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The Fine Structure Constant

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Origin of fine structure constant

- Introduced by Sommerfeld in 1916, as relativistic correction of Bohr model.
- non-relativistic $En = -\alpha^2 mc^2 \left(\frac{1}{2n^2} \right) = \frac{-13.6eV}{n^2}$
- relativistic correction $-\alpha^4 mc^2 \frac{1}{4n^2} \left[\frac{2n}{(l+1/2)} - \frac{3}{2} \right]$

$\alpha = e^2 / \hbar c \approx 1/137$ Speed of electron / speed of light

Fine structure splitting

- spin-orbit interaction (L·S, fine structure) $-\alpha^4 mc^2 \frac{1}{4n^2}\left[\frac{2n}{(j+1/2)} - \frac{3}{2}\right]$

The same magnitude as relativistic correction

$$\alpha = e^2/\hbar c \approx 1/137$$

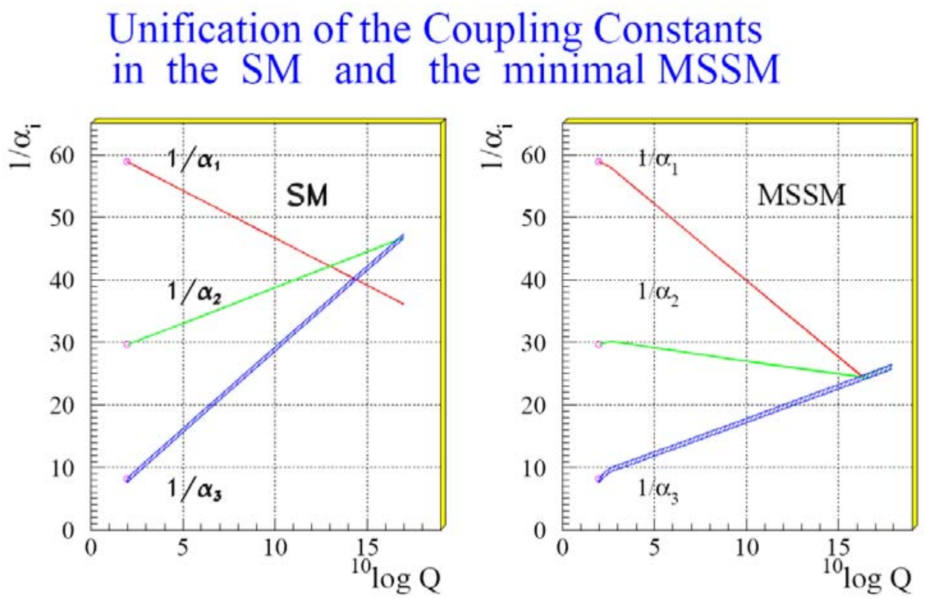
The strength of electromagnetic interaction
→ Irrelevant to strong, weak and gravitational forces

Standard Model description

- Usually use coupling constant g to denote the strength of an interaction
- Electromagneic: $g_e = (4\pi\alpha)^{1/2}$
- Strong: g_s
- Weak: g_W and g_Z
- Interestingly, $g_W = g_e/\sin\theta_W$ and $g_Z = g_e/\sin\theta_W \cos\theta_W$ where θ_W =Weinberg Angle
- Electro-weak unification

Unification of all forces

- Supersymmetry vs Standard Model



EM force

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- QED the most well-tested theory
- Fine structure constant α the most precisely determined constant among four forces

$$\alpha = e^2/\hbar c \approx 1/137$$

Need to know e^2/h

α^{-1} = 137.035 999 070 (98)

Gabrielse, Hanneke, Kinoshita, Nio, and Odom, *Phys Rev Lett* 97, 30802 (2006)

How to measure α

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- Neutron h/m_n measurement
- h/m_A measurement of atom A
- Magnetic moment of electron
- Helium fine structure
- Muonium and hydrogen hyperfine structure
- Quantum Hall effect
- Ac Josephson effect

Neutron h/m_n measurement

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$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_n}{m_e}\frac{h}{m_n}$$

- R_∞, c, m_n, m_e very precisely known.
- h/m_n this is the quantity to measure
- De Broglie wavelength $\lambda = h/m_n v$, so measure λ and v will do
- V: rotating polarization chopper, λ: Bragg diffraction

Neutron h/m_n measurement

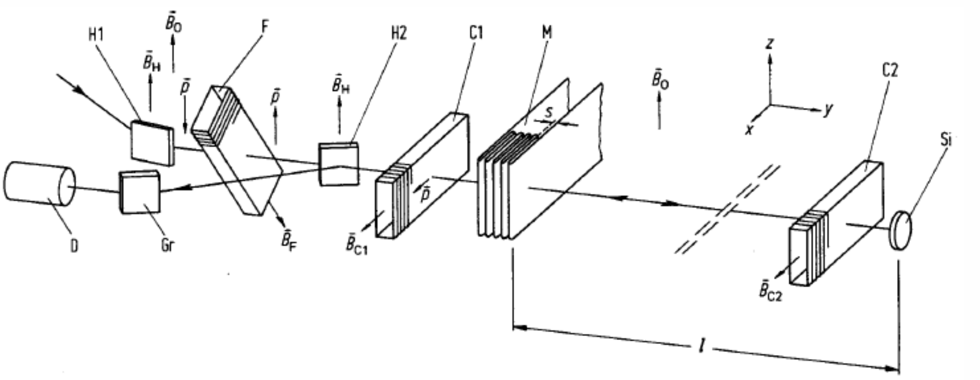


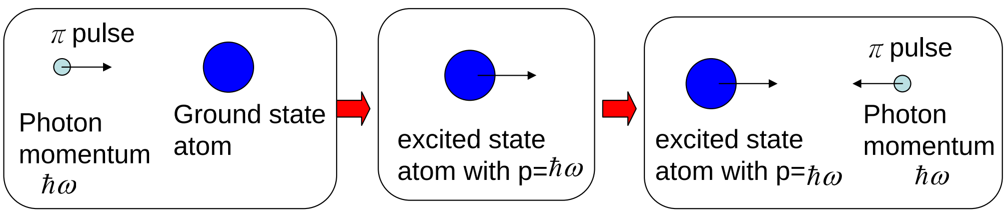
Figure 1. Arrangement for measuring h/m_n. H1, H2, Heusler crystals; F, flipping coil; C1, C2, $\pi/2$ coils; M, meander coil; Si, silicon crystal; Gr, oriented graphite; D, detector; B_0, magnetic induction of the guide field; B_H, magnetic induction magnetizing a Heusler crystal; p, polarization vector.

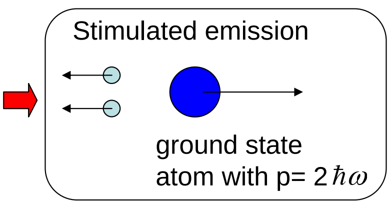
Krueger E, Nistler W and Weirauch W 1995 *Metrologia* **32** 117

h/m_A measurement of atom A

$$\alpha^2 = \frac{2R_\infty}{c}\frac{m_A}{m_e}\frac{h}{m_A}$$

- h/m_A this is the quantity to measure
- Concept of photon recoil





Input two photon, output two photon
But atom gain kinetic energy
$p^2/2m=2h^2\omega^2/m$

Photon recoil

- So input photon and output photon have different energy $\Delta\omega=2\hbar k^2/m$
- h/m measured, so is α
- In real experiment, a technique called atom interferometer is employed

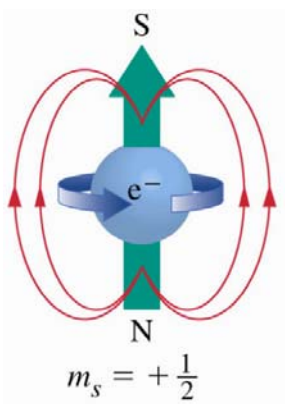
example: Weiss D S, Young B C and Chu S 1993 *Phys. Rev. Lett.* **70** 2706

g factor of electron

- what is g factor?

$$\frac{|\vec{\mu}|}{\mu_B} = g \cdot \frac{|\vec{s}|}{\hbar}$$

- Classical non-relativistic g=1
- QM, Dirac Theory predict g=2
- In reality, g= 2.002319304



g factor of electron

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- g can be expanded as a function of fundamental constant α
- measure g → measure α if all the required information known

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + \ldots\ldots$$
$$+ a_{\mu\tau} + a_{hadronic} + a_{weak}$$

How to measure g

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Spin precession (Larmor) frequency

$$\hbar\omega_L = m_s g \cdot \mu_B \cdot B \qquad \mu_B = \frac{e\hbar}{2m}$$

Calibration of the magnetic field by cyclotron frequency:

$$\hbar\omega_C = \frac{q}{M} B$$

$$g = 2\frac{\omega_L}{\omega_C}\frac{q}{e}\frac{m}{M} = 2\frac{\omega_L}{\omega_C}$$

Measurement on a single electron in a Penning trap

α^{-1} = 137.035 999 070 (98)

Gabrielse, Hanneke, Kinoshita, Nio, and Odom, *Phys Rev Lett* 97, 30802 (2006)

Helium fine structure

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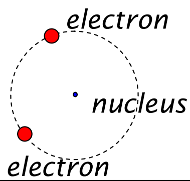
- helium: simple atom one can calculate



Contribution	Magnitude	
Nonrelativistic energy	Z^2	4
Relativistic correction	$Z^4\alpha^2$	9×10^{-4}
Anomalous magnetic moment	$Z^4\alpha^3$	7×10^{-4}
Mass polarization (SMS)	$Z^2\mu/M$	5×10^{-4}
Second-order mass polarization	$Z^2(\mu/M)^2$	8×10^{-8}
Finite mass correction (NMS)	$Z^4\alpha^2\mu/M$	1×10^{-7}
QED correction (Lamb shift)	$Z^4\alpha^3\ln\alpha$	6×10^{-3}
Finite Nuclear Size	$Z^4(R_N/a_o)^2$	2×10^{-9}

Helium spectroscopy

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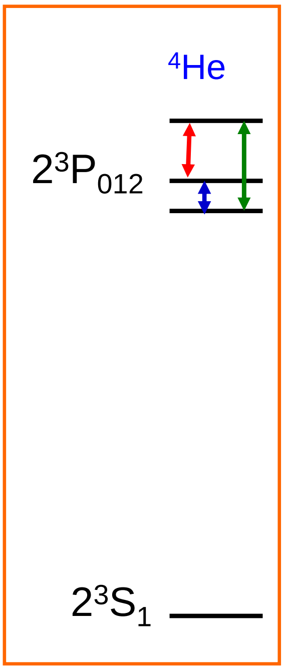
- Fine structure splitting
  $\rightarrow$test theory, determine α
- Measure fine structure splitting to 1kHz $\rightarrow$ α to 1 part per 10^8
- Some unsolved problems in theoretical side



Previous attempt

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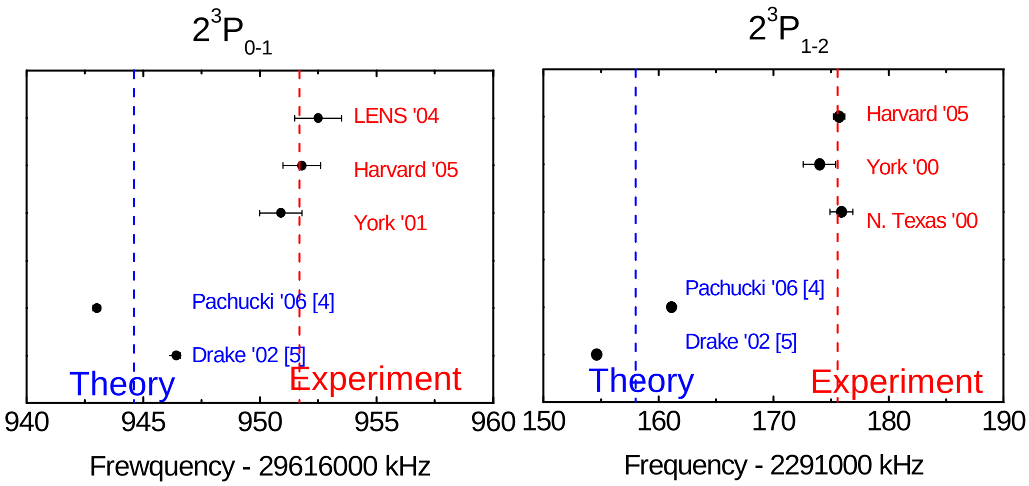
- uncertainty of theory and exp < 1kHz
- difference = 10kHz and 20kHz



Hydrogen and muonium

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- Hydrogen, simpler than helium
- Theory 100% correct, but…
- Proton is spin ½ particle, (helium spin 0)
- Hyper fine structure more sensitive to probe magnetic moment
- Muonium? anti-muon+electron or muon + positron, difficult

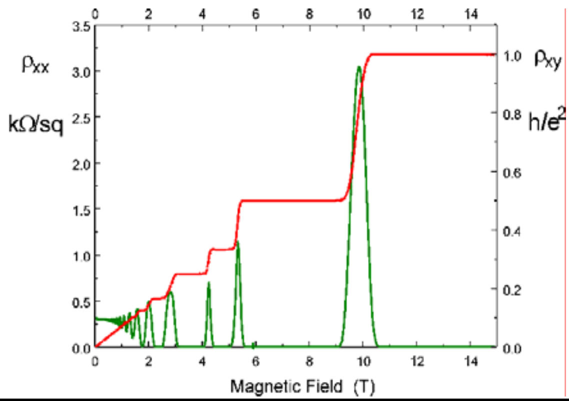
Quantum Hall effect

- Quantum Hall effect $R=h/ne^2$

R: resistance, n: an integer

$$\alpha = e^2 / \hbar c$$

- Can be very precise in principle
- The difficulty is to calibrate a standard Ohm



Josephson effect

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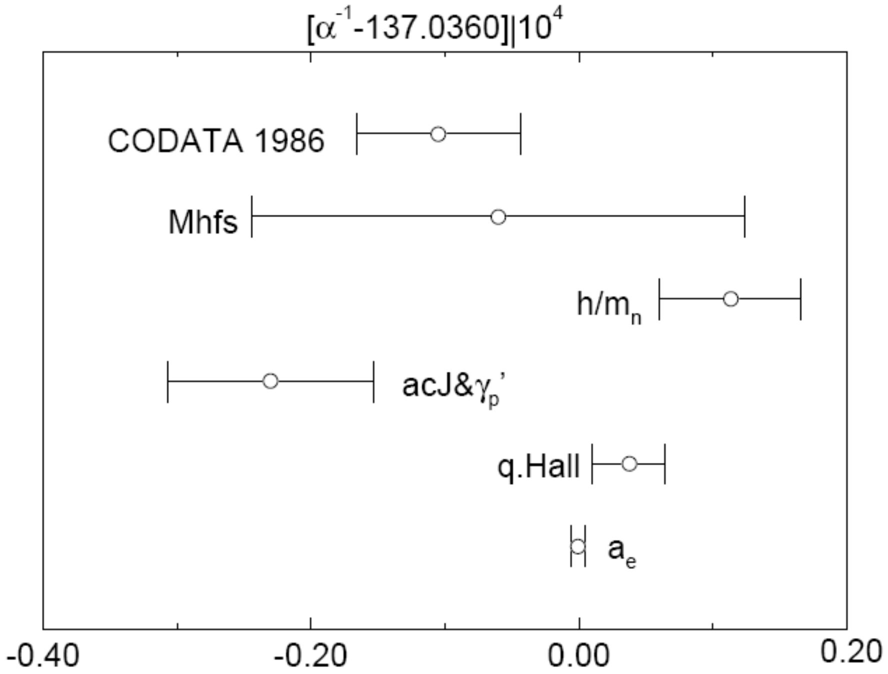
- DC Josephson effect: current through an insulating junction due to Tunneling (no E)
- AC Josephson effect: DC voltage V across the junction may also give rise to a AC current with frequency ω
- $\omega=2eV/h$, one can measure ω and V very precisely, and determine e/h to 10^{-16}
- Unfortunately α proportional to e^2/h

summary



The constancy of constant

Is the constant really constant?