Dep. of Physics, NTHU

Oct. 31 (2011)

# From Cavity QED
# to
# Quantum Transport


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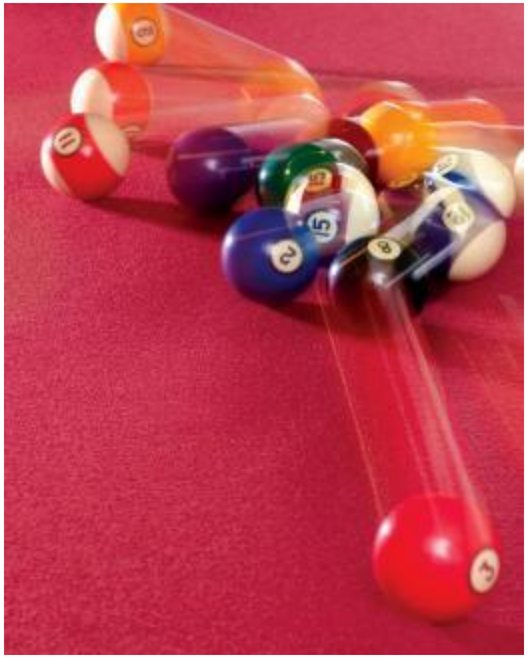
Unified single-photon and single-electron counting statistics: From cavity QED to electron transport

**Phys. Rev. A 82**, 063840 (2010), N. Lambert, Y. N. Chen*, and F. Nori

Distinguishing quantum and classical transport through nanostructures

**Phys. Rev. Lett. 105**, 176801 (2010), N. Lambert, C. Emary, Y. N. Chen, and F. Nori.

**Quantum or not? Mathematical equations resolve nanostructures behavior**



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# Outline

- Coherence and entanglement
- Quantum Teleportation
- Cavity QED
- Time-adjusted Photon Counting
- Quantum Transport in Organism

Quantum Information

**Teleportation**

Dense coding

Secret sharing

Key distribution

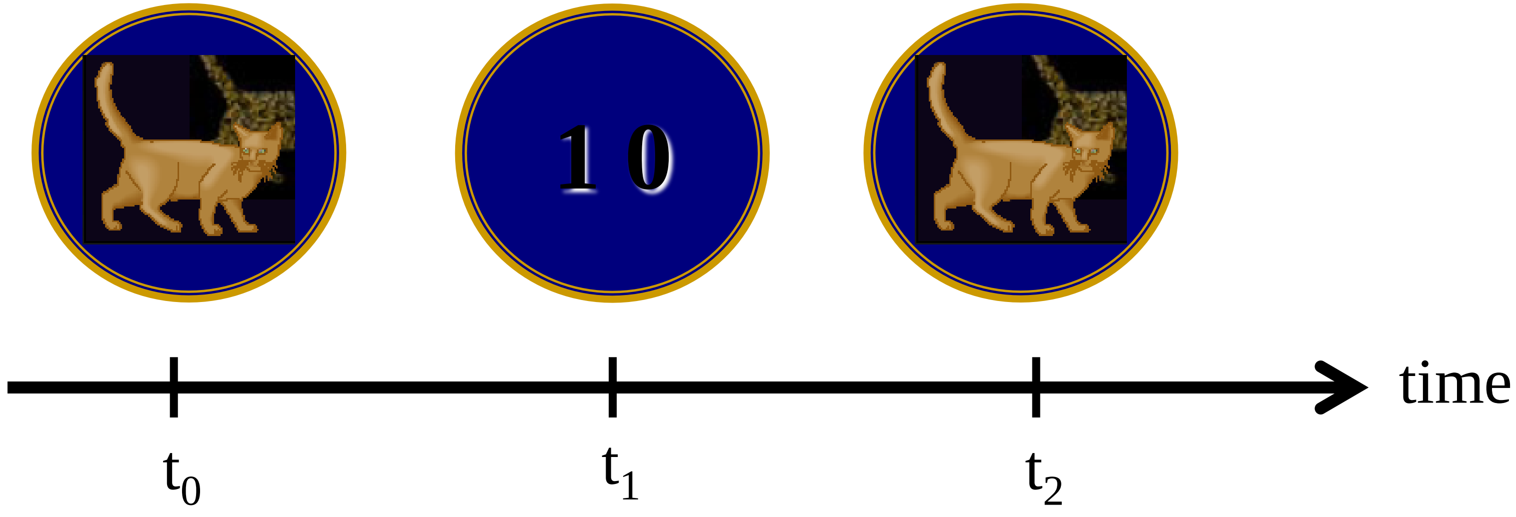
coherence and entanglement

Quantum Computation

Algorithms

Bit : 0, 1 or +, - or boy, girl....

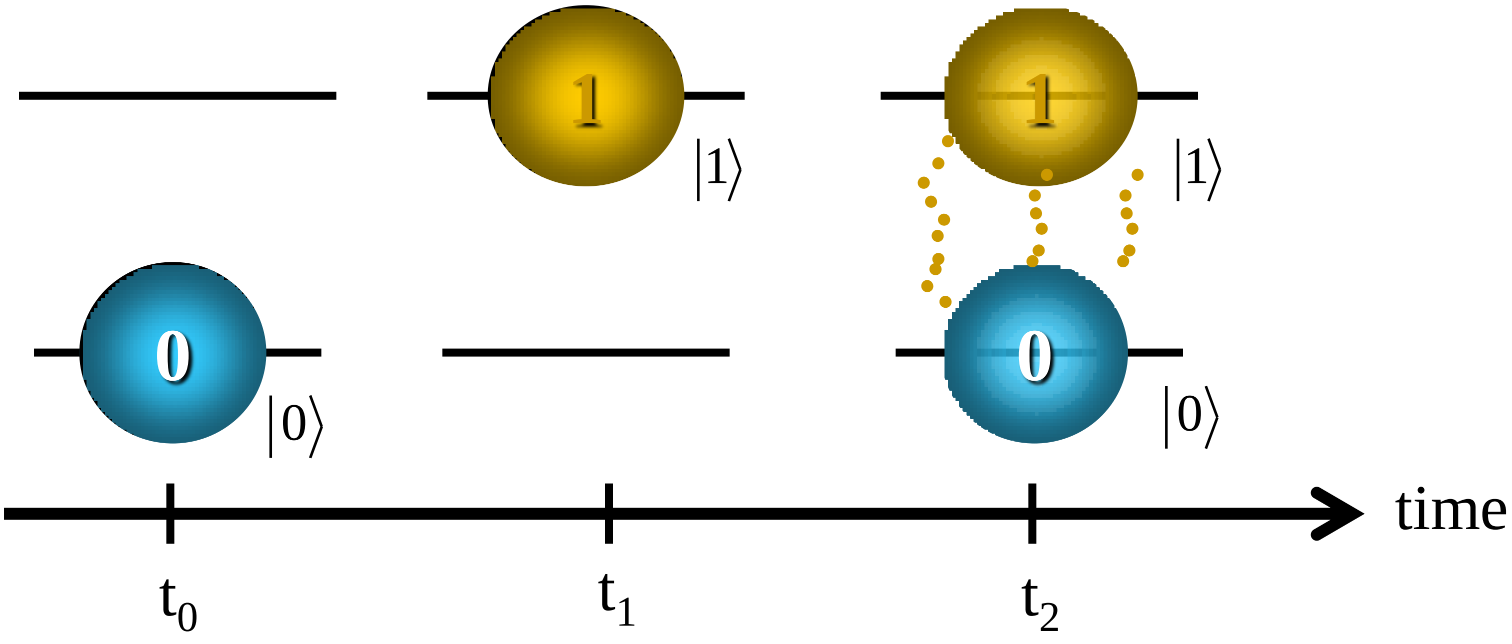
Any two-level system



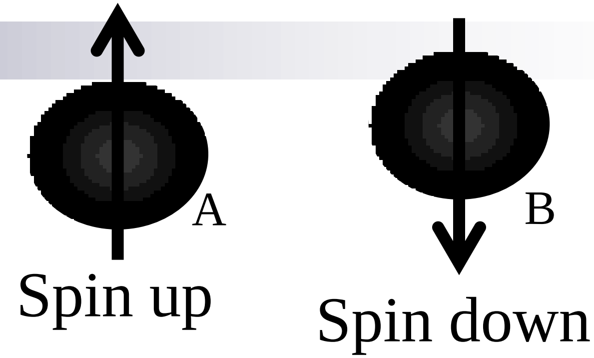
# Q-bit: Any two-level and physical system

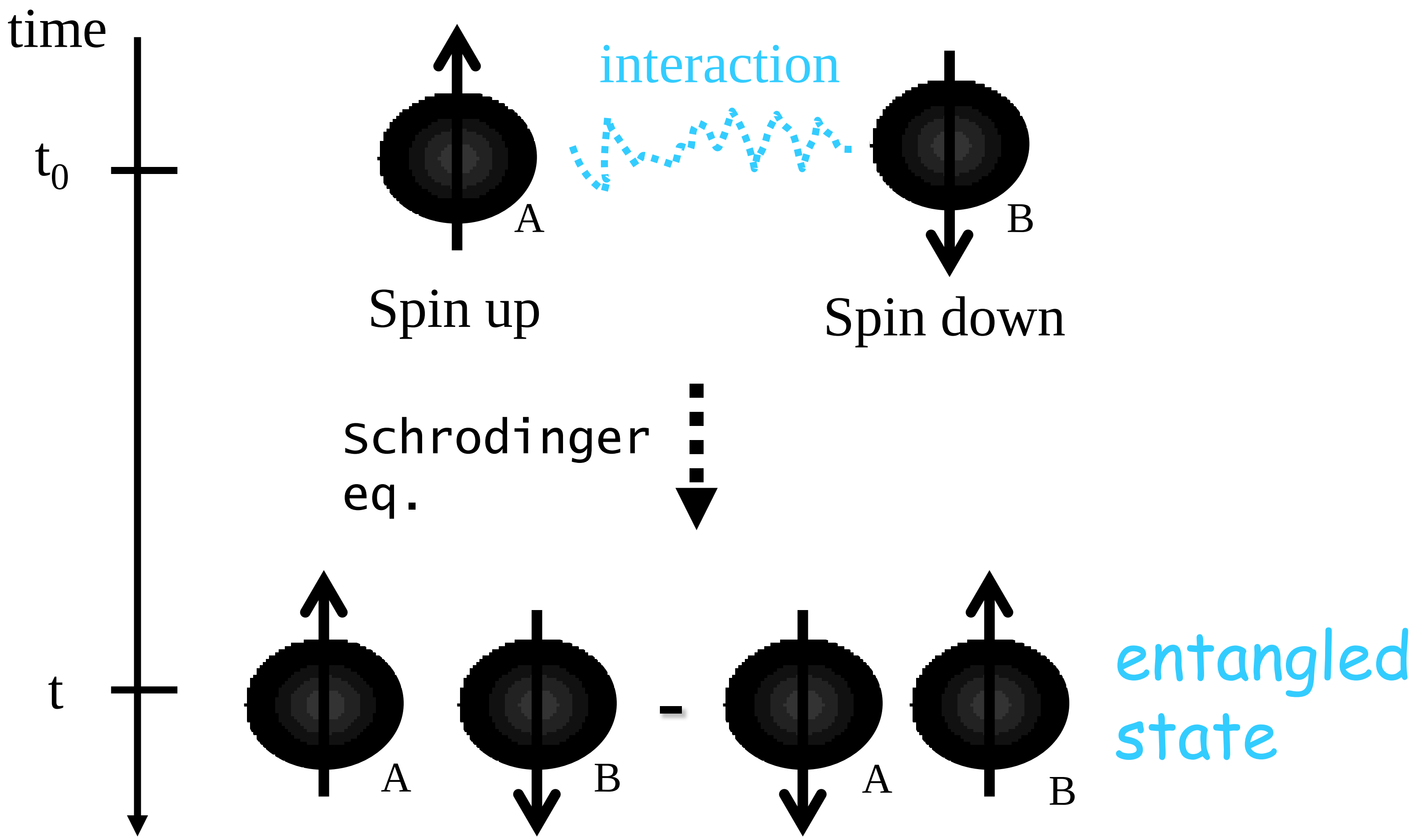
(Quantum bit)

Two-level atom



# Two qbits : two spins

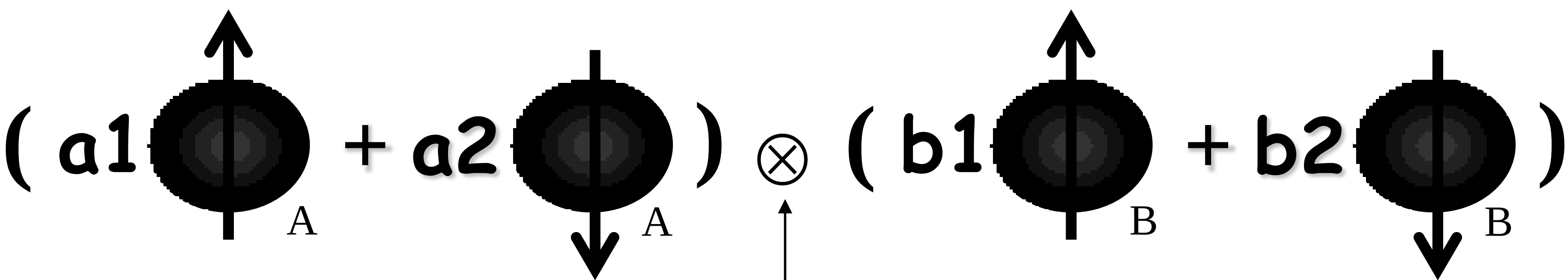


# Entanglement

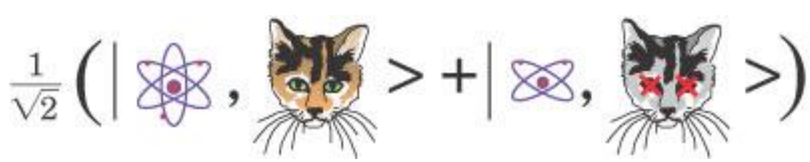
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow_A \downarrow_B\rangle - \frac{1}{\sqrt{2}}|\downarrow_A \uparrow_B\rangle$$

*.*? Impossible to factory




Symbol of connecting to independent system

# 薛丁格的貓：To be or not to be?

# Quantum Teleportation

量子遠傳(隱形傳送)

# Teleportation: Science fiction or science?


From Prof. Beenakker's web-page

# Quantum Teleportation

(top, left) Richard Jozsa, William K. Wootters, Charles H. Bennett. (bottom, left) Gilles Brassard, Claude Crépeau, Asher Peres. Photo: André Berthiaume.

In 1993 an international group of six scientists, including IBM fellow Charles H. Bennett, confirmed the intuitions of the majority of science fiction writers by showing that perfect teleportation is indeed possible in principle, but only if the original is destroyed.

# PREPARING FOR QUANTUM TELEPORTATION . . .

Scientific American, April 2000; by Zeilinger

QUANTUM TELEPORTATION OF A PERSON (impossible in practice but a good example to aid the imagination) would begin with the person inside a measurement chamber (*left*) alongside an equal mass of auxiliary material (*green*).The auxiliary matter has previously been quantum-entangled with its counterpart, which is at the faraway receiving station (*right*).

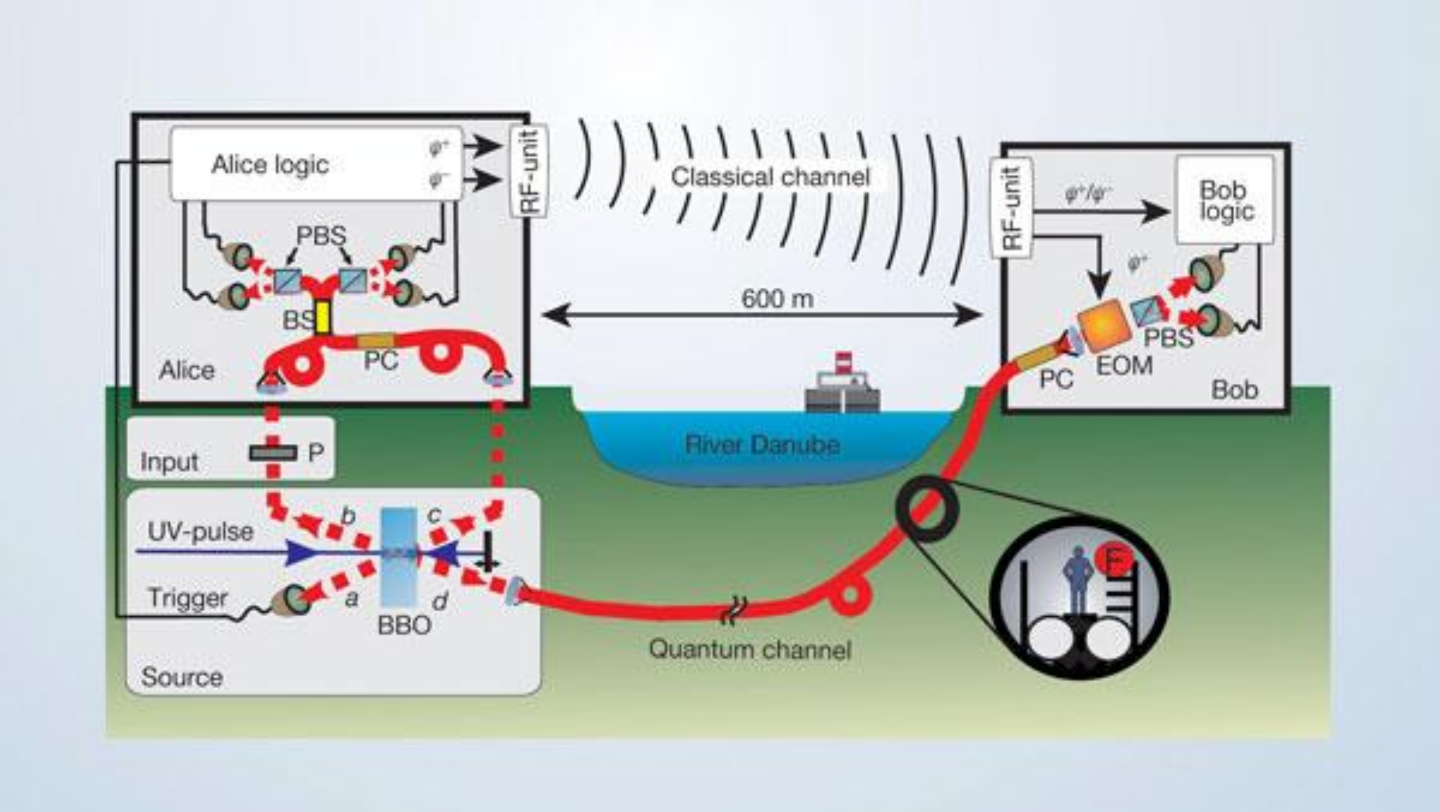
# … TRANSMISSION OF RANDOM DATA …

MEASUREMENT DATA must be sent to the distant receiving station by conventional means.This process is limited by the speed of light, making it impossible to teleport the person faster than the speed of light.

## … RECONSTRUCTION OF THE TRAVELER

RECEIVER RE-CREATES THE TRAVELER, exact down to the quantum state of every atom and molecule, by adjusting the counterpart matter’s state according to the random measurement data sent from the scanning station.

# Quantum teleportation across the Danube




R. Ursin *et al.* describe the high-fidelity teleportation of photons over a distance of 600 metes across the River Danube in Vienna.

***Nature* 430, 849 (2004)**

## Teleportation with real atoms:

1. **Deterministic quantum teleportation with atoms**

M. RIEBE et al., *Nature* **429**, 734 (17 June 2004)

With calcium ions

2. **Deterministic quantum teleportation of atomic qubits**

M. D. BARRETT et al., *Nature* **429**, 737(17 June 2004)

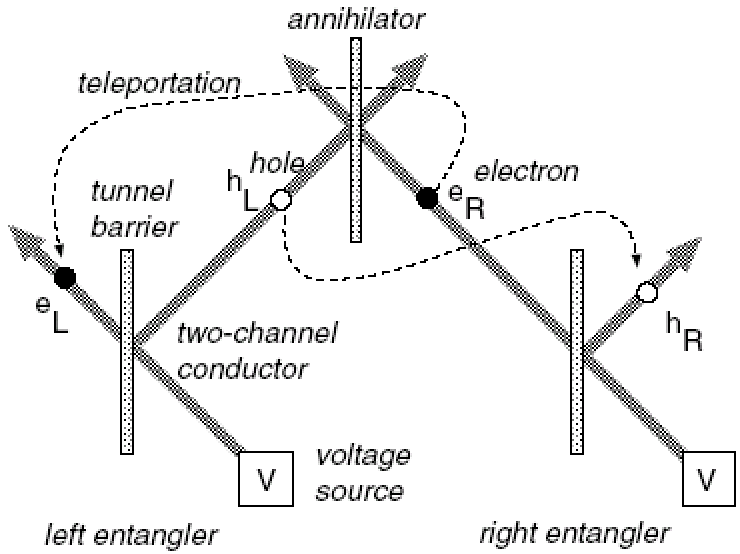
With atomic ($^{9}Be^{+}$) ions

# Proposal for teleportation in solid state system

Phys. Rev. Foucs, **6 February 2004**

**"Beam Up an Electron!"**

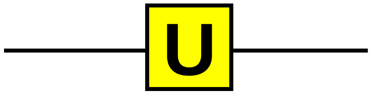
C. W. J. Beenakker and M. Kindermann, Phys. Rev. Lett. 92, 056801(2004)



# Local Unitary Operations

## NOTATION

Single-qubit
unitary transformation U :

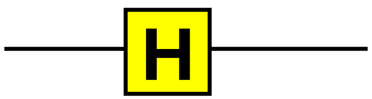



Qubit is denoted by horizontal line

## PATICULAR UNITARY OPERATIONS

Hadamard transform

$$\mathrm{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

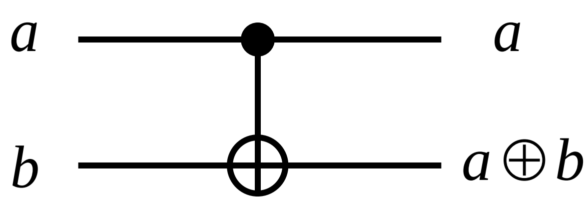



Unilateral Pauli rotations

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# Collective Unitary Operations

**controlled-NOT(XOR)**
transformation



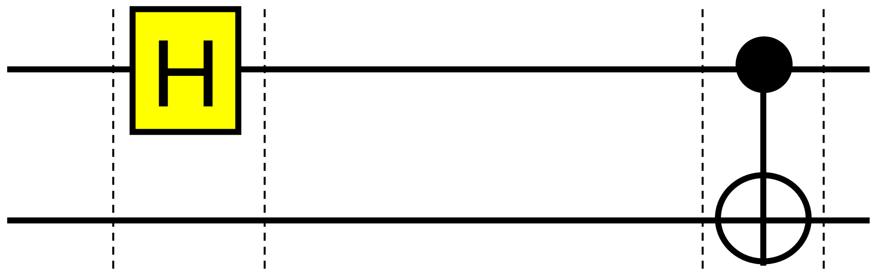

addition modulo 2

$$|0\rangle_C |0\rangle_T \xrightarrow{\text{CNOT}} |0\rangle_C |0\rangle_T$$

$$|1\rangle_C |0\rangle_T \xrightarrow{\text{CNOT}} |1\rangle_C |1\rangle_T$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

# Maximally Entanglement Generation



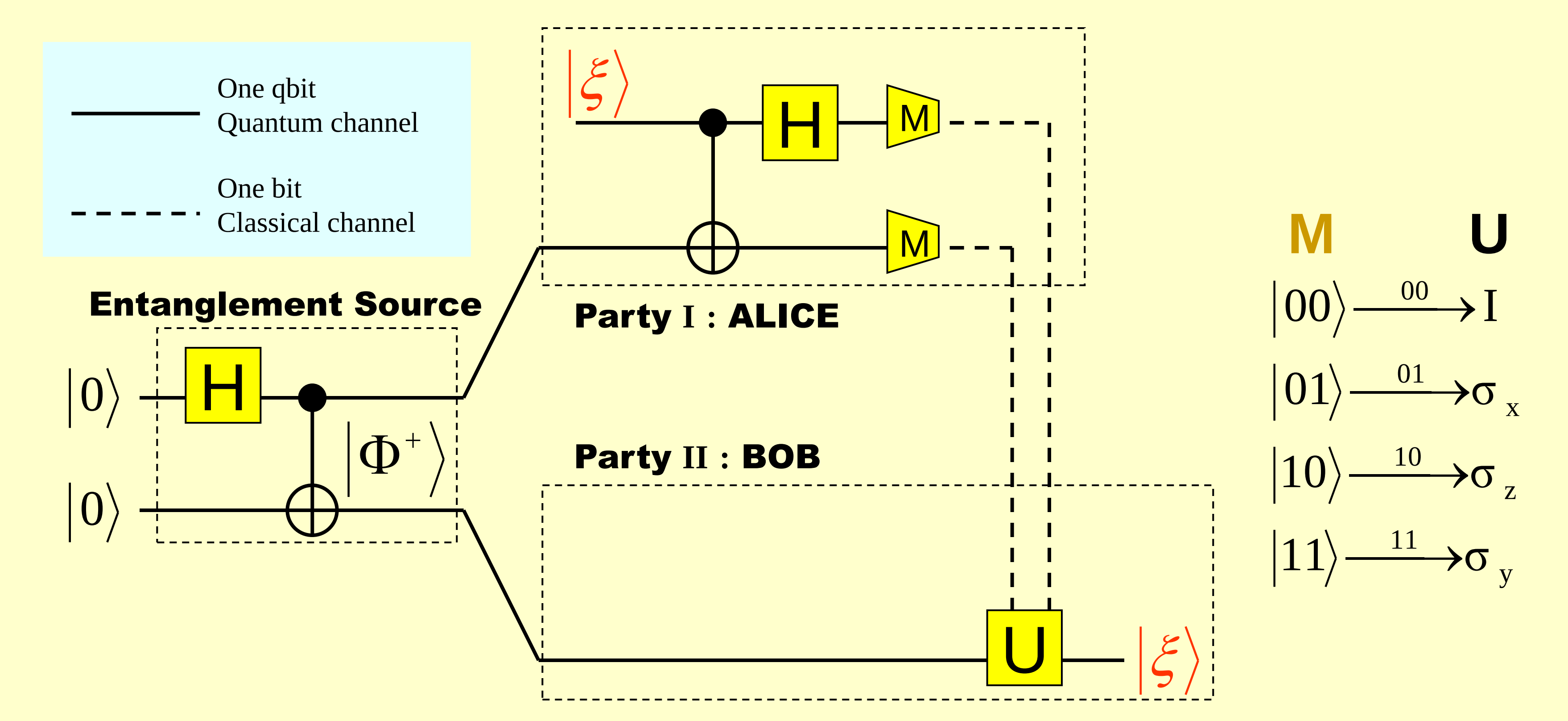

$$|00\rangle \to \quad \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle \to \quad \to |\Phi^+\rangle = \tfrac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|01\rangle \to \quad \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|1\rangle \to \quad \to |\Psi^+\rangle$$

$$|10\rangle \to \quad \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|0\rangle \to \quad \to |\Phi^-\rangle$$

$$|11\rangle \to \quad \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)|1\rangle \to \quad \to |\Psi^-\rangle$$

# Quantum Network for Teleportation




$$|\xi\rangle\otimes|\Phi^+\rangle = (\alpha|0\rangle+\beta|1\rangle)[\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)]$$

$$= \frac{1}{\sqrt{2}}(\alpha|000\rangle+\alpha|011\rangle+\beta|100\rangle+\beta|111\rangle)]$$

$$|00\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle+|\Phi^-\rangle)$$

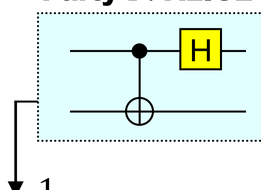
$$|01\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle+|\Psi^-\rangle)$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle-|\Psi^-\rangle)$$

$$|11\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle-|\Phi^-\rangle)$$

$$= \frac{1}{2}[|\Phi^+\rangle(\alpha|0\rangle+\beta|1\rangle)+|\Phi^-\rangle(\alpha|0\rangle-\beta|1\rangle) + |\Psi^+\rangle(\alpha|1\rangle+\beta|0\rangle)+|\Psi^-\rangle(\alpha|1\rangle-\beta|0\rangle)]$$

Party I : ALICE




$$= \frac{1}{2}[|00\rangle(\alpha|0\rangle+\beta|1\rangle)+|10\rangle(\alpha|0\rangle-\beta|1\rangle) + |01\rangle(\alpha|1\rangle+\beta|0\rangle)+|11\rangle(\alpha|1\rangle-\beta|0\rangle)]$$

**Teleportation of Nonclassical Wave Packets of Light**
N. Lee *et al*., **Science 332**, 330 (2011)

**Quantum Teleportation Between Distant Matter Qubits**
S. Olmschenk *et al*., **Science 323**, 486 (2009)

**Quantum teleportation between light and matter**
J. F. Sherson *et al*., **Nature 443**, 557 (2006)

# Cavity QED

- Spontaneous emission of single two-level atom

- Interaction between a two-level atom and the photon reservoir:

$$H' = \sum_{\vec{q}} D_{\vec{q}} b_{\vec{q}} \sigma_+ e^{i\vec{q}\cdot\vec{x}} + H.c.$$

$b_{\vec{q}}$ : *photon operator* $\quad \sigma_+$ :*creating operator of atom*

- In the interaction picture, the state vector :

$$\left|\Psi(t)\right\rangle = f_0(t)\left|+;0\right\rangle + \sum_{\vec{q}} f_{\vec{q}}(t)\left|-;1_{\vec{q}}\right\rangle$$

, where

$\left|+;0\right\rangle$ : an atom initially in the excited state

$\left|-;1_{\vec{q}}\right\rangle$ : a photon of **q** in the radiation field

# Results :

$$f_0(t) = e^{i\Delta\omega t - \gamma t},$$

where ♑ is the decay rate

👎◆ represents the *Lamb Shift*

The radiation intensity distribution :

$$\left| f_{\vec{q}}(t=\infty) \right|^2 = \frac{\left| D_{\vec{q}} \right|^2}{(\omega_0 - c|\vec{q}| + \Delta\omega)^2 + \gamma^2}$$

, where

$$\gamma = \pi \sum_{\vec{q}} \left| D_{\vec{q}} \right|^2 \delta(\omega_0 - c|\vec{q}|),$$

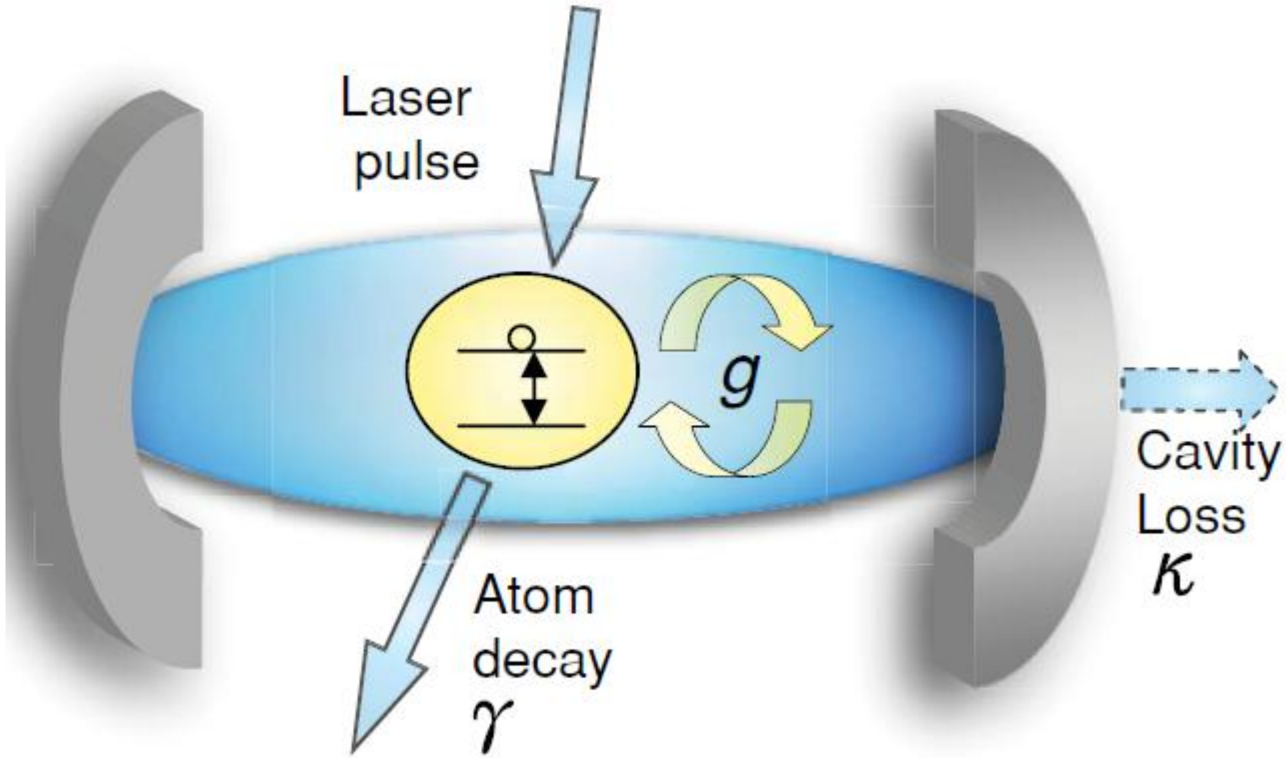
$$\Delta\omega = \sum_{\vec{q}} \mathrm{P} \frac{\left| D_{\vec{q}} \right|^2}{\omega_0 - c|\vec{q}|}$$

$\omega_0$ is the energy spacing

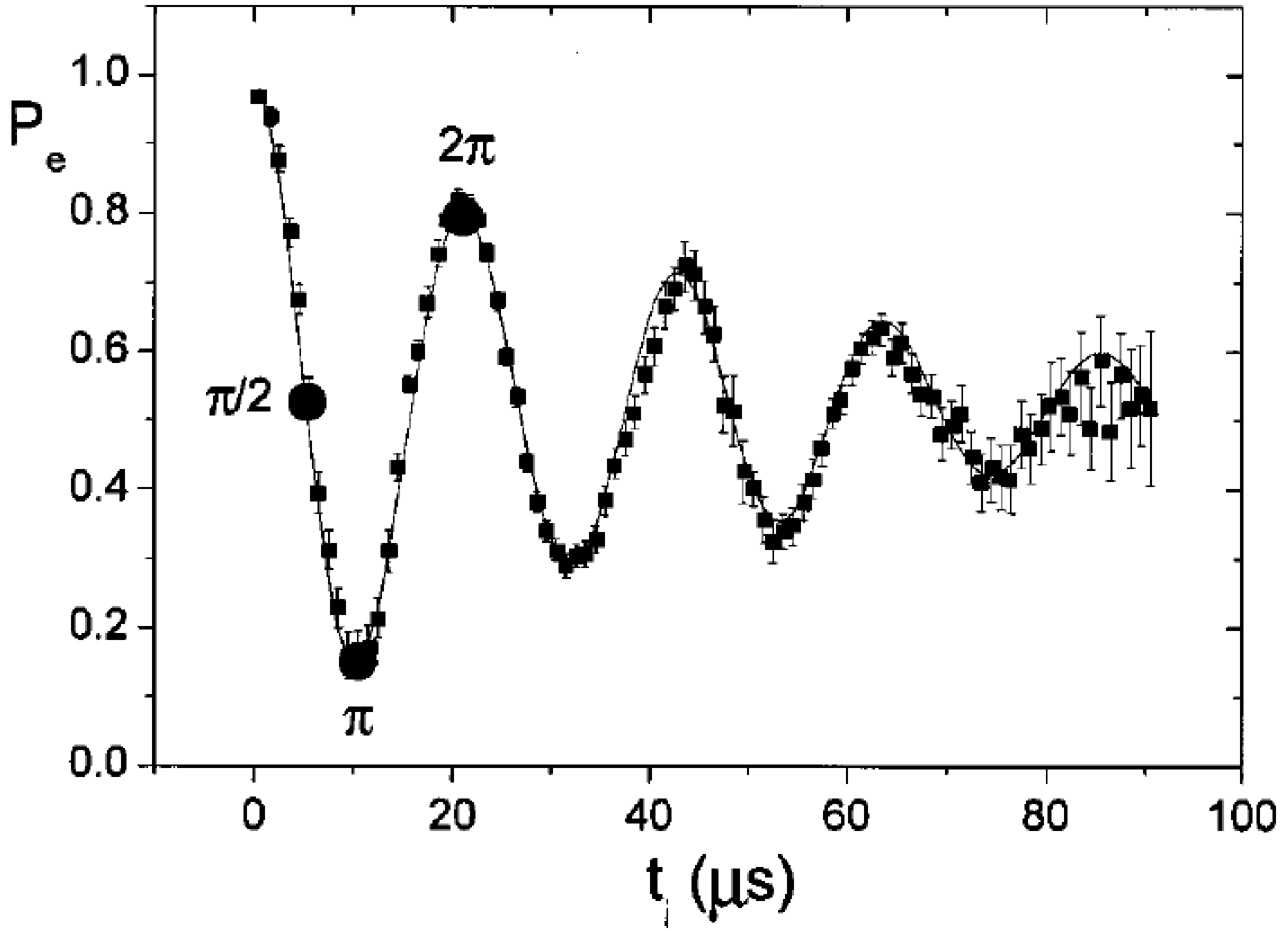
# Two-level atom inside a cavity

The interaction between the atom and single-mode cavity:

$$H' = \hbar g(\sigma_+ b^- + \sigma_- b^+) \qquad |\Psi(t)\rangle = f_+(t)|+;0\rangle + f_-(t)|-;1\rangle$$

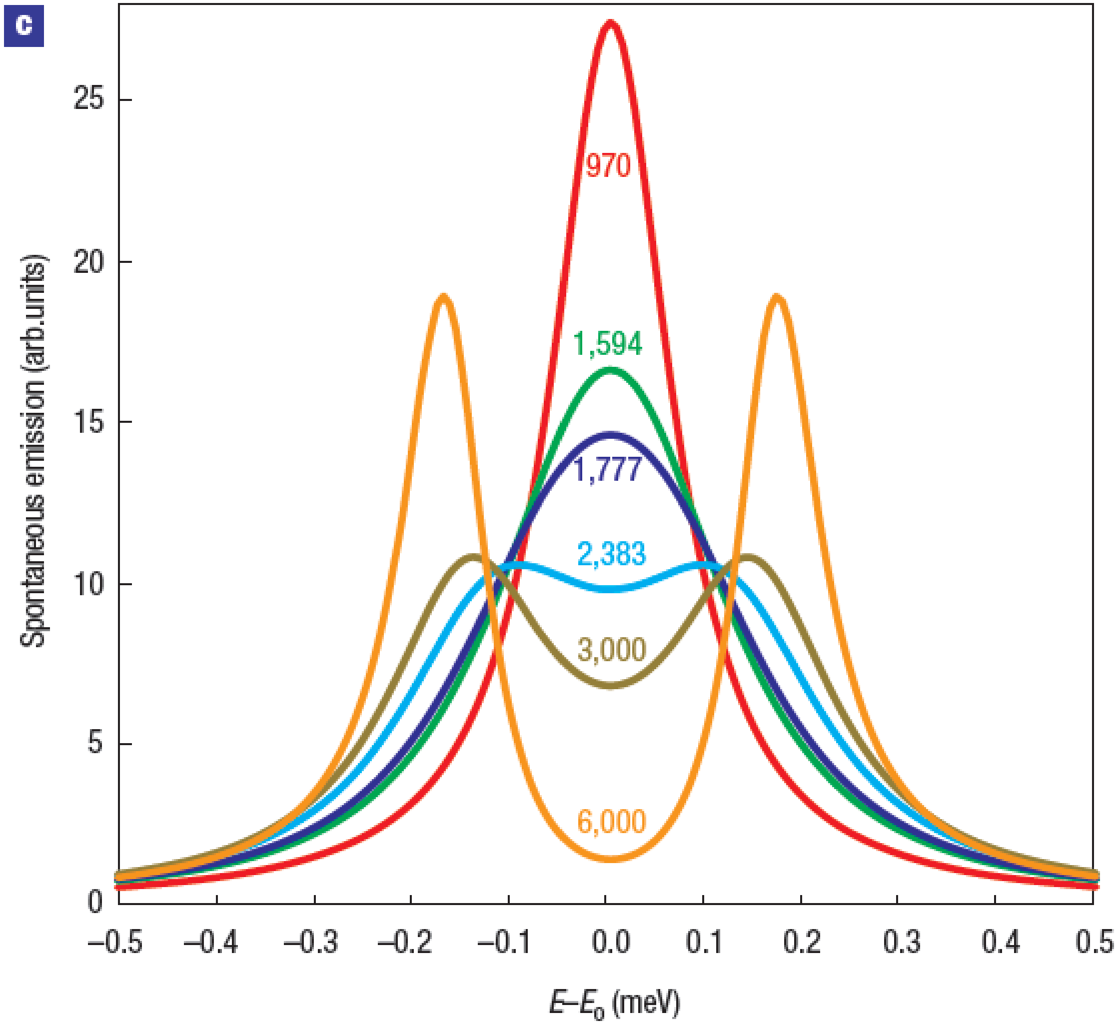


# Vacuum Rabi oscillations

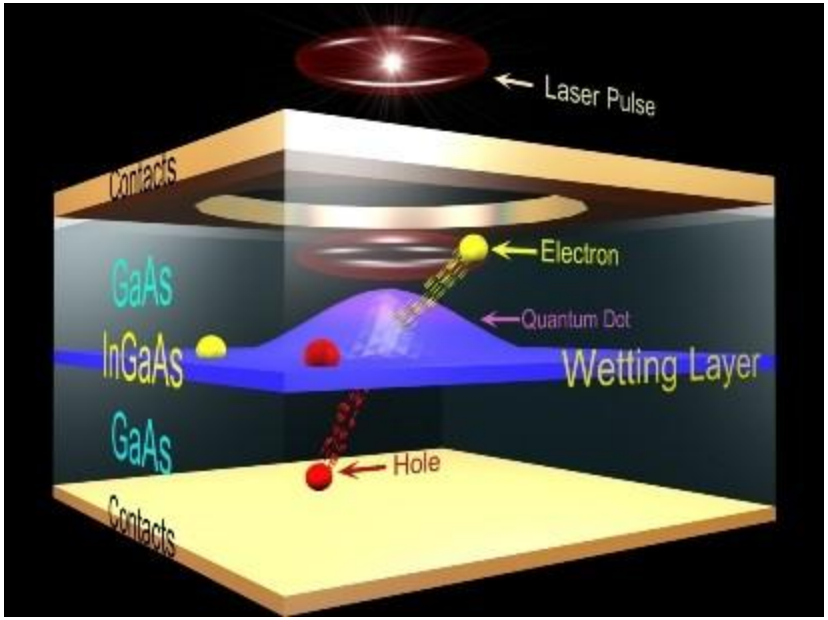


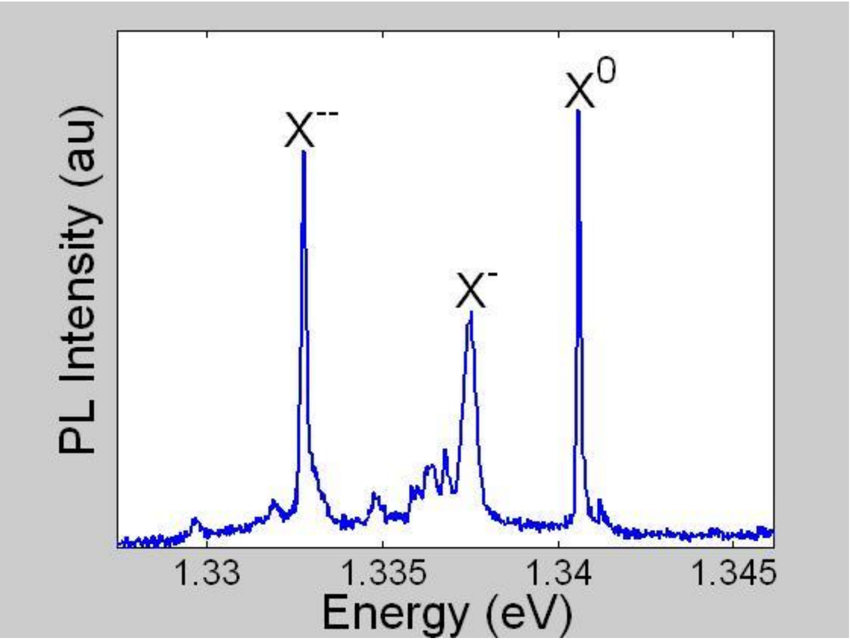

J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73,** 565 (2001).

# Vacuum Rabi splitting

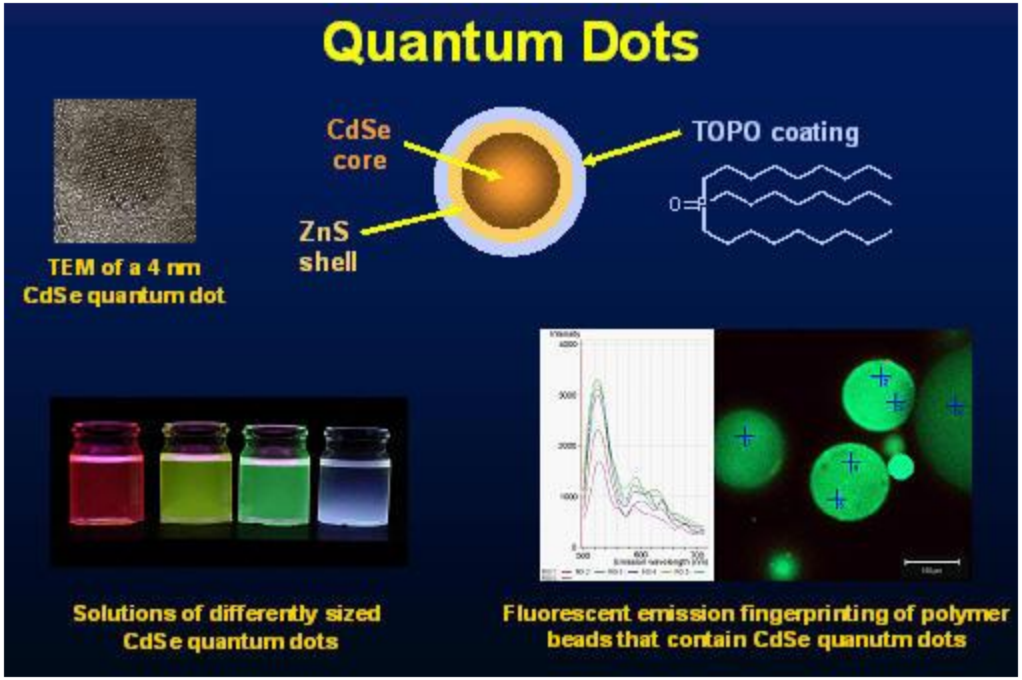


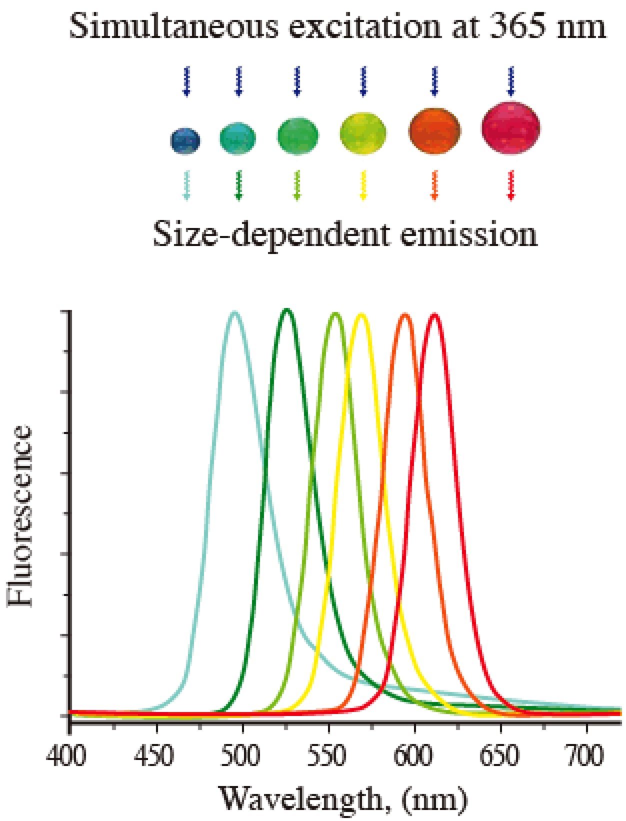
## Self-Assembled QDs



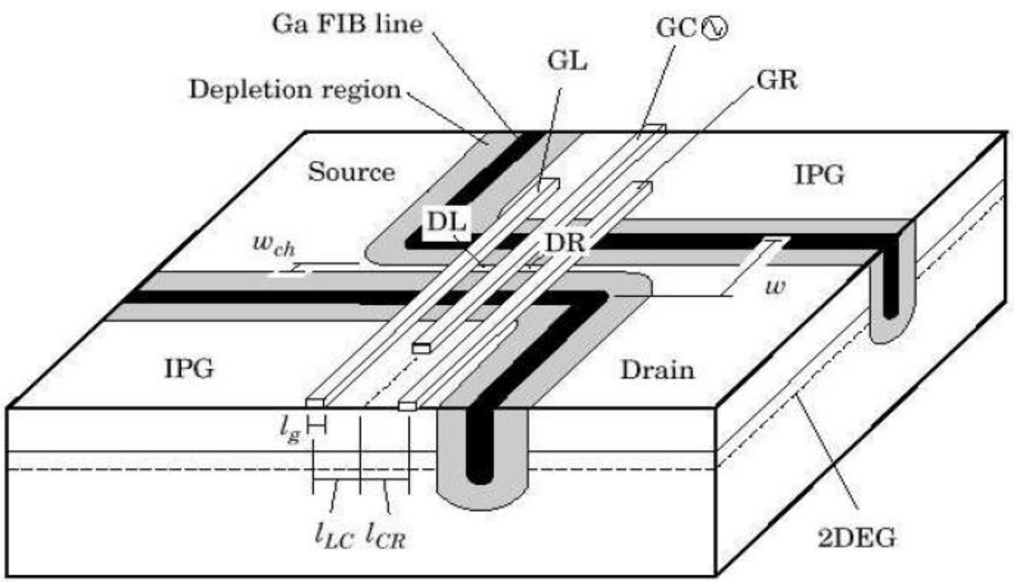



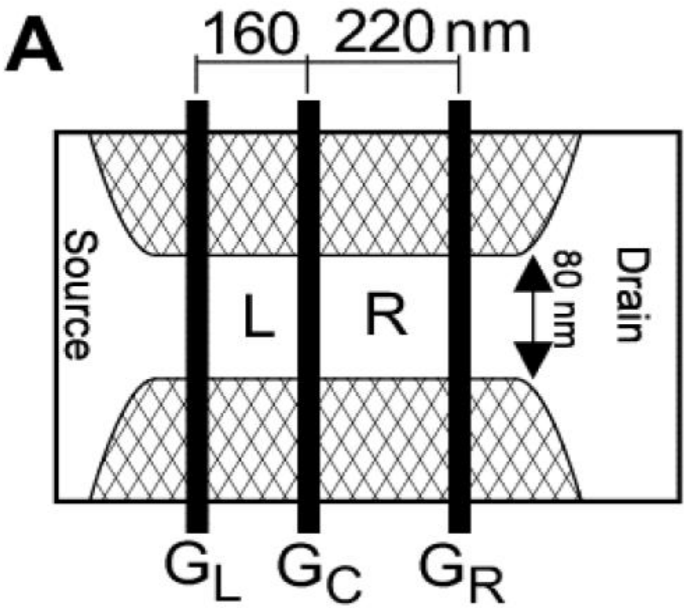
# Colloidal QDs

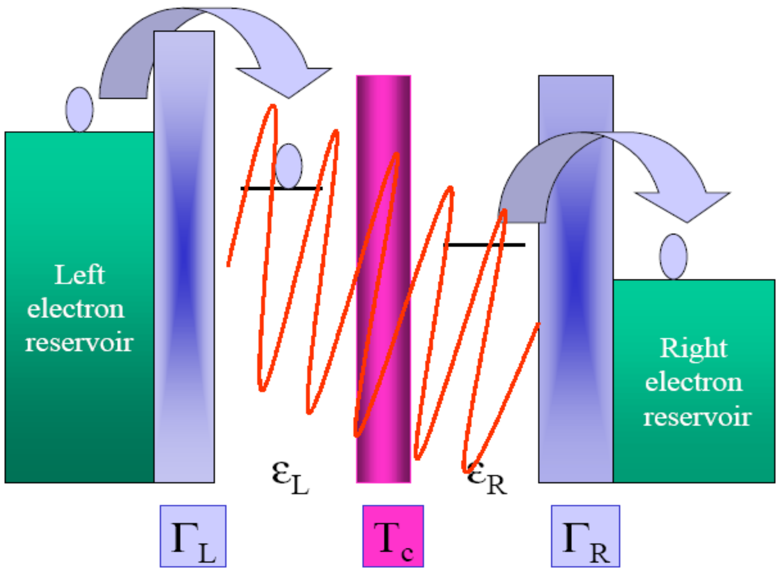


# Gate-confined Double Quantum Dots

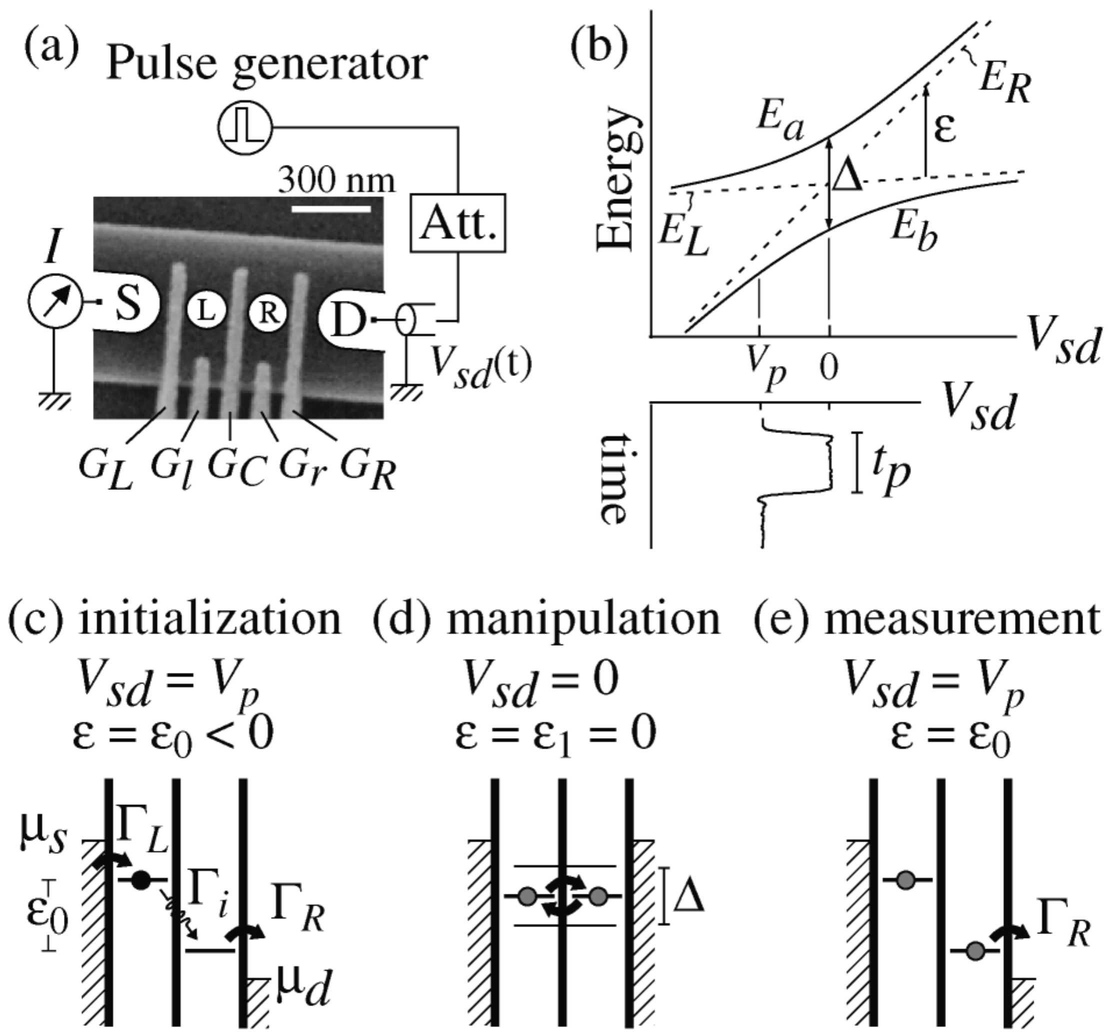


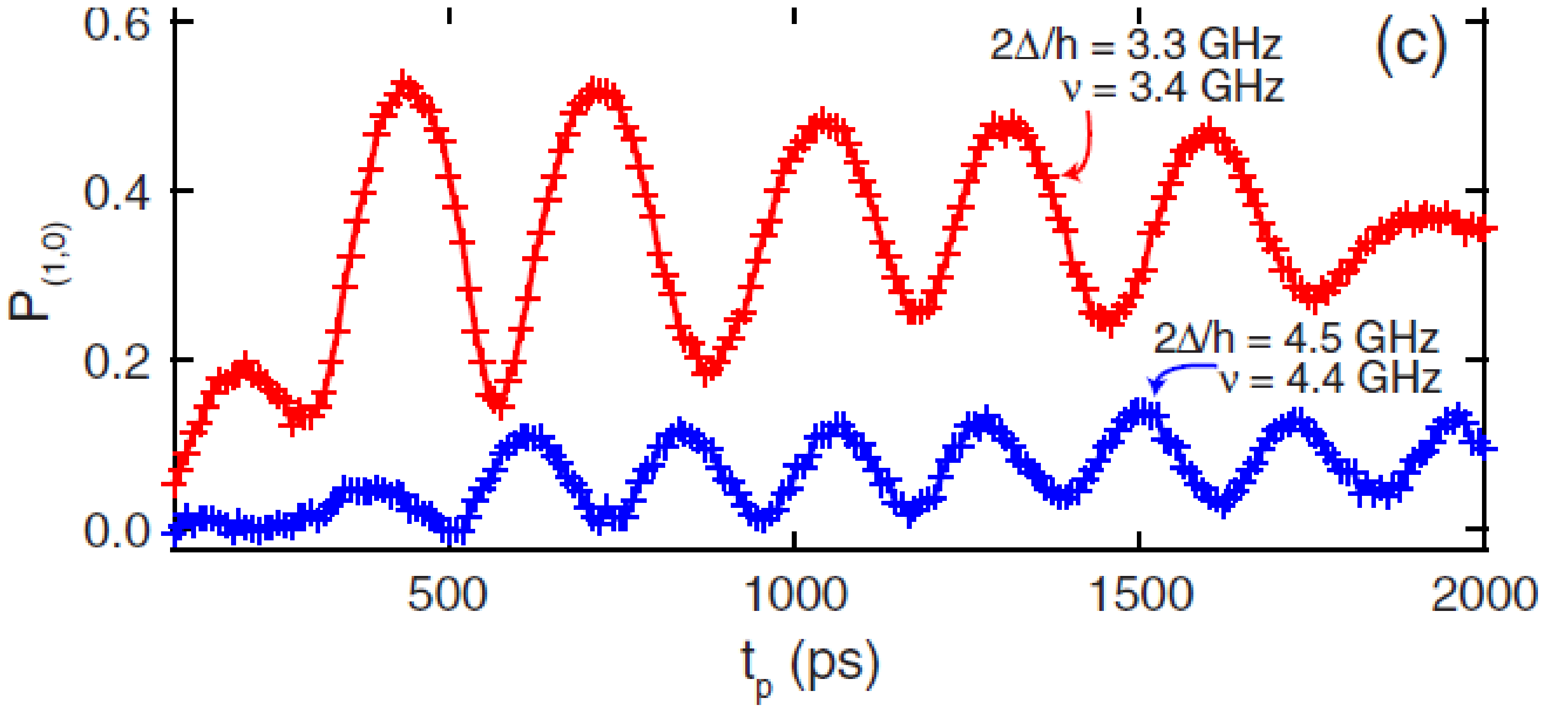



# Quantum Coherence in Double Quantum Dots

$$H = \frac{1}{2}\epsilon\sigma_z + \Delta\sigma_x$$

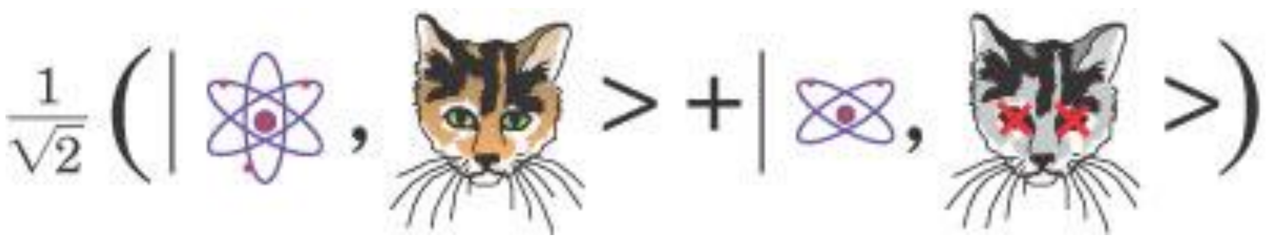





K. D. Petersson, J. R. Petta, H. Lu, and A. C. Gossard, PRL 105, 246804 (2010)

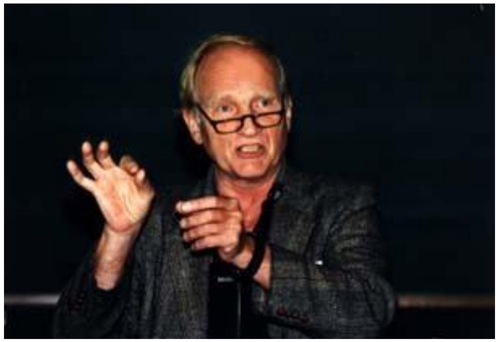
# Question:

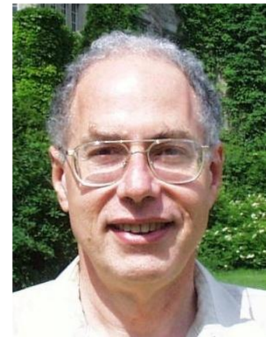
# Are they truly quantum?

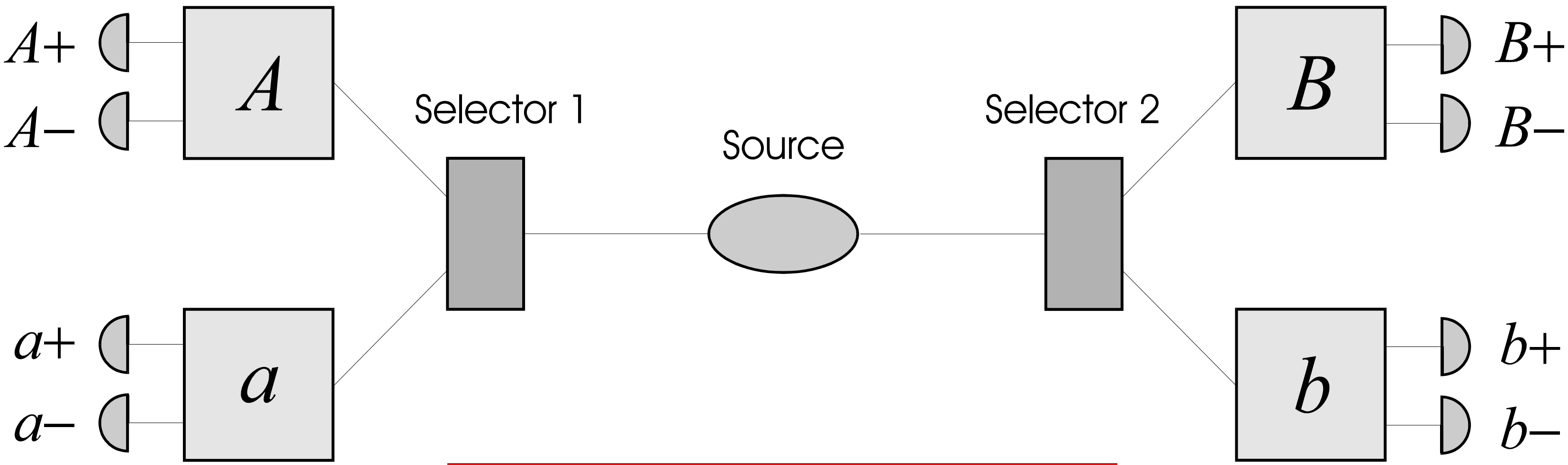


# Quantum vs Classical

## Bell's Inequality: Locality and Realism






$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

# The Bell-CHSH inequality

$$A, a, B, b \in \{-1, 1\}$$

$$(A-a, A+a) \in \{(0, \pm 2), (\pm 2, 0)\}$$

$$(A-a)B-(A+a)b \in \{-2, 2\}$$

$$-2 \leq \langle AB - Ab - aB - ab \rangle \leq 2$$

$$\left| \langle AB \rangle - \langle Ab \rangle - \langle aB \rangle - \langle ab \rangle \right| \leq 2$$

# Predictions of QM for the *singlet* state

$$\left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}\left(\left|01\right\rangle - \left|10\right\rangle\right)$$

$$\left\langle AB\right\rangle = \left\langle \psi^- \right| \hat{A}\otimes\hat{B} \left|\psi^-\right\rangle$$
$$= -\cos\theta_{AB}$$

# QM violates the Bell-CHSH inequality

$$F_{\mathrm{QM}} = \left| \langle AB \rangle - \langle Ab \rangle - \langle aB \rangle - \langle ab \rangle \right|$$

$$= \left| -\cos\theta_{AB} + \cos\theta_{Ab} + \cos\theta_{aB} + \cos\theta_{ab} \right|$$

$$\hat{A} = \sigma_x$$

$$\hat{a} = \sigma_y$$

$$\hat{B} = (\sigma_y - \sigma_x)/\sqrt{2}$$

$$\hat{b} = (\sigma_y + \sigma_x)/\sqrt{2}$$

$$F_{\mathrm{QM}} = 2\sqrt{2} > 2!!!$$

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

## Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
*Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France*
(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

VOLUME 49, NUMBER 2 PHYSICAL REVIEW LETTERS 12 JULY 1982

## Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger
*Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique, Université Paris-Sud, F-91406 Orsay, France*
(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

VOLUME 49, NUMBER 25 PHYSICAL REVIEW LETTERS 20 DECEMBER 1982

## Experimental Test of Bell's Inequalities Using Time-Varying Analyzers

Alain Aspect, Jean Dalibard,[a] and Gérard Roger
*Institut d'Optique Théorique et Appliquée, F-91406 Orsay Cédex, France*
(Received 27 September 1982)

Correlations of linear polarizations of pairs of photons have been measured with time-varying analyzers. The analyzer in each leg of the apparatus is an acousto-optical switch followed by two linear polarizers. The switches operate at incommensurate frequencies near 50 MHz. Each analyzer amounts to a polarizer which jumps between two orientations in a time short compared with the photon transit time. The results are in good agreement with quantum mechanical predictions but violate Bell's inequalities by 5 standard deviations.

# Leggett-Garg Inequality (Bell's inequality in time)

Realism and non-invasive measurement

**Quantum mechanics versus macroscopic realism:**
**Is the flux there when nobody looks?**

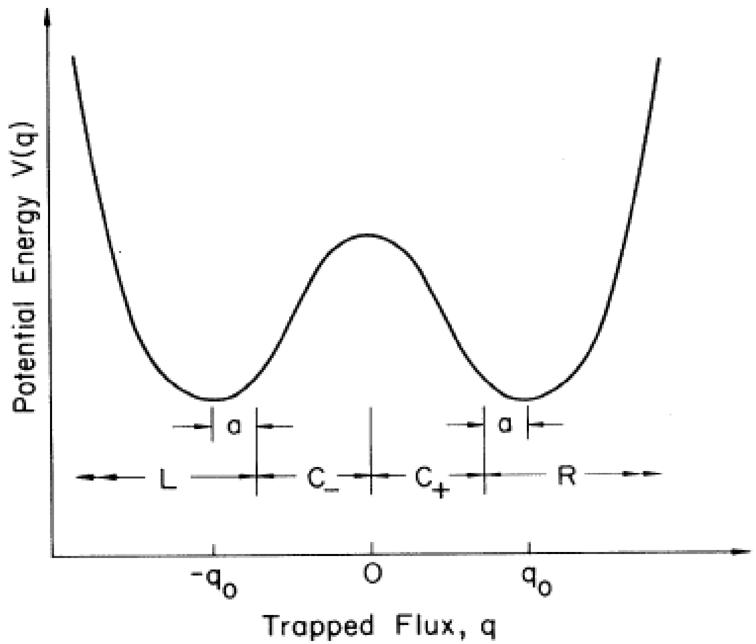



FIG. 1. The potential $V(q)$ for the trapped flux $q$. The various notations are explained in the text.

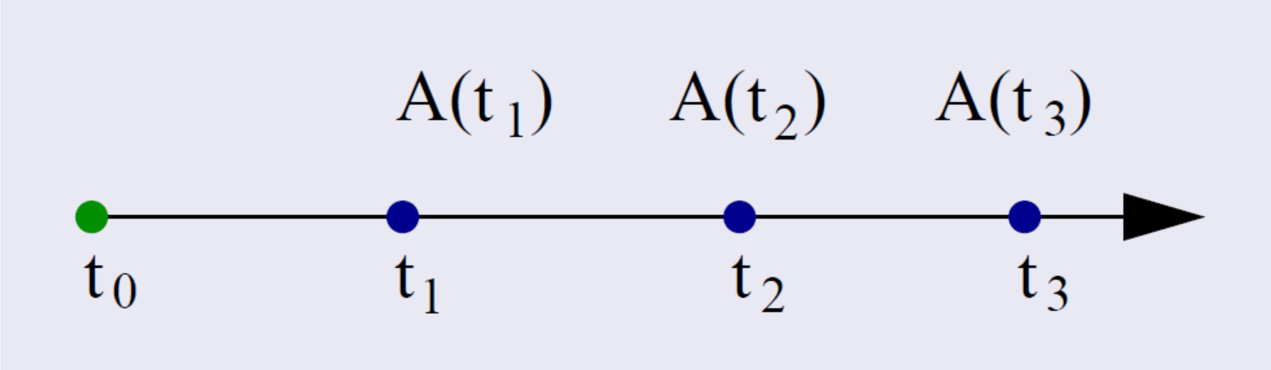
Leggett and Garg, Phys. Rev. Lett. 54, 857–860 (1985)

Given an observable $A(t)$, bound above and below by $|A(t)| \leq 1$, the assumption of:

- macroscopic realism, and
- non-invasive measurement,

implies the inequality,

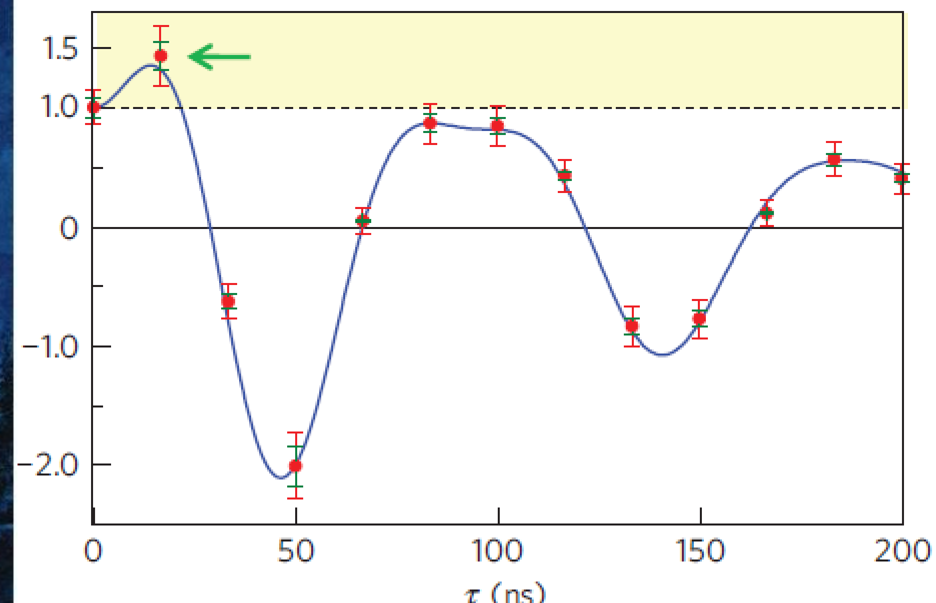
$$|\langle A(t_2)A(t_1)\rangle + \langle A(t_3)A(t_2)\rangle - \langle A(t_3)A(t_1)\rangle| \leq 1$$




$\Rightarrow$ This can be violated by QM systems!

# No moon there

An experiment reveals that micrometre-sized superconducting circuits follow the laws of quantum mechanics, and thus defy common experience of how macroscopic objects should behave.

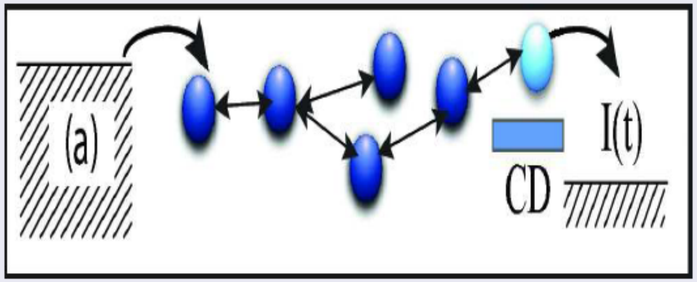



Palacios-Laloy, A. *et al.*
*Nature Phys.* ***6, 442–447 (2010).***

# Distinguishing Quantum and Classical Transport through Nanostructures

## Transport Charge Inequality:

**Open, nonequillibrium system**




cf. Ruskov '06, weak measurement of closed system

**Charge detection**

measure charge, $Q$

- non-invasive
- system in state $n$: $Q_n \geq 0$
- max value: $Q_N = Q_{\max}$

e.g. QPC

**Charge inequality**

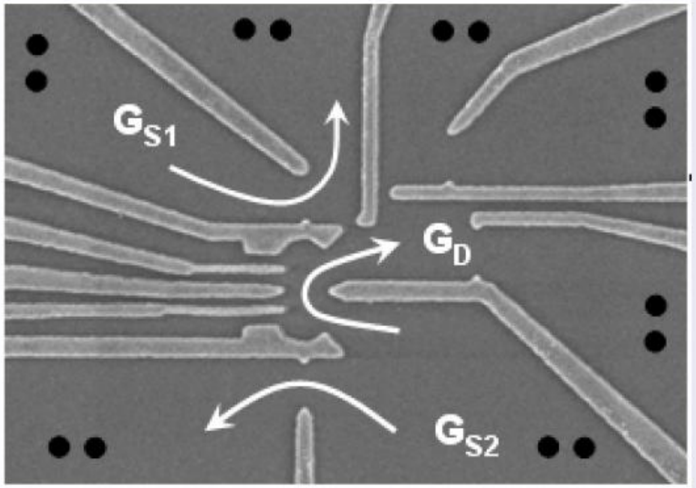
Stationary LG with $A = 2Q/Q_{\max} - 1$; $t_2 - t_1 = t_3 - t_2 = t$:

$$|2\langle Q(t)Q\rangle - \langle Q(2t)Q\rangle| \leq Q_{\max}\langle Q\rangle$$

N. Lambert, C. Emary, Y. N. Chen, and F. Nori, **Phys. Rev. Lett. 105**, 176801 (2010)

# Double Quantum Dot

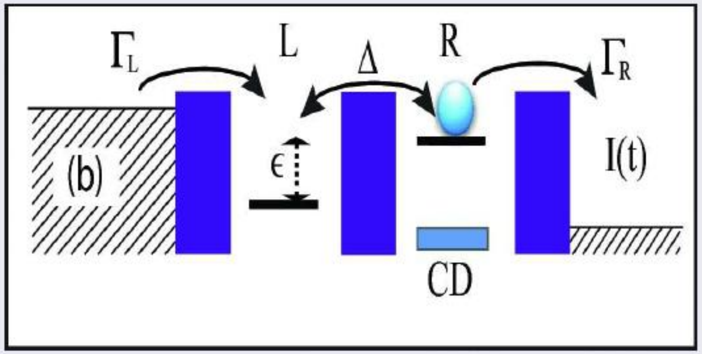
## Experiment




Manipulation of a single charge in a double quantum dot

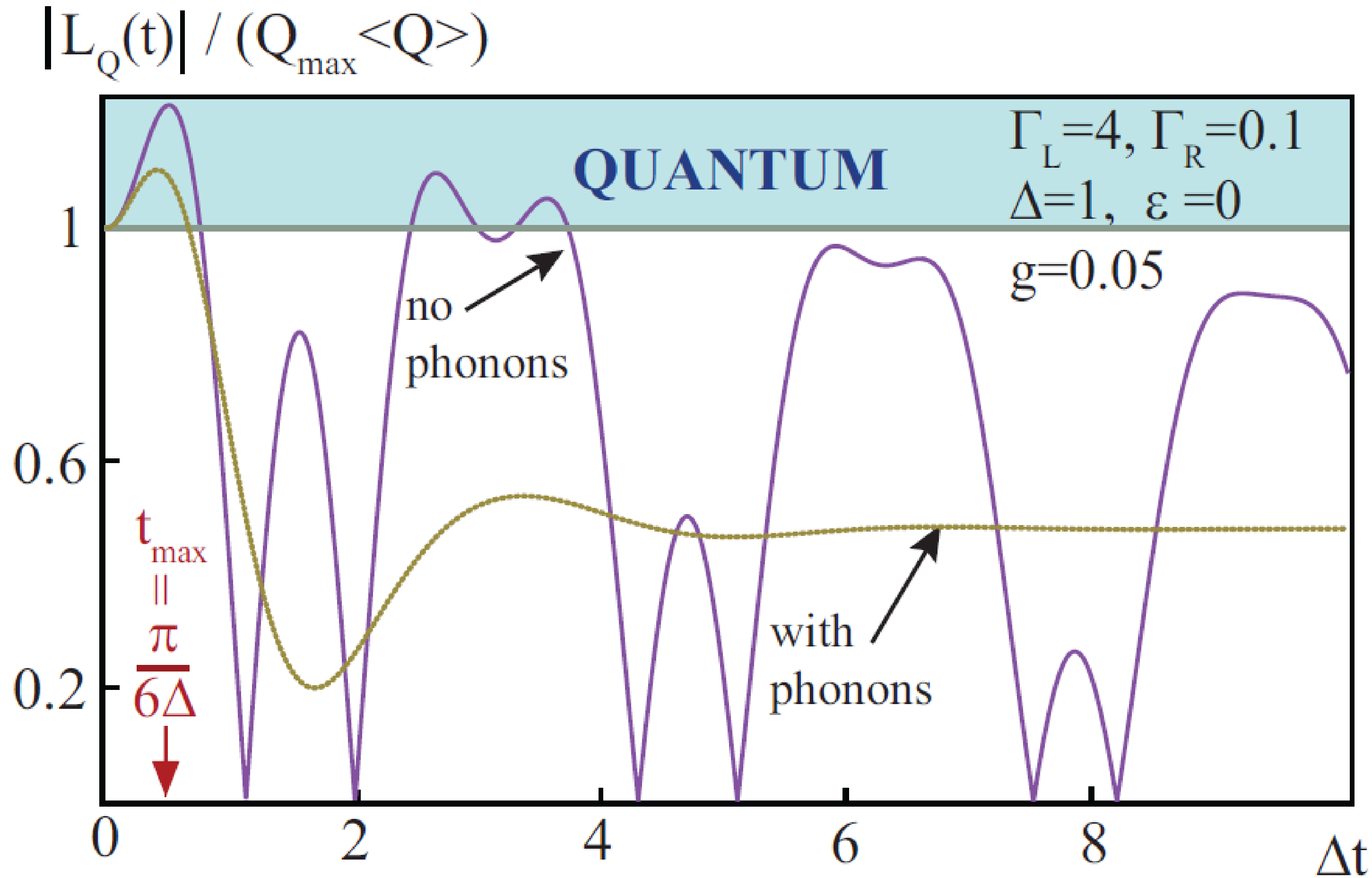
Petta et al. PRL '04

## Model




- strong Coulomb blockade
- 0-1 excess electrons
- large bias limit

# Violation of charge inequality for DQD




$$|L_Q(t)| \equiv |2\langle Q(t)Q\rangle - \langle Q(2t)Q\rangle| \leq Q_{\max}\langle Q\rangle$$

# DQD current inequality

## Current measurements

Current operator unbounded

$\Rightarrow$ can not use LG ineq ☹

## Master Equation

mean current is bounded:

$$\langle I^R \rangle_{\max}(t) = e\Gamma_R \text{ for DQD}$$

## Quantum Jumps

However, quantum jumps are 'invasive':

$$\langle I^R(\tau) I^R \rangle = \langle\langle \mathcal{J}^R \Omega(\tau) \mathcal{J}^R \rangle\rangle$$

$\mathcal{J}^R$: jump operator transfers population from dot $R$ to lead

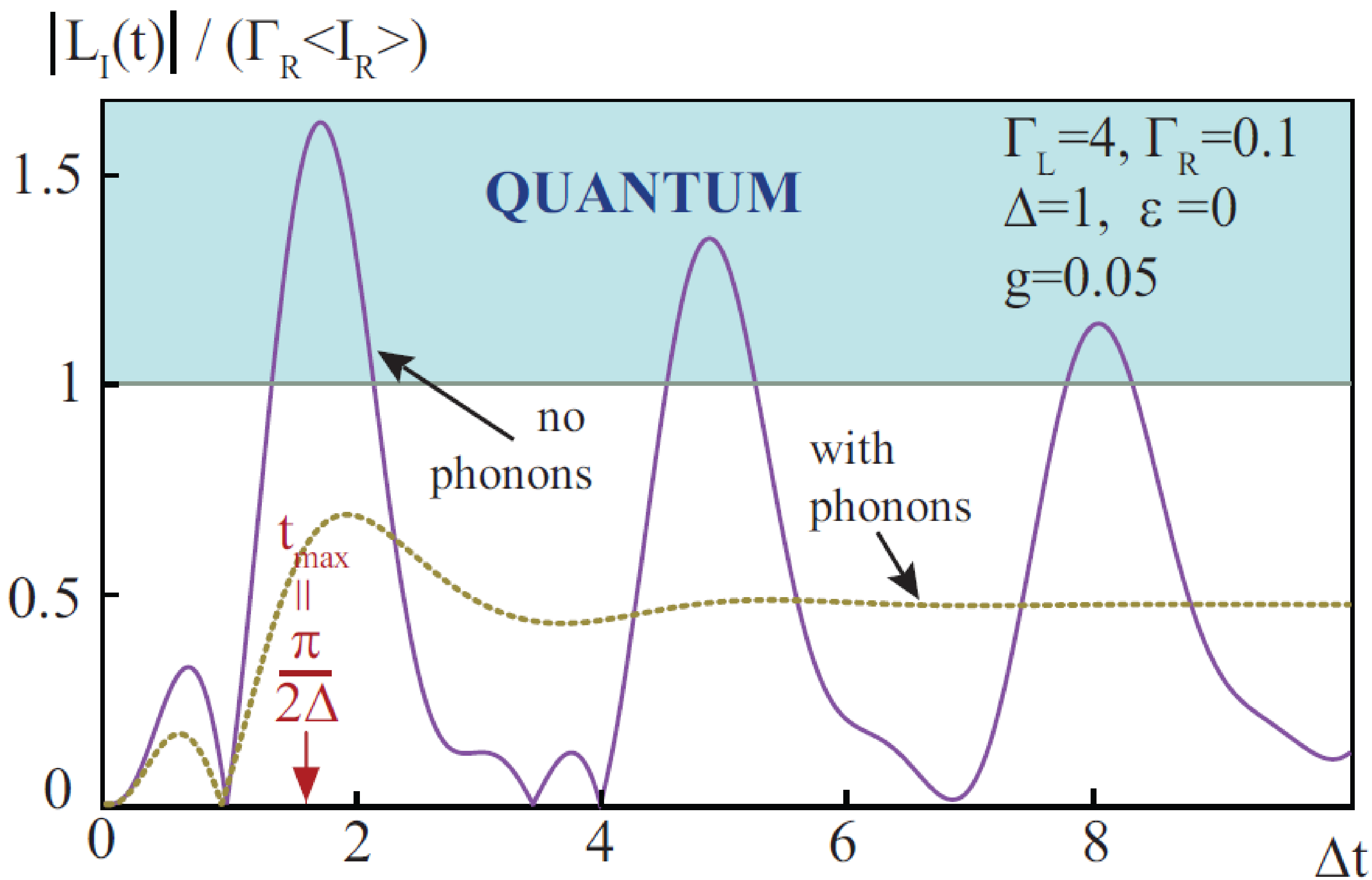
## Current inequality

Assume

- Macroscopic realsim
- Markovian ME
- DQD geometry

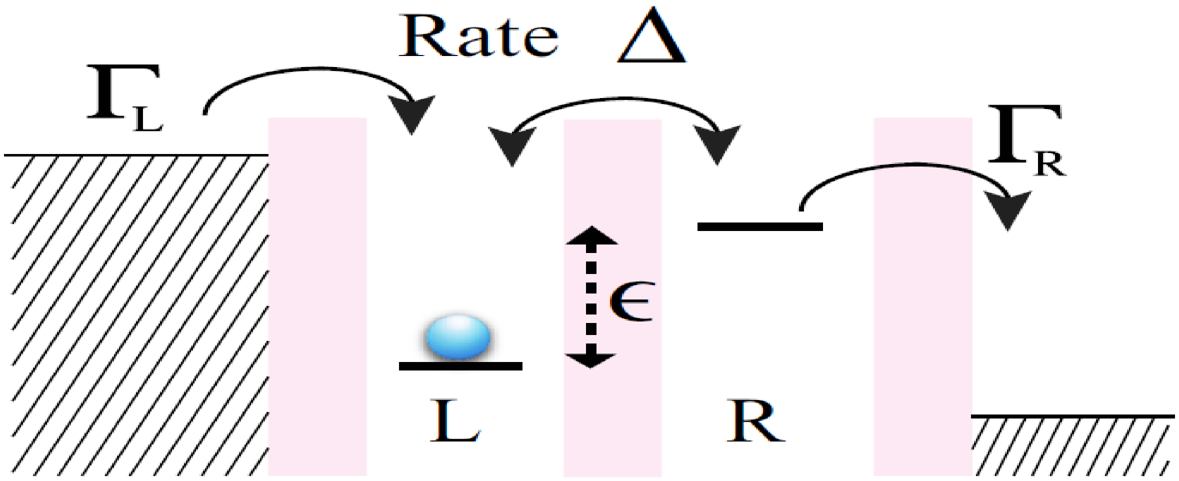
$$\boxed{|2\langle I(t) I \rangle - \langle I(2t) I \rangle| \leq \Gamma_R \langle I \rangle}$$

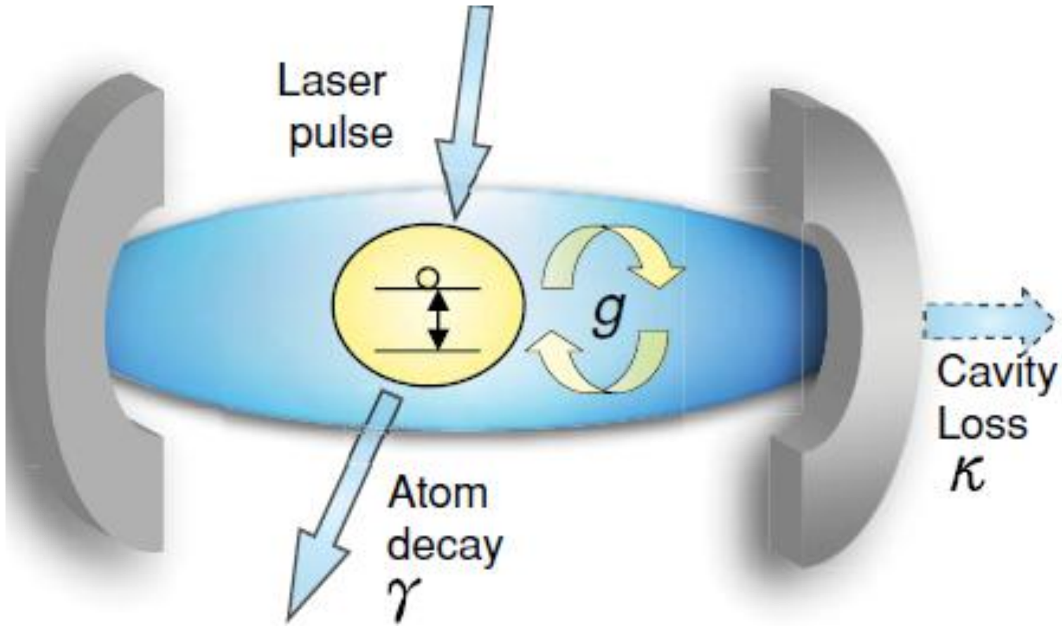
# Violation of DQD current inequality



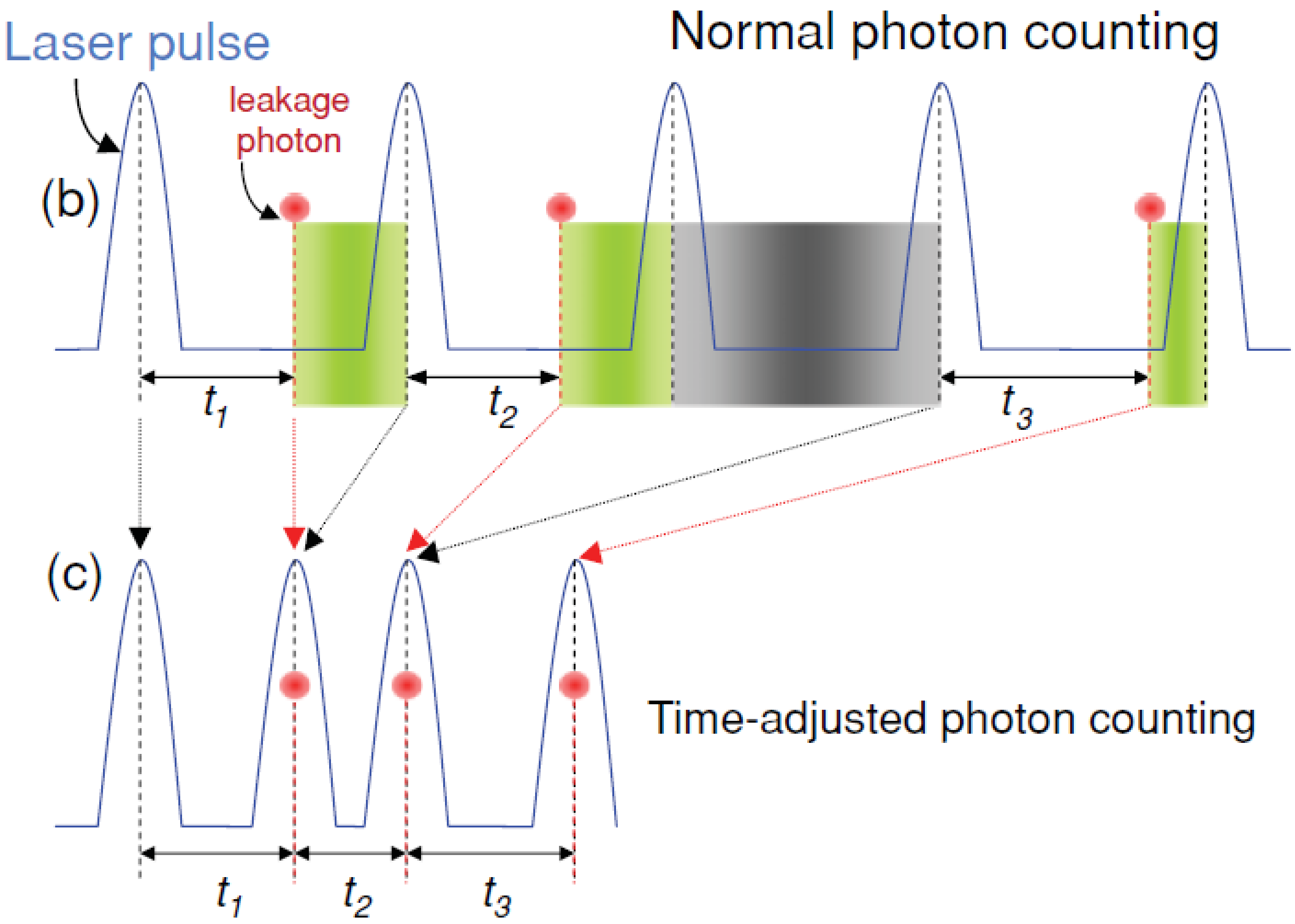

$$|L_I(t)| \equiv |2\langle I(t)I\rangle - \langle I(2t)I\rangle| \leq \Gamma_R\langle I\rangle$$

# Comparison between DQD and CQED systems



# Time-adjusted photon counting

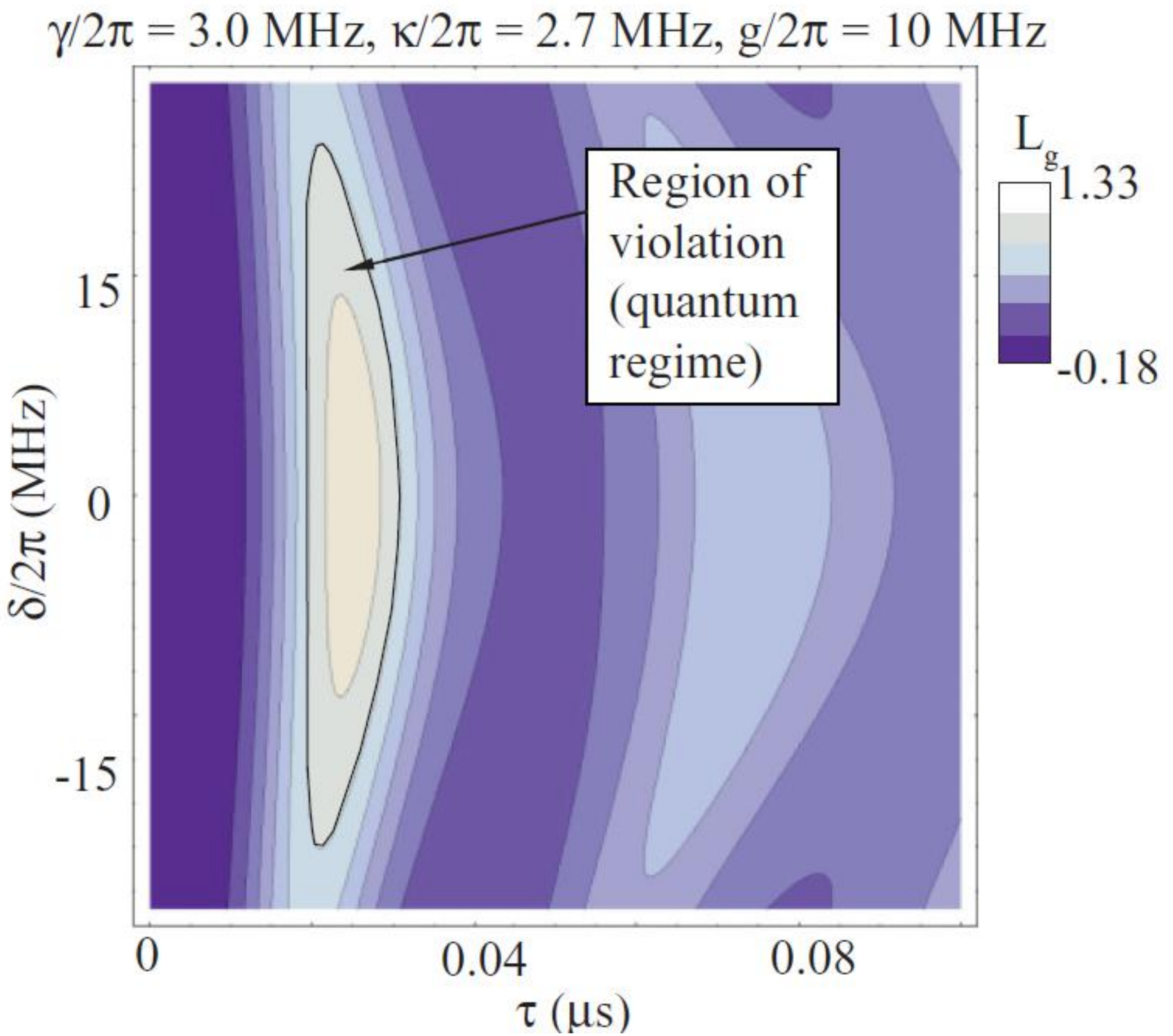


# Comparison between the properties of the cavity-QED system a double quantum dot

| Double quantum dot | Cavity QED |
|---|---|
| Electrons | Photons |
| $\|R\rangle$ = electron in the right dot | $\|g,1\rangle = \|$ground state atom, 1 photon$\rangle$ |
| $\|L\rangle$ =electron in the left dot | $\|e,0\rangle = \|$excited atom, 0 photons$\rangle$ |
| $E_L - E_R$ | $\delta/2 = (\omega - \mu)/2$ |
| Tunneling amplitude $T$ | Atom-Photon coupling $g$ |
| Tunneling rate $\Gamma_L \to \infty$ | Laser pulses with time-adjusted shift |
| Tunneling rate $\Gamma_R$ | Cavity loss rate $\kappa$ |

N. Lambert, Y. N. Chen*, and F. Nori, **Phys. Rev. A 82**, 063840 (2010)

# Violation of the extended LG inequality for typical parameters in single-photon cavity-QED experiments



What else?

Cavity QED without Cavity

# Dicke's superradiance for two atoms

The interaction : $H' = \sum_{j=1,2} \sum_{\vec{q}} D_{\vec{q}} b_{\vec{q}} c_j^{+} e^{i\vec{q}\cdot\vec{x}_j} + H.c.$

$\vec{x}_j$ : position of the $j$ th atom

$c_j^{+}$ : raising operator of the $j$ th atom

One can define the so-called Dicke states :

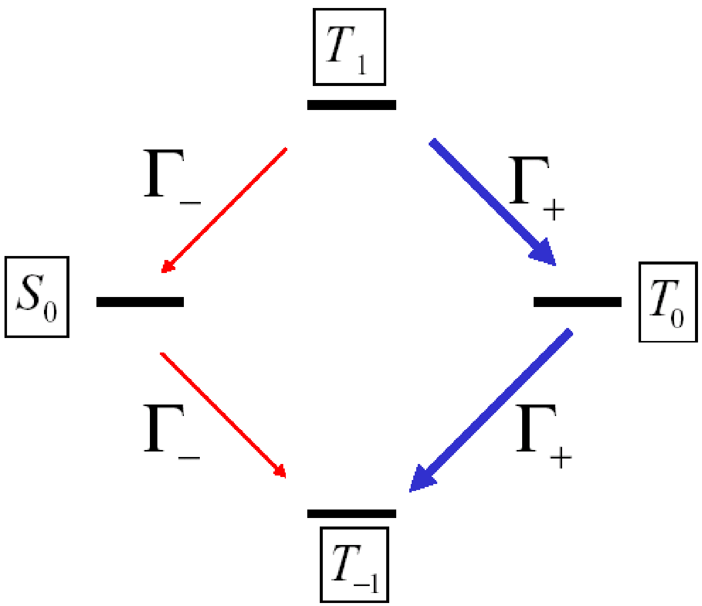
$$|T_1\rangle = |++\rangle$$

$$|T_0\rangle = \frac{1}{\sqrt{2}}|+-\rangle + \frac{1}{\sqrt{2}}|-+\rangle$$

$$|S_0\rangle = \frac{1}{\sqrt{2}}|+-\rangle - \frac{1}{\sqrt{2}}|-+\rangle$$

$$|T_{-1}\rangle = |--\rangle$$

# Decay scheme for two-atom system




Superradiant channel $\Gamma_+$ :

$$\Gamma_+ = 2\pi \sum_{\vec{q}} \frac{\left| D_{\vec{q}} e^{i\vec{q}\cdot\vec{x}_1} + D_{\vec{q}} e^{i\vec{q}\cdot\vec{x}_2} \right|^2}{2} \delta(\omega_0 - c|\vec{q}|)$$

Sub-radiant channel $\Gamma_-$ :

$$\Gamma_- = 2\pi \sum_{\vec{q}} \frac{\left| D_{\vec{q}} e^{i\vec{q}\cdot\vec{x}_1} - D_{\vec{q}} e^{i\vec{q}\cdot\vec{x}_2} \right|^2}{2} \delta(\omega_0 - c|\vec{q}|)$$

Limiting case : $d = |\vec{x}_1 - \vec{x}_2|$ << wavelength of the photon ●

⇓ $\Gamma_+ = 2\gamma_0$, $\Gamma_- = 0$

# Measurements of superradiance

Experiment in real atoms: $6\,^2P_{1/2}$ to $6\,^2S_{1/2}$ transition in $Ba_{138}{}^+$

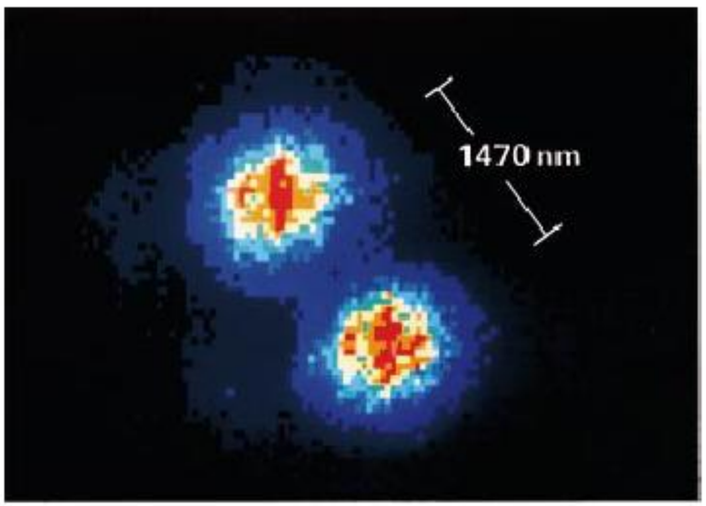



FIG. 4. (color) Diffraction-limited image of a two-ion crystal with $R = 1470$ nm. This determines the orientation of the interatomic vector $\vec{R}$ enabling a no-free-parameter fit.

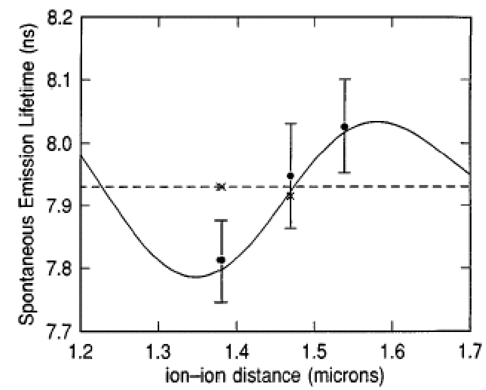
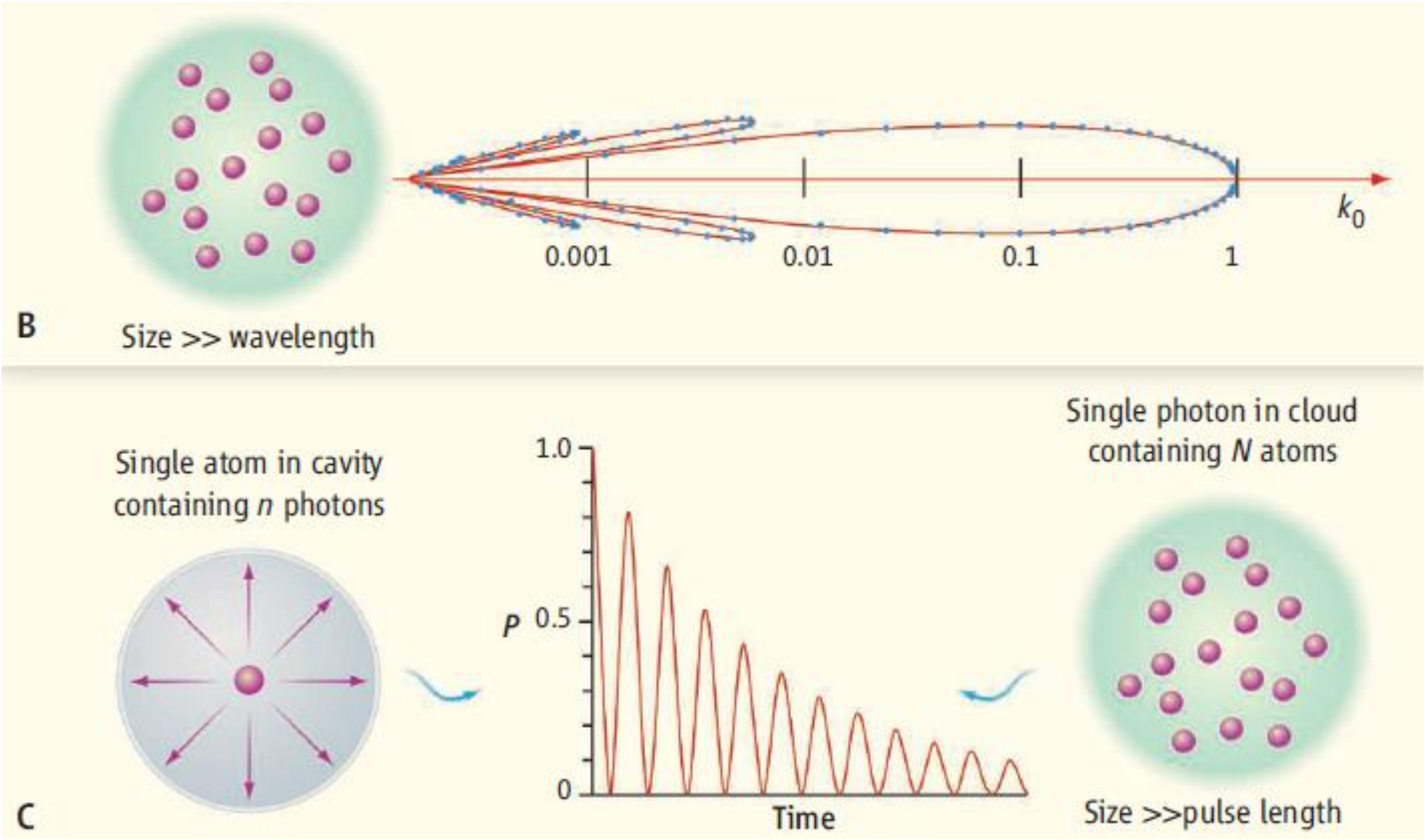



FIG. 6. Comparison of theory to experimental points at 1380, 1470, and 1540 nm (see text). The ion-ion distance is independently known by measuring the secular oscillation frequency of one ion. The lifetime is calibrated by comparison to $7.930 \pm 0.03$ ns measured for a single ion in the same apparatus. Note the polarization sensitivity (crosses, with error bars omitted for clarity).

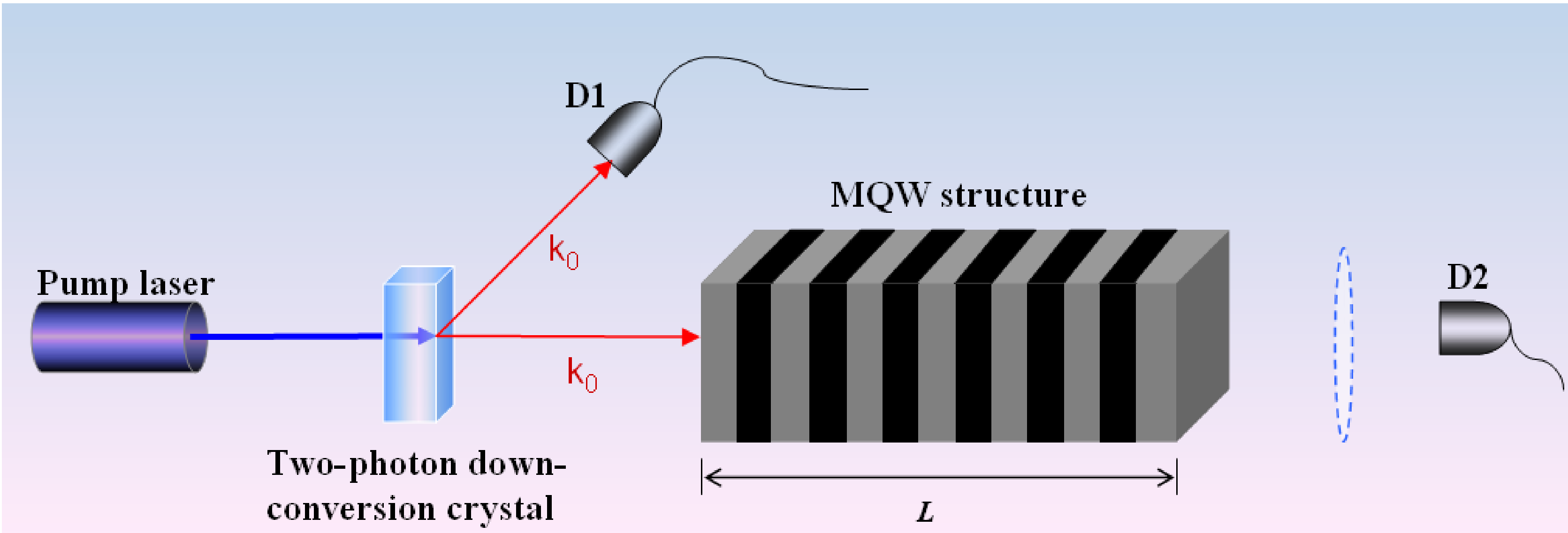
[R. G. DeVoe and R. G. Brewer, P. R. L. **76**, 2049 (1996)]

# Cavity QED without cavity

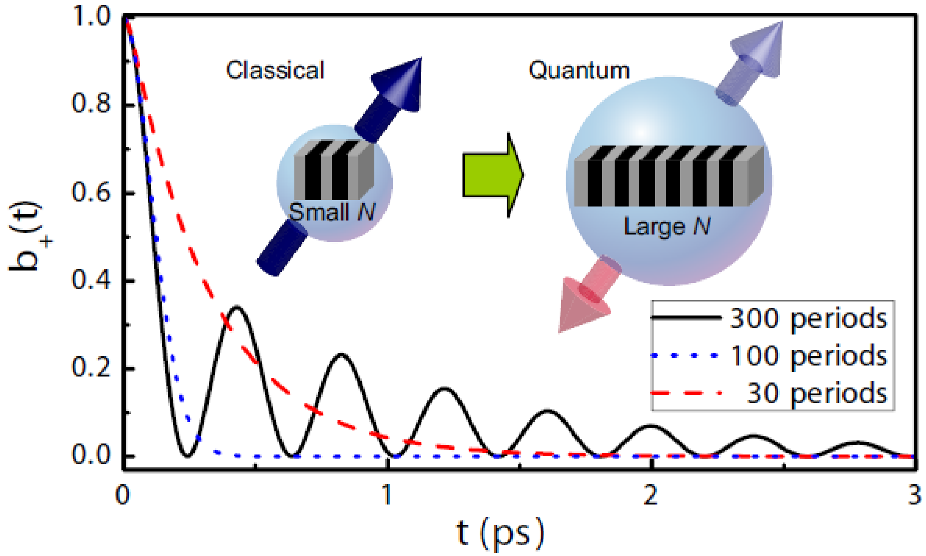



M. O. Scully, A. A. Svidzinsky, Science **325**, 1510 (2009)

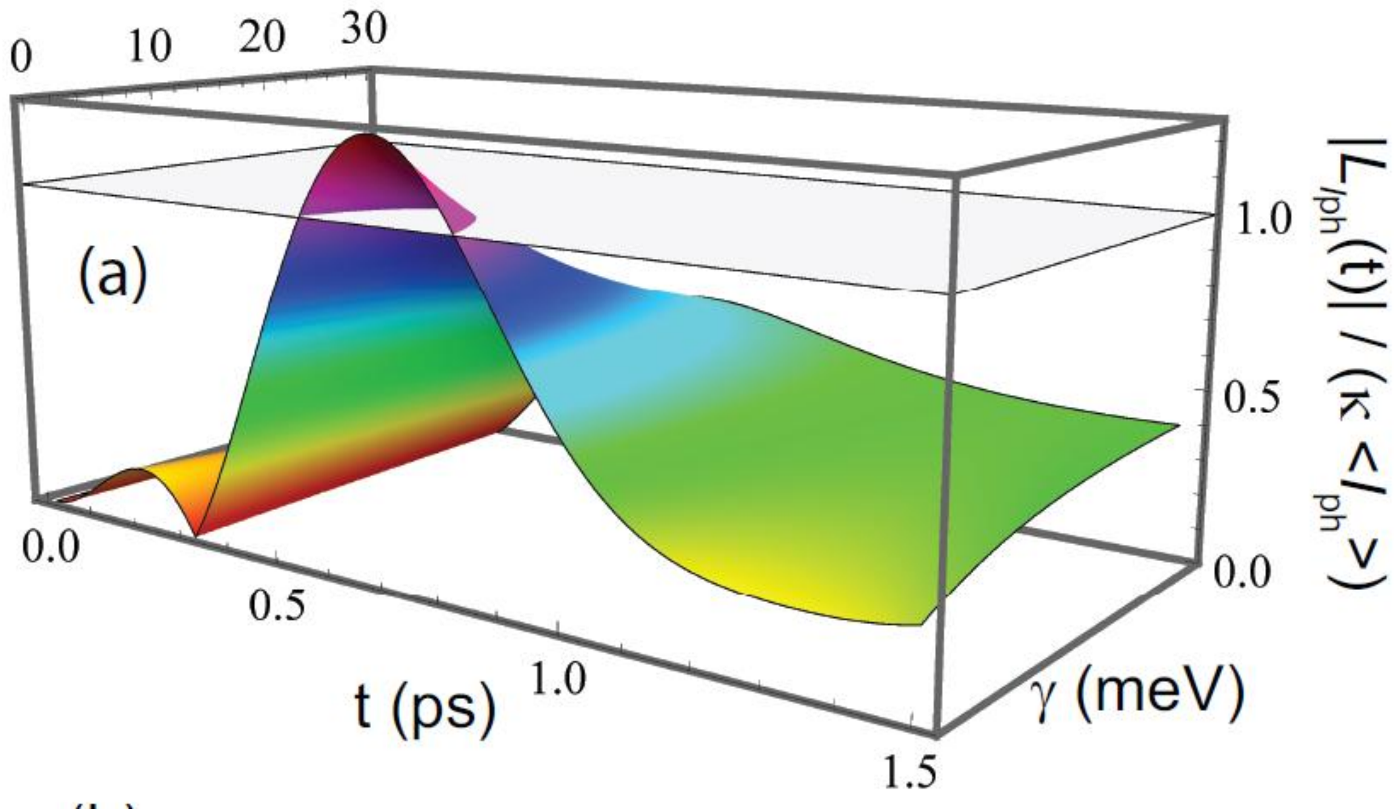
# Quantum Signatures in a Macroscopic Dipole Moment




$$|+\rangle_{k_0} = \frac{1}{\sqrt{N}} \sum_j e^{ik_0 z_j} |j\rangle \qquad |j\rangle = |g_1, g_2, \ldots, g_{j-1}, e_j, g_{j+1}, \ldots, g_N\rangle$$



# Violation of the effective photon-current inequality




G. Y. Chen, N. Lambert, C. M. Li, Y. N. Chen*, and F. Nori, in preparation (2011)

# Quantum Transport in Organism ?

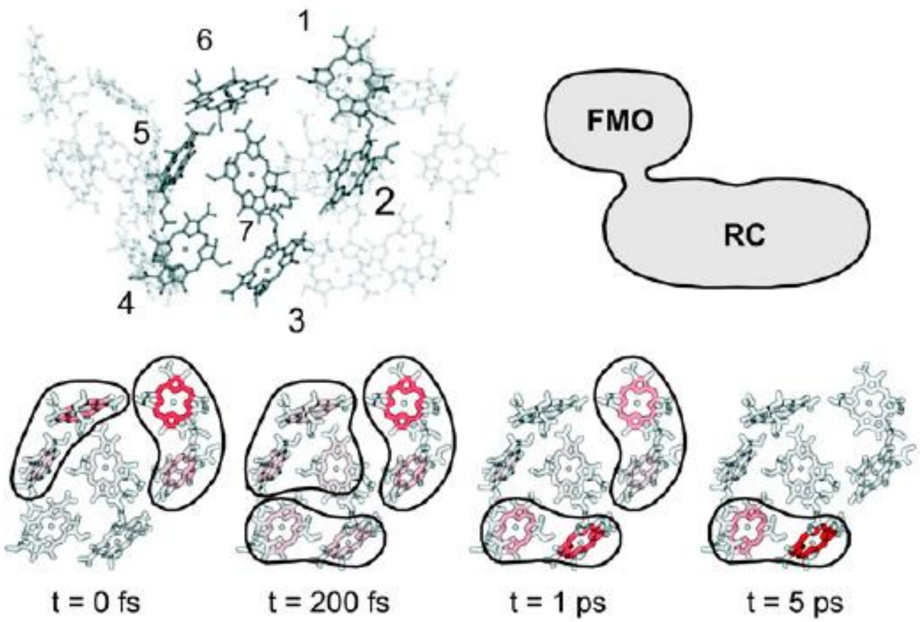
# The Quantum Dimension Of Photosynthesis

nature Vol 446|12 April 2007|doi:10.1038/nature05678

LETTERS

## Evidence for wavelike energy transfer through quantum coherence in photosynthetic systems

Gregory S. Engel[1,2], Tessa R. Calhoun[1,2], Elizabeth L. Read[1,2], Tae-Kyu Ahn[1,2], Tomáš Mančal[1,2]†, Yuan-Chung Cheng[1,2], Robert E. Blankenship[3,4] & Graham R. Fleming[1,2]


1
6
5
2
7
4
3
FMO
RC
t = 0 fs
t = 200 fs
t = 1 ps
t = 5 ps

nature | Vol 463 | 4 February 2010 | doi:10.1038/nature08811

# LETTERS

# Coherently wired light-harvesting in photosynthetic marine algae at ambient temperature

Elisabetta Collini[1]*†, Cathy Y. Wong[1]*, Krystyna E. Wilk[2], Paul M. G. Curmi[2], Paul Brumer[1] & Gregory D. Scholes[1]

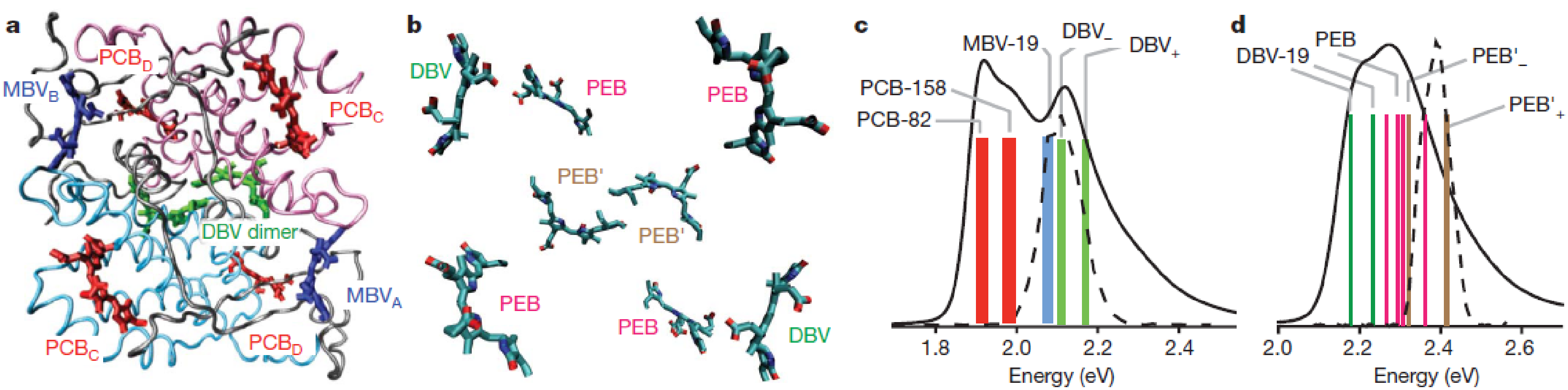
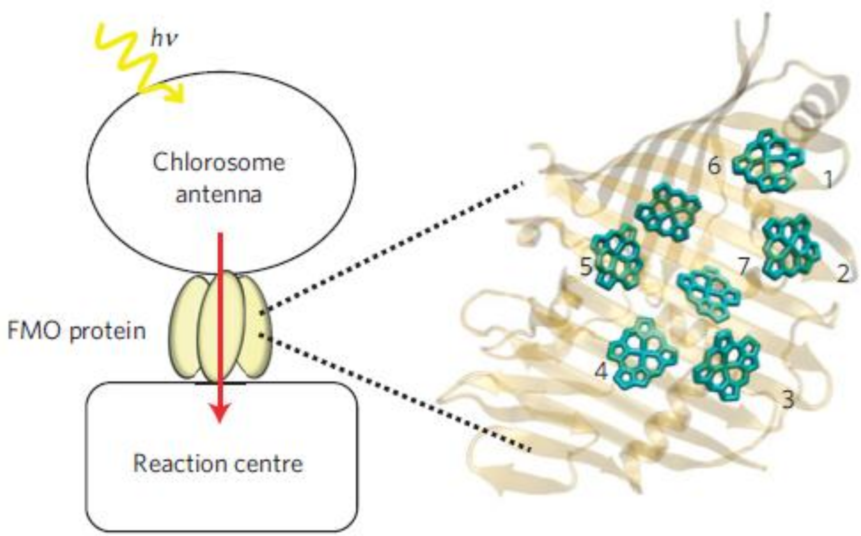


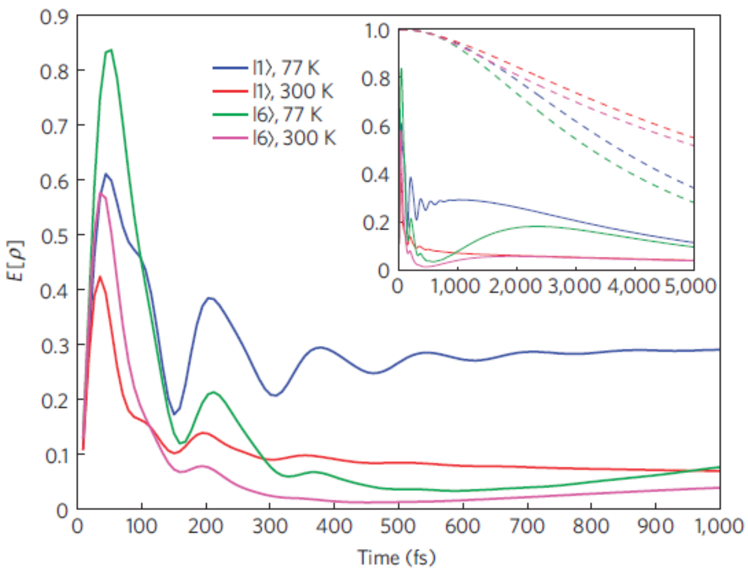

**Figure 1 | Structure and spectroscopy of cryptophyte antenna proteins.** **a**, Structural model of PC645. The eight light-harvesting bilin molecules are coloured red (PCB), blue (MBV) and green (DBV). **b**, Chromophores from the structural model for PE545 showing the different chromophore incorporation. **c**, Electronic absorption spectrum of isolated PC645 protein in aqueous buffer (294 K). The approximate absorption energies of the bilin molecules are indicated as coloured bars. **d**, Electronic absorption spectrum of isolated PE545 protein in aqueous buffer (294 K) with approximate absorption band positions indicated by the coloured bars. The spectrum of the ultrafast laser pulse is plotted as a dashed line in **c** and **d**.

nature physics
ARTICLES
PUBLISHED ONLINE: 25 APRIL 2010 | DOI: 10.1038/NPHYS1652

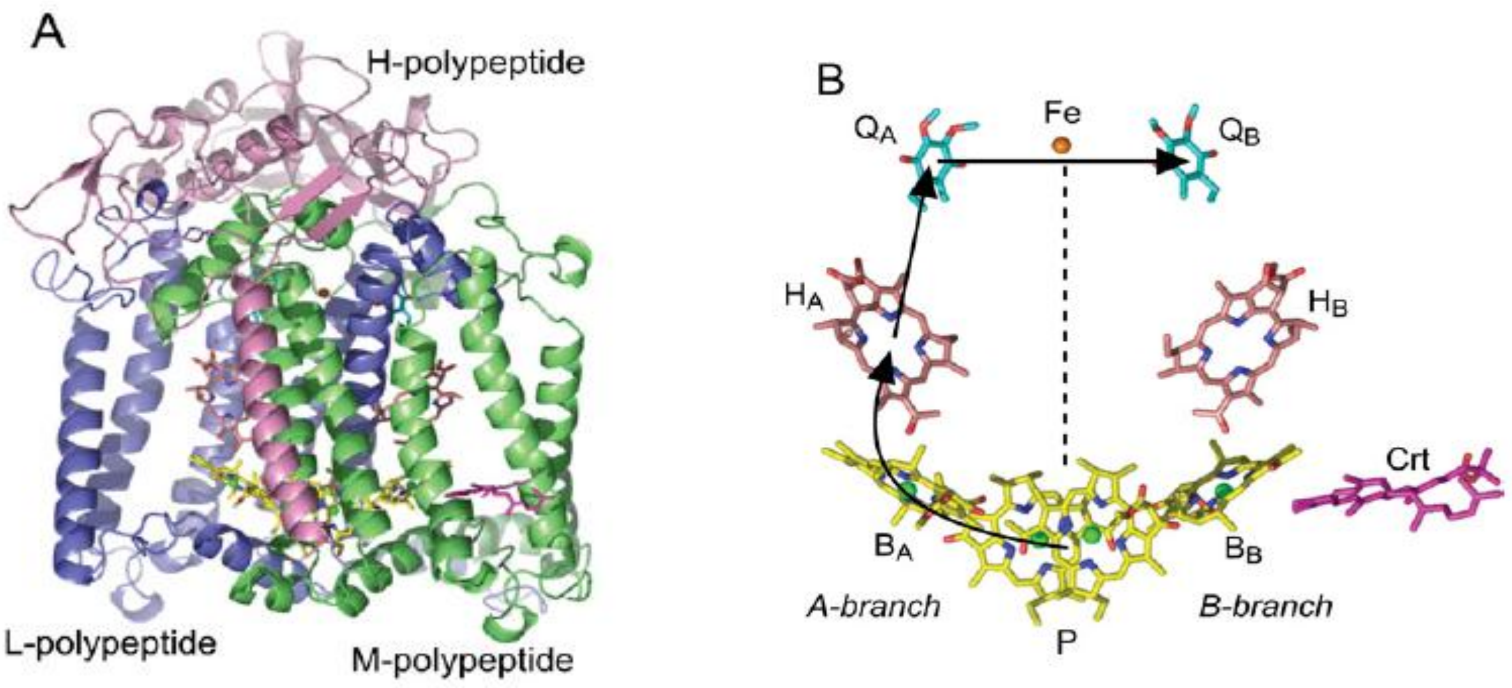
# Quantum entanglement in photosynthetic light-harvesting complexes

Mohan Sarovar[1,2]★, Akihito Ishizaki[2,3], Graham R. Fleming[2,3] and K. Birgitta Whaley[1,2]

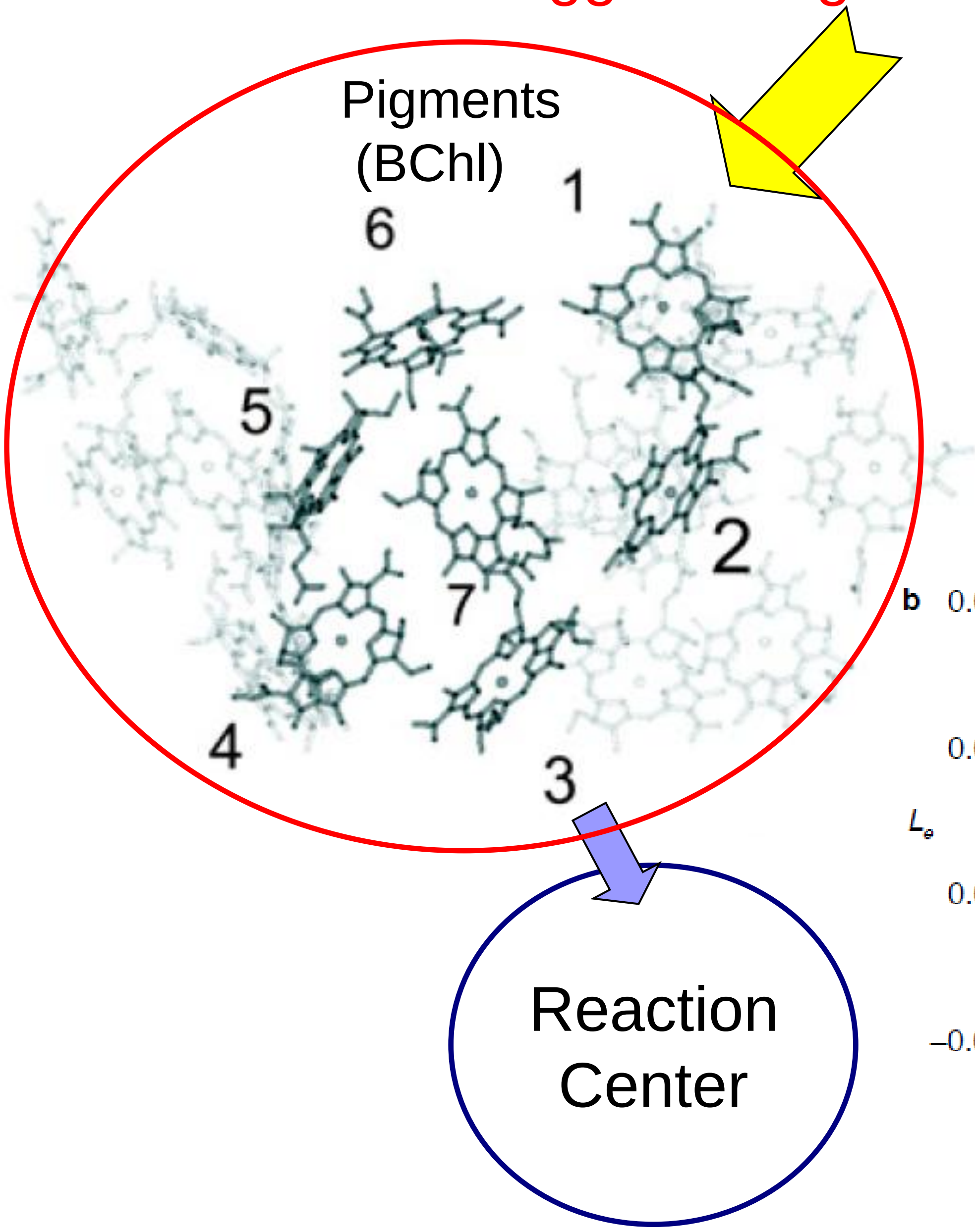


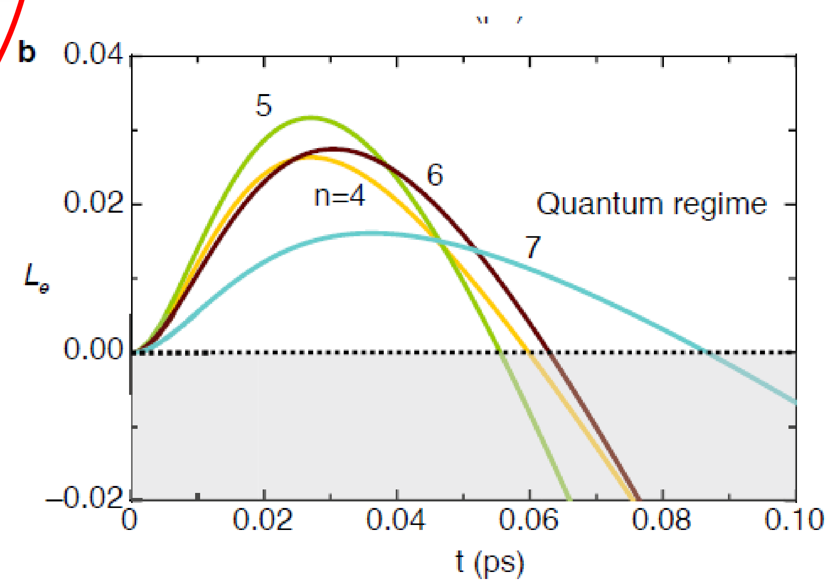



**Coherence Dynamics in Photosynthesis: Protein Protection of Excitonic Coherence**
Hohjai Lee, *et al.*
*Science* **316**, 1462 (2007);
DOI: 10.1126/science.1142188



# Leggett-Garg inequality ?



# Summary

1. Coherence and Entanglement

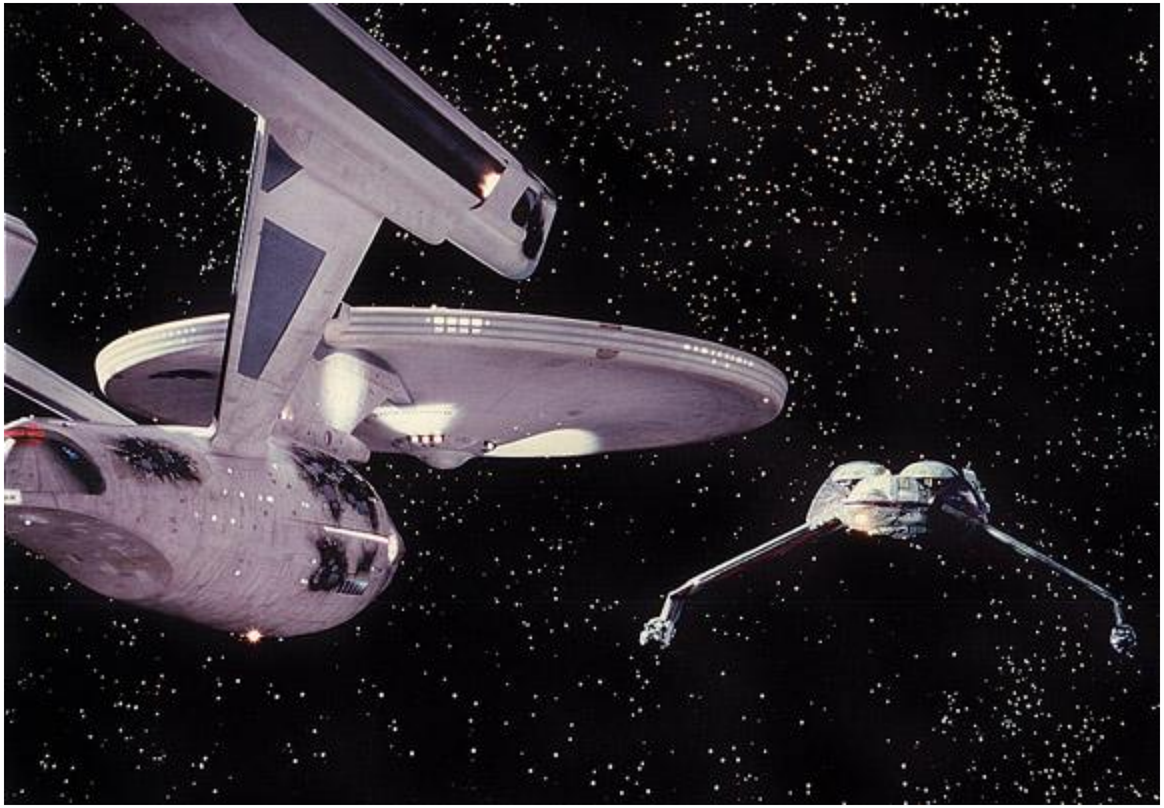
2. Teleportation

3. Extended LG inequalities

4. Time-adjusted photon counting

5. Quantumness in Biological Systems

# Thank you for your attention!



To boldly go where no man has gone before!