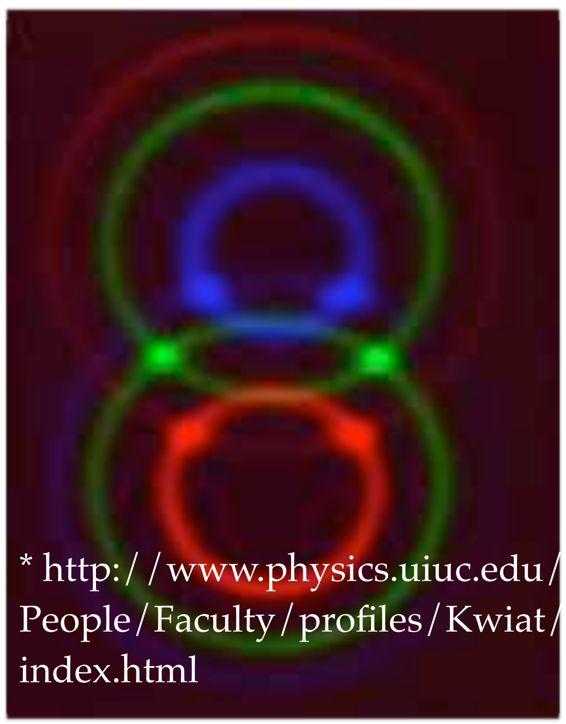
Outline

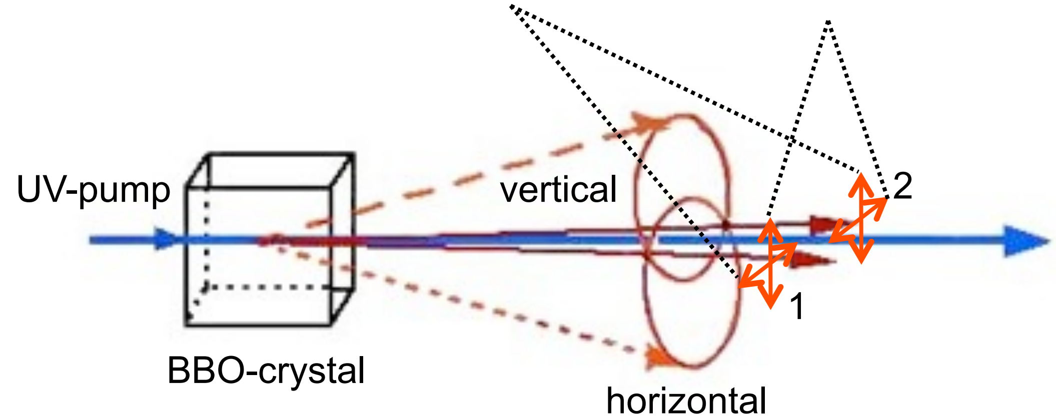
- Quantum Entanglement & Quantum Information Processing
- Entanglement Detection
- Summary

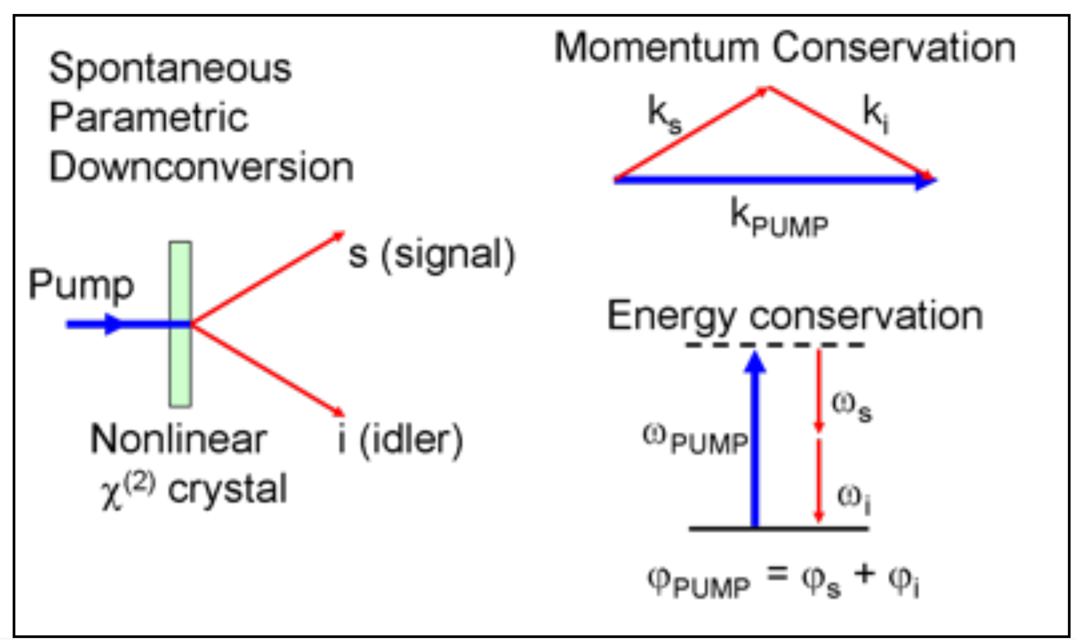
Quantum Entanglement & Quantum Information Processing

- **Source**: Polarization-entangled photon pairs (SPDC source)

$$|\psi\rangle = |H\rangle_1 |V\rangle_2 + e^{i\theta} |V\rangle_1 |H\rangle_2$$







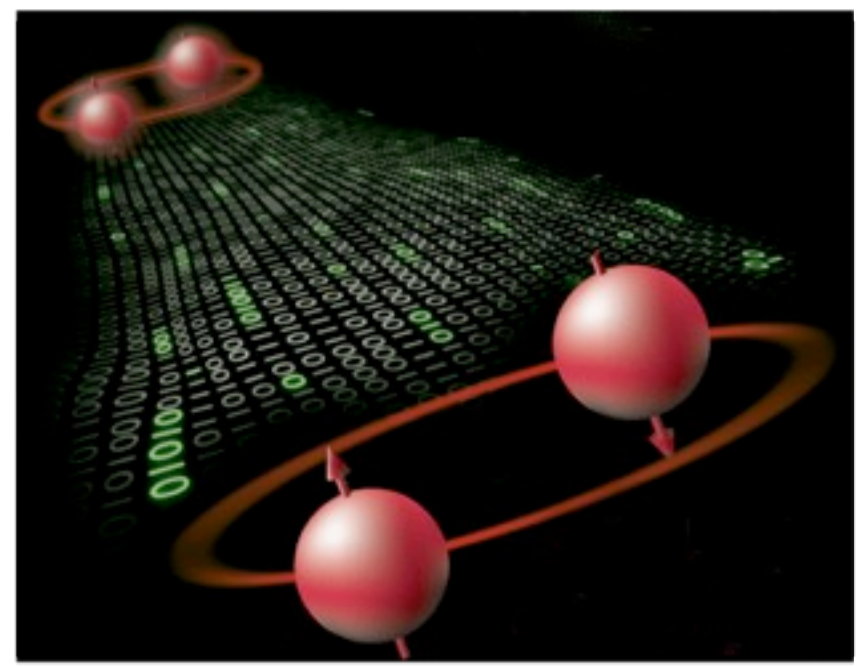
Kwiat, P. G., Mattle, K., Weinfurter, H., Zeilinger, A., Alexander, V. S. & Shih, Y. H. *New high-intensity source of polarization-entangled photon pairs.* Phys. Rev. Lett. **75**, 4337-4341 (1995).

Quantum Entanglement & Quantum Information Processing

- Quantum Teleportation

(top, left) Richard Jozsa, William K. Wootters, Charles H. Bennett. (bottom, left) Gilles Brassard, Claude Crépeau, Asher Peres. Photo: André Berthiaume.

C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W. K. Wootters, *Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels*, Phys. Rev. Lett. **70**, 1895-1899 (1993)

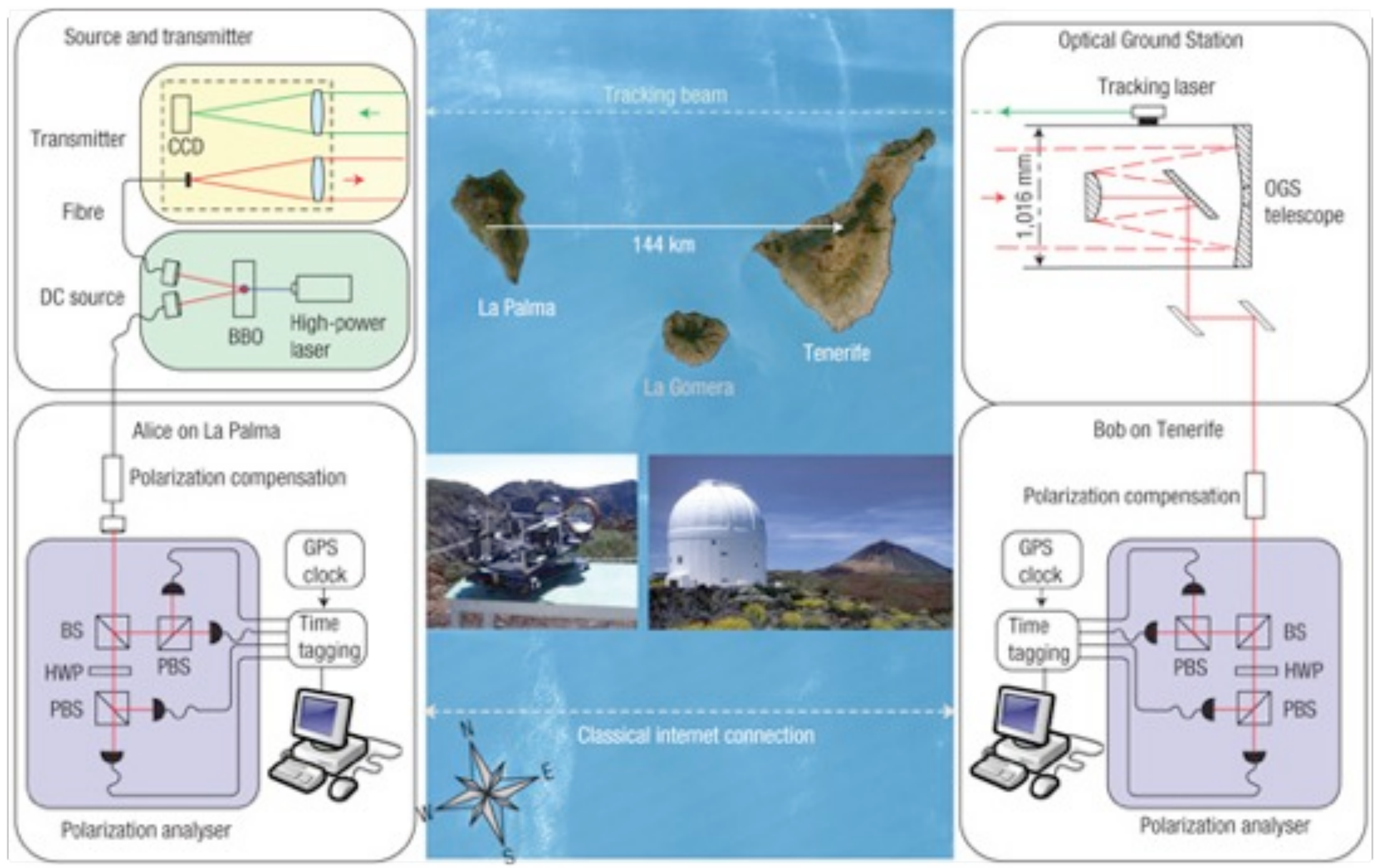


Q. Zhang et al., *Experimental quantum teleportation of a two-qubit composite system*, Nature Phys. **2**, 678-682 (2006)

Quantum Entanglement & Quantum Information Processing

- Quantum Cryptography

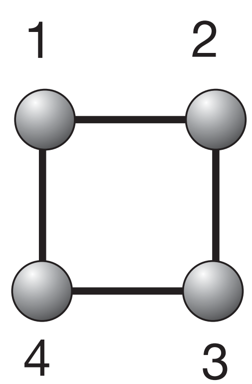
A. K. Ekert, *Quantum cryptography based on Bell's theorem*, Phys. Rev. Lett. 67, 661–663 (1991).



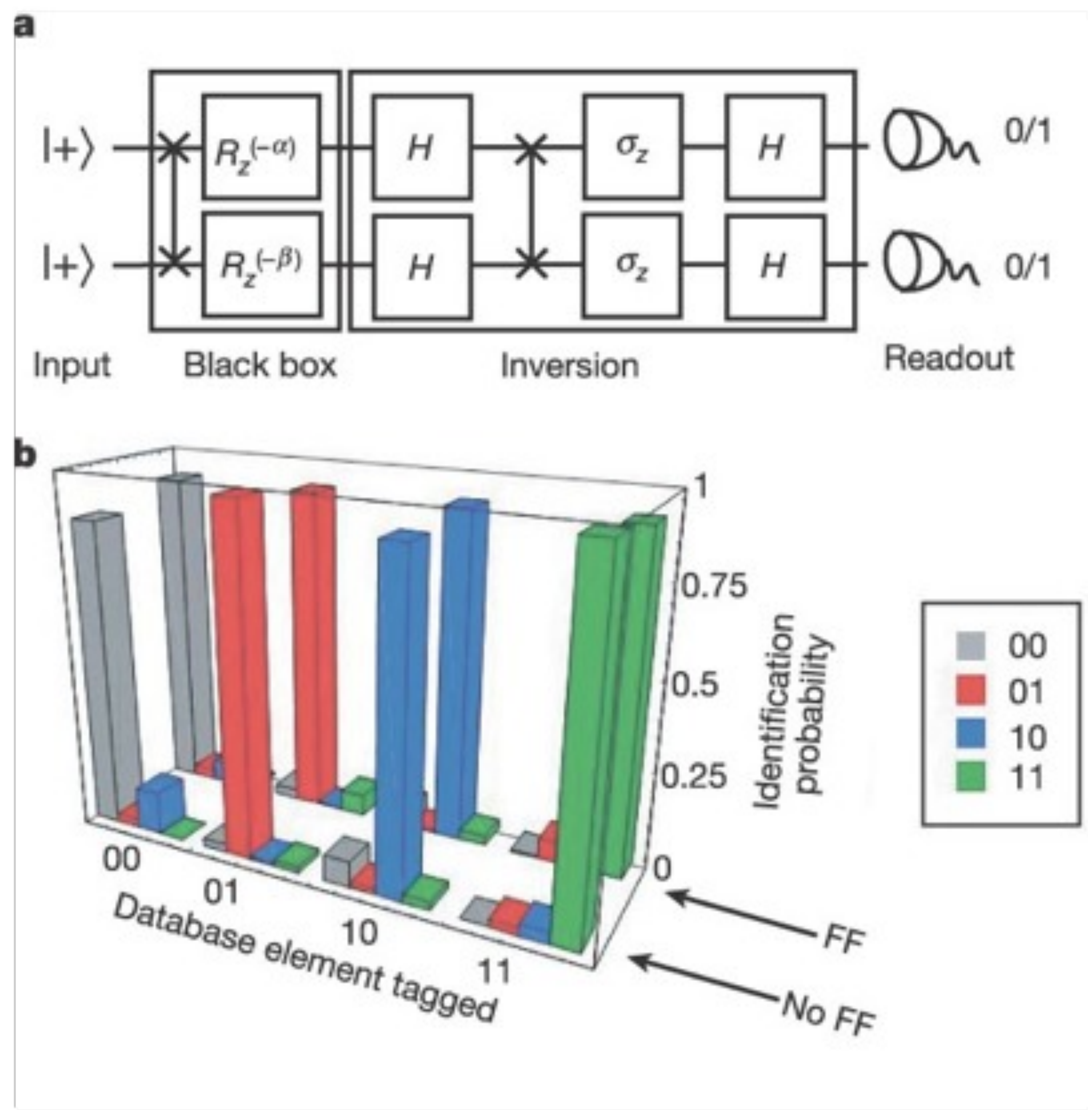
R. Ursin et al., *Entanglement-based quantum communication over 144 km*, Nature Phys. **3**, 481-486 (2007).

Quantum Entanglement & Quantum Information Processing

- Quantum Computation -One-way quantum computer



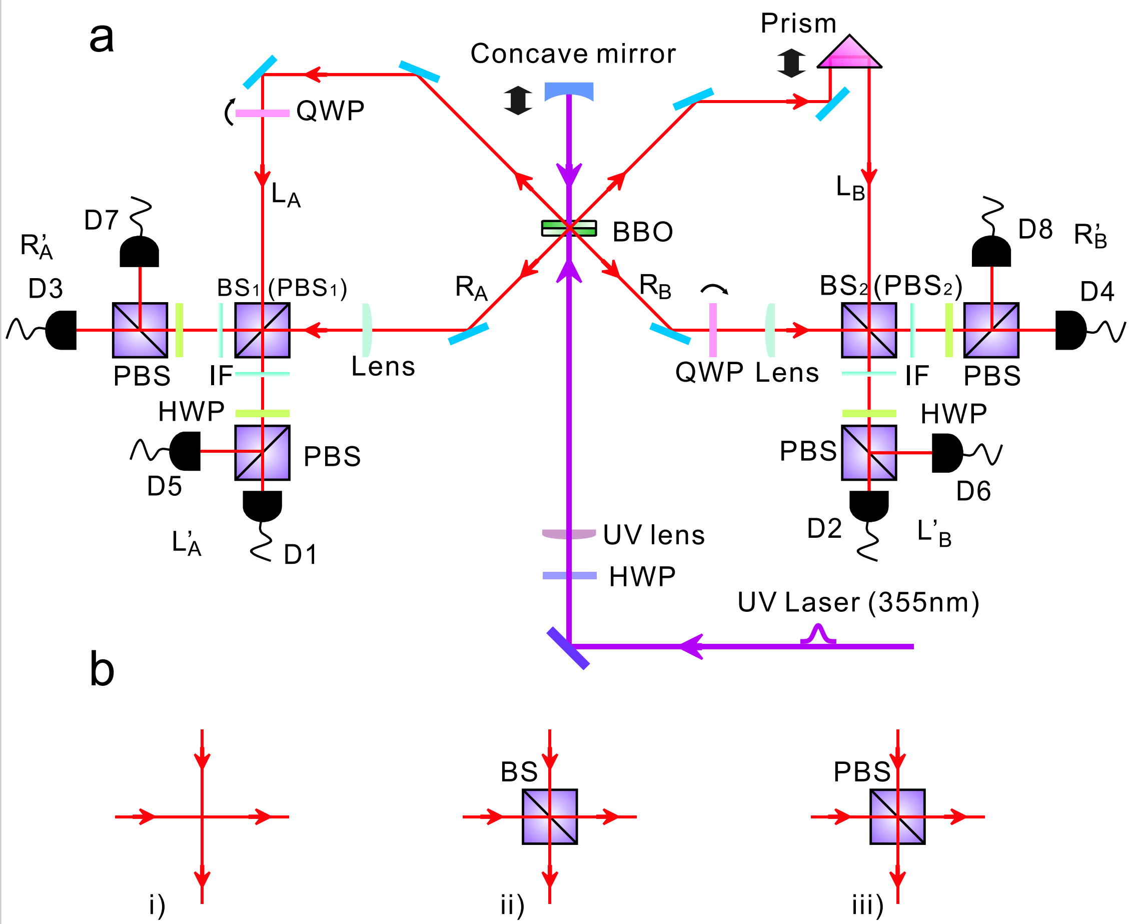
R. Raussendorf & H. J. Briegel, *A one-way quantum computer*, Phys. Rev. Lett. **86**, 5188–5191 (2001)

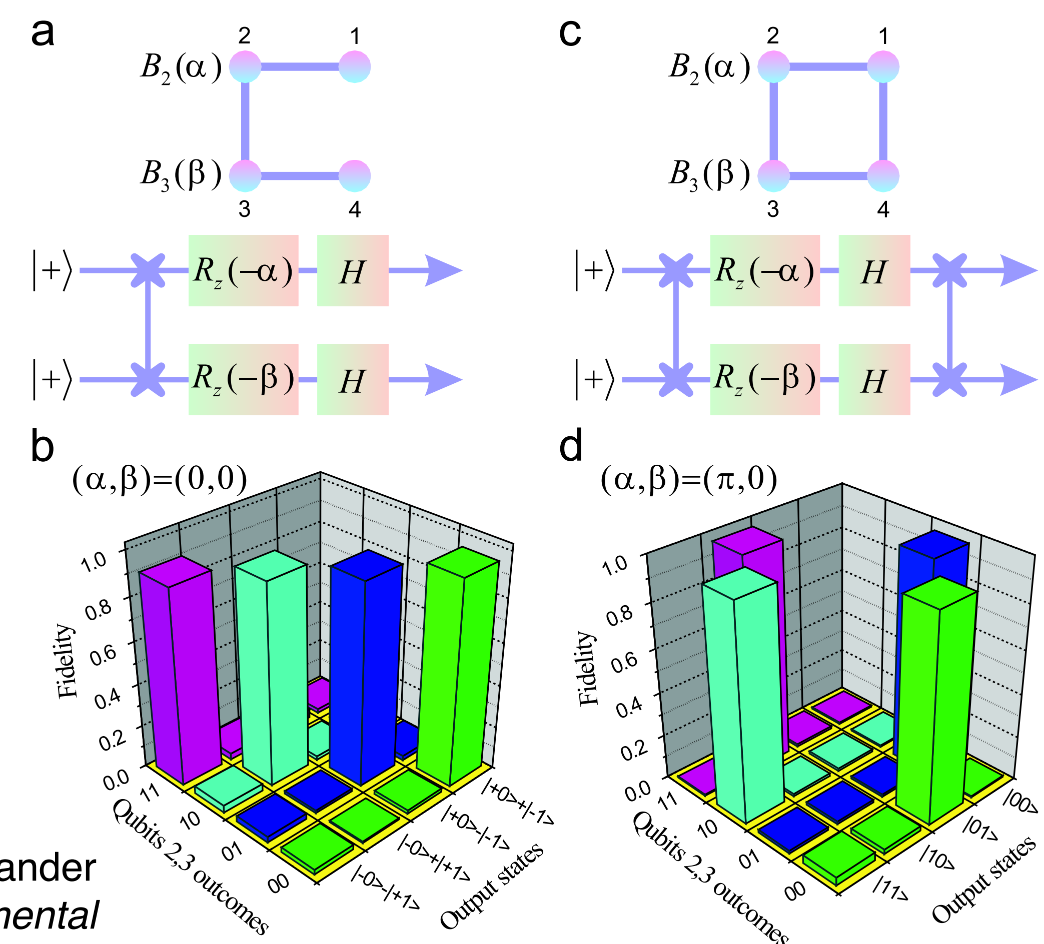


P. Walther et al., *Experimental one-way quantum comuting*, Nature **434**, 169–176(2005)

Quantum Entanglement & Quantum Information Processing

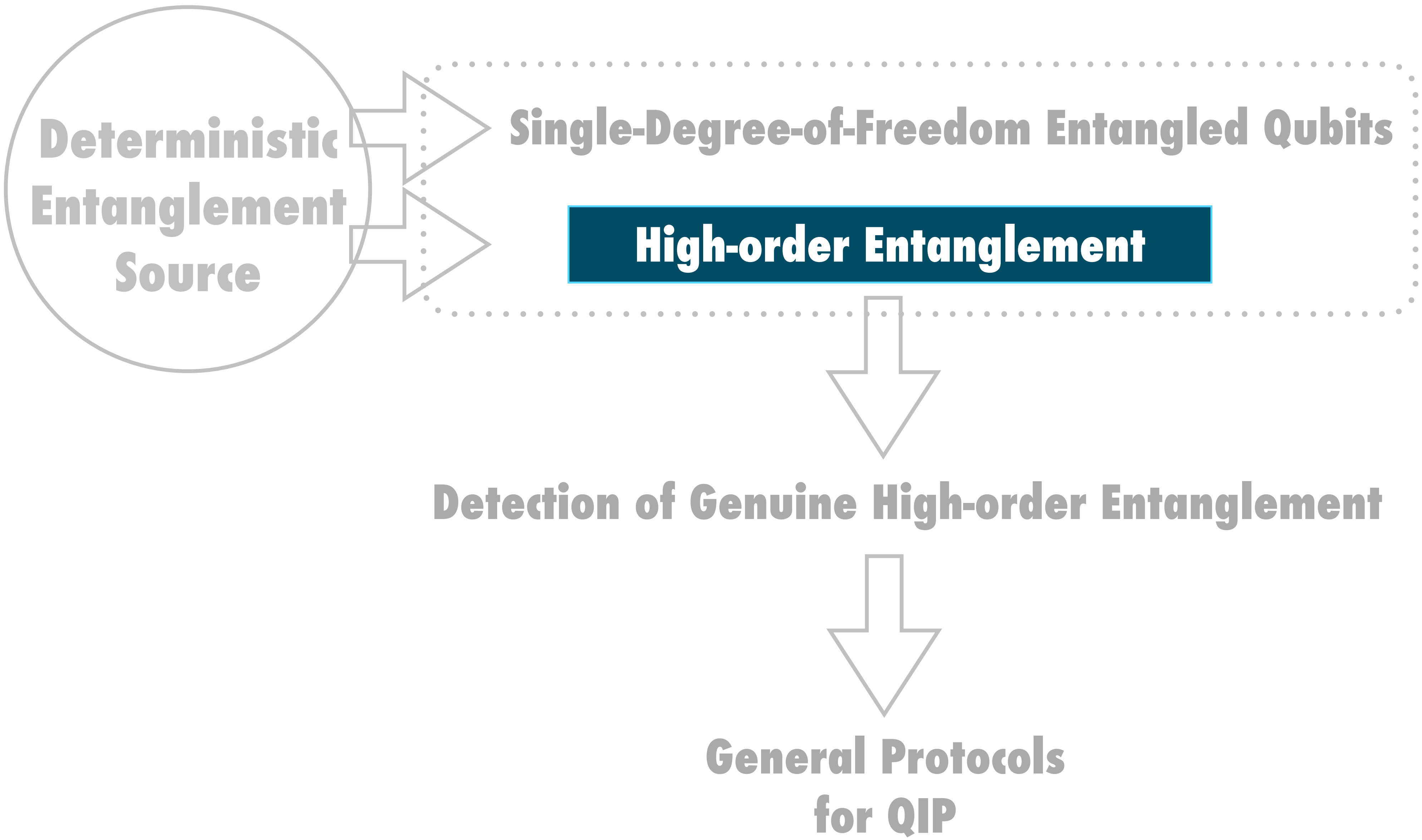
- Quantum Computation -One-way quantum computer





Kai Chen, Che-Ming Li, Qiang Zhang, Yu-Ao Chen, Alexander Goebel, Shuai Chen, Alois Mair, and Jian-Wei Pan, *Experimental Realization of One-Way Quantum Computing with Two-Photon Four-Qubit Cluster States*. Phys. Rev. Lett. **99**, 120503 (2007).

General Trend towards Quantum Information Processing



High-order Entanglement

- Orbital Angular Momentum (OAM)-entangled photon pairs

Encoding Qutrit (3-level Quantum System) in OAM of light:

- The OAM is associated with the *transverse phase front* of light beam

OAM = -1 hbar *OAM = 0 hbar* *OAM = +1 hbar*

The phase fronts of light beams in OAM eigenstates rotate

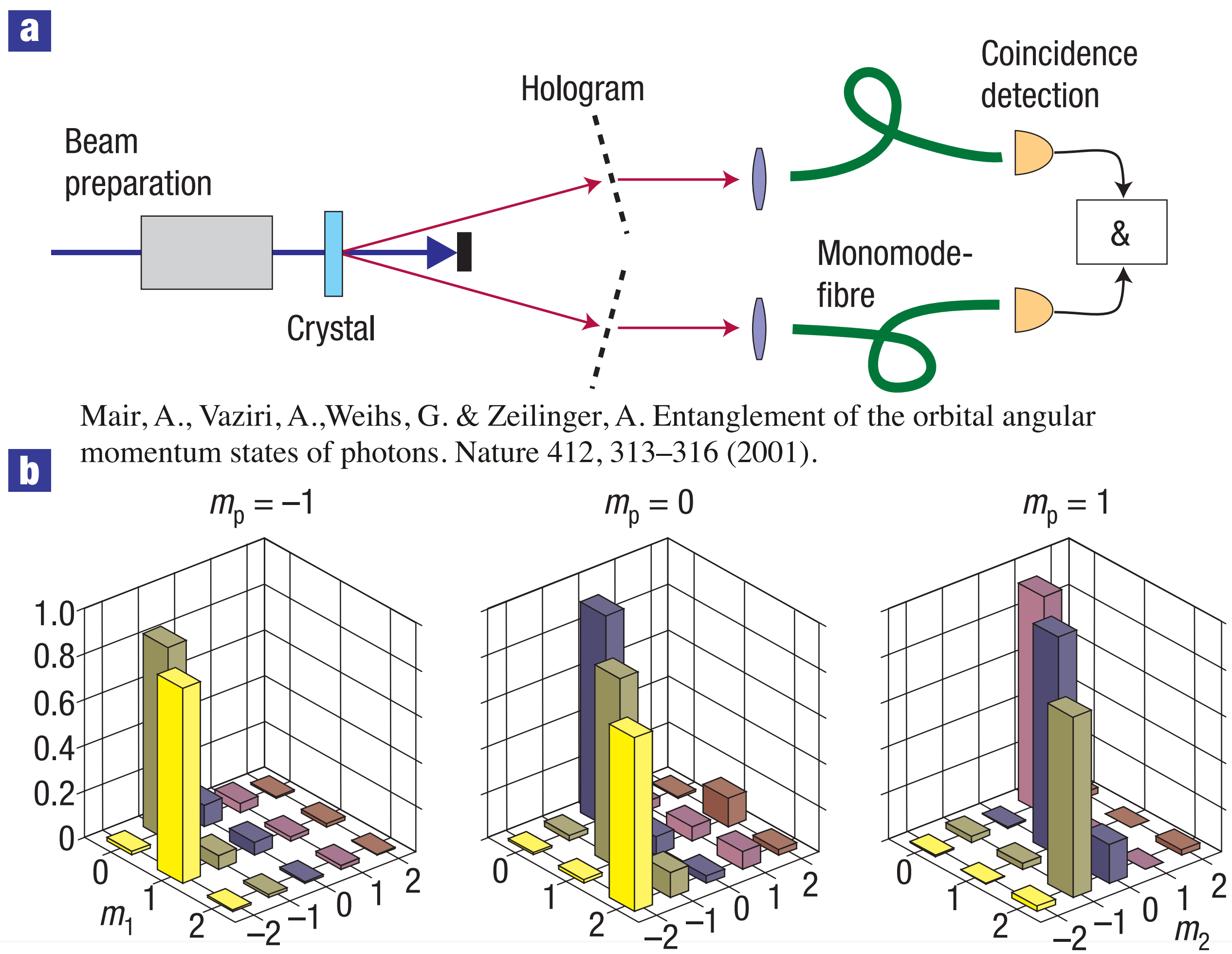
http://www.physics.gla.ac.uk/Optics/

- Such light beams are conveniently described in terms of *Laguerre-Gaussian modes*

High-order Entanglement

- Orbital Angular Momentum (OAM)-entangled photon pairs

The quantum correlation between Laguerre-Gaussian modes can be created in a down-conversion experiment:



Mair, A., Vaziri, A.,Weihs, G. & Zeilinger, A. Entanglement of the orbital angular momentum states of photons. Nature 412, 313–316 (2001).

a, Experimental configuration used to detect the quantum correlations in OAM of paired photons generated in an SPDC

b, Experimental data demonstrating that the OAM of the pump beam (m_p) is transferred to the sum of OAM of the generated photons (m_1 and m_2). In this particular case, the state of the down-converted photons is a coherent quantum superposition of all the different possibilities for the OAM state of the photons fulfilling the condition $m_p = m_1 + m_2$.
Ex: $m_p = 0$

$$|+\hbar, -\hbar\rangle + |-\hbar, +\hbar\rangle + |0, 0\rangle$$

General Trend towards Quantum Information Processing

Deterministic Entanglement Source

Single-Degree-of-Freedom Entanglement

High-order Entanglement

Detection of Genuine High-order Entanglement

General Protocols for QIP

Detecting Genuine High-order Entanglement

Detection of Genuine High-order Entanglement

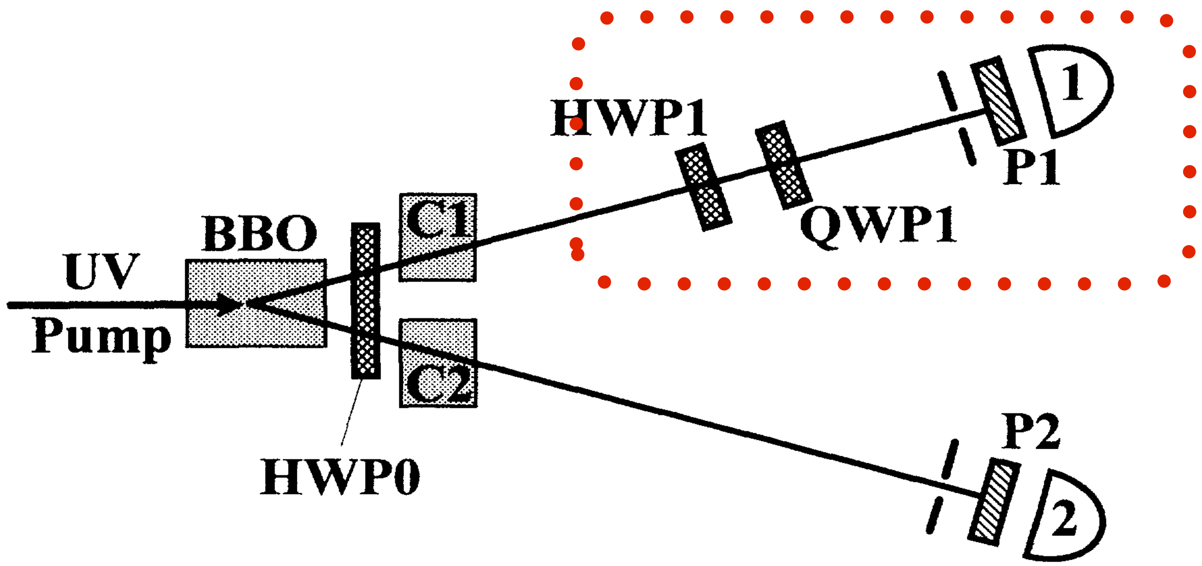
Local Measurement Settings

Correlation Criterion

Detecting Genuine High-order Entanglement

Local Measurement Settings (for qubits)

Ex: Settings for Polarization-Entangled Photon Pairs



- HWP + QWP: Single Qubit Rotation
- PBS or Polarizer: Qubit State Selection
- Single Photon Detector: Qubit Detection

polarization analyzer

P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, "New high-intensity source of polarization-entangled photon pairs", Phys. Rev. Lett. 75, 4337 (1995).

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Bell-type Inequality

$$S = E(\theta_1, \theta_2) + E(\theta_1', \theta_2) + E(\theta_1, \theta_2') - E(\theta_1', \theta_2'),$$ Bell kernel

and $E(\theta_1, \theta_2)$ is given by

$$\frac{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) - C(\theta_1, \theta_2^\perp) - C(\theta_1^\perp, \theta_2)}{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) + C(\theta_1, \theta_2^\perp) + C(\theta_1^\perp, \theta_2)}.$$

S>2: Quantum Correlation

P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, "New high-intensity source of polarization-entangled photon pairs", Phys. Rev. Lett. 75, 4337 (1995).

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Bell-type Inequality

$$S = E(\theta_1, \theta_2) + E(\theta_1', \theta_2) + E(\theta_1, \theta_2') - E(\theta_1', \theta_2'),$$ Bell kernel

and $E(\theta_1, \theta_2)$ is given by **4 Local Measurement Settings**

$$\frac{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) - C(\theta_1, \theta_2^\perp) - C(\theta_1^\perp, \theta_2)}{C(\theta_1, \theta_2) + C(\theta_1^\perp, \theta_2^\perp) + C(\theta_1, \theta_2^\perp) + C(\theta_1^\perp, \theta_2)}.$$

S>2: Quantum Correlation

P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. H. Shih, "New high-intensity source of polarization-entangled photon pairs", Phys. Rev. Lett. 75, 4337 (1995).

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Entanglement Witness

Target state: $|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$

Experimental State (output): ρ_Φ

Entanglement Witness: $\hat{W}_\Phi = \frac{1}{2}I - |\Phi\rangle\langle\Phi|$

Correlation Criterion:

$Tr[\hat{W}_\Phi \rho_\Phi] < 0 \Rightarrow$ ρ_Φ **is a Entangled state close to** $|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Entanglement Witness

Correlation Criterion:

$$Tr[\hat{W}_\Phi \rho_\Phi] < 0 \quad \Rightarrow$$

ρ_Φ is a Entangled state close to

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$$

Local Operator Decomposition (LOD) i.e. Measuring State Fidelity

$$\hat{W}_\Phi = \frac{1}{2}I - \boxed{|\Phi\rangle\langle\Phi|} \quad \Rightarrow \quad |\Phi\rangle\langle\Phi| = \frac{1}{4}(I + \sigma_z \otimes \sigma_z + \sigma_x \otimes \sigma_x - \sigma_y \otimes \sigma_y)$$

$$\sigma_z = |H\rangle\langle H| - |V\rangle\langle V| \qquad \sigma_x = |+\rangle\langle +| - |-\rangle\langle -|$$

$$\sigma_y = |R\rangle\langle R| - |L\rangle\langle L|$$

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Entanglement Witness

Correlation Criterion:

$$Tr[\hat{W}_\Phi \rho_\Phi] < 0$$

ρ_Φ is a Entangled state close to

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$$

Quantum State Tomography (QST)

$$\rho_\Phi = \begin{bmatrix} \rho_{00} & \rho_{01} & \rho_{02} & \rho_{03} \\ \rho_{10} & \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{20} & \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{30} & \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix}$$

Detecting Genuine High-order Entanglement

Correlation Criterion (for qubits)

Ex: Entanglement Witness

Correlation Criterion:

$$Tr[\hat{W}_{\Phi}\rho_{\Phi}] < 0 \Rightarrow$$

ρ_{Φ} is a Entangled state close to

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle)$$

Quantum State Tomography (QST)

Local Operator Decomposition (LOD) i.e. Measuring State Fidelity

	QST	LOD
# of Local Measurement Settings	15	3

Detecting Genuine High-order Entanglement

Detection of Genuine High-order Entanglement

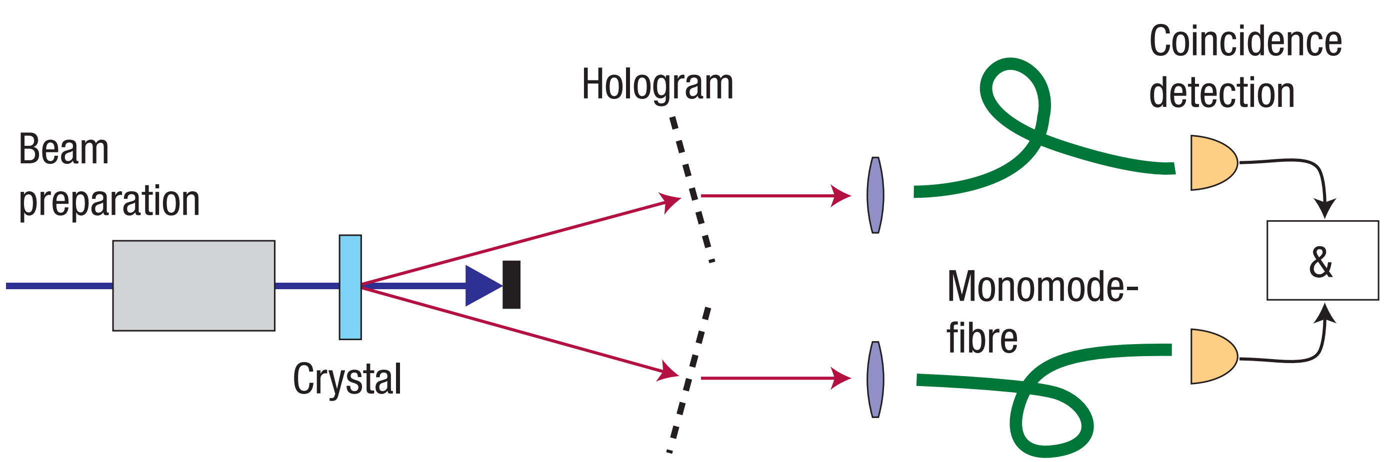
Local Measurement Settings

Correlation Criterion

Detecting Genuine High-order Entanglement

Local Measurement Settings (for qutrits)

Ex: Settings for OAM-Entangled Photon Pairs



Mair, A., Vaziri, A.,Weihs, G. & Zeilinger, A. Entanglement of the orbital angular momentum states of photons. Nature 412, 313–316 (2001).

- **Hologram:** Single Qutrit Rotation
- **Single-mode Fiber:** Qutriit State Selection (lowest order spatial component)
- **Single Photon Detector:** Qutrit Detection

spatial mode analyzer

Detecting Genuine High-order Entanglement

Correlation Criterion (for qutrits)

Ex: Bell-type Inequality (4 Local Measurement Settings)

$$\begin{aligned} S_3 = &+ P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) \\ &+ P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) \\ &- P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1), \end{aligned}$$

Bell kernel

where

$$P(A_a = B_b + k) = \sum_{j=0}^{2} P(A_a = j, B_b = j + k\text{mod}3)$$

S>2: Quantum Correlation

A. Vaziri, G. Weihs, & A. Zeilinger, *Experimental two-photon, three dimensional entanglement for quantum communications*. Phys. Rev. Lett. 89, 240401(2002).

Detecting Genuine High-order Entanglement

Correlation Criterion (for qutrits)

Ex: Bell-type Inequality (4 Local Measurement Settings)

$$\begin{aligned} S_3 = &+ P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) \\ &+ P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) \\ &- P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1), \end{aligned}$$ Bell kernel

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S>2: Quantum Correlation

Genuine 3-level entanglement??

A. Vaziri, G. Weihs, & A. Zeilinger, *Experimental two-photon, three dimensional entanglement for quantum communications*. Phys. Rev. Lett. 89, 240401(2002).

Detecting Genuine High-order Entanglement

Correlation Criterion (for qutrits)

Ex: Bell-type Inequality (4 Local Measurement Settings)

$$
\begin{aligned}
S_3 = &+ P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) \\
&+ P(B_2 = A_1) - P(A_1 = B_1 - 1) - P(B_1 = A_2) \\
&- P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1),
\end{aligned}
$$

Bell kernel

where

$$P(A_a = B_b + k) = \sum_{j=0}^{2} P(A_a = j, B_b = j + k \mathrm{mod} 3)$$

4 Local Measurement Settings:

(A_1, B_1) (A_1, B_2)

(A_2, B_1) (A_2, B_2)

S>2: Quantum Correlation

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Detecting Genuine High-order Entanglement

Correlation Criterion (for qutrits)

Ex: Entanglement Witness

Target state: $|\mathrm{MES}\rangle = (e^{i\alpha\pi}|L\rangle|r\rangle + |G\rangle|g\rangle + e^{i\beta\pi}|R\rangle|l\rangle)/\sqrt{3}$

Inoue, R., Yonehara, T., Miyamoto, Y., Koashi, M. & Kozuma, M. *Measuring Qutrit-Qutrit Entanglement of Orbital Angular Momentum States of an Atomic Ensemble and a Photo*n. Phys. Rev. Lett. **103**, 110503 (2009).

Sanpera, A., Bruss, D. & Lewenstein, M. Schmidt-number witnesses and bound entanglement. Phys. Rev. A **63**, 050301(R) (2001).

Thew, R. T., Nemoto, K., Weihs, A. G. & Munro, W. J. Qudit quantum-state tomography. Phys. Rev. A **66**, 012303 (2002).

Detecting Genuine High-order Entanglement

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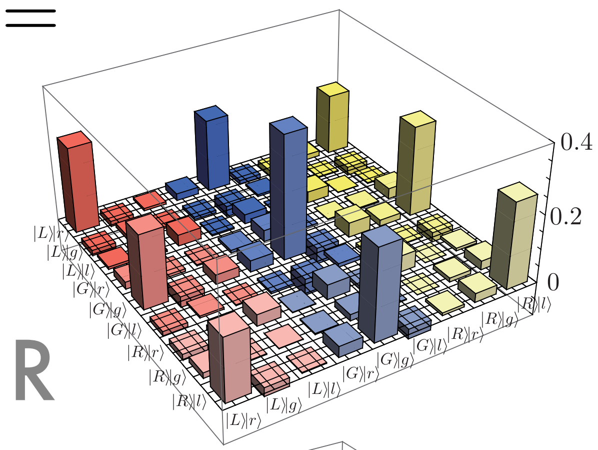
(1) Quantum State Tomography

$\hat{\rho}_{\mathrm{exp}} =$



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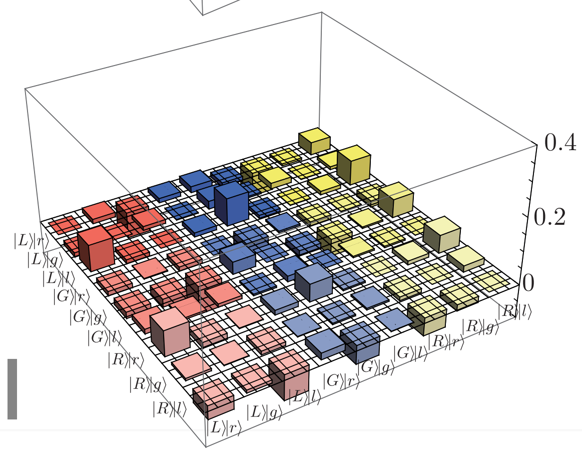

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Inoue, R., Yonehara, T., Miyamoto, Y., Koashi, M. & Kozuma, M. *Measuring Qutrit-Qutrit Entanglement of Orbital Angular Momentum States of an Atomic Ensemble and a Photo*n. Phys. Rev. Lett. **103**, 110503 (2009).

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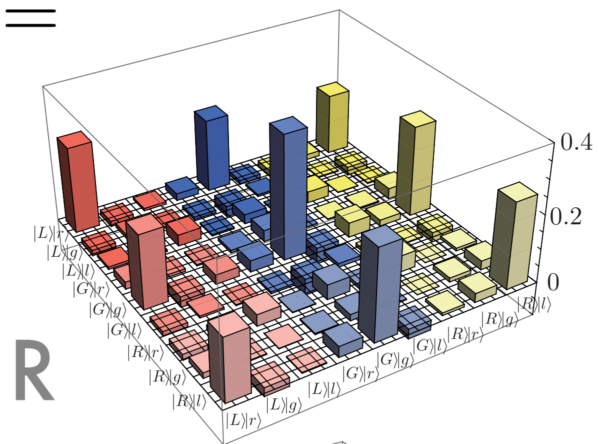

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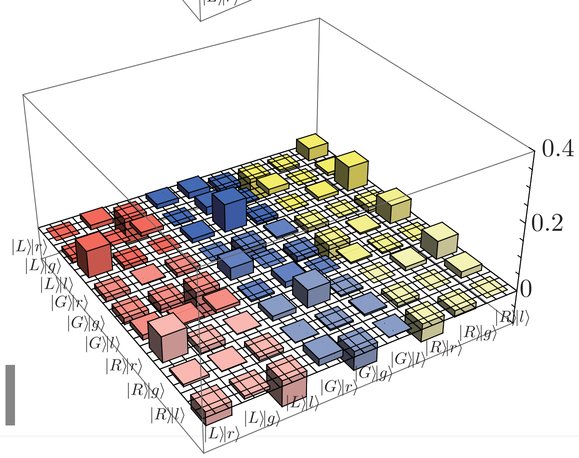

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(2) Experimental State Fidelity:

$$F_{\mathrm{exp}} \equiv \langle \mathrm{MES}|\hat{\rho}_{\mathrm{exp}}|\mathrm{MES}\rangle = 0.74 \pm 0.02$$

Inoue, R., Yonehara, T., Miyamoto, Y., Koashi, M. & Kozuma, M. *Measuring Qutrit-Qutrit Entanglement of Orbital Angular Momentum States of an Atomic Ensemble and a Photo*n. Phys. Rev. Lett. **103**, 110503 (2009).

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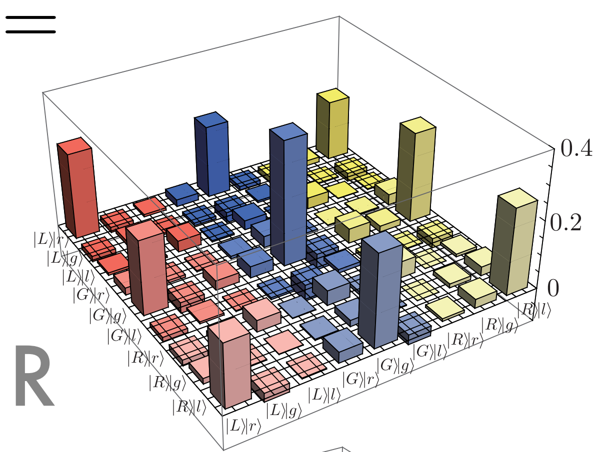
$\hat{\rho}_{\mathrm{exp}} =$

R



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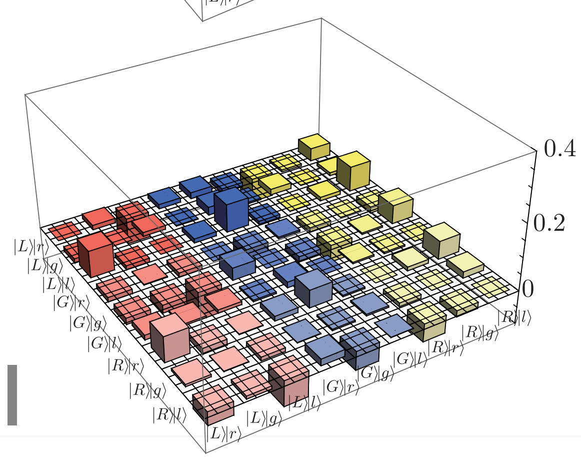

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(3) The experimental state is identified as **Genuine three-level entangled** through the correlation criterion:

$$F_{\mathrm{exp}} > 2/3$$

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(1) Quantum State Tomography

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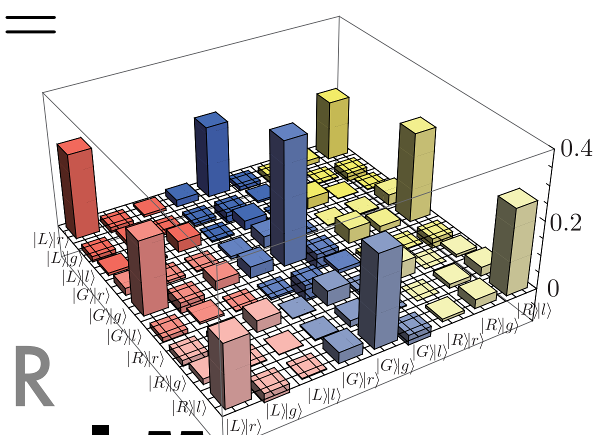
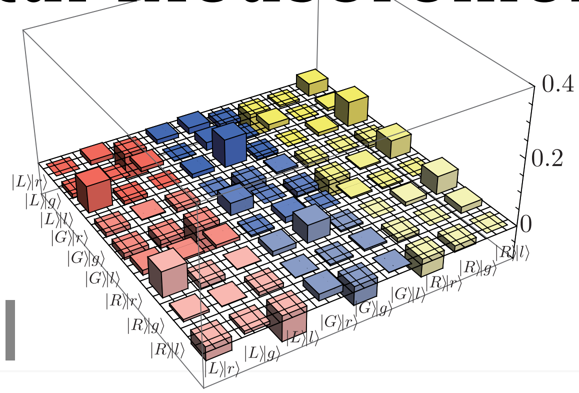
R



80 Local Measurement Settings



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Detecting Genuine High-order Entanglement

- Generic multilevel Bell-type Inequalities (BI)

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d: level #; N: qudit #	BI	QST+CC	LOD+CC
Multilevel & Multipartite Genuineness	X	☺	☺
# of Local Measurement Setting	2^N	$d^{2N} - 1$	$2(d^N - 1)$

Detecting Genuine High-order Entanglement

Q1: **2**-local measurement settings are enough for detecting, e.g., states close to

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|+\hbar, -\hbar\rangle + |0,0\rangle + |-\hbar, +\hbar\rangle) \quad ?$$

Q2: **2**-local measurement settings are enough for estimating experimental state fidelity without full **Quantum State Tomography**??

Q3: **2**-local measurement settings are enough for detecting **Genuine General High-order Entanglement** and for estimating experimental state fidelity without taking full **Quantum State Tomography**???

Efficient Detection of Genuine High-order Entanglement

- Detecting states close to $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zz} + |11\rangle_{zz} + |22\rangle_{zz})$

Observation:

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zx} + |12\rangle_{zx} + |21\rangle_{zx})$$

$$|ab\rangle_{zx} : a + b \doteq 0$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{xz} + |12\rangle_{xz} + |21\rangle_{xz})$$

$$|ab\rangle_{xz} : a + b \doteq 0$$

z: normal basis
x: fourier basis

Efficient Detection of Genuine High-order Entanglement

- Detecting states close to $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zz} + |11\rangle_{zz} + |22\rangle_{zz})$

(1) Correlator 1

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zx} + |12\rangle_{zx} + |21\rangle_{zx})$$

(2) Correlator 2

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{xz} + |12\rangle_{xz} + |21\rangle_{xz})$$

where $\hat{k}_o = |k\rangle_{oo}\langle k|\,,\ k \in \{0,1,2\},\ o \in z, x$

Efficient Detection of Genuine High-order Entanglement

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(2) Correlator 2

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{xz} + |12\rangle_{xz} + |21\rangle_{xz})$$

$$\begin{aligned}\hat{c}_1 = & (\hat{0}_z - \hat{1}_z) \otimes \hat{0}_x \\ & +(\hat{1}_z - \hat{2}_z) \otimes \hat{2}_x \\ & +(\hat{2}_z - \hat{0}_z) \otimes \hat{1}_x\end{aligned}$$

where $\hat{k}_o = |k\rangle_{oo} \langle k|\,,\ k \in \{0,1,2\},\ o \in z, x$

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(2) Correlator 2

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{xz} + |12\rangle_{xz} + |21\rangle_{xz})$$

$$\begin{aligned}\hat{c}_2 = & (\hat{0}_x - \hat{1}_x) \otimes \hat{0}_z \\ & +(\hat{1}_x - \hat{2}_x) \otimes \hat{2}_z \\ & +(\hat{2}_x - \hat{0}_x) \otimes \hat{1}_z\end{aligned}$$

where $\hat{k}_o = |k\rangle_{oo}\langle k|, \; k \in \{0,1,2\}, \; o \in z, x$

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Efficient Detection of Genuine High-order Entanglement

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$$|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{xz} + |12\rangle_{xz} + |21\rangle_{xz})$$

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where $\hat{k}_o = |k\rangle_{oo}\langle k|,\ k \in \{0,1,2\},\ o \in z, x$

Efficient Detection of Genuine High-order Entanglement

Correlation Criterion

Target state: $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zz} + |11\rangle_{zz} + |22\rangle_{zz})$

Experimental State: ρ_ψ

Entanglement Witness:

$$\hat{W}_\psi = 3I - (\hat{c}_1 + \hat{c}_2)$$

Correlation Criterion:

$$Tr[\hat{W}_\psi \rho_\psi] < 0$$

ρ_ψ is a Genuine 3-level Entangled State close to $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zz} + |11\rangle_{zz} + |22\rangle_{zz})$

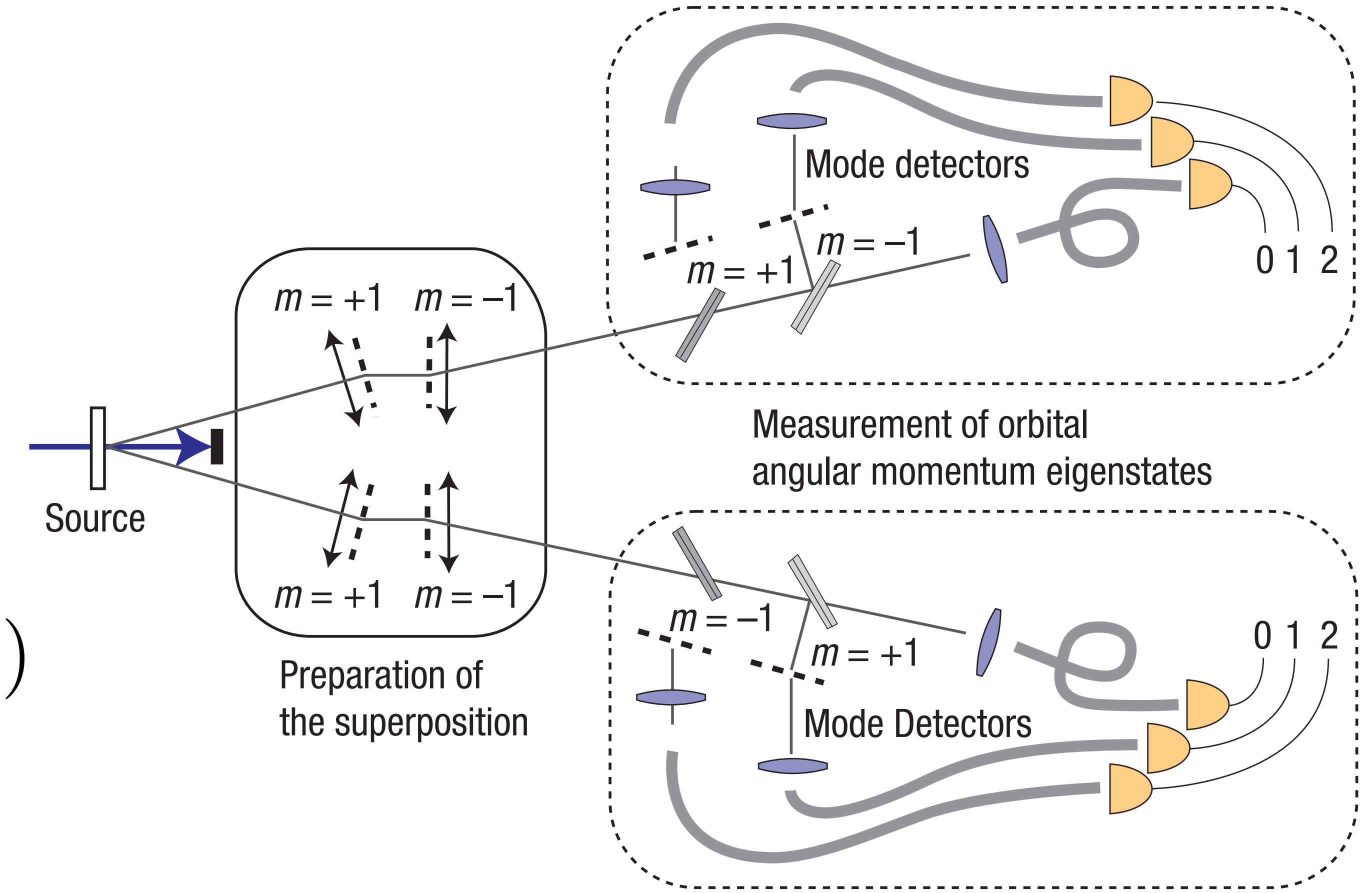
Efficient Detection of Genuine High-order Entanglement

- Detecting states close to $|\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle_{zz} + |11\rangle_{zz} + |22\rangle_{zz})$

Experimental Implementations

2 Local Measurement Settings are sufficient to implement

$$\hat{W}_\psi = 3I - (\hat{c}_1 + \hat{c}_2)$$



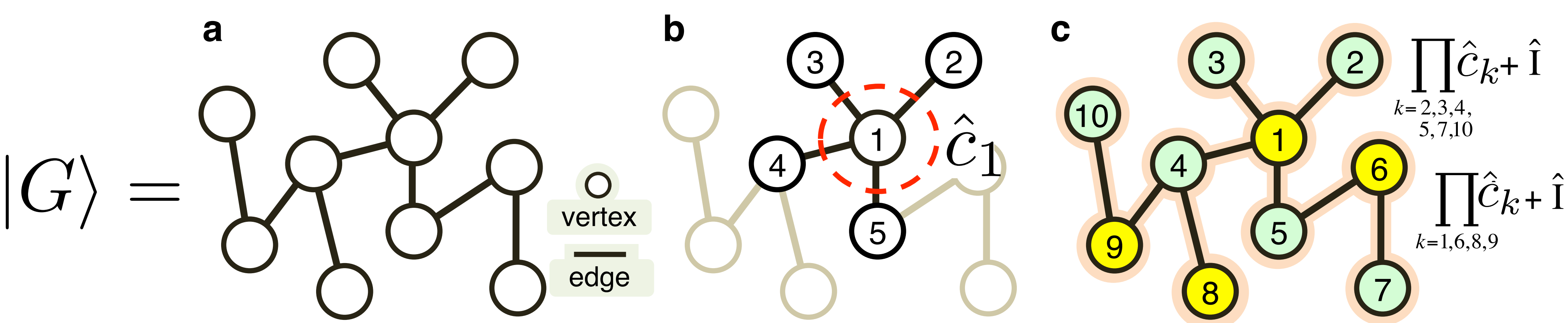
Molina-Terriza, G., Torres, J. P. & Torner, L. *Twisted photons*. Nature Phys. **3**, 305 (2007).

A. Vaziri, G. Weihs, & A. Zeilinger, *Experimental two-photon, three dimensional entanglement for quantum communications*. Phys. Rev. Lett. 89, 240401(2002).

Efficient Detection of Genuine High-order Entanglement

- Efficient Entanglement Criterion:

$$\sum_{j=1}^{q}\left\langle \prod_{k\in Y_j}\frac{\hat{c}_k+\hat{\mathrm{I}}}{t_d+1}\right\rangle > \frac{1}{d}(l-1)(q-\eta_q)+\eta_q$$



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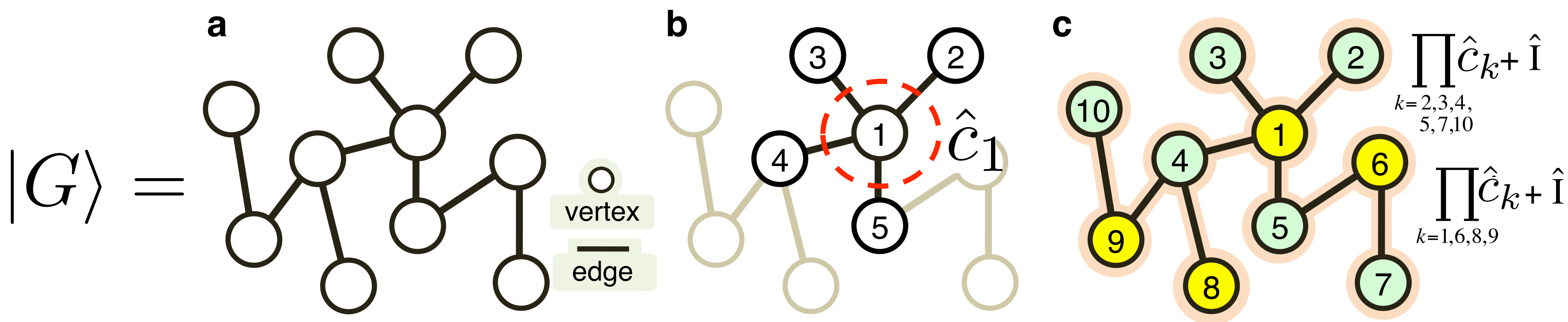

q-colorable Graph states -> *q* local measurement settings are sufficient for detection, independent of the qudit # and *d*

Efficient Detection of Genuine High-order Entanglement

- $$\sum_{j=1}^{q}\left\langle \prod_{k\in Y_j}\frac{\hat{c}_k+\hat{\mathrm{I}}}{t_d+1}\right\rangle > \frac{1}{d}(l-1)(q-\eta_q)+\eta_q$$

Here $\hat{c}_k$, called *correlator*, is an new type of correlation operators such that

$$\hat{c}_k\left|G\right\rangle = t_d\left|G\right\rangle ,$$

where $t_2=1, t_3=2, t_4=9,\ldots$.

Efficient Detection of Genuine High-order Entanglement

- Robustness of the Criterion (Witness)

The capability of the witness to identify an originally pure state, $|\Psi\rangle$, in the presence of white noise as a Genuine High-order Entanglement.

Here the contaminated state is of the form

$$\rho_{\mathrm{w}} = \frac{p}{d^N} I + (1-p)|\Psi\rangle\langle\Psi|,$$

where p is the probability of uncolored noise.

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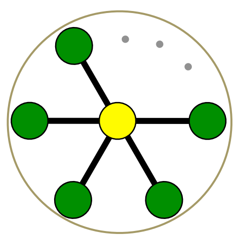
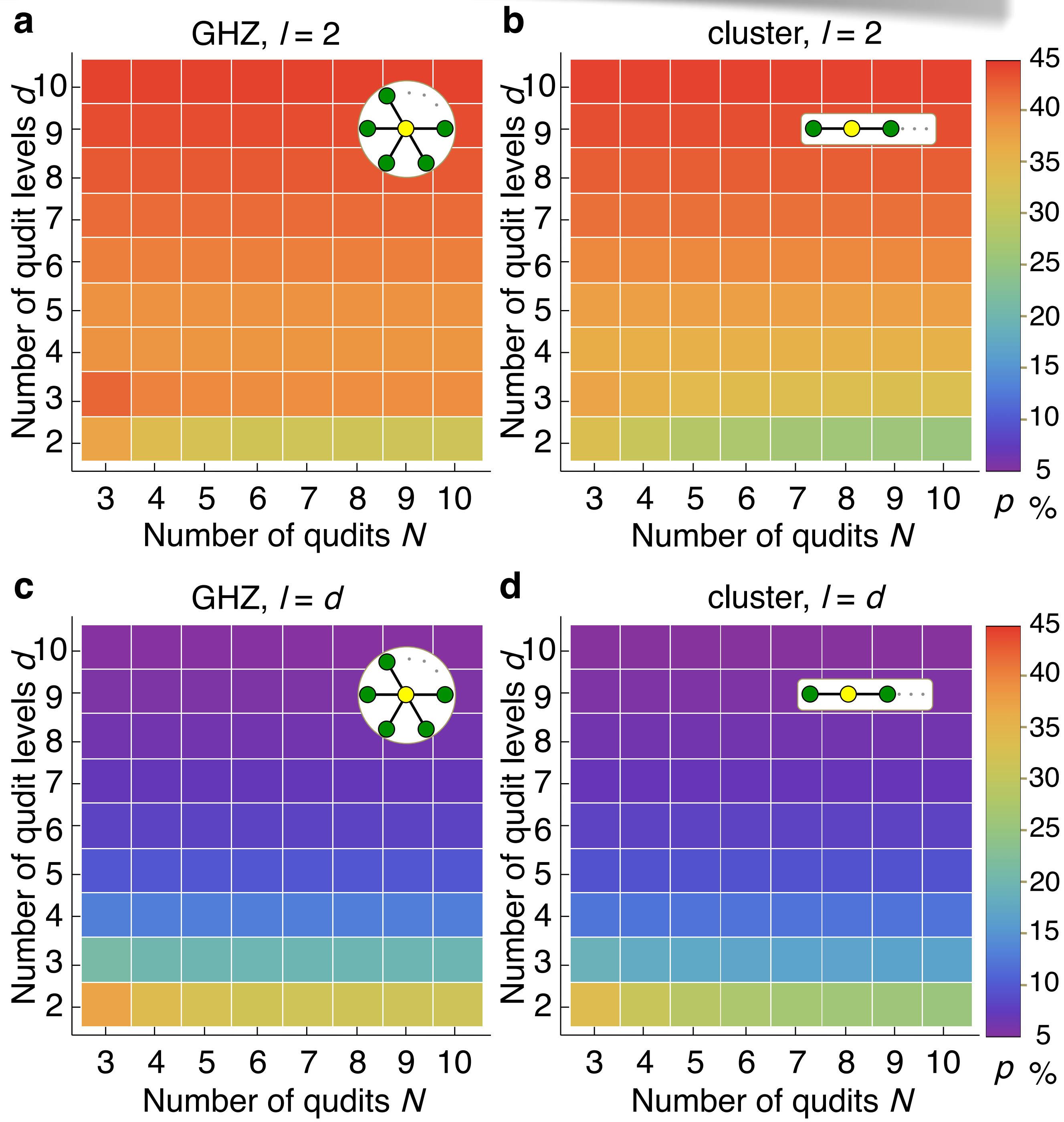
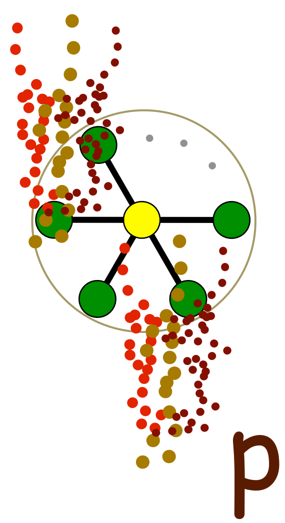
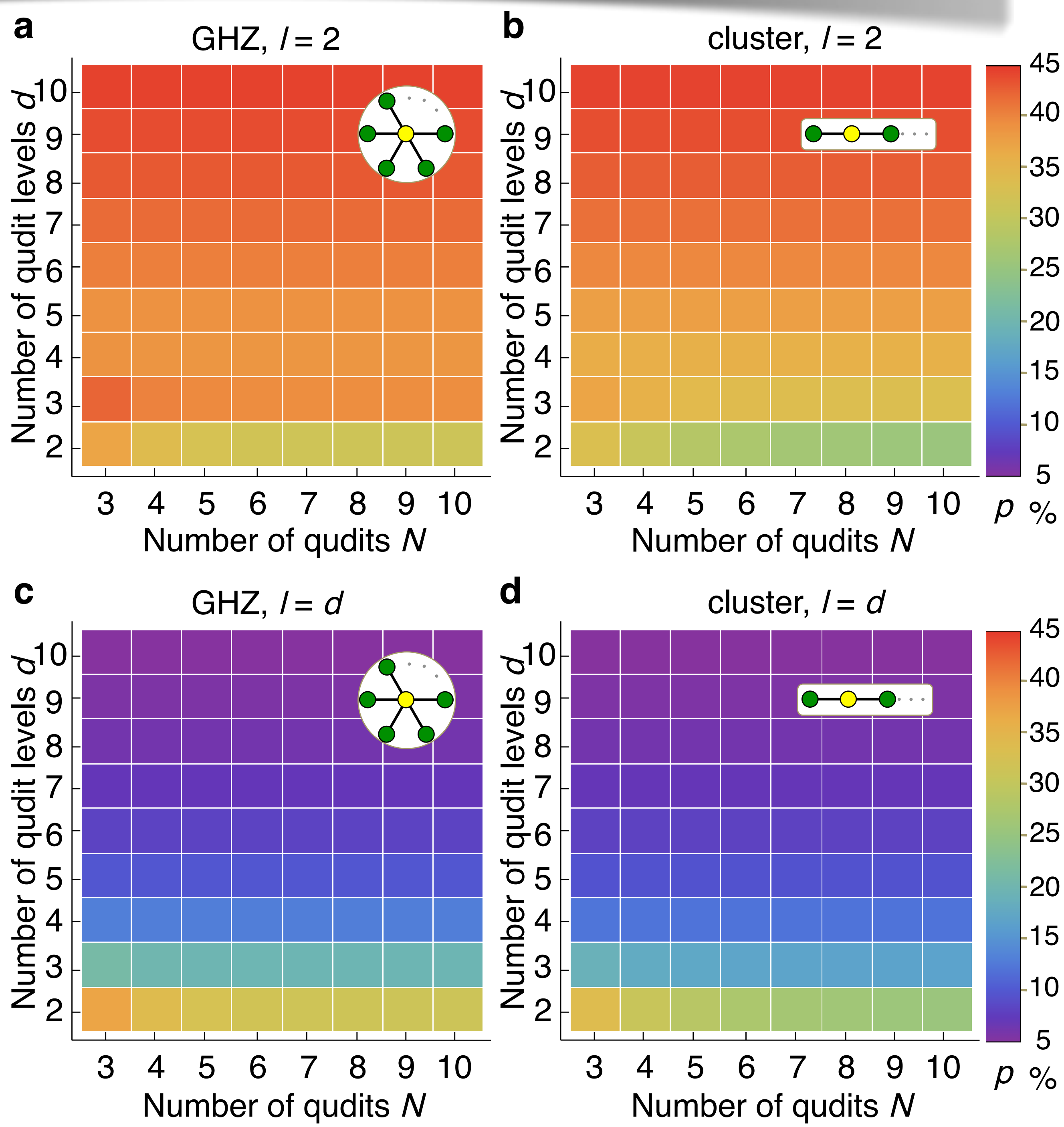
Efficient Detection of Genuine High-order Entanglement





Efficient Detection of Genuine High-order Entanglement

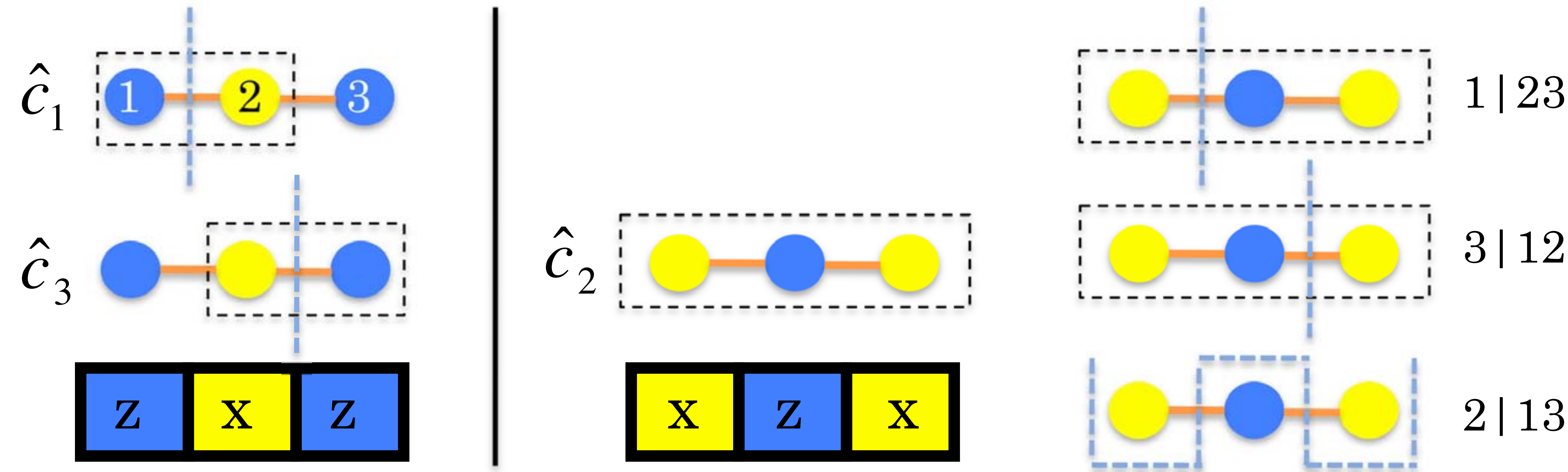




More about CORRELATORS $\hat{c}_k$

- *Q: What are correlators?*

They detect correlations between two groups of qudits:



Local Measurement Setting

X : Fourier basis

Z : normal basis

: considered system

: bipartite splitting

More about CORRELATORS $\hat{c}_k$

- *Q: How do we derive correlators from the state vector?*

E.g. d=3: $$|C\rangle = \frac{1}{\sqrt{9}} \sum_{\substack{\alpha,\beta,\gamma=0 \\ \alpha+\beta+\gamma \doteq 0}}^{2} |\alpha\rangle_z |\beta\rangle_x |\gamma\rangle_z$$

z	x	z

$$\begin{aligned}\hat{c}_2 = \; & (\hat{0}_z - \hat{1}_z) \otimes (\hat{0}_x \hat{0}_z + \hat{1}_x \hat{2}_z + \hat{2}_x \hat{1}_z) \\ & +(\hat{1}_z - \hat{2}_z) \otimes (\hat{0}_x \hat{2}_z + \hat{2}_x \hat{0}_z + \hat{1}_x \hat{1}_z) \\ & +(\hat{2}_z - \hat{0}_z) \otimes (\hat{0}_x \hat{1}_z + \hat{1}_x \hat{0}_z + \hat{2}_x \hat{2}_z)\end{aligned}$$

$$\begin{aligned}+ \; & (\hat{0}_z - \hat{2}_z) \otimes (\hat{0}_x \hat{0}_z + \hat{1}_x \hat{2}_z + \hat{2}_x \hat{1}_z) \\ & +(\hat{1}_z - \hat{0}_z) \otimes (\hat{0}_x \hat{2}_z + \hat{2}_x \hat{0}_z + \hat{1}_x \hat{1}_z) \\ & +(\hat{2}_z - \hat{1}_z) \otimes (\hat{0}_x \hat{1}_z + \hat{1}_x \hat{0}_z + \hat{2}_x \hat{2}_z)\end{aligned}$$

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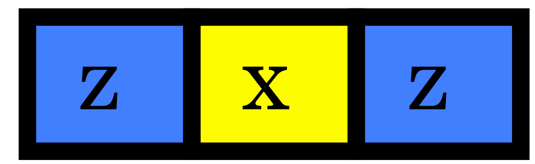


$$\begin{aligned}\hat{c}_2 = \ & (\hat{0}_z - \hat{1}_z) \otimes (\hat{0}_x\hat{0}_z + \hat{1}_x\hat{2}_z + \hat{2}_x\hat{1}_z) = \hat{p}_0 \\ & +(\hat{1}_z - \hat{2}_z) \otimes (\hat{0}_x\hat{2}_z + \hat{2}_x\hat{0}_z + \hat{1}_x\hat{1}_z) = \hat{p}_1 \\ & +(\hat{2}_z - \hat{0}_z) \otimes (\hat{0}_x\hat{1}_z + \hat{1}_x\hat{0}_z + \hat{2}_x\hat{2}_z) = \hat{p}_2\end{aligned}$$

$$\langle \hat{p}_0 \rangle, \langle \hat{p}_1 \rangle, \langle \hat{p}_2 \rangle > 0$$

Sufficient condition for dependent systems

$$+ \quad \begin{aligned} & (\hat{0}_z - \hat{2}_z) \otimes (\hat{0}_x\hat{0}_z + \hat{1}_x\hat{2}_z + \hat{2}_x\hat{1}_z) = \hat{p}_0' \\ & +(\hat{1}_z - \hat{0}_z) \otimes (\hat{0}_x\hat{2}_z + \hat{2}_x\hat{0}_z + \hat{1}_x\hat{1}_z) = \hat{p}_1' \\ & +(\hat{2}_z - \hat{1}_z) \otimes (\hat{0}_x\hat{1}_z + \hat{1}_x\hat{0}_z + \hat{2}_x\hat{2}_z) = \hat{p}_2'\end{aligned}$$

Sufficient condition for dependent systems

$$\langle \hat{p}_0' \rangle, \langle \hat{p}_1' \rangle, \langle \hat{p}_2' \rangle > 0$$

where $\hat{k}_o = |k\rangle_{oo}\langle k|,\ k \in \{0,1,2\}, o \in \{z,x\}.$

More about CORRELATORS $\hat{c}_k$

- *Q: Why* $\langle \hat{p}_1 \rangle, \langle \hat{p}_2 \rangle, \langle \hat{p}_3 \rangle > 0$ *is a sufficient condition for dependent systems?*

$$\hat{p}_0 = (\hat{0}_z - \hat{1}_z) \otimes (\hat{0}_x \hat{0}_z + \hat{1}_x \hat{2}_z + \hat{2}_x \hat{1}_z)$$
$$\hat{p}_1 = (\hat{1}_z - \hat{2}_z) \otimes (\hat{0}_x \hat{2}_z + \hat{2}_x \hat{0}_z + \hat{1}_x \hat{1}_z)$$
$$\hat{p}_2 = (\hat{2}_z - \hat{0}_z) \otimes (\hat{0}_x \hat{1}_z + \hat{1}_x \hat{0}_z + \hat{2}_x \hat{2}_z)$$

Ans: For any biseparable states, e.g.,  we have

$$\langle \hat{p}_0 \rangle = \left(\langle \hat{0}_z \rangle - \langle \hat{1}_z \rangle \right) \langle \hat{0}_x \hat{0}_z + \hat{1}_x \hat{2}_z + \hat{2}_x \hat{1}_z \rangle,$$
$$\langle \hat{p}_1 \rangle = \left(\langle \hat{1}_z \rangle - \langle \hat{2}_z \rangle \right) \langle \hat{0}_x \hat{2}_z + \hat{2}_x \hat{0}_z + \hat{1}_x \hat{1}_z \rangle,$$
$$\langle \hat{p}_2 \rangle = \left(\langle \hat{2}_z \rangle - \langle \hat{0}_z \rangle \right) \langle \hat{0}_x \hat{1}_z + \hat{1}_x \hat{0}_z + \hat{2}_x \hat{2}_z \rangle.$$

It is impossible to have $\langle \hat{p}_1 \rangle, \langle \hat{p}_2 \rangle, \langle \hat{p}_3 \rangle > 0$.

Whereas a contradiction reveals the dependent systems.

ONLY THE MINIMAL TWO MEASUREMENT SETTINGS ARE NEEDED

- Witness for N-qudit cluster state

$$W_C = \left[\frac{(d-1)\eta+2}{d}\right] I + \left[\prod_{odd\ k}\frac{\hat{c}_k + I}{t_d + 1} + \prod_{even\ k}\frac{\hat{c}_k + I}{t_d + 1}\right]$$

2 local measurement settings: Z X Z X   X Z X Z

- Witness for N-qudit GHZ state $\left|GHZ_N^d\right\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\left|k\right\rangle^{\otimes N}$

$$W_{GHZ} = \left[\frac{(d-1)\eta+2}{d}\right] I + \left[\frac{\hat{g}_0 + I}{t_d + 1} + \prod_{k=1}^{N-1}\frac{\hat{g}_k + I}{t_d + 1}\right]$$

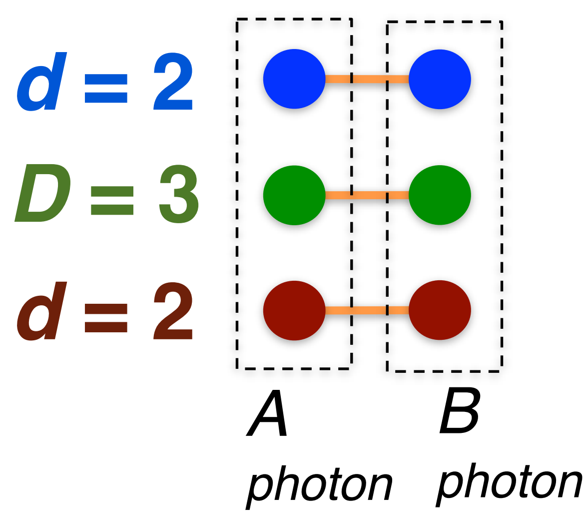
2 local measurement settings: X X X X   Z Z Z Z

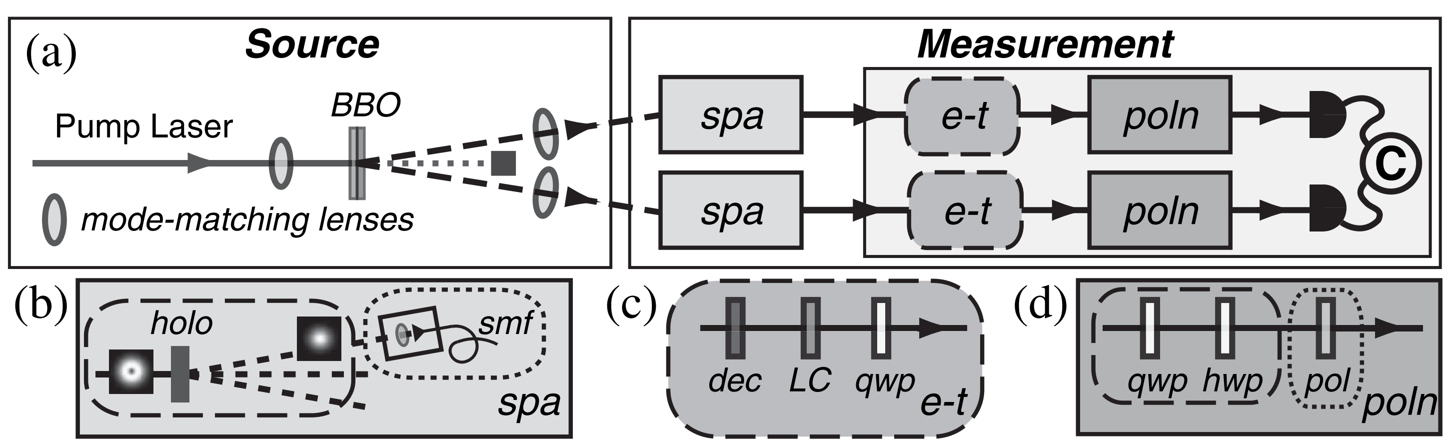
Efficient Detection of Genuine High-order Entanglement

Remarks

- Detecting (2x2 x 3x3 x 2x2)-dimensional Hyperentangled photons

$$\left\langle \bigotimes_{k=1}^{N} \frac{\hat{a}_1^{(k)} + \hat{a}_2^{(k)} + t_{d_k}\hat{\mathrm{I}}}{3t_{d_k}} \right\rangle > 1 - \frac{D(d-1)}{3d(D-1)}$$





polarization ( ), spatial mode ( ), and energy time ( )

J. T Barreiro, N. K. Langford, N. A. Peters,& P. G. Kwiat, *Generation of Hyperentangled Photon Pairs*, Phys. Rev. Lett. **95**, 260501 (2005).

Efficient Detection of Genuine High-order Entanglement

Remarks

- Estimating Quantum State Fidelity without Quantum State Tomography

Qudit Graph State:

$$\langle G|\,\rho\,|G\rangle \geq \frac{1}{q-\eta_q}\left(\sum_{j=1}^{q}\left\langle \prod_{k\in Y_j}\frac{\hat{a}_k+\hat{\mathrm{I}}}{t_d+1}\right\rangle-\eta_q\right)$$

Hyperentangled State:

$$\langle H|\,\rho\,|H\rangle \geq 3d\left(1-\frac{1}{D}\right)\left\langle \bigotimes_{k=1}^{N}\frac{\hat{a}_1^{(k)}+\hat{a}_2^{(k)}+t_{d_k}\hat{\mathrm{I}}}{3t_{d_k}}\right\rangle+\frac{3d}{D}-2d$$

Efficient Detection of Genuine High-order Entanglement

Remarks

- Speed up the present Experimental Verification of High-order Entanglement

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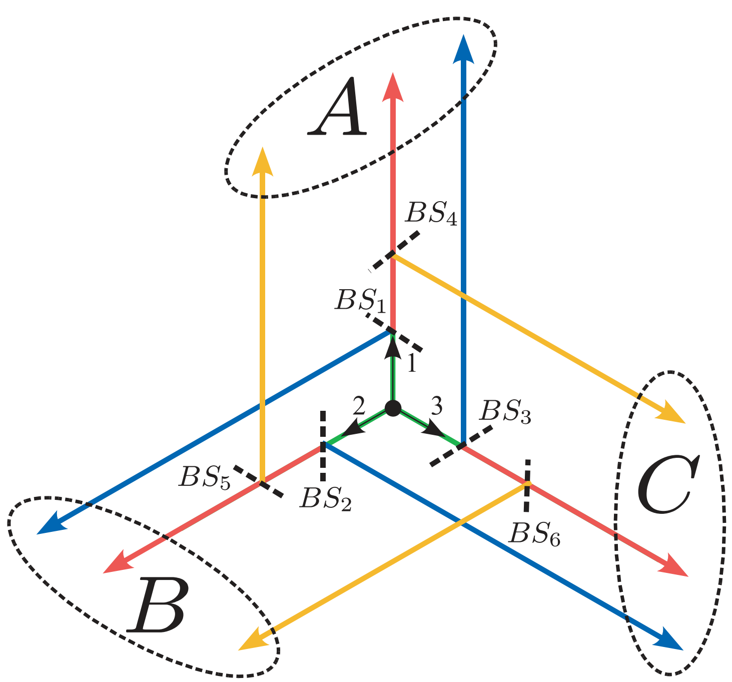
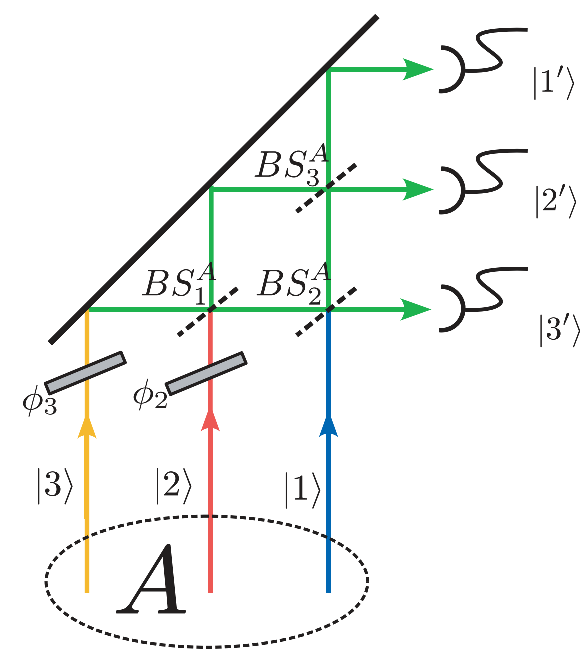
Efficient Detection of Genuine High-order Entanglement

Remarks

- Speed up the future Experimental Verification of High-order Entanglement





Vallone, G., Mataloni, P. & Cabello, A. *Multiparty multilevel energy-time entanglement.* Phys. Rev. A **81**, 032105 (2010).

Detecting Genuine High-order Entanglement

Q1: **2**-local measurement settings are enough for detecting, e.g., states close to

$$|\psi\rangle = \frac{1}{\sqrt{3}}(|+\hbar, -\hbar\rangle + |0,0\rangle + |-\hbar, +\hbar\rangle) \quad ?$$

Yes

Q2: **2**-local measurement settings are enough for estimating experimental state fidelity without full **Quantum State Tomography**??

Yes

Q3: **2**-local measurement settings are enough for detecting **Genuine General High-order Entanglement** and for estimating experimental state fidelity without taking full **Quantum State Tomography**???

Yes

Summary

- Genuine High-order Entanglement can be efficiently detected without complicated local measurements.
- The present detection schemes can be applied for the present and future experiments.
- Estimating Quantum State Fidelity without full Quantum State Tomography is possible.

Thanks for your attention!!