Geometric modes and their potential applications

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Outline

- Introduction
- Motivation
- Results
- Potential Applications
- Conclusions

Introduction: Gaussian Beams

$$\nabla^2 E(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2 E(\mathbf{r},t)}{\partial t^2} = 0 \Rightarrow \nabla^2 E(\mathbf{r}) + k^2 E(\mathbf{r}) = 0$$

$$E(\mathbf{r}) = E_0 \psi(x,y,z) e^{-jkz} \Rightarrow \nabla_t^2 \psi - j2k\frac{\partial \psi}{\partial z} = 0$$

$$\psi_{00} = [\frac{w_o}{w(z)} e^{-\frac{r^2}{w^2(z)}}]\{e^{-jkz} e^{j\tan^{-1}(z/z_o)}\} e^{-j\frac{kr^2}{2R_c}}$$

$$\psi_{mn} = [\frac{w_o}{w(z)} H_m(\frac{\sqrt{2}x}{w(z)}) H_n(\frac{\sqrt{2}y}{w(z)}) e^{(-\frac{x^2+y^2}{w^2(z)})}] \{e^{-jkz} e^{j(m+n+1)\tan^{-1}(z/z_o)}\} e^{-j\frac{k(x^2+y^2)}{2R_c}}$$

Phase: frequency

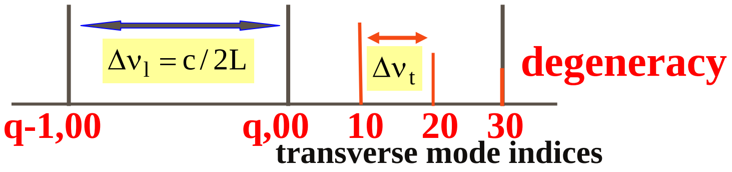
$$kz_2 - kz_1 - (m+n+1)[\tan^{-1}(z_2/z_o) - \tan^{-1}(z_1/z_o)] = q\pi$$

$$\Rightarrow \nu_{qmn} = \frac{c}{2L}[q + \frac{m+n+1}{\pi}\cos^{-1}(\sqrt{g_1 g_2})]$$

Introduction : Degenerate cavities

$$\nu_{qmn} = \frac{c}{2L}[q + \frac{m+n+1}{\pi}\cos^{-1}(\sqrt{g_1 g_2})]$$

Example: $g_1 g_2$ = 1/4, 1/2, 3/4



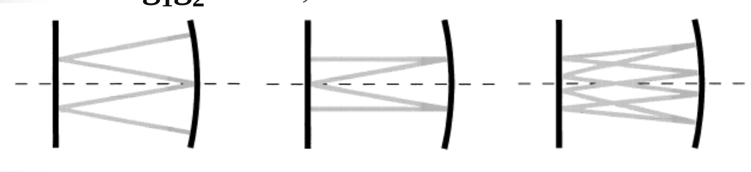
Transfer matrix

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

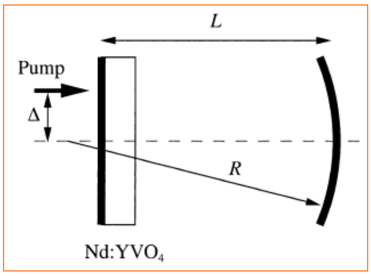
$$T^N = I$$

Motivation: geometric mode correspondence principle

- $g_1 g_2 = 1/2$, R = 2L

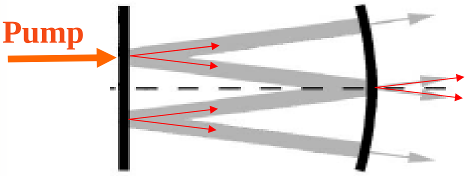


- End-pumped solid-state lasers

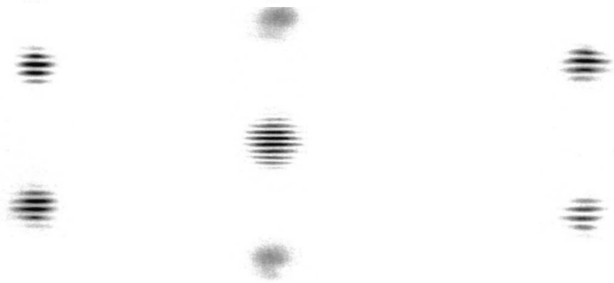


W or M mode?

Geometric W mode: multi-bouncing fundamental Gaussian beam (MBFGB) model



Opt. Commun. 188, 345 (2001)



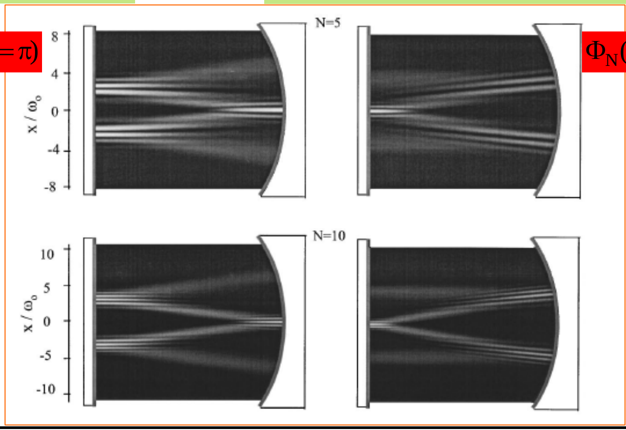
Wave representation

SU(2)

$$\Phi_N(x,y,z;\tau) = \frac{1}{(1+|\tau|^2)^{N/2}} \sum_{p=0}^{N} \binom{N}{p}^{1/2} \tau^p \Phi^{(HG)}_{4p,0}(x,y,z),$$

$\tau = \exp(i\phi)$ Phys. Rev. A 69, 053807 (2004)

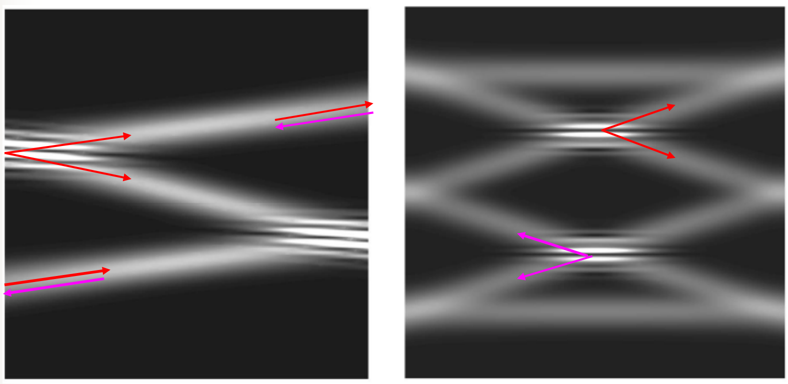
$\Phi_N(\phi=\pi)$ $\Phi_N(\phi=0)$



Correspondence principle

Operator method

Symmetrical cavity R = 2L, $g_1 g_2$ = 1/4

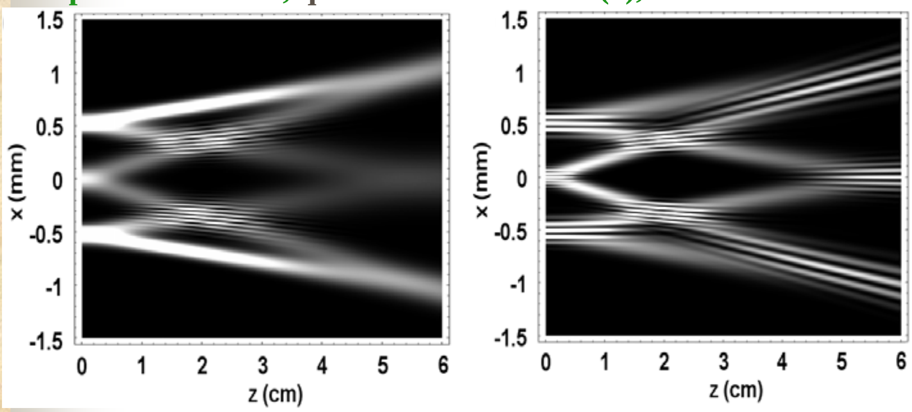


J. Opt. Soc. Am. A 22, 1559 (2005)

Results: contradiction between two methods

Plano-concave cavity $g_1 g_2 = 1/4$: VW mode

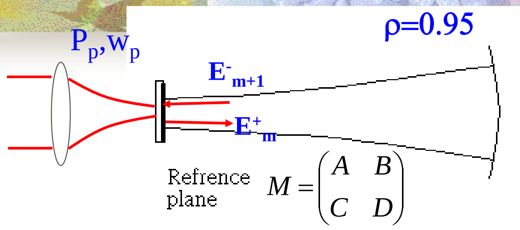
Operator method, $q/k = \pi/180$ SU(2), N=20



$\Phi_{20}(\phi = \pi/2) + \Phi_{20}(\phi = -\pi/2)$

Simulation

Fox-Li approach



Collin integral

$$E^-_{m+1}(r) = \frac{-2\pi i}{B\lambda}\int_0^a \exp(ik2L)\, E^+_m(r')\exp\{(i\pi/B\lambda)(A r'^2 + D r^2)\}\, J_0(2\pi r r'/B\lambda) r'\, dr'$$

$$E^+_{m+1}(r) = \rho E^-_{m+1}(r)\exp[\sigma \Delta N(r) d]\Pi(r/a)$$

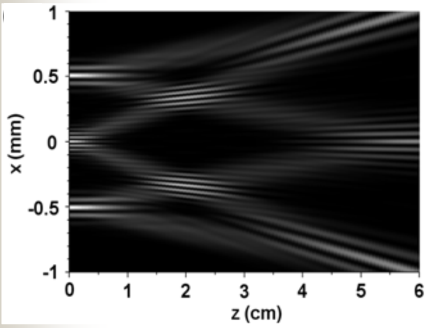
Rate equation

$$\Delta N_{m+1}(r) = \Delta N_m(r) + R_{pm}(r)\Delta t - \gamma \Delta N_m(r)\Delta t - \frac{|E_m(r)|^2}{E_s^2}\gamma \Delta N_m(r)\Delta t$$

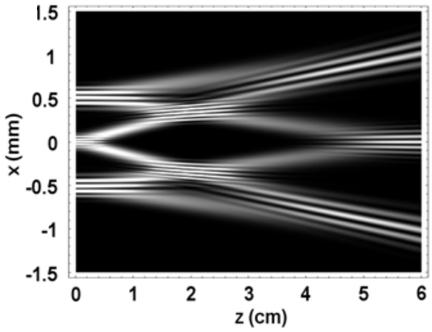
$$\int_V R_{pm} dV = P_p / h\nu_p$$

Numerical results: VW mode

Fox-Li

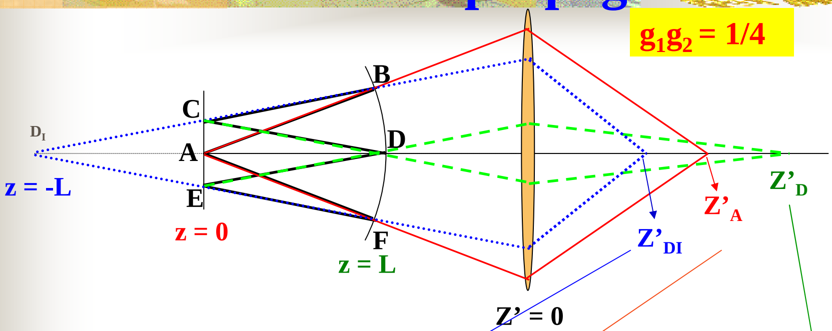


SU(2)

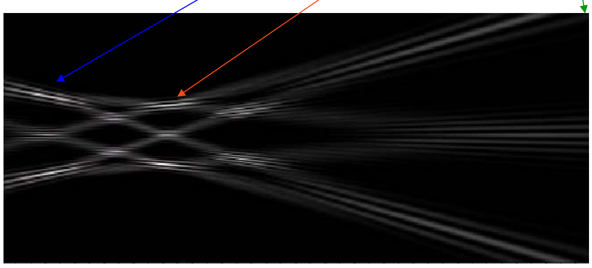


Numerical beam propagation

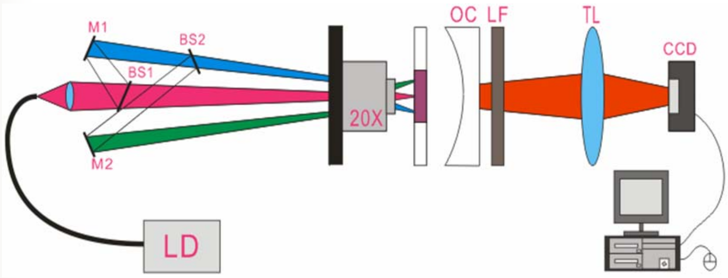
$g_1 g_2 = 1/4$



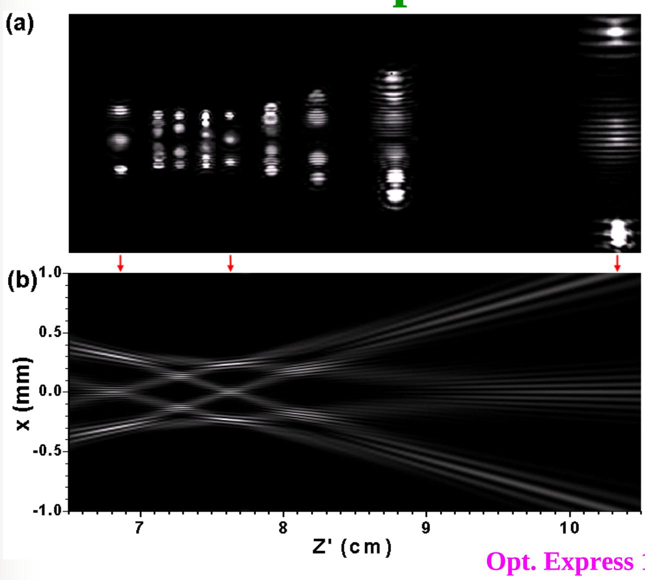
Satisfy Gaussian lens law



Experimental setup



VW mode pattern

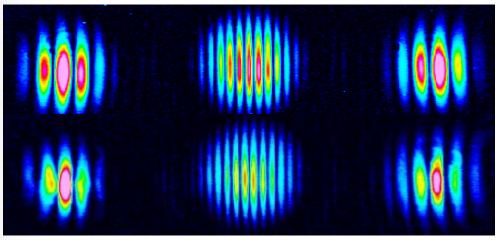


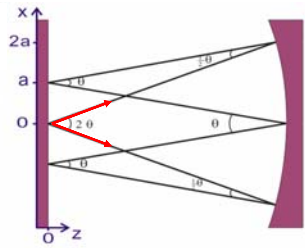
Experiment

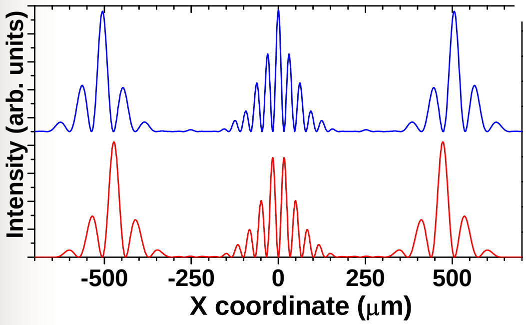
Fox-Li calculation

Opt. Express 15, 12693 (2007)

Outside the MBFGB model: dark-centered profile



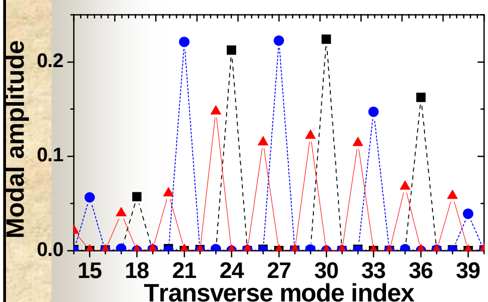




Bright-centered

Dark-centered

Mode expansion



p	0	1	2	3	4	5	6	7	8	9
HG_{6p}	0	π	0	π	0	π	0	π	0	π
HG_{6p+3}	0	π	0	π	0	π	0	π	0	π

Transverse mode locking

Cooperative frequency locking

● $E_{sp}(x) = 0.21\Psi_{24,q}(x) + 0.22e^{i\pi}\Psi_{30,q-2}(x) + 0.16\Psi_{36,q-4}(x)$

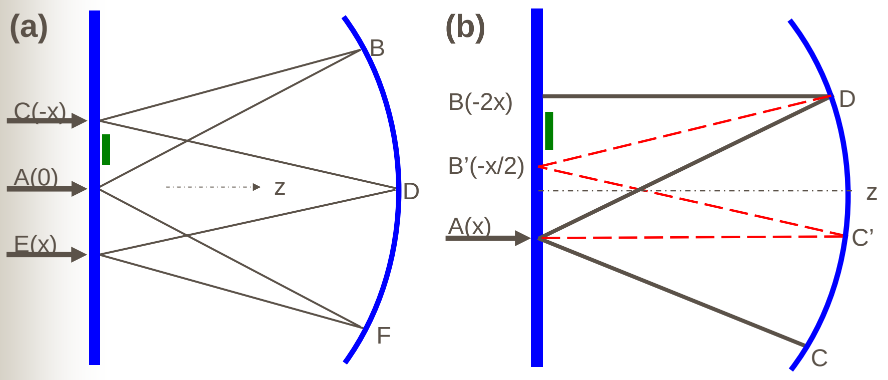
■ $E_{sp}(x) = 0.22e^{i\pi}\Psi_{21,q}(x) + 0.22\Psi_{27,q-2}(x) + 0.15e^{i\pi}\Psi_{33,q-4}(x)$

Compared with SU(2)

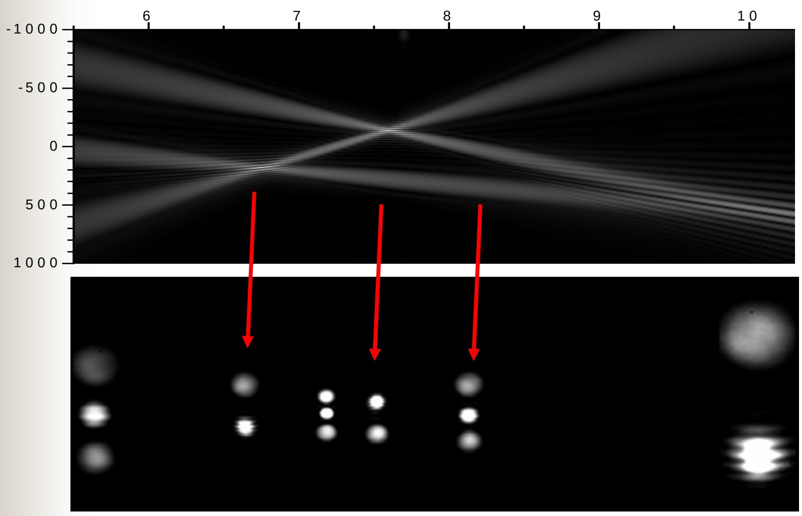
$$\Phi_N(x,y,z;\tau) = \frac{1}{(1+|\tau|^2)^{N/2}} \sum_{p=0}^{N} \binom{N}{p}^{1/2} \tau^p \Phi_{3p,0}^{(HG)}(x,y,z),\ \tau = e^{i\phi}$$

$$\Phi_N(x,y,z;\phi=\pi/2) + \Phi_N(x,y,z;\phi=-\pi/2), N=20$$

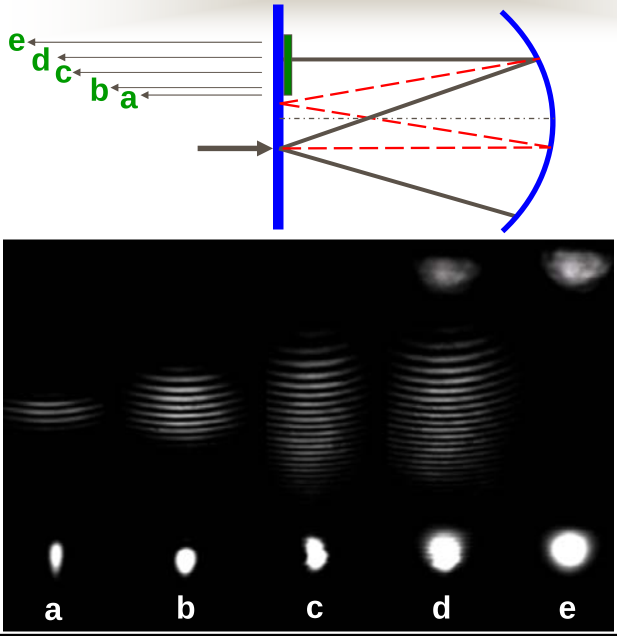
Outside the MBFGB model: wide- & narrow-N modes



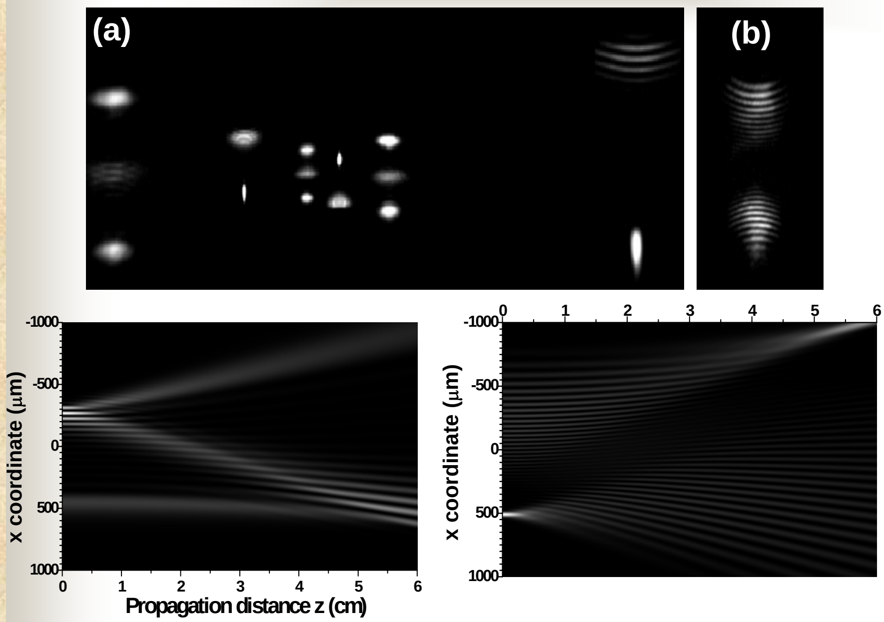
Wide-N mode propagation



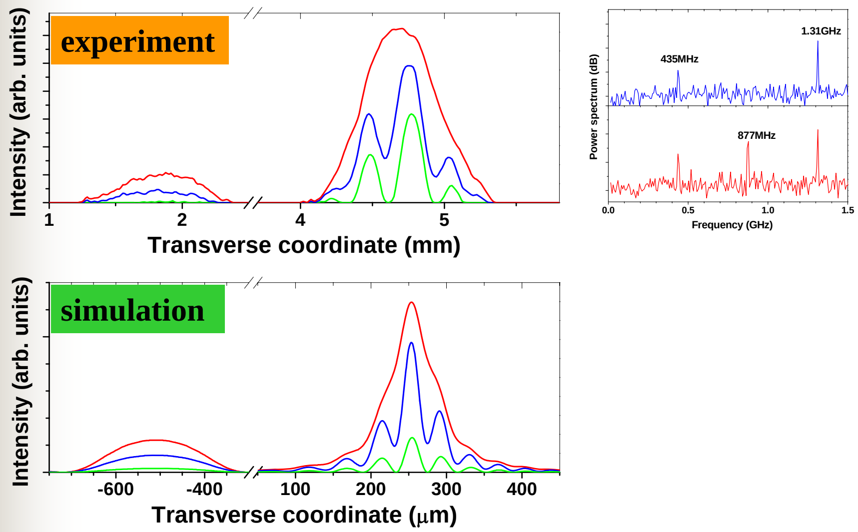
Narrow-N to wide-N mode



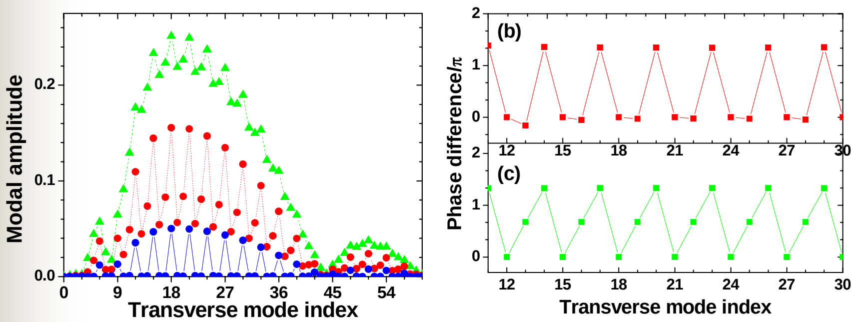
Narrow-N mode propagation



Outside the MBFGB model :
multifrequencies wide-N mode



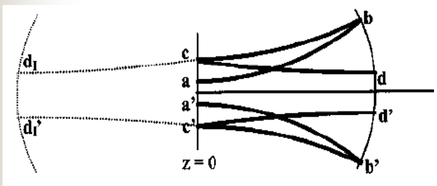
N mode expansion



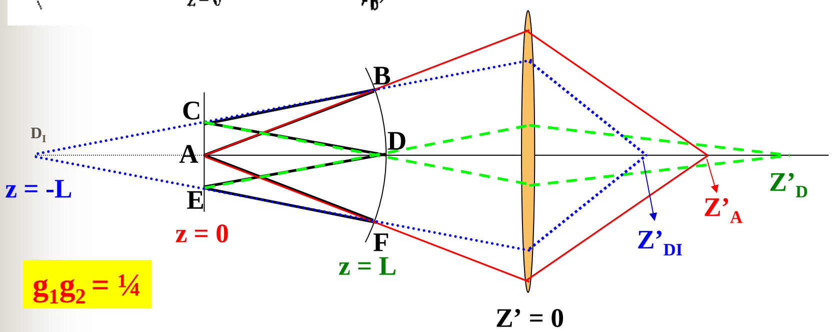
Summary

1. We generate a geometric VW mode and clarify the differences between the operator method and SU(2) wave representation
2. We indicate that both the operator method and the SU(2) wave representation are insufficient
3. We demonstrate the evidences for geometric modes outside the MBFGB model

Another geometric mode: multi-beam-waist (MBW) mode

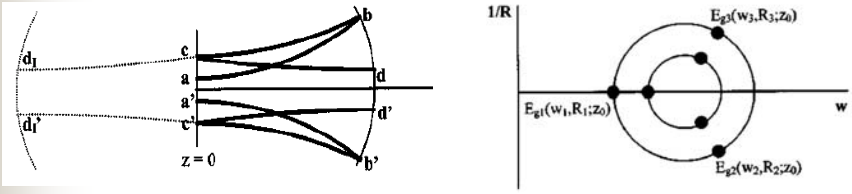


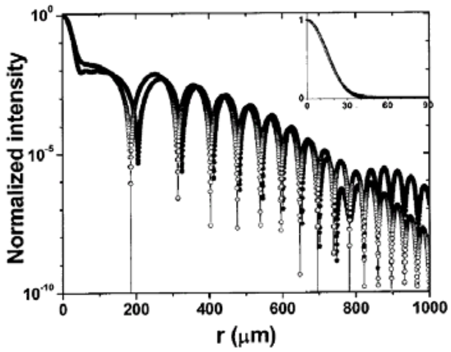
Tightly-focused pumping



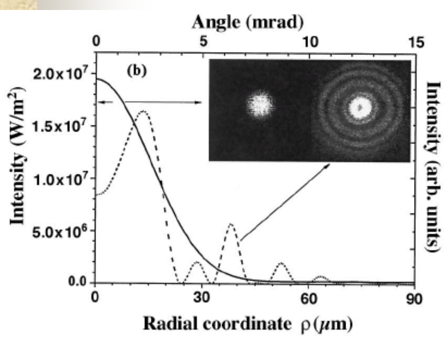
$g_1g_2 = ¼$

Geometric picture: three-round-trip superposition



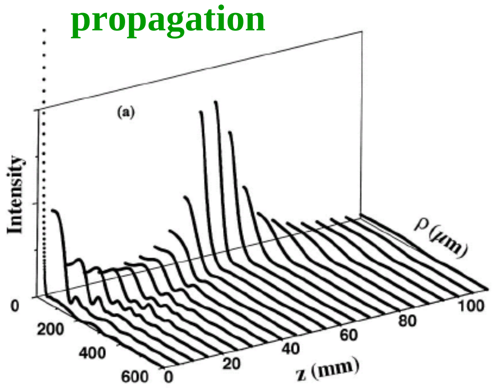


MBW mode propagation



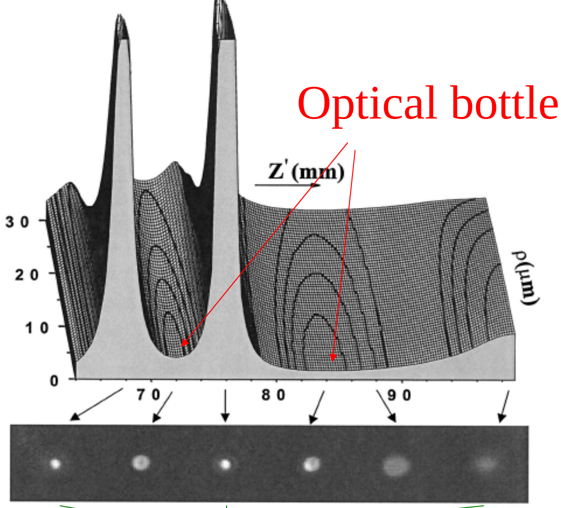
Far-field profile

Intra-cavity propagation



Propagation after transform lens

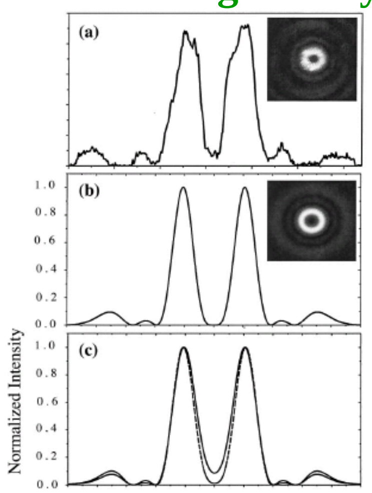
Optical bottles



Three beam waists

Deeper bottles by smaller pump size

At 1/3-degeneracy



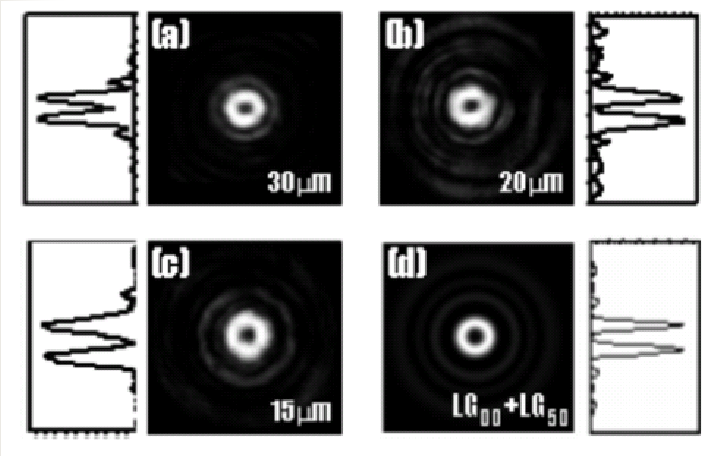
Add 350 μm aperture

$LG_{00} + \pi\, LG_{30}$

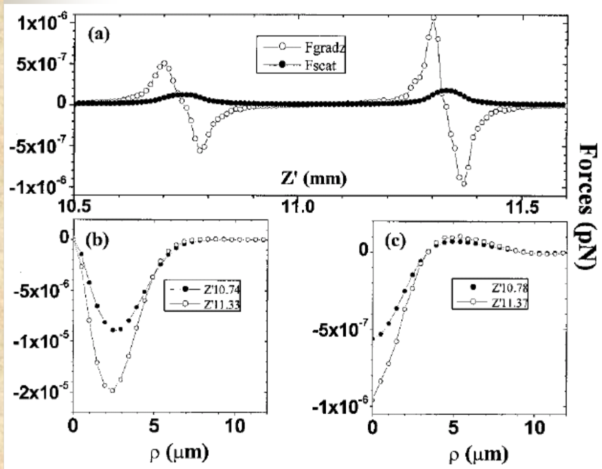
350 μm aperture

Pump size 15 μm

Bottles at 1/5-degeneracy



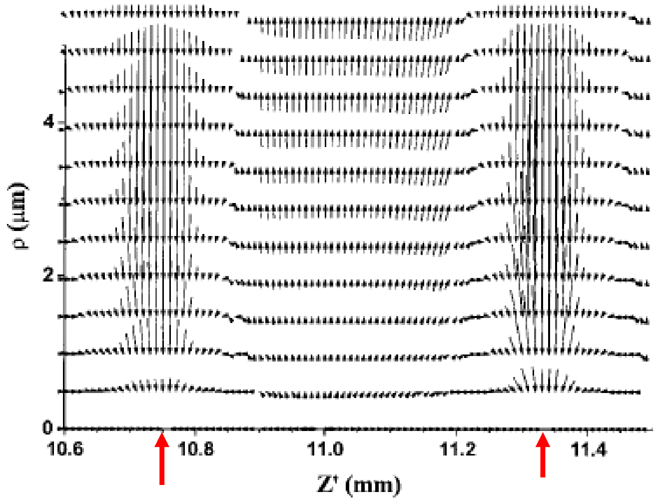
Application 1: laser trapping



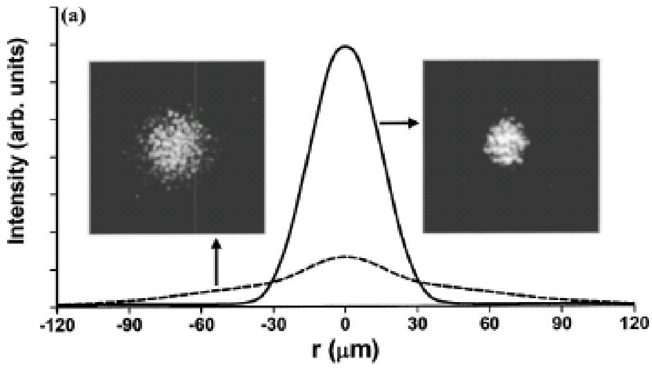
$$\mathbf{F}_{scat}(\mathbf{r}) = \frac{n_2}{c}\frac{8}{3}\pi k^4 a^6 \left(\frac{n_r^2 - 1}{n_r^2 + 2}\right)^2 I(\mathbf{r})\mathbf{k}$$

$$\mathbf{F}_{grad}(\mathbf{r}) = \frac{2\pi n_2 a^3}{c}\left(\frac{n_r^2 - 1}{n_r^2 + 2}\right)\nabla I(\mathbf{r})\mathbf{k}$$

Vector plot



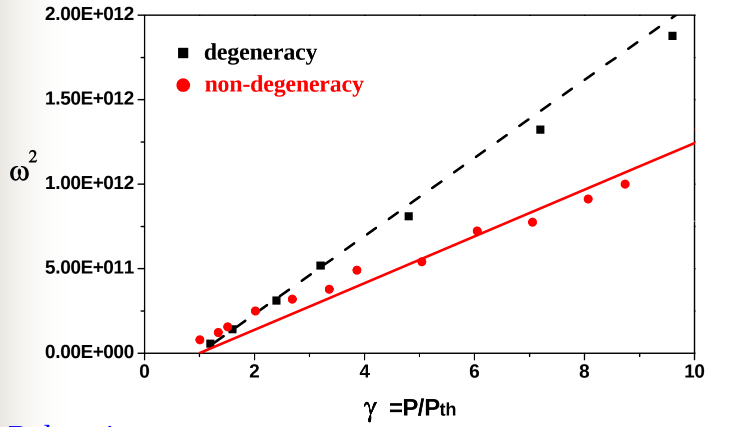
Application 2: subwavelength focusing



Application 3: atom trap

Application 4: enhanced spon. emission?

Spontaneous emission rate may be altered in resonators



Relaxation Osc. Freq. $\omega^2 = \frac{\gamma - 1}{\tau_c \tau_f}, \quad \tau_c = \frac{2l'}{c}[-\ln R_1 R_2 + 2\alpha l]^{-1} \approx const.$

Rate equation

$$\tau \frac{\partial}{\partial t} N(z) = \tau R(z) - (1 + \frac{I(z)}{I_s}) N(z) + \tau D \frac{\partial^2}{\partial z^2} N(z) - A\tau N^2(z)$$

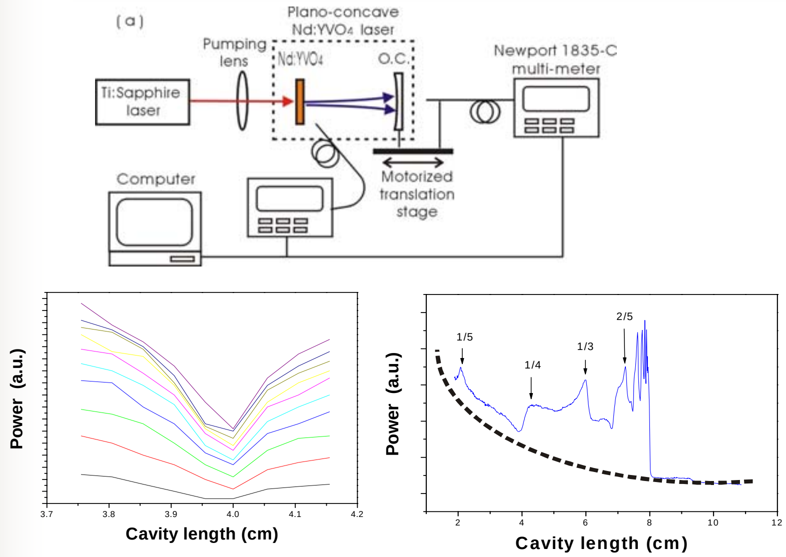
Energy diffusion: diffusion of electrons from the transfer of the energy of an excited ion to a neighboring ion in the ground state

Auger upconversion: an energy transfer between two ions in the upper level of the laser transition

Good overlap integral of pump and mode *3/4

Spatial hole burning (single mode) *2/3

Cavity decay rate (photon lifetime)



Summary

- We generated bottle beams from a bare laser in different degenerate cavities.
- Multi-beam waists can be explained by a geometric picture.
- The deep bottles can be obtained by a small pump size and a proper aperture.
- Such beams may be used as an optical tweezers, sub-wavelength focusing, atom trap…

Thanks for Listening

Laser dynamics:

nonlinear dynamics of lasers

$$F_x(x,t) = m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -ax + bx^2$$

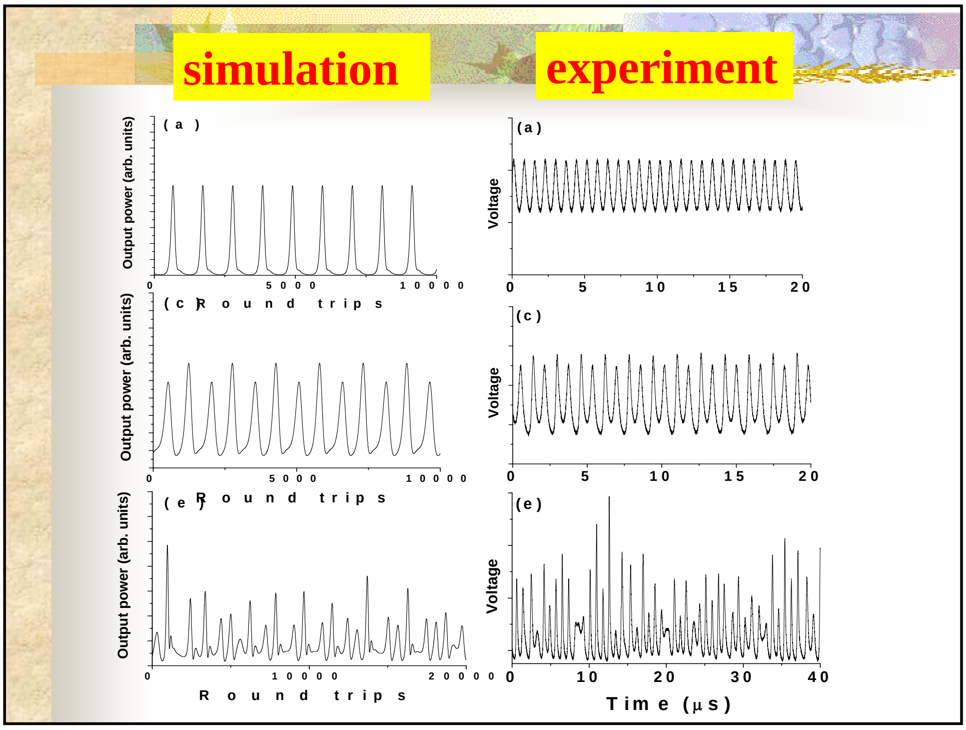
Deterministic system

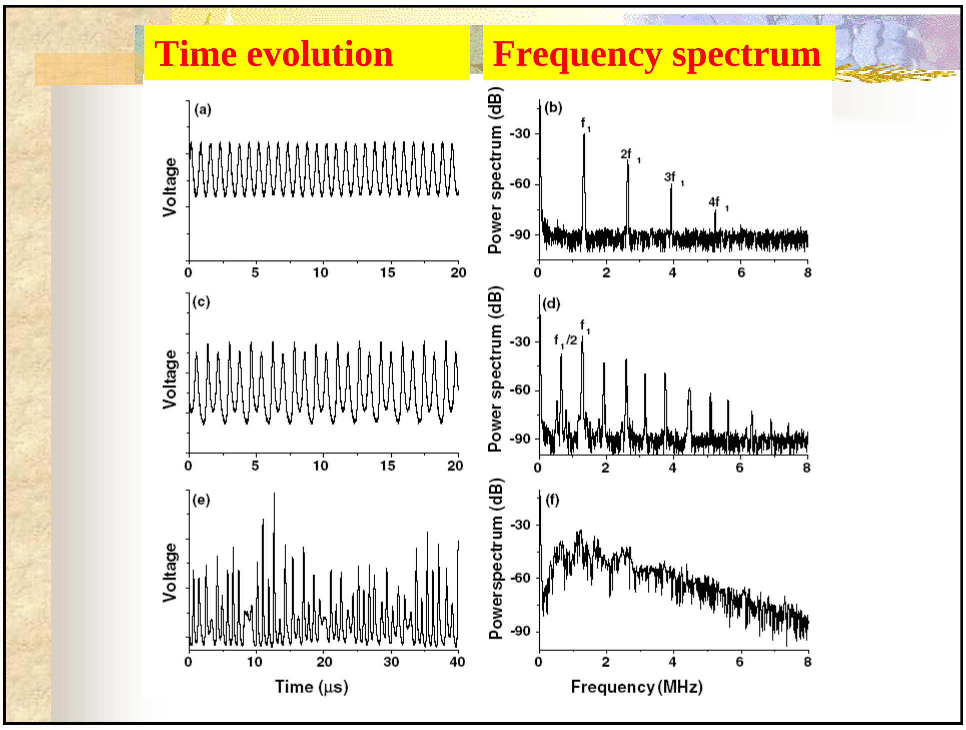
- **The time evolution equation**
- **The values of the parameters**
- **The initial conditions**

1. What kind of nonlinear system exhibit chaos?

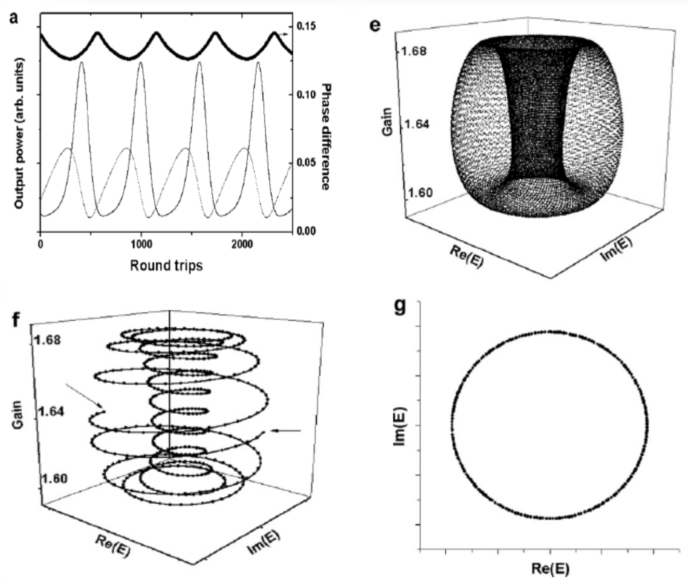
2. How does the behavior of a NL system change if the parameters change?

3. How do we describe chaos?


simulation
experiment
Output power (arb. units)
Round trips
Voltage
Time (μs)

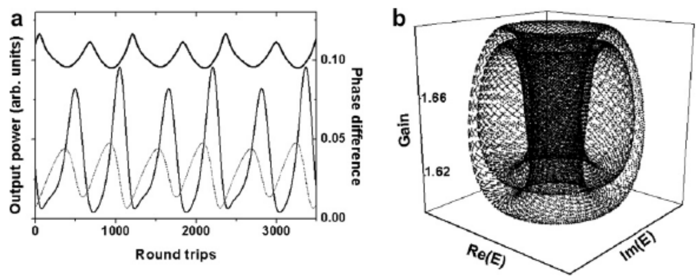

Time evolution
Frequency spectrum
Voltage
Time (μs)
Power spectrum (dB)
Frequency (MHz)

Period-1 evolution

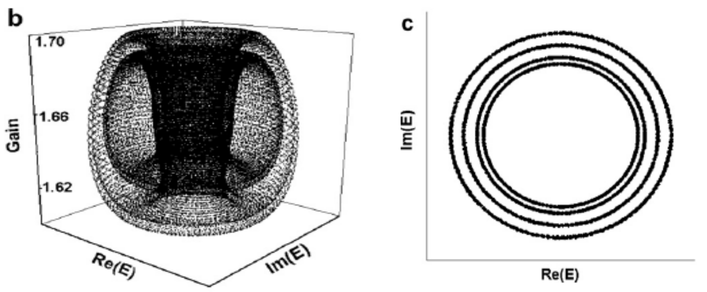


Period-2 and 4

Period -2



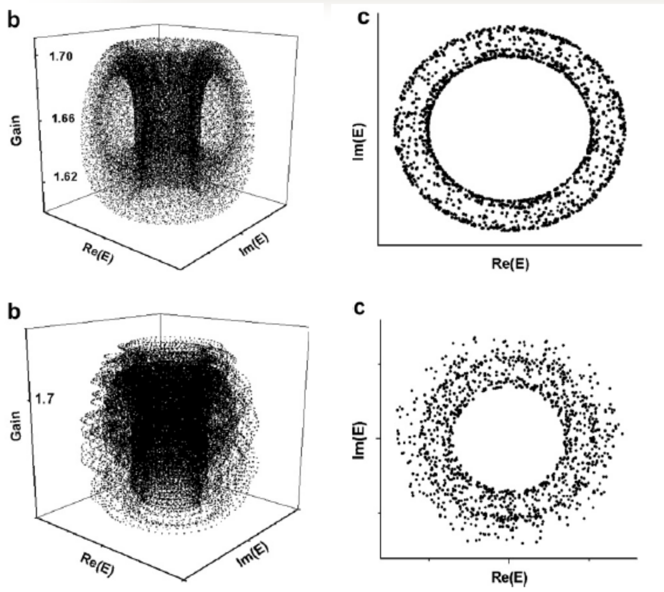
Period -4



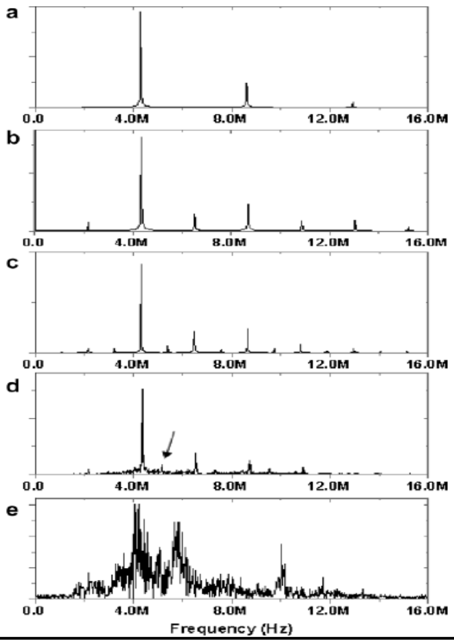
Trajectory in phase space

Quasi-period

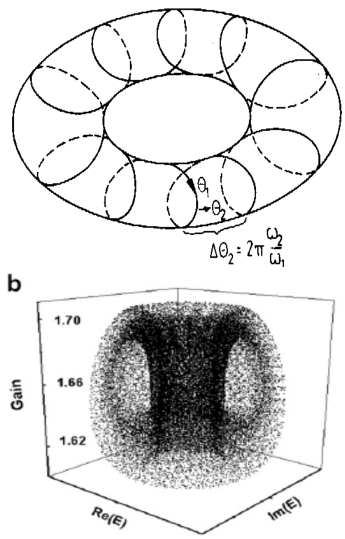
Chaos



Frequency spectrum



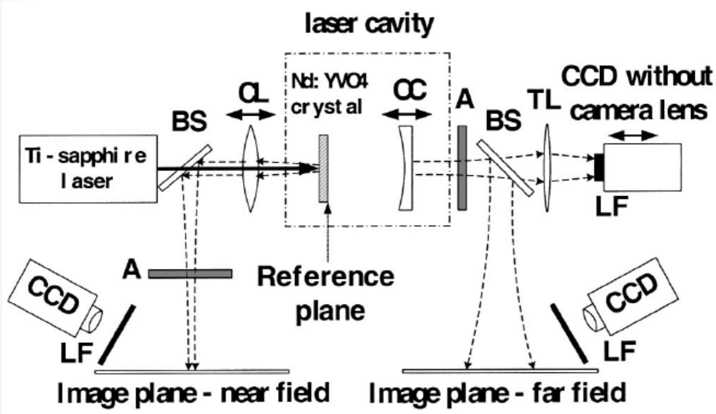
Quasi-periodic torus in phase space

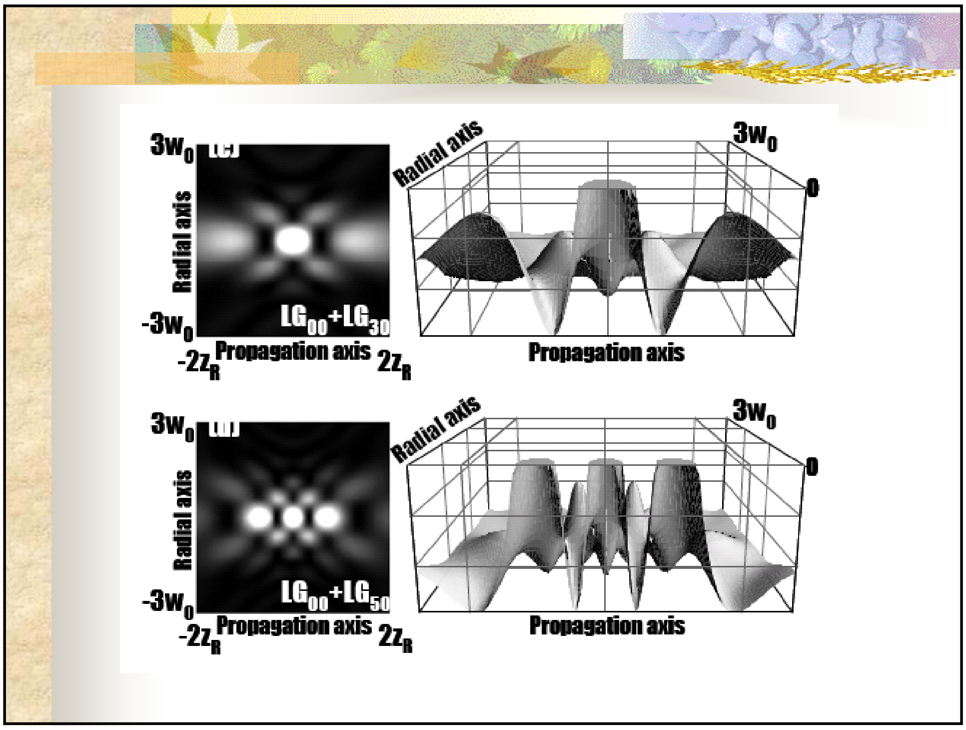


Summary

- Deterministic chaos are found in a tightly-focused end-pumped solid-state laser
- New torus in phase space is described for quasi-periodic behavior
- The route to chaos is the mixed effect of period doubling and quasi-periodic

Thanks for Listening




3w0
Radial axis
-3w0
LG00+LG30
-2zR
Propagation axis
2zR
Radial axis
3w0
0
Propagation axis
3w0
Radial axis
-3w0
LG00+LG50
-2zR
Propagation axis
2zR
Radial axis
3w0
0
Propagation axis