From Dancing WavePacket to the Frictionless Atom Cooling

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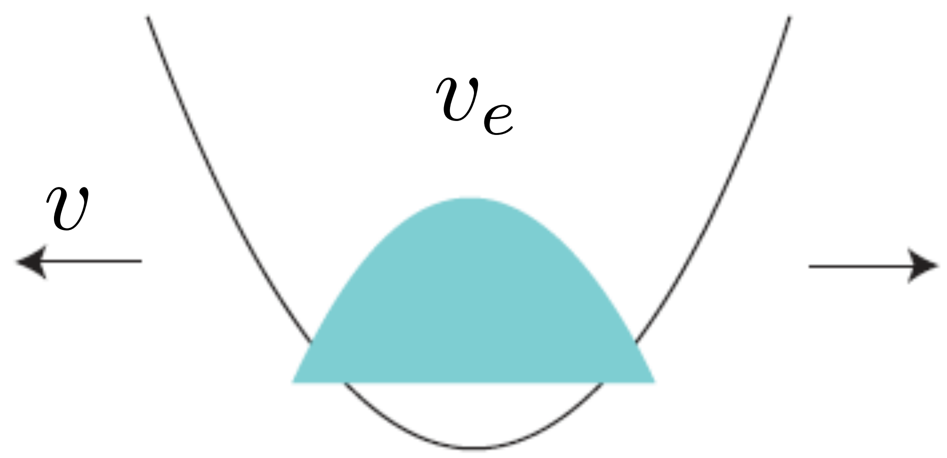
outline

- Motivation
- Quantum friction and the classical picture
- The frictionless atom cooling

Initial problem:



$$\omega(t) = \frac{2}{mL^2(t)}$$

$$L(t) = L_0 + v\ t$$

$$v_e \sim \frac{1}{mL}$$

$v \ll v_e$: ADIABATIC APPROXIMATION.

$v \gg v_e$: SUDDEN APPROXIMATION.

$v \approx v_e$: ?

Simulation Method

M.V. Berry & G. Klein, J. Phy. A **17**, 1805(1984),
C. H. Chang, personal note.

- For a linear expansion or contraction potential, the wavefunction will be

$$H(t)\varphi_n(x,t) = i\frac{\partial}{\partial t}\varphi_n(x,t)$$

$$\varphi_n(x,t) = N(t)\phi_n(\frac{x}{L(t)})e^{i\frac{mvx^2}{2(L_0+vt)}-i\int_0^t E_n(t')dt'},$$

where $L(t) = L_0 + vt$ and $N(t) = N(0)/\sqrt{L(t)/L(0)}$

$\varphi_n(\frac{x}{L(t)})$ THE INSTANTS WAVEFUNCTION,

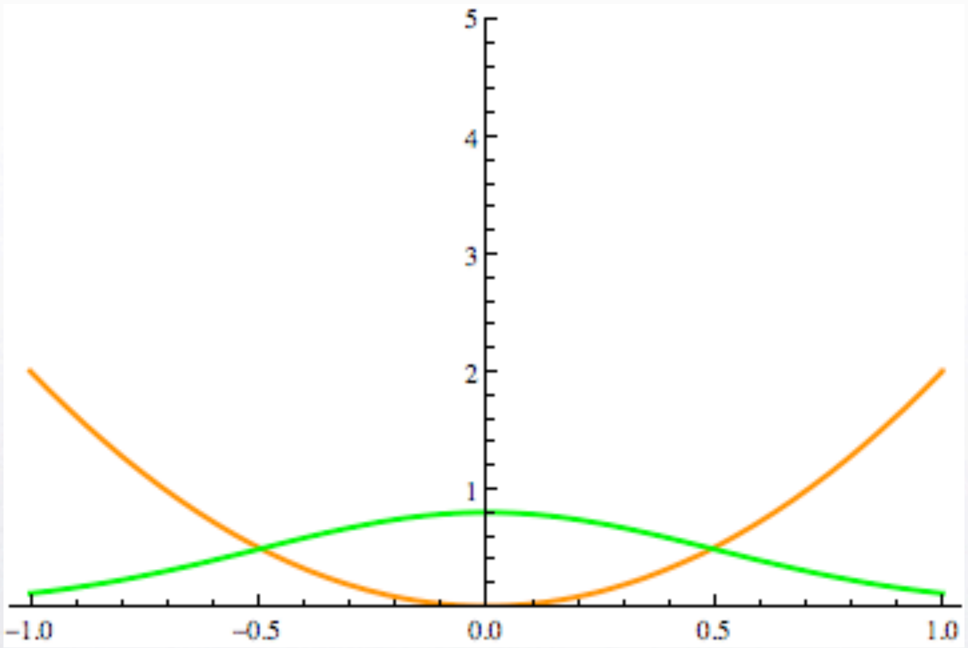
AND

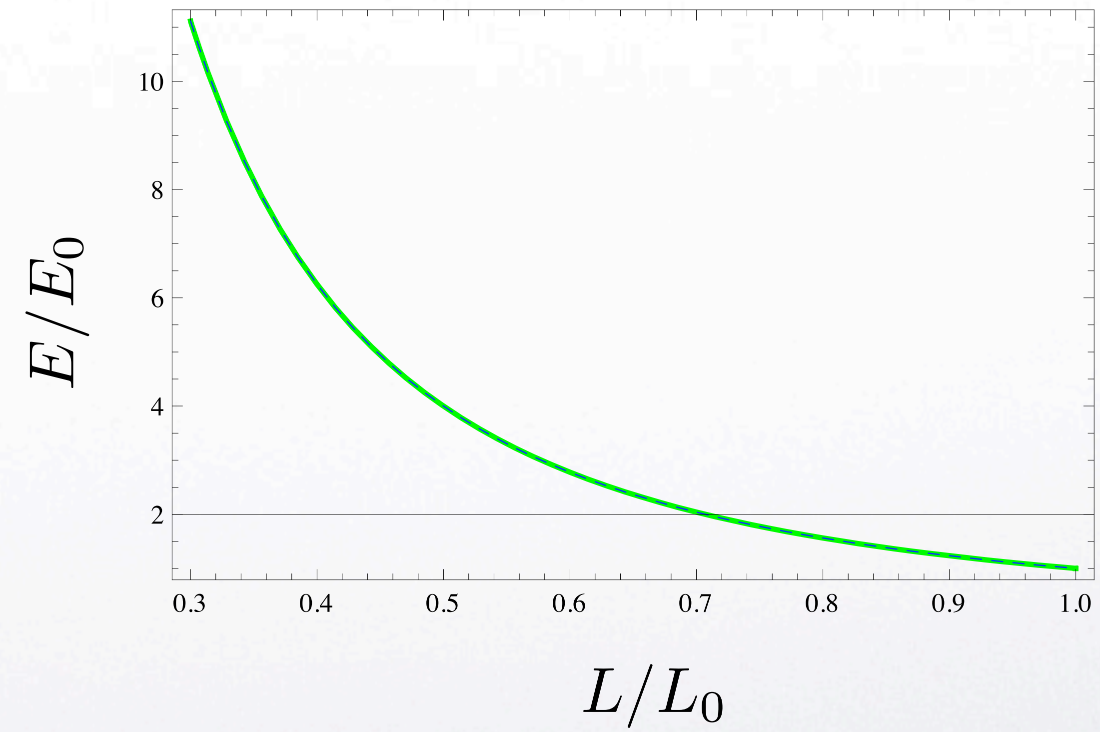
$E_n(t)$ THE INSTANTS ENERGY.

Simulation Result I: adiabatic limit

Set $v_e = 1/mL_0$,

$v = v_e/100.$

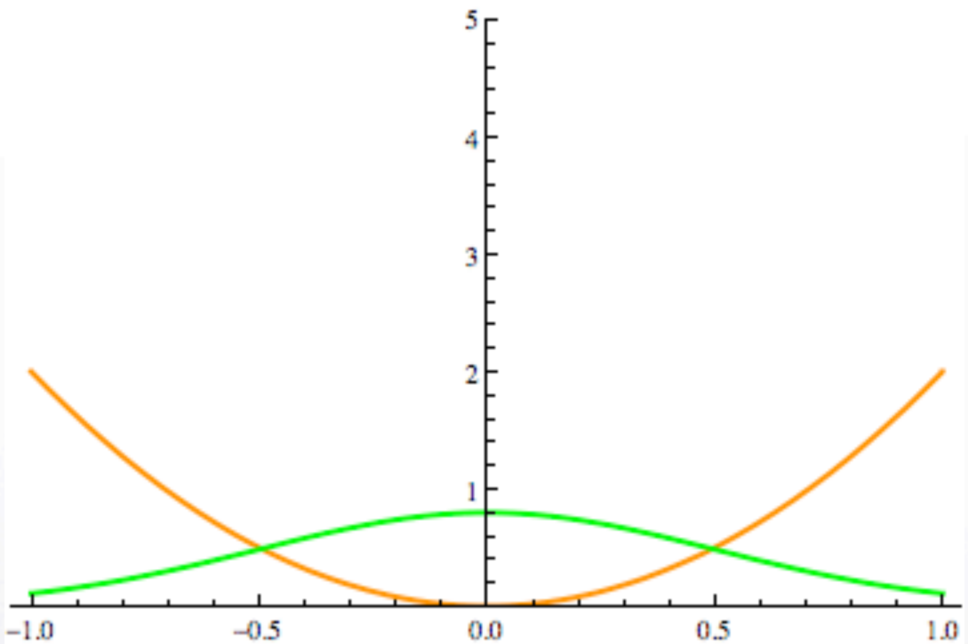


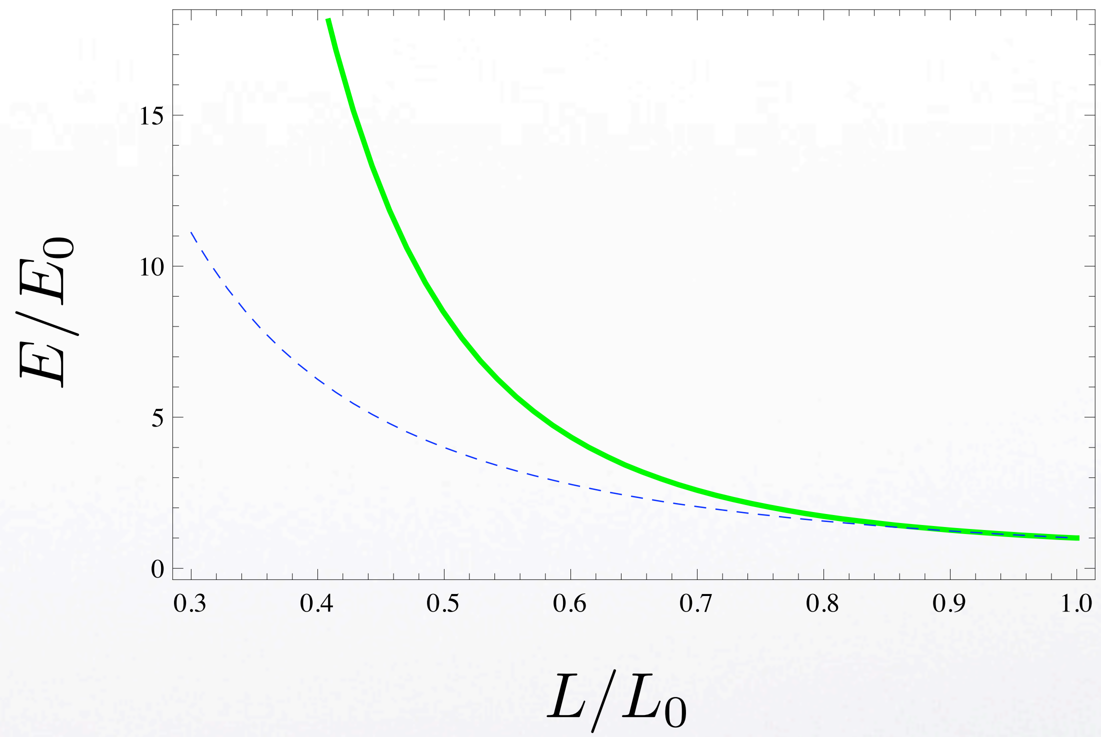


$$E(t) \approx \frac{\omega(t)}{\omega_0} E_0$$

Simulation Result II: sudden limit

$v = 10v_e$

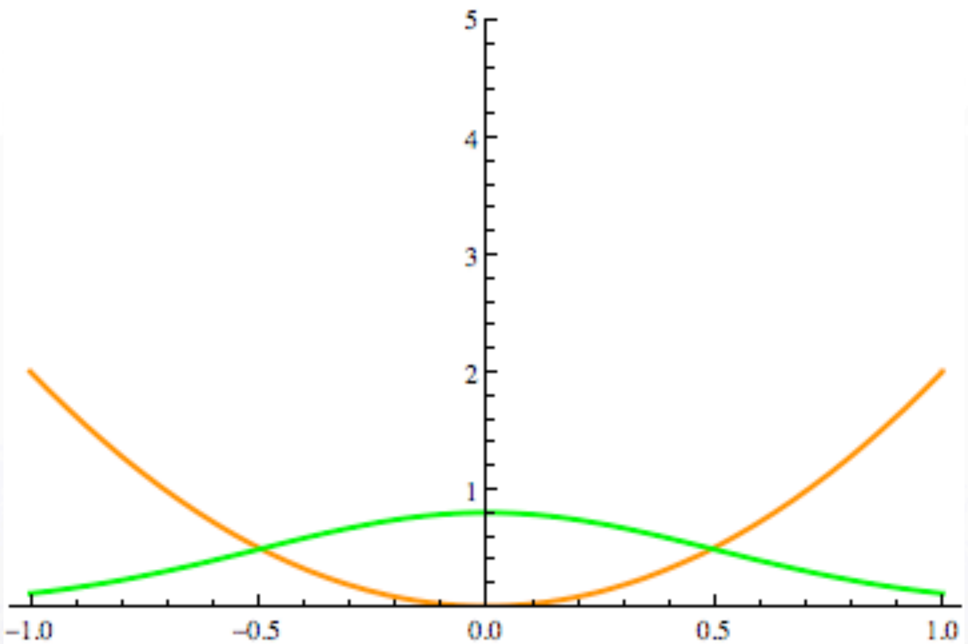


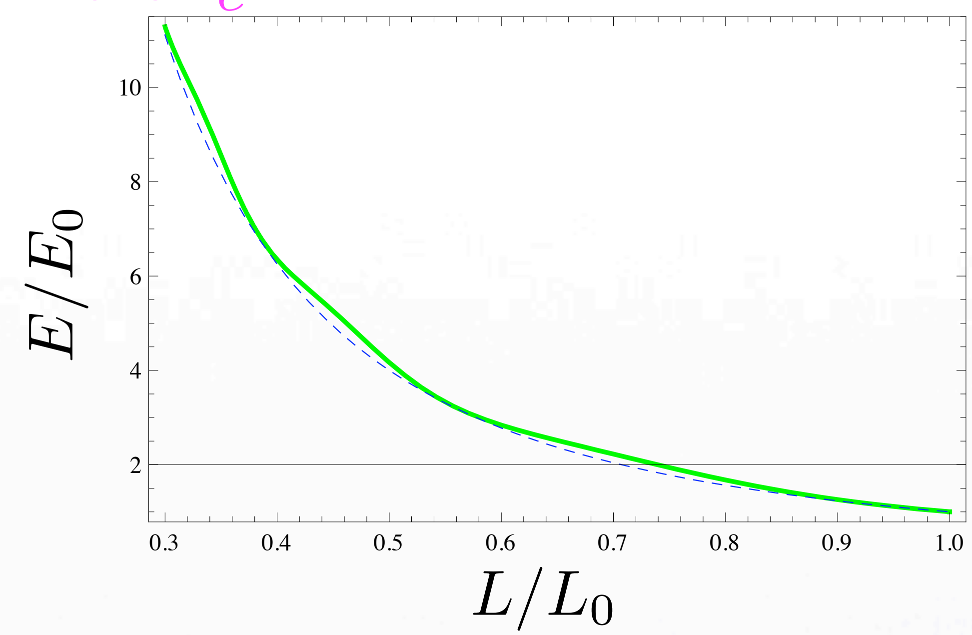


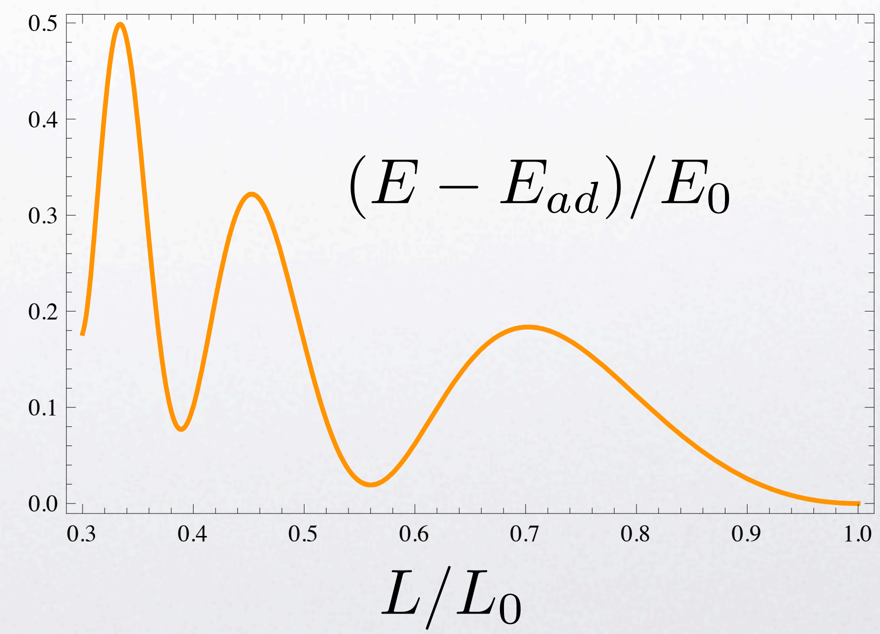
$$E(t) \approx E_0 + \frac{1}{2}m\left(\omega_t^2 - \omega_0^2\right)\langle\Psi_0|x^2|\Psi_0\rangle.$$

Simulation Result III

$v = 0.5v_e$

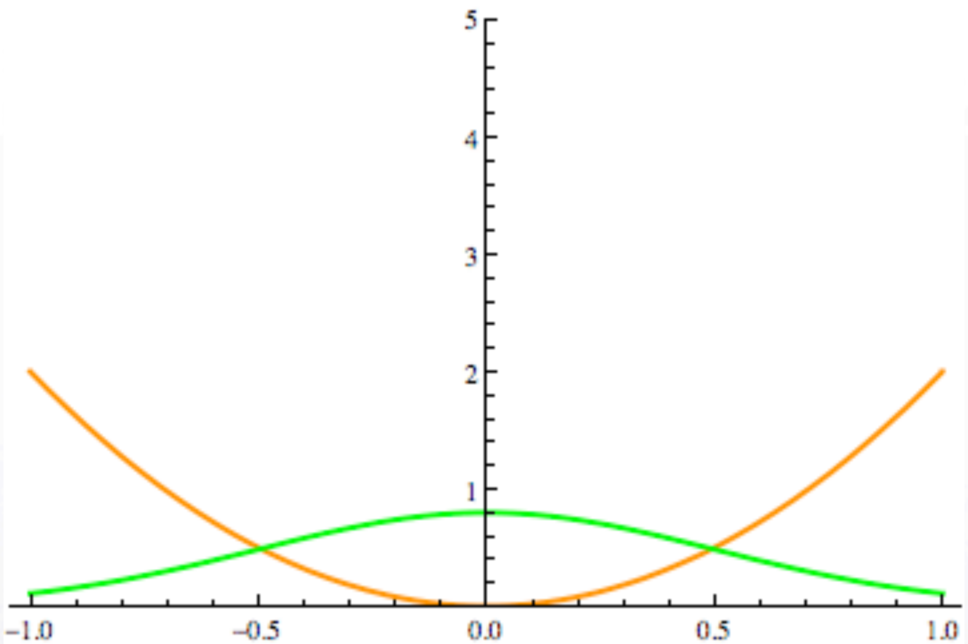


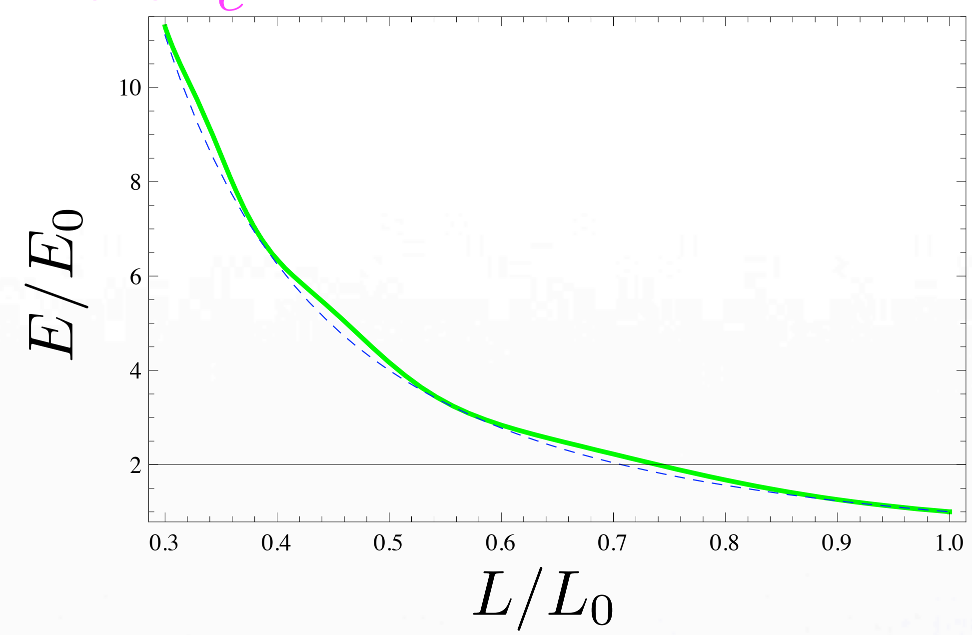


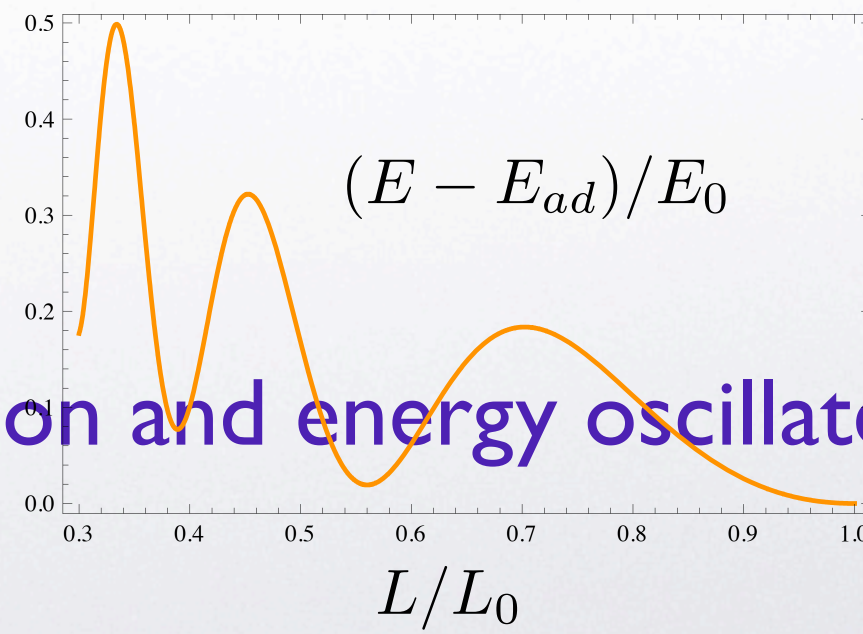


Simulation Result III

$v = 0.5 v_e$





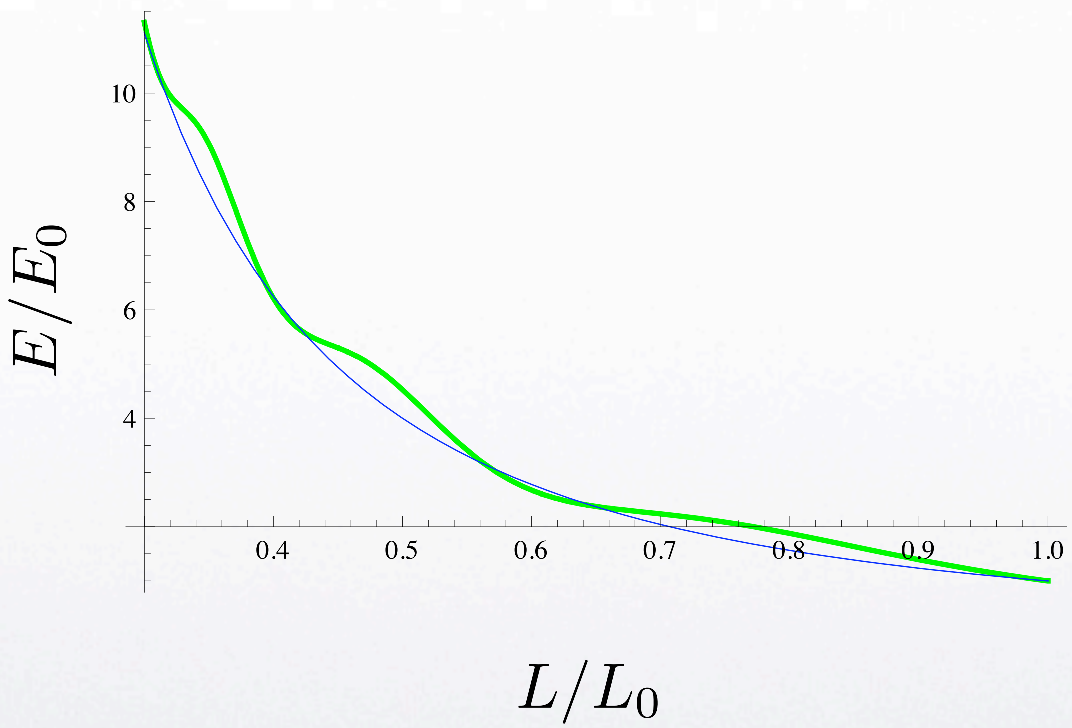


Why the wavepacket distribution and energy oscillate during the contraction?

Simulation Result III

$v = 0.5v_e$

$$\psi_0 = a_0\phi_0 + a_2\phi_2$$



Quantum Breathing

Breathing oscillation induced by sudden change

A. Minguzzi & D. M. Gangardt, Phys. Rev. Lett. **94**, 240404 (2005).

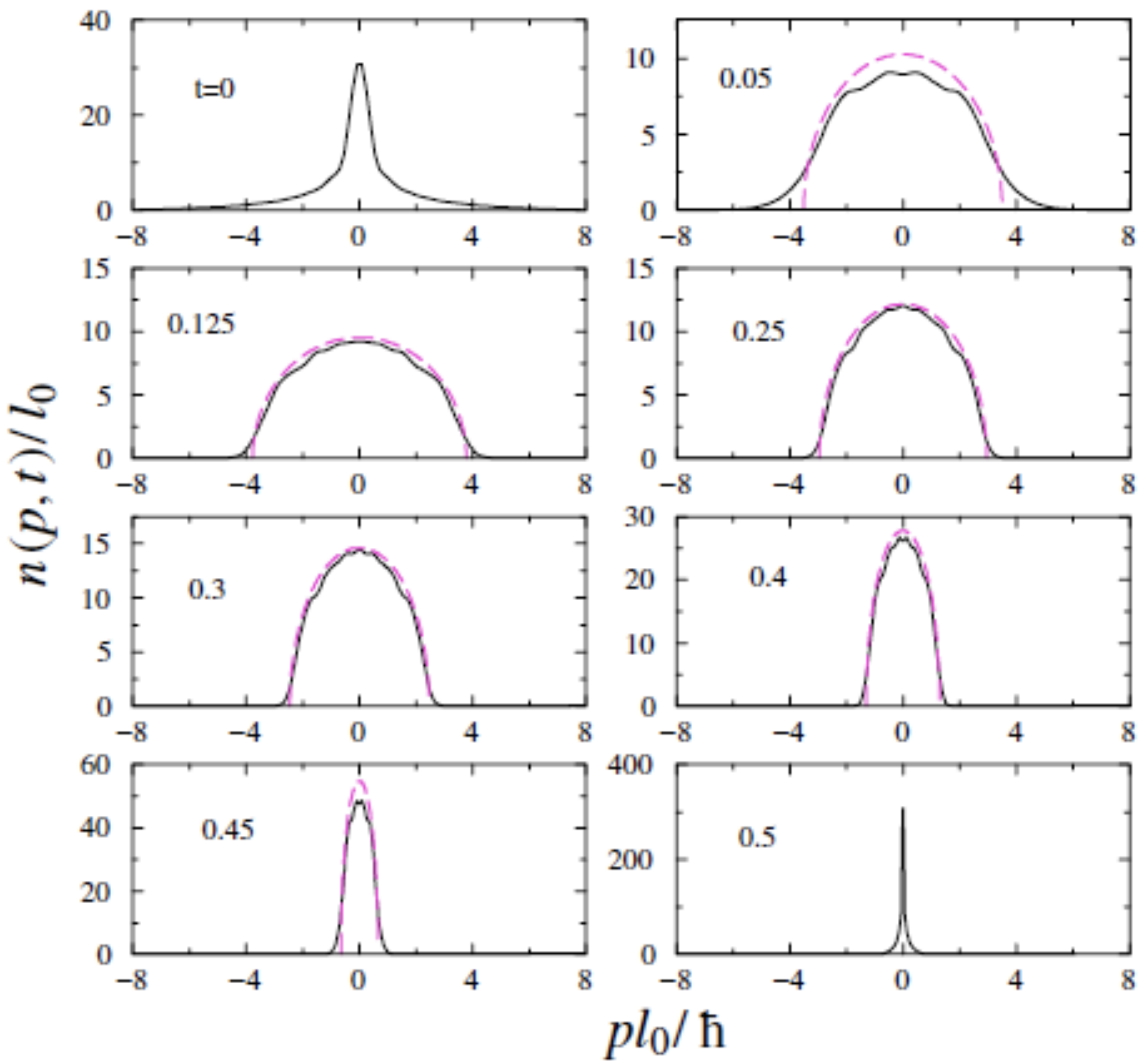


FIG. 2 (color online). Momentum distribution of an oscillating Tonks gas with $N = 9$ and $\omega_0/\omega_1 = 10$ at different times (in units of $T = \pi/\omega_1$) indicated on the panels, from numerical solution (solid lines) and Thomas-Fermi approximation (dashed lines). The units are indicated on the axis labels.

$$\omega_0 \longrightarrow \omega_1$$

$$T = \pi/\omega_1$$

Quantum Breathing:

Experimental Result

J. F. Schaff *et al*., Phys. Rev. A **82,** 033430(2010).

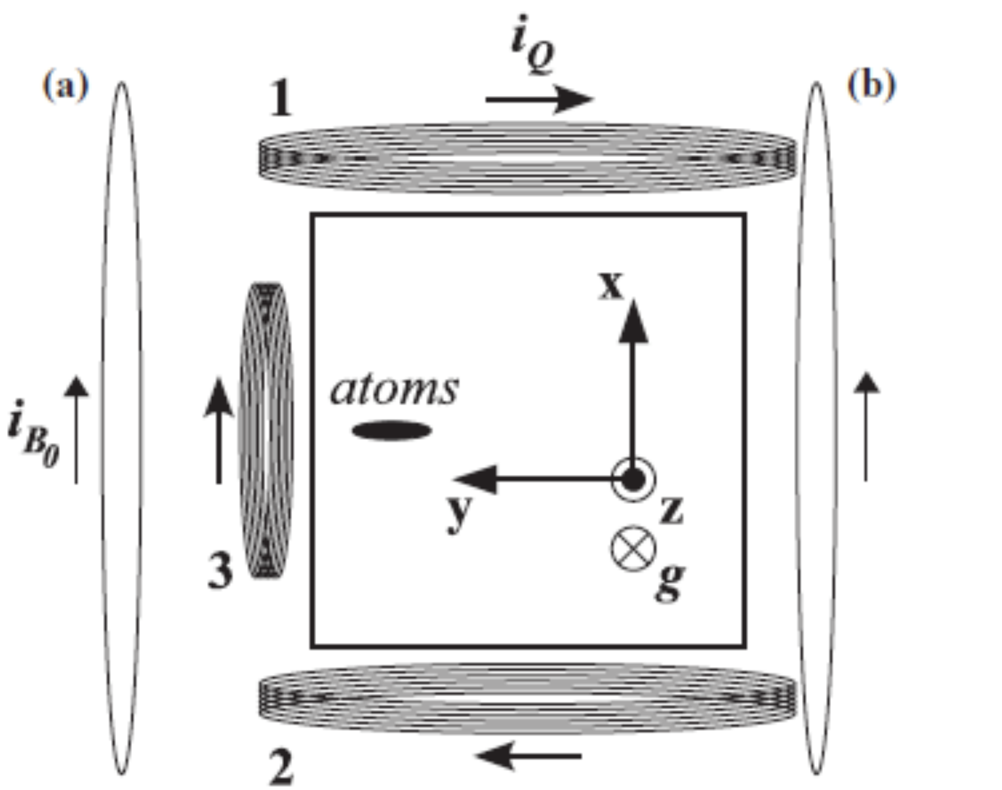
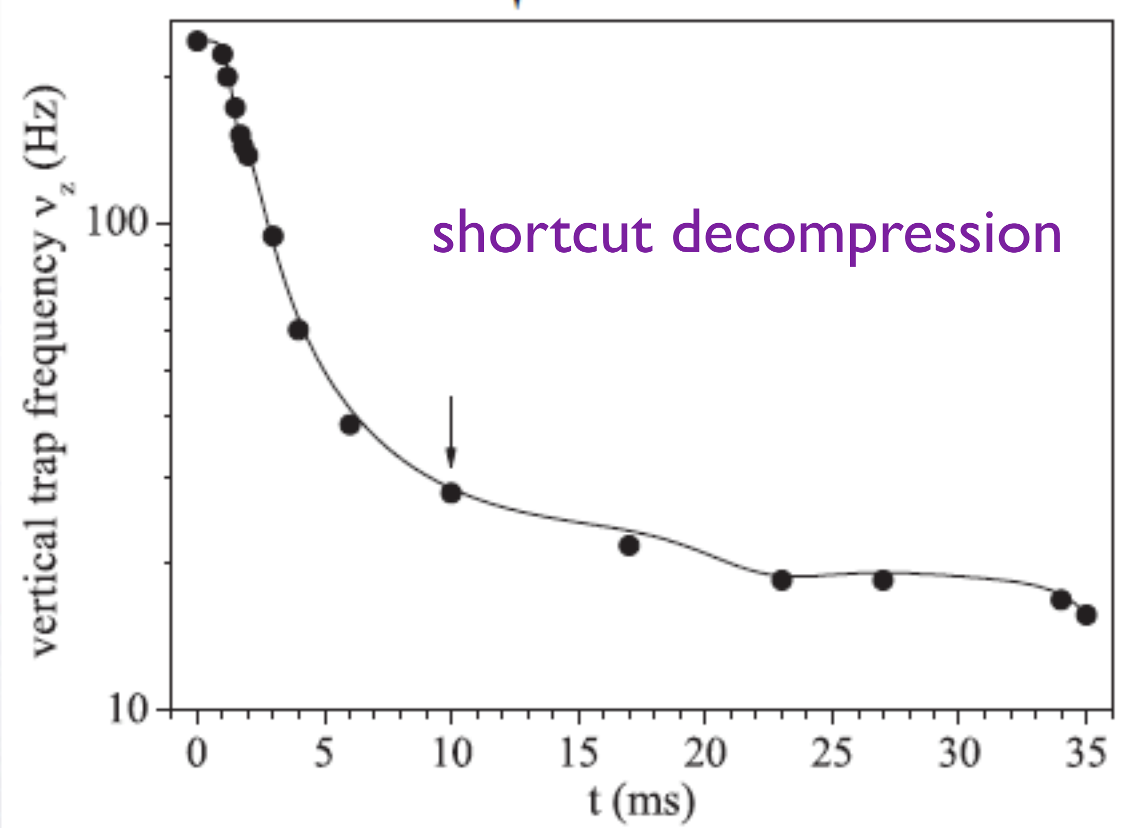


FIG. 1. Trapping geometry (figure in the horizontal plane). Ultracold ^{87}Rb atoms are trapped in an Ioffe-Pritchard-type magnetic trap created by current i_Q running through the three QUIC coils **1**, **2**, and **3**. An additional pair of coils (**a** and **b**) produces a homogeneous field along y, which allows an independent tuning of the trap minimum field B_0 via the current i_{B_0}.

$$\omega_{x,z} \approx \sqrt{\frac{\mu}{m}} \frac{B'(i_Q)}{\sqrt{B_0(i_Q, i_{B_0})}},$$

$$\omega_y = \sqrt{\frac{\mu}{m}} \sqrt{B''(i_Q)}.$$



Quantum Breathing:
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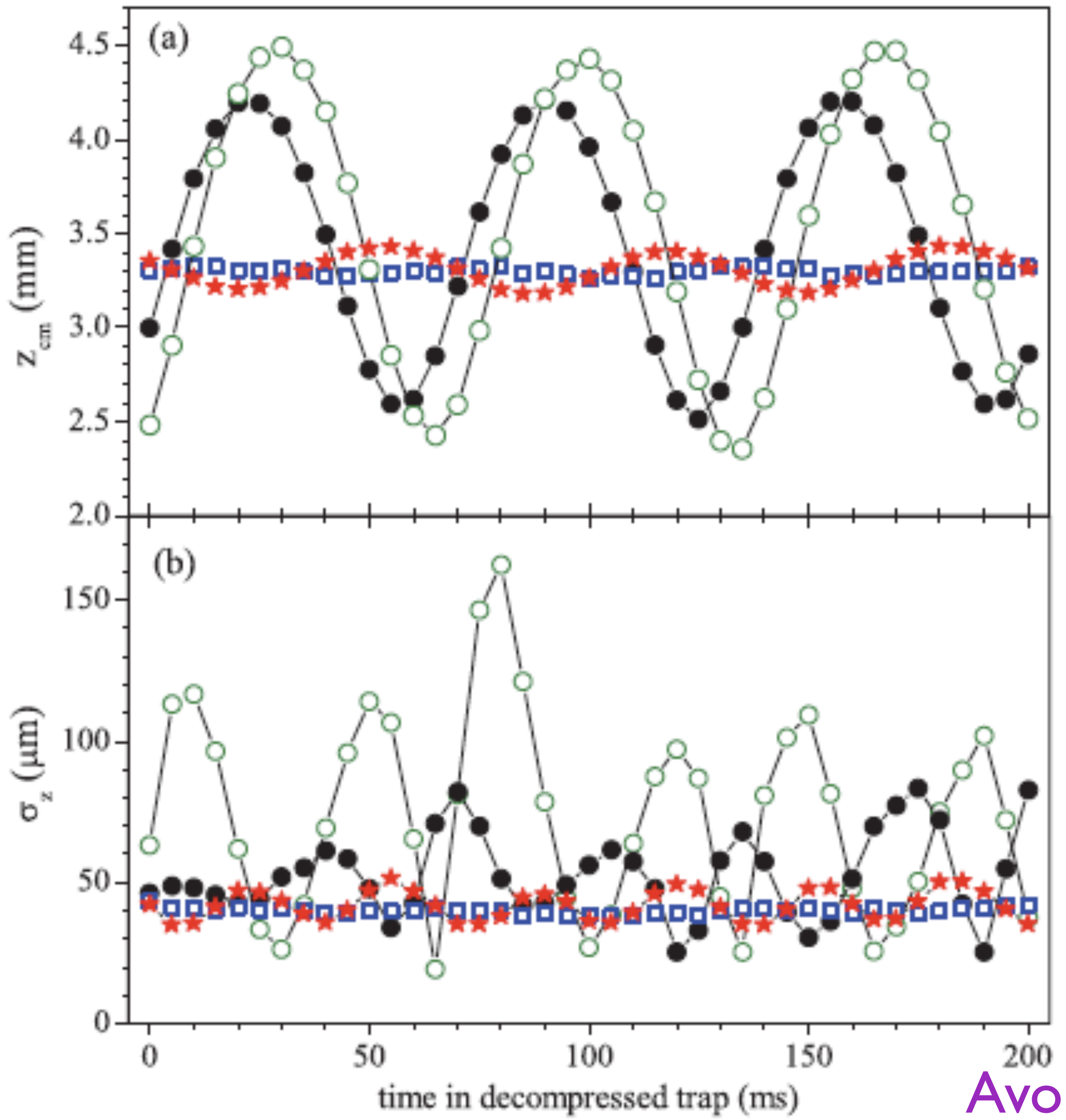

FIG. 3. (Color online) Vertical trap decompression: comparison between different schemes. We report in (a) and (b), respectively, the cloud's vertical center-of-mass position z_{cm} and size σ_z versus time after decompression for four different sequences. Open circles (green): abrupt decompression; solid circles (black): linear decompression in 35 ms; stars (red): shortcut decompression in 35 ms; squares (blue): linear decompression in 6 s.

$$E_{\rm exc} = E_{\rm dip} + E_{\rm breath}$$

$$E_{\rm dip} = 1/2 m\omega_{fz}^2 \Delta z_{\rm cm}{}^2$$

induced by trap center shifting!!

$$E_{\rm breath} \approx 2m\omega_{fz}^2 \Delta\sigma_z{}^2$$

Avoid quantum breathing can enhance cooling efficiency!!

Analytical analysis:
Method of time dependent eigenvectors

M. A. Lohe, J. Phy. A **42**, 035307(2009),
C. H. Chang & T. M. Hong, in preparation (2010).

For $\hat{H}(t) = \hat{p}^2/2m + m\omega^2(t)\hat{x}^2/2,$

The time-dep eigenvector is

$$\Psi_n(t,x) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} \frac{e^{-i(n+1/2)\int_0^t dt'(\omega_0/b^2)}}{(2^n n! b)^{1/2}} \times e^{i(m/2\hbar)(\dot{b}/b + i\omega_0/b^2)x^2} H_n\left[\left(\frac{m\omega_0}{\hbar}\right)^{1/2}\frac{x}{b}\right],$$

with $\ddot{b} + \omega^2(t)b = \omega_0^2/b^3,\ b(0) = 1,\ \ddot{b}(0) = 0.$

b(t)=L(t)/L(0).

Analytical analysis:
The energy change for packet in the time-dep trap I

C. H. Chang & T. M. Hong, in preparation (2010).

$$\langle \hat{H}(t)\rangle = \sum_{nl} c_n c_l^* \langle \Psi_l(t)|\hat{H}(t)|\Psi_n(t)\rangle$$

$$= \langle \psi_0| e^{im\dot{b}(0)\hat{x}^2/2\hbar} e^{i\frac{\hat{H}(0)}{\hbar}\int_0^t dt'/b^2} \left[\left(\frac{\hat{H}(0)}{b^2} - \frac{m}{2} b\ddot{b}\hat{x}^2\right) + \frac{m}{2}\dot{b}^2\hat{x}^2 + \frac{\dot{b}}{2b}\{\hat{x},\hat{p}\}\right] e^{-i\frac{\hat{H}(0)}{\hbar}\int_0^t dt'/b^2} e^{-im\dot{b}(0)\hat{x}^2/2\hbar}|\psi_0\rangle,$$

$|\psi_0\rangle$ initial wavepacket

$e^{-im\dot{b}(0)\hat{x}^2/2\hbar}$ projection operator

$e^{-i\frac{\hat{H}(0)}{\hbar}\int_0^t dt'/b^2}$ evolution operator

Analytical analysis:

The energy change for packet in the time-dep trap II

C. H. Chang & T. M. Hong, in preparation (2010).

The energy contributed from

1. Instaneous energy for trap $\omega(t)$

$$\left(\frac{\hat{H}(0)}{b^2} - \frac{m}{2} b\ddot{b}\hat{x}^2\right)$$

2. The kinetic energy

$$\frac{m}{2}\dot{b}^2\hat{x}^2$$

3. The quantum friction

T. Feldmann & R. Kosloff, Phys. Rev. E, **61**, 4774(2000),
R. Kosloff & T. Feldmann, Phys. Rev. E, **65**, 0551021(2002).

$$\frac{\dot{b}}{2b}\{\hat{x}, \hat{p}\} \propto [\hat{H}(0), \hat{H}(t)]$$

Analytical analysis:

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C. H. Chang & T. M. Hong, in preparation (2010).

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T. Feldmann & R. Kosloff, Phys. Rev. E, **61**, 4774(2000),
R. Kosloff & T. Feldmann, Phys. Rev. E, **65**, 0551021(2002).

$$\frac{\dot{b}}{2b}\{\hat{x},\hat{p}\} \propto [\hat{H}(0), \hat{H}(t)]$$

The energy depend on the packet follows or resists to the trap motion !!

Analytical analysis:

The energy change for packet in the time-dep trap III

C. H. Chang & T. M. Hong, in preparation (2010).

$$
\begin{aligned}
\langle \hat{H}(t) \rangle =& \hbar \Big(\frac{\omega_0}{b^2} + \frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b}) \Big) \\
& \times \Big[\frac{1}{2} + \langle \psi_0 | \Lambda^+ \Lambda | \psi_0 \rangle \Big] \\
& + \hbar \mathrm{Re} \Big[\Big(\frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b}) - i\frac{\dot{b}}{b} \Big) \\
& \times e^{-i2\int_0^t dt' \omega_0 / b^2} \langle \psi_0 | \Lambda\Lambda | \psi_0 \rangle \Big],
\end{aligned}
$$

with $\Lambda = a - i\frac{\dot{b}(0)}{2\omega_0}(a^+ + a)$

Analytical analysis:

The energy change for packet in the time-dep trap III

C. H. Chang & T. M. Hong, in preparation (2010).

$$\langle \hat{H}(t) \rangle = \hbar\Big(\frac{\omega_0}{b^2} + \frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b})\Big) \times \Big[\frac{1}{2} + \langle \psi_0 | \Lambda^+ \Lambda | \psi_0 \rangle\Big]$$

$$+ \hbar \mathrm{Re}\Big[\Big(\frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b}) - i\frac{\dot{b}}{b}\Big) \times e^{-i2\int_0^t dt'\omega_0/b^2} \langle \psi_0 | \Lambda\Lambda | \psi_0 \rangle\Big],$$

with

$$\Lambda = a - i\frac{\dot{b}(0)}{2\omega_0}(a^+ + a)$$

Analytical analysis:

The energy change for packet in the time-dep trap III

C. H. Chang & T. M. Hong, in preparation (2010).

$$\langle \hat{H}(t)\rangle = \hbar\Big(\frac{\omega_0}{b^2} + \frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b})\Big) \times \Big[\frac{1}{2} + \langle\psi_0|\Lambda^+\Lambda|\psi_0\rangle\Big]$$

$$+ \hbar\mathrm{Re}\Big[\Big(\frac{1}{2\omega_0}(\dot{b}^2 - b\ddot{b}) - i\frac{\dot{b}}{b}\Big) \times e^{-i2\int_0^t dt'\omega_0/b^2}\langle\psi_0|\Lambda\Lambda|\psi_0\rangle\Big],$$

Quantum breathing term

with $$\Lambda = a - i\frac{\dot{b}(0)}{2\omega_0}(a^+ + a)$$

Quantum breathing during the contraction

C. H. Chang & T. M. Hong, in preparation (2010).

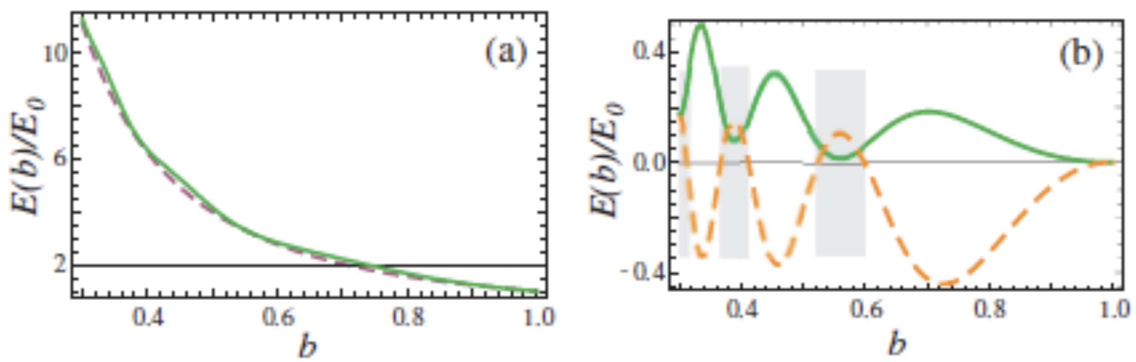


Figure 1: (color online). (a) The average energies of the wave packet for different contraction processes: linear contraction for $\dot{b} = -\omega_0/4$ and $\ddot{b} = 0$ (solid line), adiabatic process for $E(b) = E_0/b^2$(dashed line). (b) The solid line is the difference between linear contraction and adiabatic energies. The dashed line is the expection value for $E_0\{\hat{x}, \hat{p}\}/\hbar$ and the gray area denote the region of wave-packet diffusion.

The period is determined by

$$\int_0^T dt'\omega_0/b^2(t) = \pi,$$

$$T = b\pi/\omega_0$$

Corresponding effect in classical field

C. H. Chang & T. M. Hong, in preparation (2010).

The harmonic trap can be constructed by electromagnetic field

$$\vec{A} = \frac{1}{2} B(-y,\ x,\ 0),$$

$$\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + \frac{e^2 B^2}{8mc^2}(x^2 + y^2) + \hat{H}_z$$

We chose $\frac{eB}{c} = 2m\omega = \frac{4}{L^2(t)},$

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t},\ \vec{B} = \nabla \times \vec{A},$$

The electron in the classical field

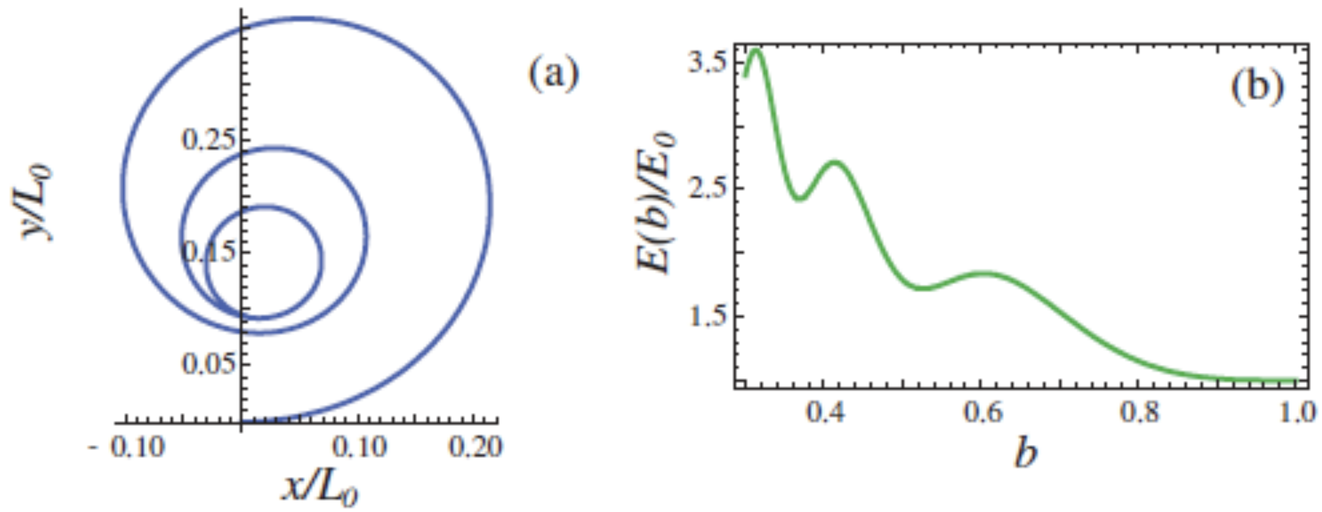


Figure 2: (color online). The electron with initial conditions $x = 0$, $y = 0$, $v_x = \omega_0 L_0/2$, and $v_y = 0$ in the EM fields. (a) The moving trajectory with the lengthscale $L_0 = (2\hbar/m\omega_0)^{1/2}$. (b) The energy change.

Why the packet dancing in the time-dependent harmonic trap

C. H. Chang & T. M. Hong, in preparation (2010).

- The oscillation appear as the projecting from the initial state to the eigenvectors.
- The harmonic trap decide the breathing frequency induced by the quantum friction.
- The quantum breathing modify the energy from the friction term in the trap.

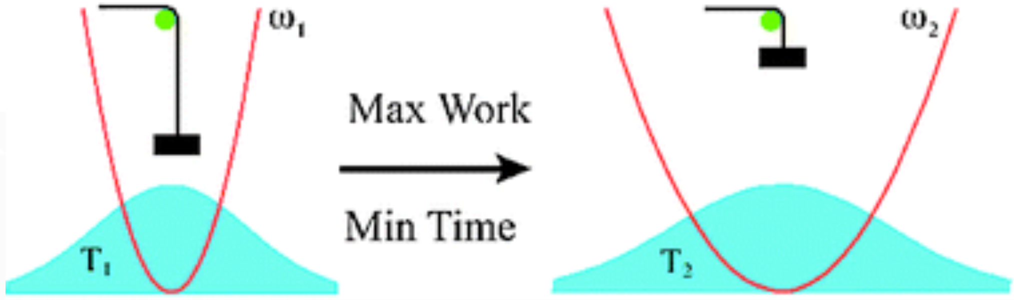
Conclusion I

- The general solution which include sudden and adiabatic approximation is found .
- The quantum breathing will modify the energy in the time-dep trap.

PartII

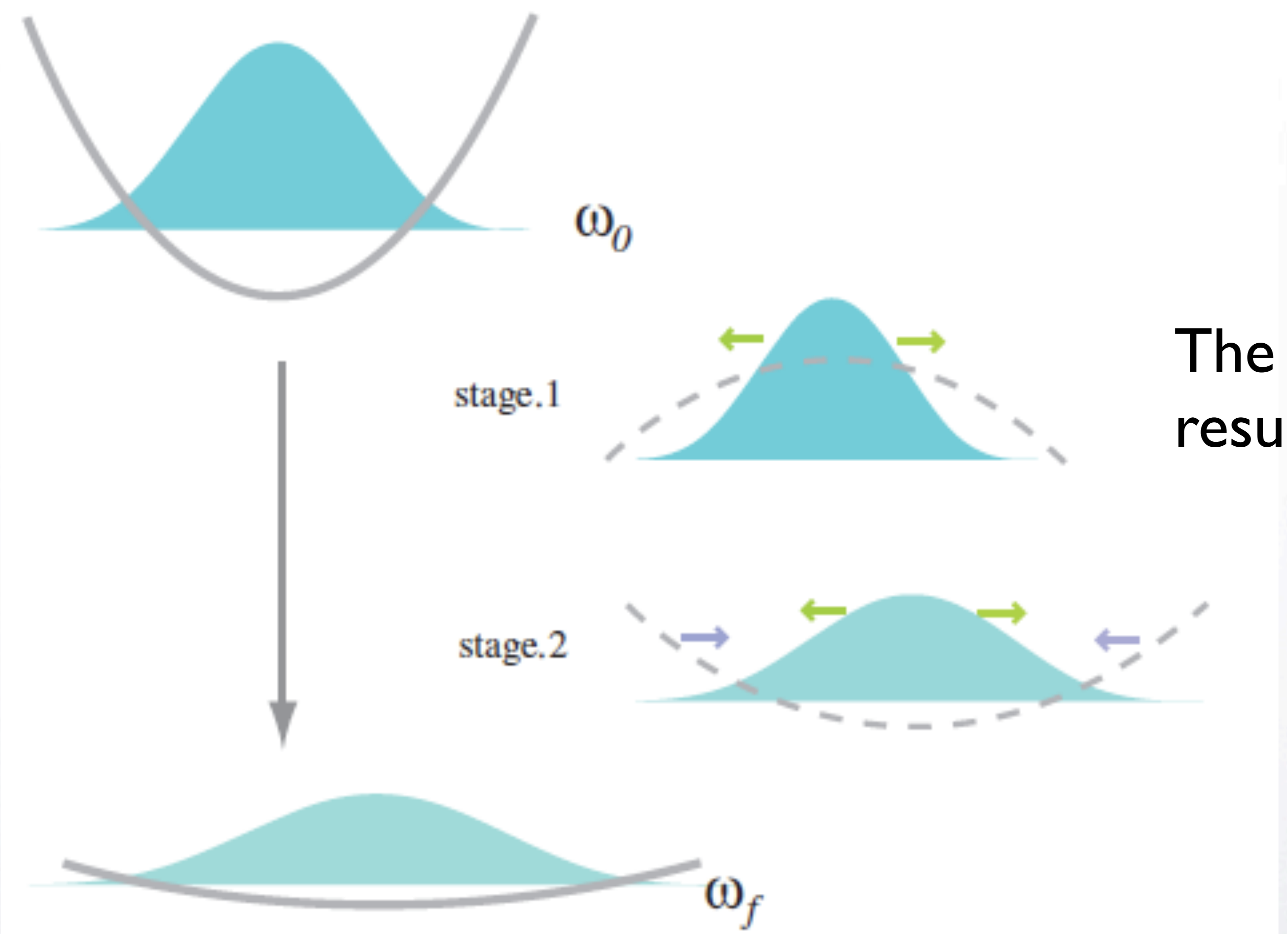
Frictionless Cooling in the time-dependent harmonic trap

Fast Optimal Cooling between two states



P. Salaman *et al.*, Phys. Chem. Chem. Phys. **11**, 1027(2009).
Xi Chen *et al.*, Phys. Rev. Lett. **80**, 063421 (2009).
J. F. Schaff *et al.*, Phys. Rev. A **82,** 033430(2010).

Frictionless Cooling in the trap
Our Strategy



The energy change equivalent to the adiabatic result:

$$E_f = \frac{\omega_f}{\omega_0} E_0$$

Frictionless Cooling in the trap
Our Strategy II

C. H. Chang & T. M. Hong, in preparation (2010).

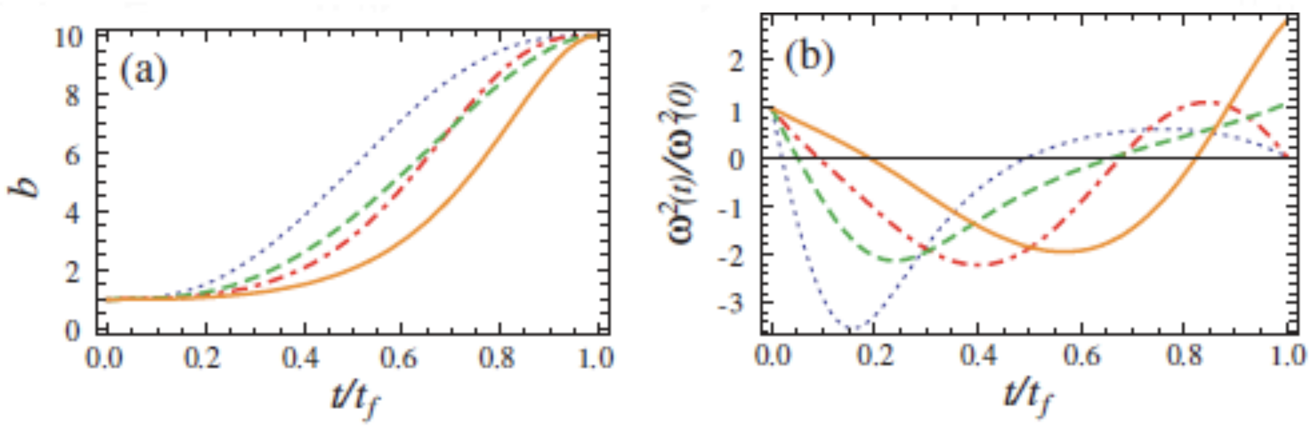


Figure 4: (color online). Cooling in $t_f = 2$ms.(a) Examples of ansatz for b. The simple polynomial ansatz for our result (dashed line) and for Ref. [1] (dotted line). The exponentials of a polynomial for our result (solid line) and Ref. [1] (dash-dotted line). (b) The corresponding squared frequency $\omega^2(t)$. $\omega(0) = 250 \times 2\pi$ Hz, $\gamma = 10$.

Besides the original condition:

$$b(0) = 1, \ \ddot{b}(0) = 0.$$

We add conditions

$$\dot{b}(0) = 0 \quad \dot{b}(t_f) = 0$$

to avoid Quantum Breathing & Kinetic Energy !!