Crescent Waves in Optical Caivties

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Outline

Crescent-Moon

http://www.hbastro.com/LunarPlanetaryThumbs.html

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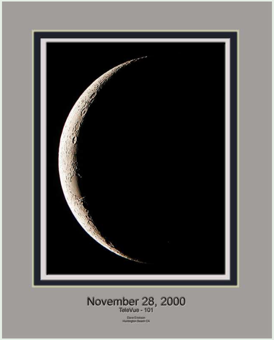
Crescent



What are Crescent Waves?

Crescent Waves

A specific localized soliton state of Crescent or Barchan sand dune.

Figure 1 A field of crescent-shaped barchan sand dunes in the desert between Chimbote and Casma on the coast of Peru.

Nature **426**, p.p. 619 (2003)

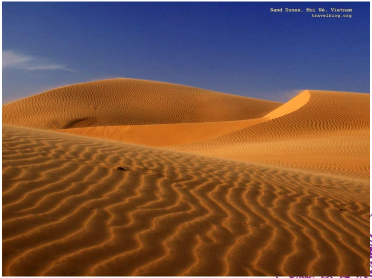


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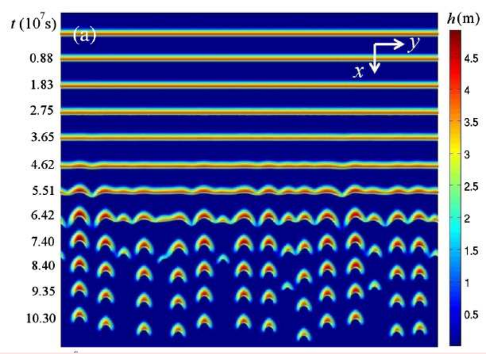
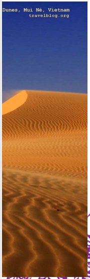
Figure 1 A field of cres
desert between Chimbote

Nature **426**, p.p. 61

Formation of Sand Dune!



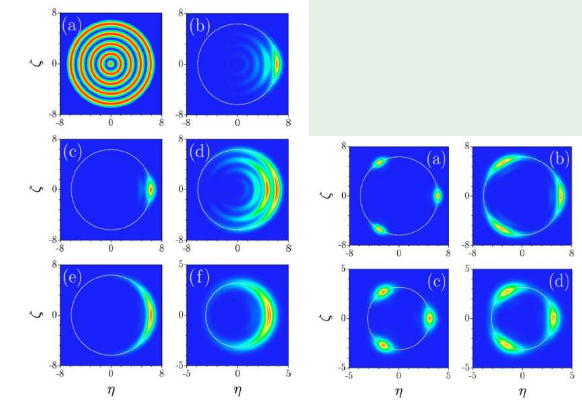
Phys. Rev. Lett. **107**, 188001 (2011).



Find Crescent Waves?

- Surface solitons pinged to circular surface boundary
- Exist in highly nonlocal media through superposition of vortices
- Introducing inhomogeneous losses
- Semiconductor micro cavity with composite optical cavities

Rotating surface solitons



Opt. Lett. **32**, 2948 (2007).

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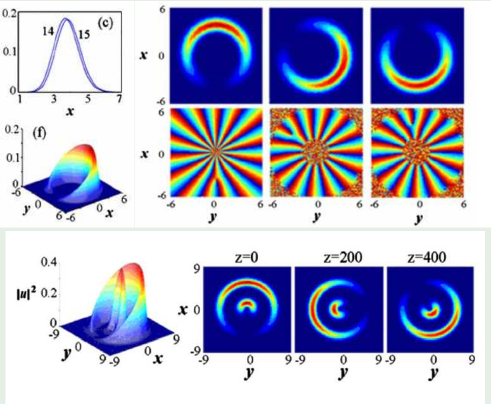
Crescent vortex solitons in strongly nonlocal nonlinear media



Phys. Rev. A **78**, 023824 (2008).

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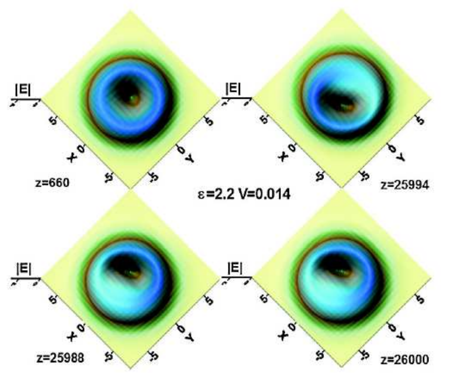
Varieties of Stable Vortical Solitons in Ginzburg-Landau Media with Radially Inhomogeneous Losses



Phys. Rev. Lett. **105**, 213901 (2007).

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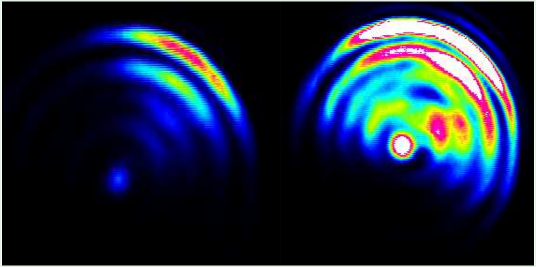
Crescent Waves in Optical Cavities



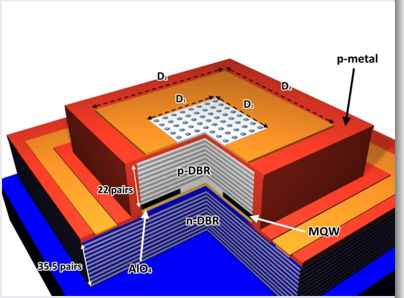
Phys. Rev. Lett. **107**, 183902 (2011).

Vertical Caivty Surface Emitting Laser

Structure of VCSEL

- integrated microcavity fabrication
- controllable confinement
- small mode volumes
- ultrahigh quality factors

- large emitting area
- study nonlinear pattern formation and soliton behavior



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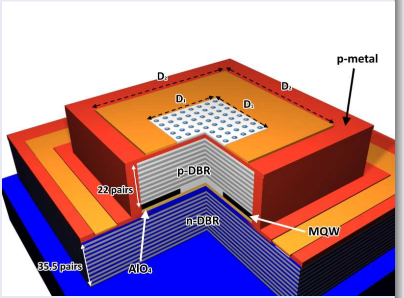
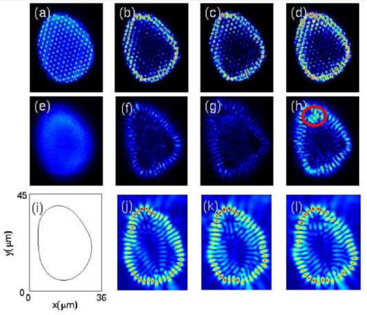
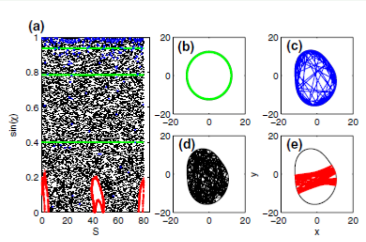


VCSEL for transverse mode tailoring

- **Dynamical Localized Mode Lasing**
- Nonlinear Bandgap Mode
- Lasing direction control
- Optical Crescent Waves

Dynamic Localized Mode in VCSEL





Phys. Rev. Lett. **101**, 084101 (2008).

VCSEL for transverse mode tailoring

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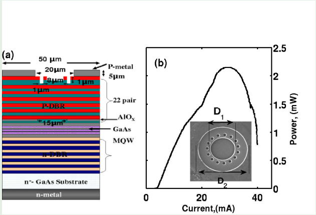
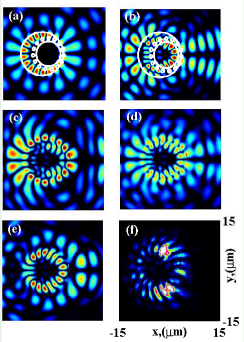
Unidirectional Lasing Cavity mode and dynamic localized mode





Appl. Phys. Lett. **94**, 221112 (2009).

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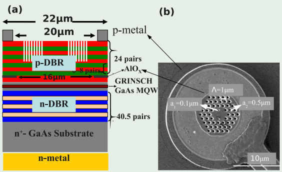
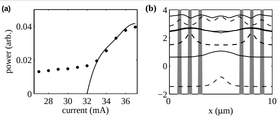
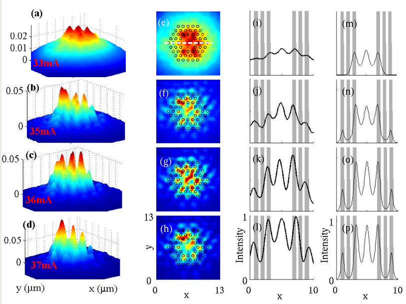
Nonlinear localized modes in bandgap microcavities







Opt. Lett. **35**, 3207 (2010).

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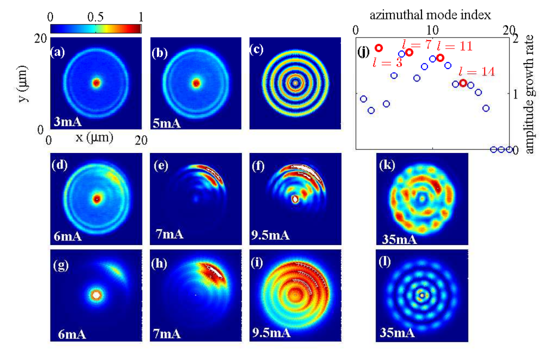
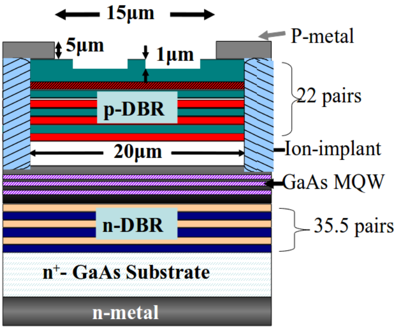
Crescent Waves in Optical Cavities

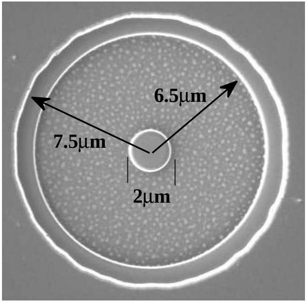


Phys. Rev. Lett. **107**, 183092 (2011).

Experimental observation of Optical Crescent Waves



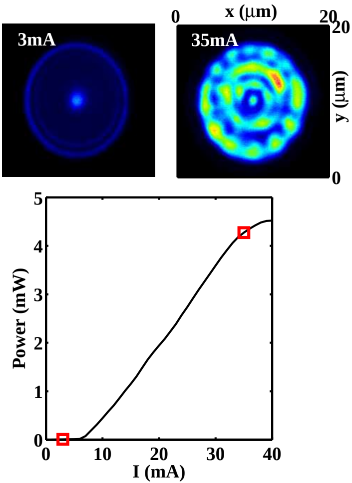
sideview



topview

- ion-implanted VCSEL(lateral) with epitaxial layers by MOCVD on a n^+-GaAs substrate
- GRINSCH active region (triple-GaAs/AlGaAs QW)
- lambda cavity (vertical)
- surface structured by FIB
- composite cavity design (a circular HI-cavity + annular HI-cavity

Experimental observation of Optical Crescent Waves



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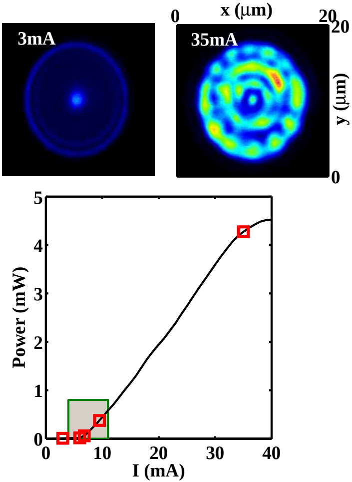

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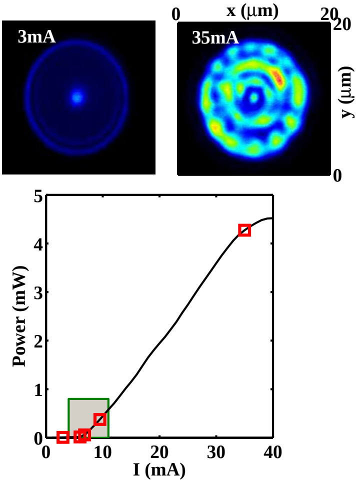

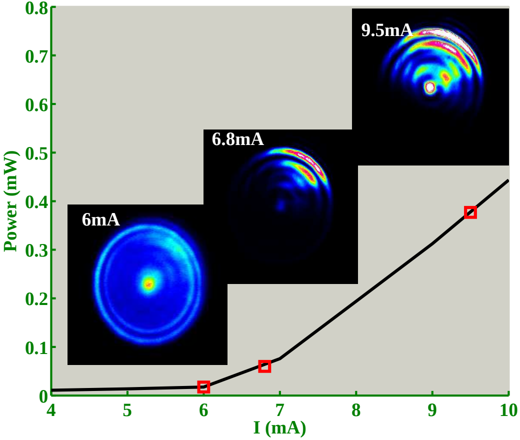

Typical Field-Carrier Model for Laser in Semiconductor

- SVEA
- spatio-temporal dynamics of: intracavity field E carrier density N

$$\partial_t E = -(1+\eta+i\theta)E - i2C\Theta(N-1)E + i\nabla_\perp{}^2 E + E_I$$

$$\partial_t N = -\gamma\left[N + \beta N^2 - I + |E|^2(N-1) - d\nabla_\perp{}^2 N\right]$$

- C is the absorption(coupling) strength scaled to the resonator. transmission.
- η is the linear absorption coefficient.
- θ is the cavity detuning.
- $\Theta = (i+\alpha)$ represents the absorptive and refractive nonlinear response of materisl
- α is its linewidth engencement factor.
- γ and β are the normalized decay rate of the carrier density that describe the nonradiative and radiative carrier recombination, respectively.
- d is the diffusion constant of the carrier
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To Reduced Wave Equation: $E_I = 0, d \ll 1, \beta = 0$

We can then expand the carrier density to the first order in Fourier space,

$$N - 1 \approx \int_{-\infty}^{t} \underbrace{\exp\left[-\gamma(1 + |E(t-t')|^2 - d\partial_x^2)(t-t')\right]}_{\text{Response}} \underbrace{\gamma(I-1)}_{\text{Source}} dt'$$

When the system reaches equivbrium, $|E|^2$ is time-invariant and

$$N - 1 = \frac{I-1}{1+|E|^2} - \frac{I-1}{\gamma(1+|E|^2)^2} d\partial_x^2$$

It is obvious that this system is nonlocal,non-instantaneous and dissipative in nature.

To Reduced Wave Equation (cont.)

Reduced Wave Equation

$$
\begin{aligned}
0 \;=\; & i\partial_\tau E + \frac{\bar\theta}{\alpha}E - \left[-(1+\eta) + \frac{2C(I-1)}{1+|E|^2}\right]E + \partial_{\xi\xi}E \\
& -\frac{i}{\alpha}\left[-(1+\eta) + \frac{2C(I-1)}{1+|E|^2}\right]E - \frac{i}{\alpha}\frac{I-1}{\gamma(1+|E|^2)^2}d\partial_{\xi\xi}E
\end{aligned}
$$

- substitute $N - 1$
- new corrdinate system with $\tau = \alpha t$ and $\xi = \sqrt{\alpha} r$.
- Define $\bar\theta = \theta - \theta_0$ with $\theta_0 + \alpha(1+\eta) = 0$.

To Reduced Wave Equation (cont.)

Reduced Wave Equation

$$0 = i\partial_\tau E + \frac{\bar{\theta}}{\alpha} E - \left[-(1+\eta) + \frac{2C(I-1)}{1+|E|^2} \right] E + \partial_{\xi\xi} E$$

$$- \frac{i}{\alpha} \left[-(1+\eta) + \frac{2C(I-1)}{1+|E|^2} \right] E - \frac{i}{\alpha} \frac{I-1}{\gamma(1+|E|^2)^2} d\partial_{\xi\xi} E$$

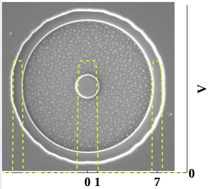
- **Dissipation vanishes**
- **SPM dominates**

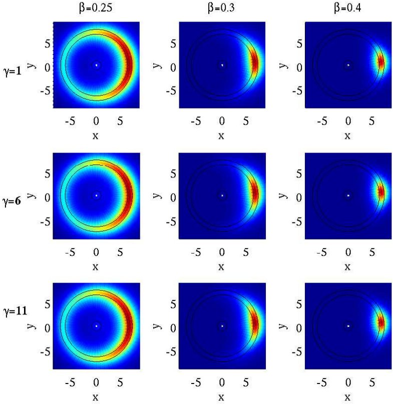
- substitute $N - 1$
- new corrdinate system with $\tau = \alpha t$ and $\xi = \sqrt{\alpha} r$.
- Define $\bar{\theta} = \theta - \theta_0$ with $\theta_0 + \alpha(1+\eta) = 0$.
- Approx. with $\alpha \gg 1$

Modeling Crescent Waves in Optical Cavties

- $\partial_\tau E = 0$
- $\xi \to (r, \theta)$
- surface induced potential $V(r, \theta)$

$$0 = \frac{\bar{\theta}}{\alpha} E - \left[-(1+\eta) + \frac{2C(I-1)}{1+|E|^2} \right] E + \nabla_\perp^2 E + V(r,\theta) E$$





- $\beta = \frac{\bar{\theta}}{\alpha} + (1+\eta)$
- nonlinearity: $\gamma = 2C(I-1)$

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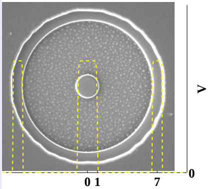

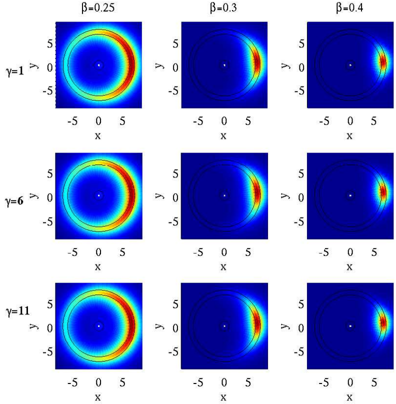

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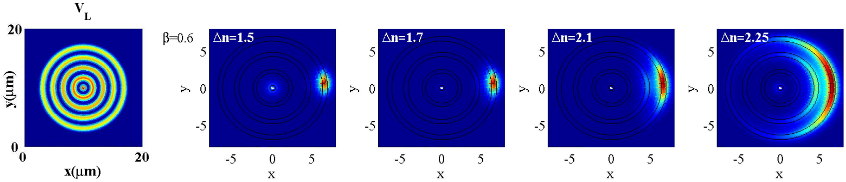
- nonlinearity: $\gamma = 2C(I-1)$

Injection $\nearrow \not\Rightarrow$ multiple, elongated

Modeling Crescent Waves in Optical Cavties (cont.)

- nonlasing background potential $V_L(r,\theta)$

$$0=\frac{\bar{\theta}}{\alpha}E-\left[-(1+\eta)+\frac{2C(I-1)}{1+|E|^2}\right]E+\nabla_\perp^2E+V(r,\theta)E+V_L(r,\theta)E$$



- $\Delta n=\mathbf{max}\,\{|V_L|\}$
- $\Delta n\propto 2C(I-1)$

Modeling Crescent Waves in Optical Cavties (cont.)

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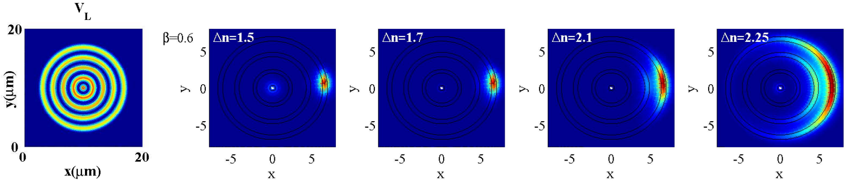


- $\Delta n = \mathbf{max}\{|V_L|\}$
- $\Delta n \propto 2C(I-1)$

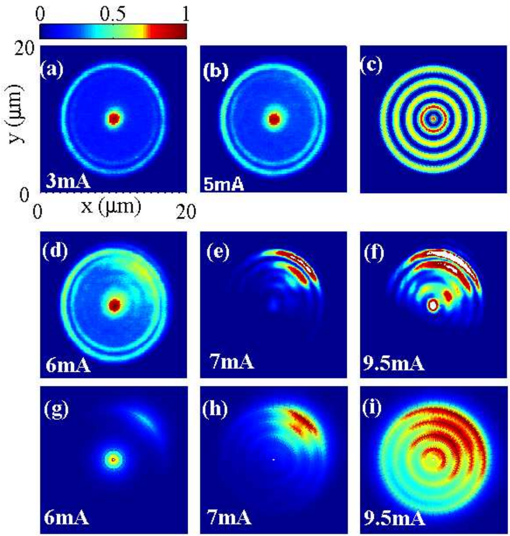
- $I \nearrow$
- elongated crescent
- multiple humped crescents

NUM VS EXP - Near Threshold Injection

- experimental demonstration of a mode transition
- (c) numerical result for linear LG_{04}-like cavity mode.

Experiments⇒

Simulation ⇒



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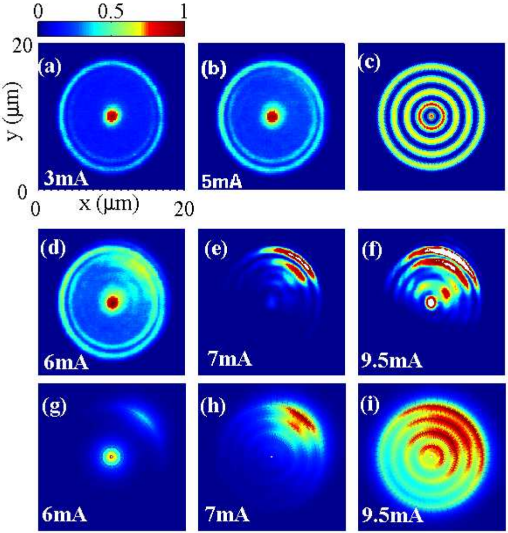
Experiments ⇒

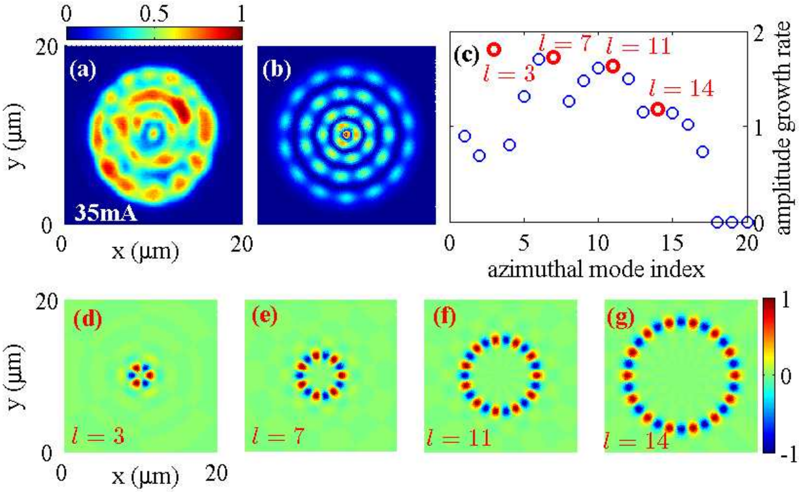
AGREES

Simulation ⇒



NUM VS EXP - High-Current Injection

- $I = 35$mA
- Very strong nonlinearity
- Azimuthal Instability



Discussion

Effect of Composite Cavity

- LG_{04}-like mode is supported by a large cavity
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Discussion

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Intrinsic linear cavity LG_{04}-like modes as a background potential V_L

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- injection currents increase, this supported LG_{04}-like mode turns to lasing too. (Azimuthally unstable)
- Our NUM are in good agreement with EXP

Summary

- Fabricated VCSEL and reported the observation of crescent surface waves near the threshold lasing condition.
- Introducing the concept: intrinsic nonlasing cavity mode as a background potential
- Stationary crescent mode transition to soliton rings is demonstrated in NUM and EXP.
- EXP and SIM: an alternative but effective approach to access optical surface modes in a variety of microcavities.

Thank you

Questions are welcome