Physics behind

Quantum Photonic Dissipative Transport Theory

Wei-Min Zhang

Physics @ National Cheng Kung University, Tainan

wzhang@mail.ncku.edu.tw

Talk at AMO seminar, NTHU **2011/11/17**

Outline

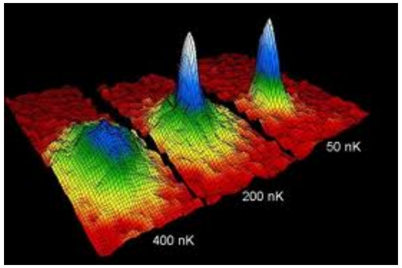
- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Outline

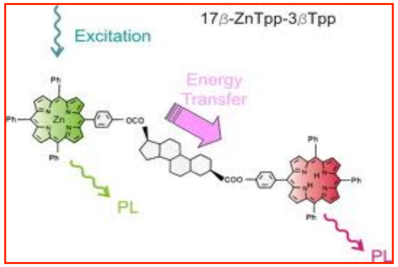
- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Typical phenomena for dissipative transport dynamics

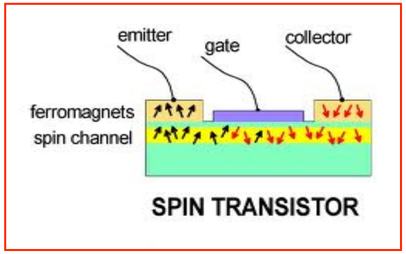
- Bose-Einstein Condensation (BEC):



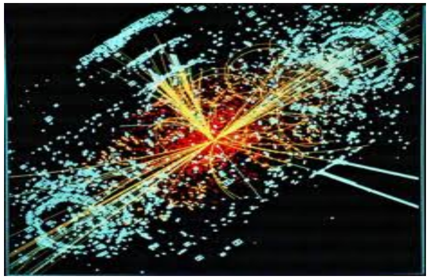
- Molecular Dynamics:



- Spintronics:



- Early Universe:



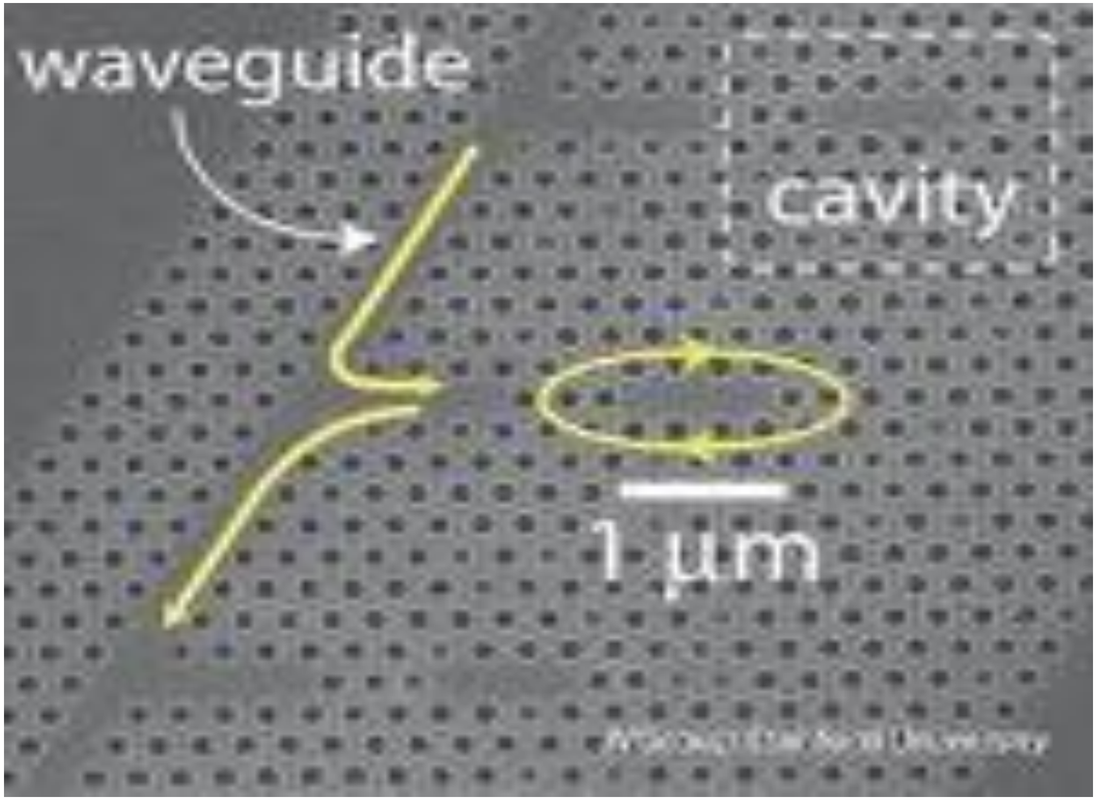
Dissipative Dynamics

⬇

Open Quantum Systems

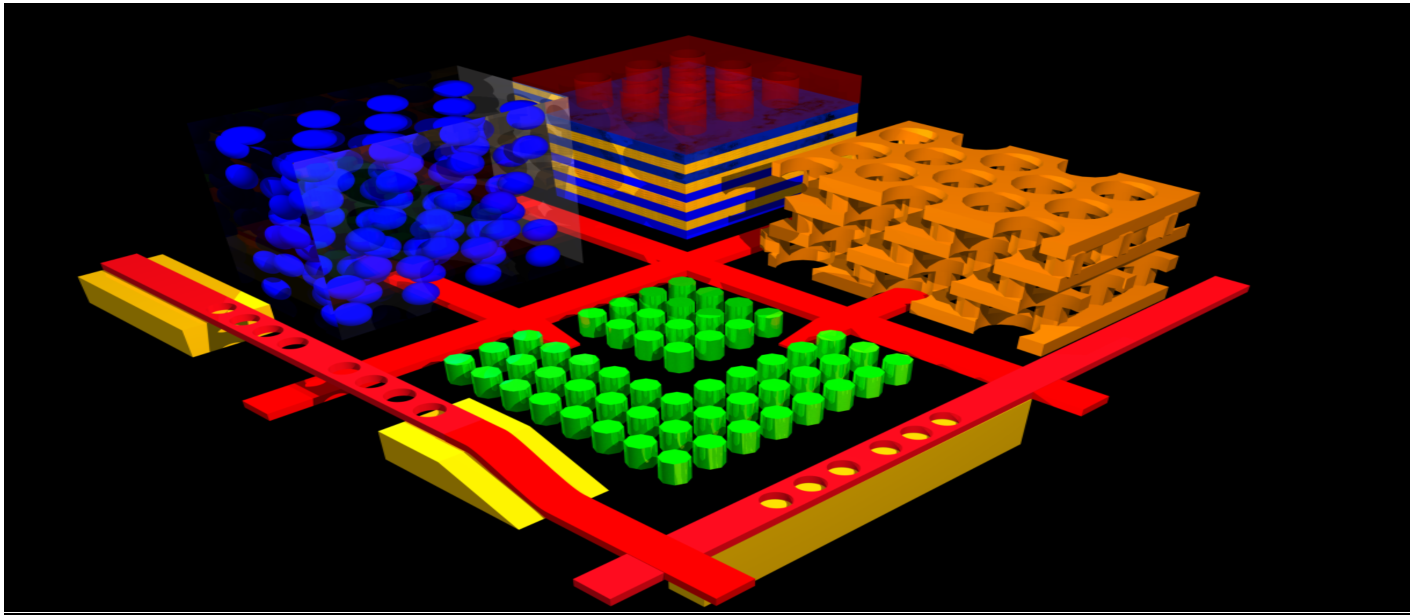
Dissipative photonic transport in photonic networks:

D. Englund *et al.*, Nature, 450, 857 (2007)



The nanophotonic networks we concerned consist of all-optical circuits incorporating photonic bandgap waveguides and driven resonators embedded in nanostructured photonic crystals.

Nanostructured photonic crystals: a lossless material

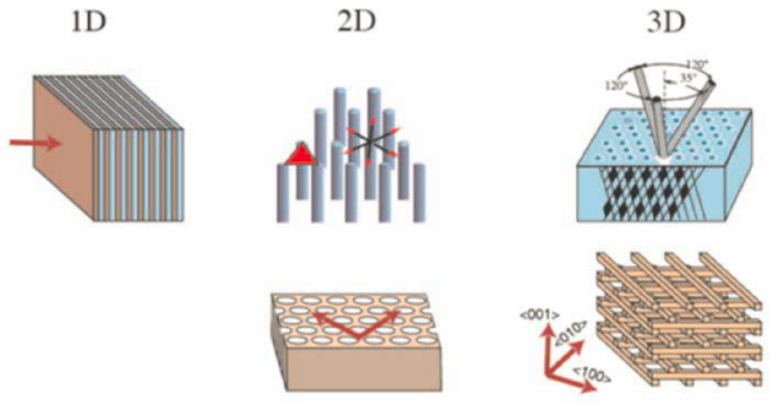


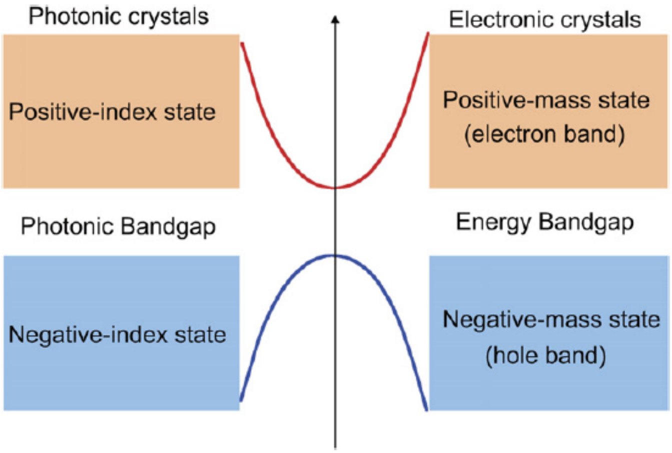
Photonic Crystals : Molding the Flow of Light,
By Johns D. Joannopoulos

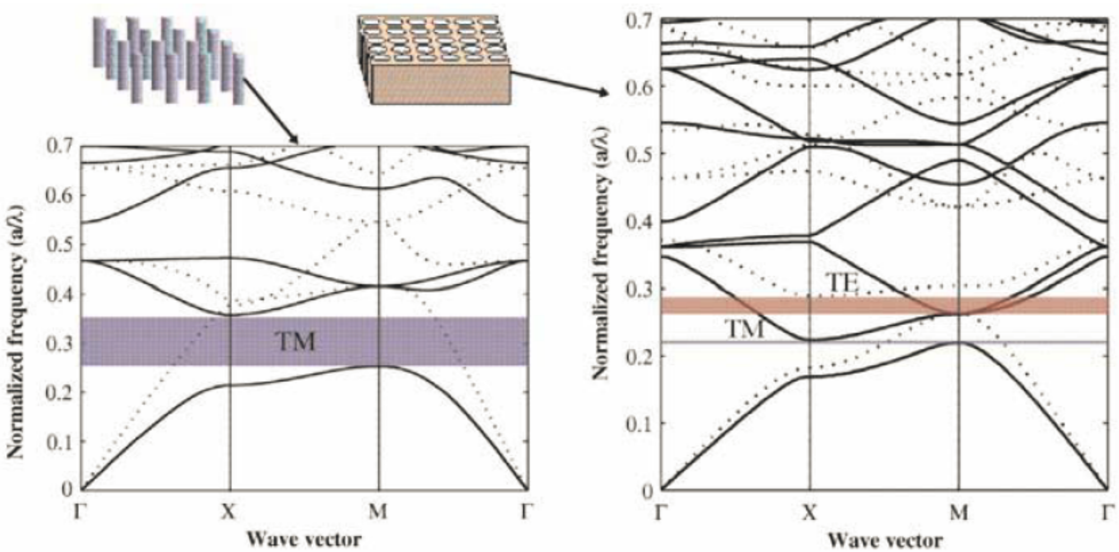
$$H = \frac{1}{2}\int d^3\mathbf{r}\left[\frac{c^2\Pi(\mathbf{r},t)^2}{\epsilon(\mathbf{r})} + (\nabla \times \mathbf{A}(\mathbf{r},t))^2\right]$$

Photonic crystals are artificial materials with periodic refractive index, its photonic band gap (PBG) structure together with its characteristic dispersion properties make the light manipulation and transmission much more efficient through the nanocavities and waveguides.

Photonic Crystals



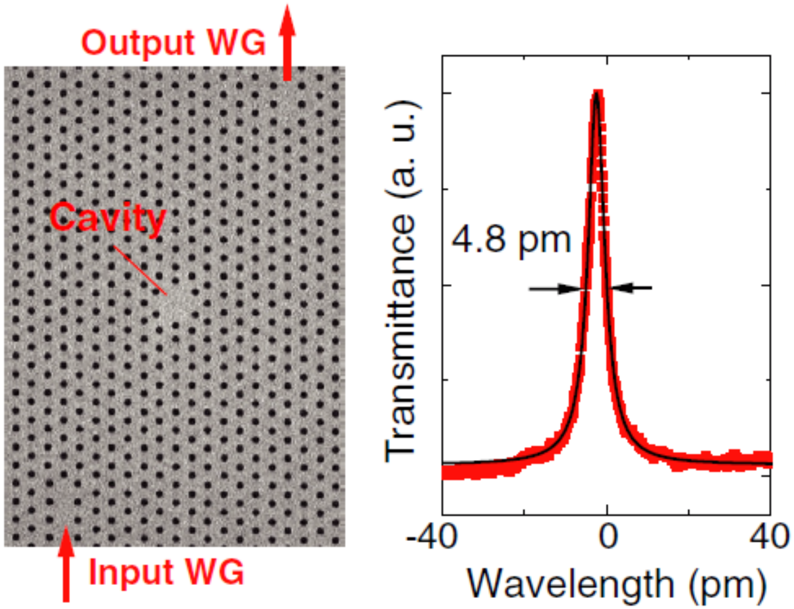


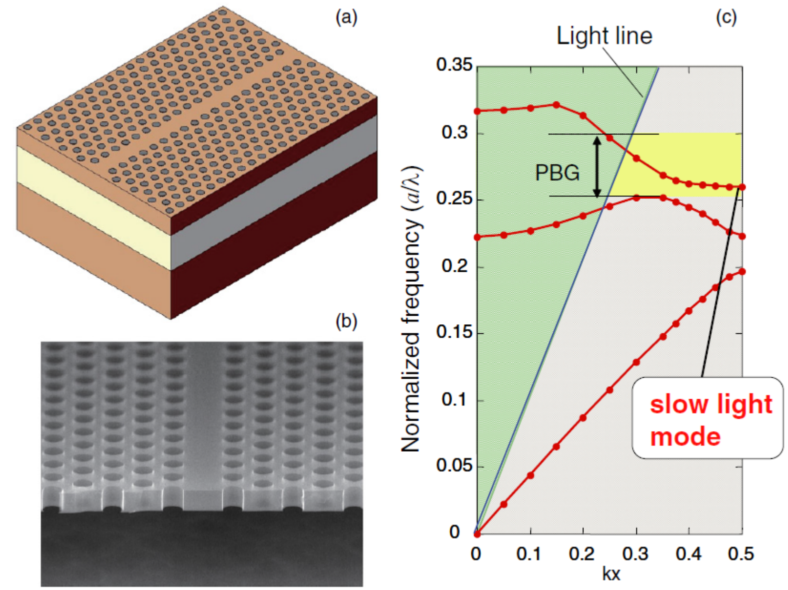


- Photonic crystals are artificial materials with periodic refractive index.
- **Photonic Band Gap**

M. Notomi, Rep. Prog. Phys. **73** (2010) 096501.

Defects





✓ **Ultra high Q cavity**

⟶ **Strong light confinement**

✓ **Waveguide**

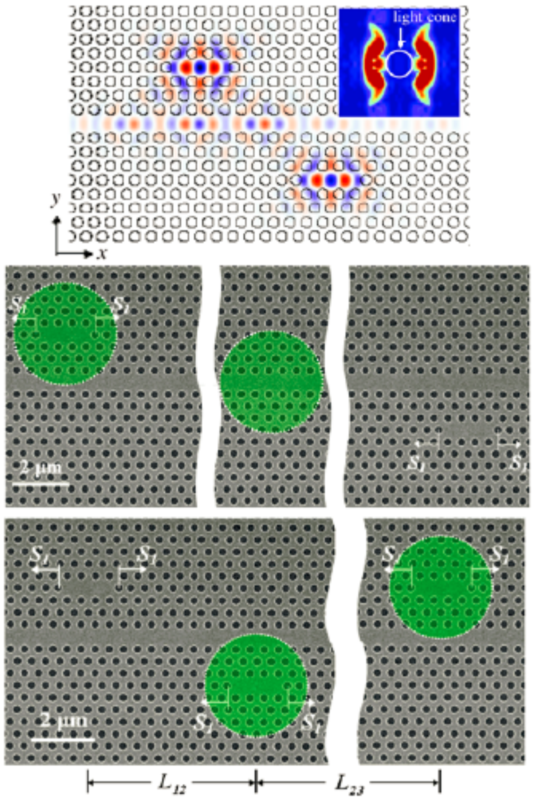
⟶ **Slow light**

➢ **Well-defined defects incorporated in photonic crystal can become functional devices!**

M. Notomi, Rep. Prog. Phys. **73** (2010) 096501

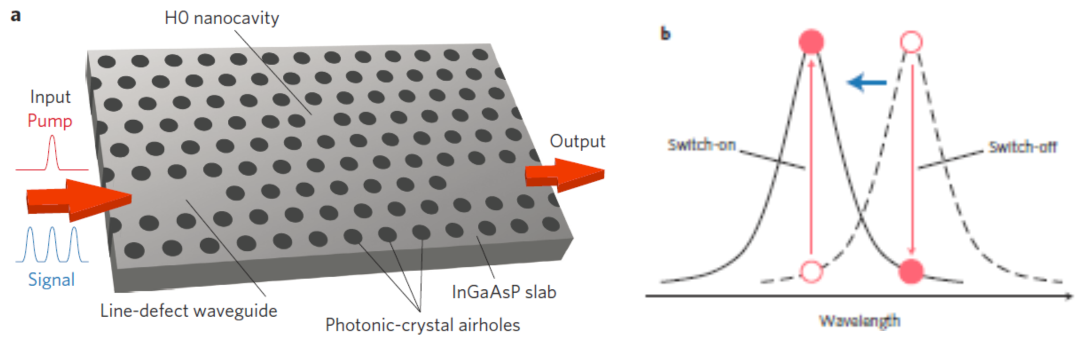
Devices

✓ All Optical EIT



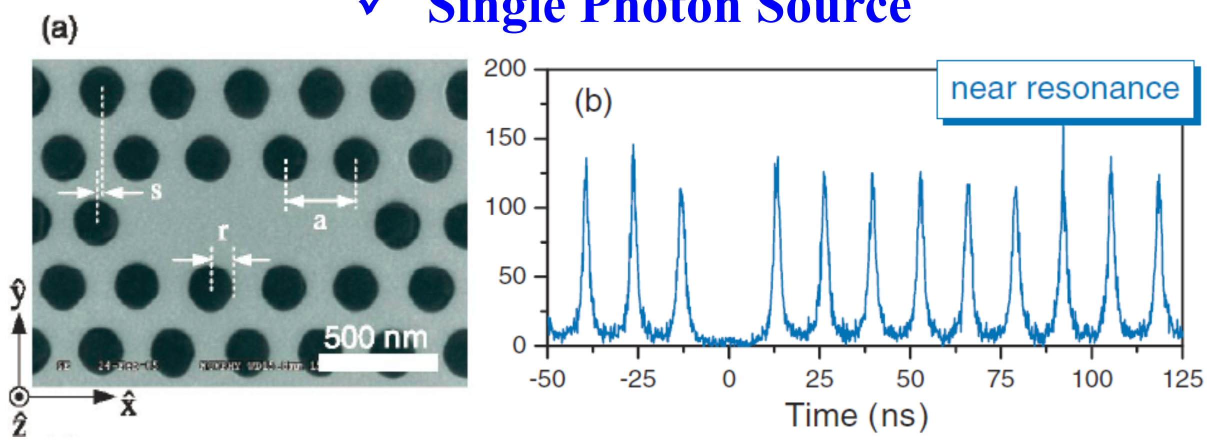
X. Yang et al. PRL **102**, 173902 (2009)

✓ All Optical Switch



K. Nozaki, et al. Nat. Photon. **4**, 477 (2010)

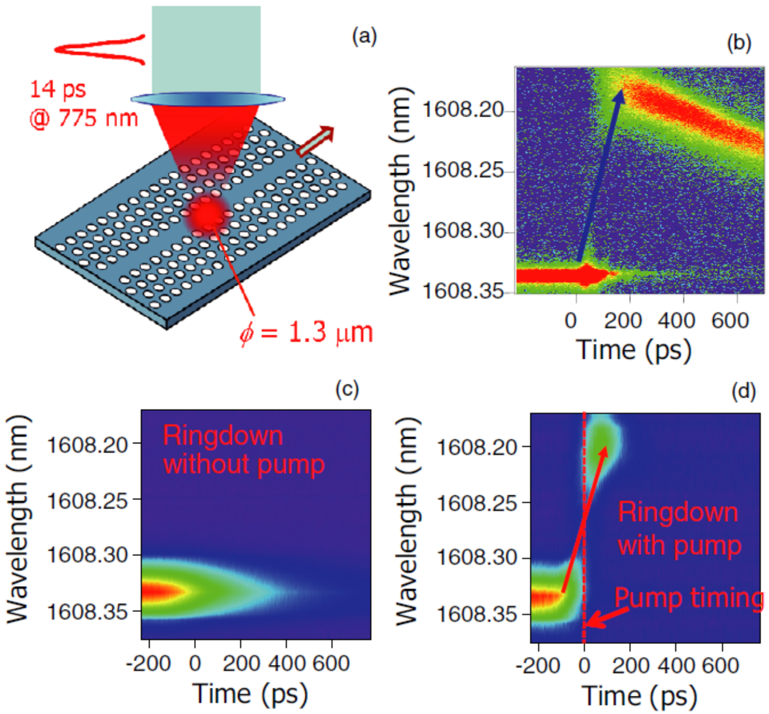
✓ Single Photon Source



W. H. Chang et al. PRL **96,** 117401 (2006)

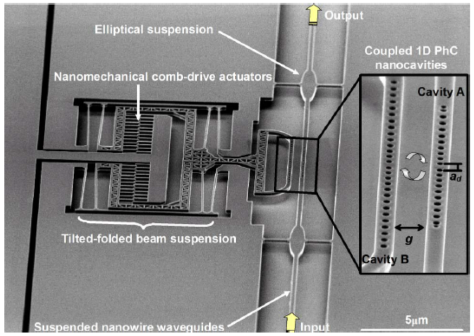
Controllability

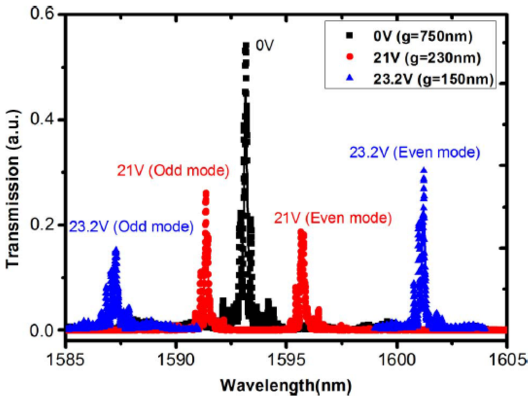
✓ **Adiabatic Wavelength Conversion**



T. Tanabe et al. PRL **102**, 043907 (2009)

✓ **Dynamical Tuning the Coupling**

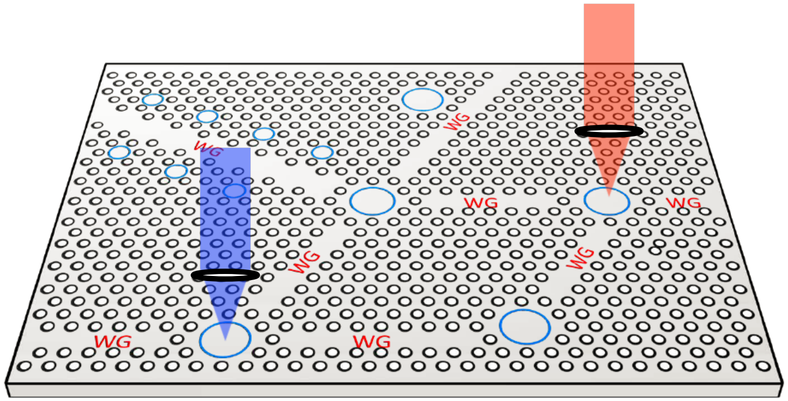




X. Chew et al. Opt. Lett. **15**, 2517(2010)

➢ **High Controllability**

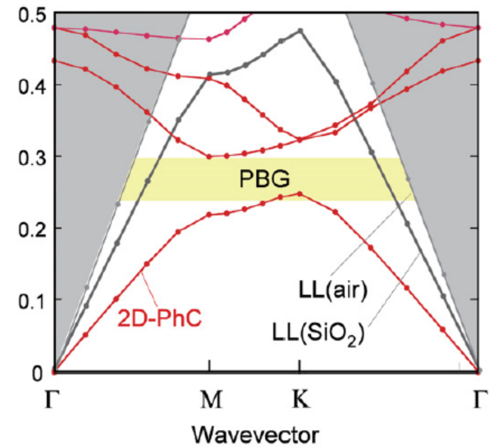
Modeling photonic circuits:



$$\mathbf{A}(\mathbf{r},t) = \sum_i \mathbf{A}_i(\mathbf{r},t) + \sum_\alpha \mathbf{A}_\alpha(\mathbf{r},t)$$

$$\mathbf{A}_i(\mathbf{r},t) = c\sqrt{\frac{\hbar}{2\omega_i}}\,\left[a_i \mathbf{u}_i(\mathbf{r})e^{-i\omega_i t} + a_i^\dagger \mathbf{u}_i^*(\mathbf{r})e^{i\omega_i t}\right]$$

$$\mathbf{A}_\alpha(\mathbf{r},t) = c\sum_k \sqrt{\frac{\hbar}{2\omega_k}}\,\left[c_{\alpha k}\mathbf{v}_{\alpha k}(\mathbf{r})e^{-i\omega_k t} + c_{\alpha k}^\dagger \mathbf{v}_{\alpha k}^*(\mathbf{r})e^{i\omega_k t}\right]$$



$$H_S(t) = \sum_i \hbar\omega_i a_i^\dagger a_i + \sum_i \left(f_i(t)a_i^\dagger + f_i^*(t)a_i\right),$$

$$H_{E\alpha} = \sum_k \hbar\omega_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k},$$

$$H_{T\alpha}(t) = \hbar\sum_{ik}\left(V_{i\alpha k}(t)a_i^\dagger c_{\alpha k} + V_{i\alpha k}^*(t)c_{\alpha k}^\dagger a_i\right).$$

- open optical systems

Lei & WMZ, arXiv: 1011.1475 (2010)

Quantum theory for open systems: a long-standing problem

Master Equation:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H_S, \rho(t)] + \int_{t_0}^{t} d\tau \mathcal{L}(t-\tau)\rho(\tau)$$

Openness

Quantum Mechanics

Non-Markovian Memory

However, it has been attempted for many years without a very satisfactory answer to find the exact master equation for an arbitrary open quantum system since Pauli first proposed the phenomenological master equation in 1928 !

Outline

- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Three basic approaches for electronic quantum transport:

- Scattering theory approach

  Büttiker, *Phys. Rev*. B **46**, 12485 (1992)

  using single particle scattering states to build up the multiparticle states with the proper symmetry

- NE Green function approach

  Wingreen, Jauho & Meir, *Phys. Rev.* B **48**,8487 (1993)

  based on Schwinger-Keldysh's Non-Equilibrium GF

  J. Schwinger, *J. Math. Phys.* **2**, 407 (1961)

  L. V. Keldysh, *Sov. Phys. JETP*, **20**, 1018 (1965)

- Master equation approach

  Jin, Tu, WMZ & Yan, *New J. Phys.* **12**, 183013 (2010)

  based on Feynman-Vernon influence functional

  Feynman and Vernon, Ann. Phys. **24**, 118 (1963)

Quantum transport based on scattering theory:

Büttiker, *Phys. Rev.* B **46**, 12485 (1992)

- Single-particle scattering matrix:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

- Transport current of each reservoir:

$$\langle I_\alpha \rangle = \frac{e}{h} \int d\varepsilon \{ (M_\alpha - \mathrm{tr}[s^\dagger_{\alpha\alpha} s_{\alpha\alpha}] f_\alpha(\varepsilon) - \sum_\beta \mathrm{tr}[s^\dagger_{\alpha\beta} s_{\alpha\beta}] f_\beta(\varepsilon) \}$$

- Landauer-Büttiker formula:

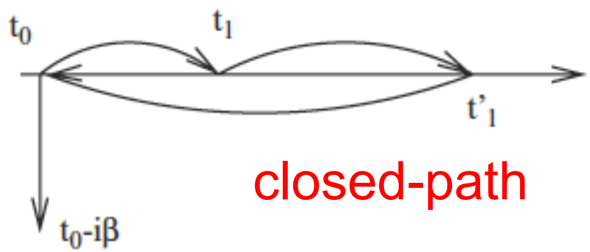
$$J = \frac{1}{2}(\langle I_L \rangle - \langle I_R \rangle) = \frac{e}{h} \int d\varepsilon \mathcal{T}(\varepsilon)[f_L(\varepsilon) - f_R(\varepsilon)]$$

with $\mathcal{T}(\varepsilon) = \mathrm{tr}[t^\dagger t]$

Limited to the simple steady transport phenomena with a “black” box.

Nonequilibrium GF approach:

L. V. Keldysh,
JETP 20, 1018 (1965)



closed-path

- closed-time Dyson equation:

$$\left\{ \mathrm{i}\frac{\partial}{\partial \tau_1} - \left[-\frac{1}{2}\nabla^2_{x_1} + U(x_1,\tau_1) \right] \right\} G(x_1,\tau_1,x_{1'},\tau_{1'}) = \delta(1-1') + \int_C \mathrm{d}\sigma \int \mathrm{d}^3 y \Sigma(x_1,\tau_1,y,\sigma) G(y,\sigma,x_{1'},\tau_{1'})$$

where

$$G(1,1') = \begin{pmatrix} G_C(1,1') & G^>(1,1') \\ G^<(1,1') & G_{\tilde{C}}(1,1') \end{pmatrix} \to \begin{pmatrix} G^r(1,1') & G^>(1,1') \\ G^<(1,1') & G^a(1,1') \end{pmatrix}$$

⇒

retarded GF

$$\left\{ i\frac{d}{d\tau} - \omega \right\} G^r(\tau,\tau') = \delta(\tau-\tau') + \int_{\tau'}^{\tau} \Sigma^r(\tau,\tau'') G^r(\tau'',\tau') d\tau''$$

$$G^< = (1 + G^{\mathrm{r}}\Sigma^{\mathrm{r}}) G_0^< (1 + \Sigma^{\mathrm{a}} G^{\mathrm{a}}) + G^{\mathrm{r}} \Sigma^< G^{\mathrm{a}}$$

lesser GF

quantum kinetic equation that can systematically explore all the nonequilibrium dynamics

e.g. see WMZ & Wilets, PRC45, 1900 (1992)

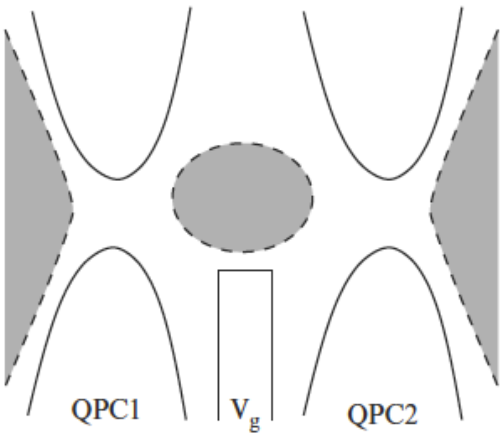
Quantum transport based on nonequilibrium GF

In terms of nonequilibrium Green functions, one has established the quantum transport theory of mesoscopic systems:

- Transient current:

Wingreen, Jauho & Meir, PRB48,8487 (1993)

$$I_\alpha(t) = -\frac{2e}{\hbar}\mathrm{Re}\int_{t_0}^{t} d\tau \, \mathrm{Tr}\Big\{\Sigma_\alpha^r(t,\tau)G^<(\tau,t) + \Sigma_\alpha^<(t,\tau)G^a(\tau,t)\Big\}$$



- *t*→∞ , one can easily obtained the famous famous: *Landauer-Buttiker* formula:

$$J = \frac{\mathrm{i}e}{\hbar}\int \frac{\mathrm{d}\varepsilon}{2\pi}[f_L(\varepsilon) - f_R(\varepsilon)]\mathcal{T}(\varepsilon) \, ,$$

$$\mathcal{T}(\varepsilon) = \mathrm{Tr}\left\{\frac{\Gamma^L(\varepsilon)\Gamma^R(\varepsilon)}{\Gamma^L(\varepsilon)+\Gamma^R(\varepsilon)}[\mathbf{G}^\mathrm{r}(\varepsilon) - \mathbf{G}^\mathrm{a}(\varepsilon)]\right\}$$

- can be applied to mesoscopic systems but still not convenient for studying transient dynamics and quantum decoherence.

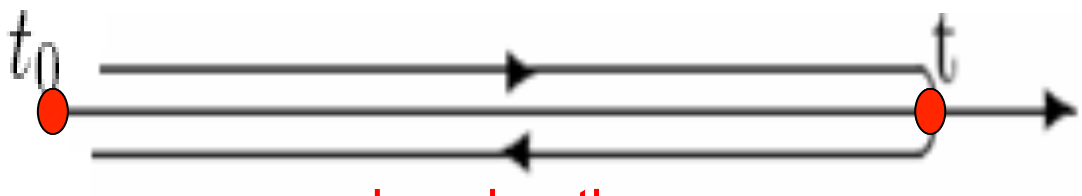
Master equation approach

Feynman & Vernon,
Ann. Phys. (1963)

- a truly nonperturbation way to fully trace over the environmental degrees of freedom, explicitly and completely:

$$\rho(t) = \mathrm{tr}_{\mathrm{B}}[U(t,t_0)\rho(t_0)\otimes\rho_{\mathrm{B}}(t_0)U^{\dagger}(t,t_0)]$$

$\rho(t_0)$



$\rho(t)$

closed-path

$$\mathcal{J}(t,t_0) = \int \mathcal{D}[\bar{\psi}\psi;\bar{\psi}'\psi']e^{i(S_A[\bar{\psi},\psi]-S_A^*[\bar{\psi}',\psi'])}\mathcal{F}[\bar{\psi},\psi;\bar{\psi}',\psi']$$

propagating function

influence functional

→ HPZ master equation for quantum Brownian motion. Hu, Paz & Zhang, PRD45, 2843 (1992)

Quantum transport with master equation.

Tu & WMZ, PRB78, 235311 (2008)
Jin, Tu, WMZ, Yan, NJP12, 183013 (2010)

$$\frac{d\rho(t)}{dt} = -i[H_S(t), \rho(t)] + \sum_\alpha [\mathcal{L}_\alpha^+(t) + \mathcal{L}_\alpha^-(t)]\rho(t)$$

Neither Born-Markov approx. nor Lindblad form

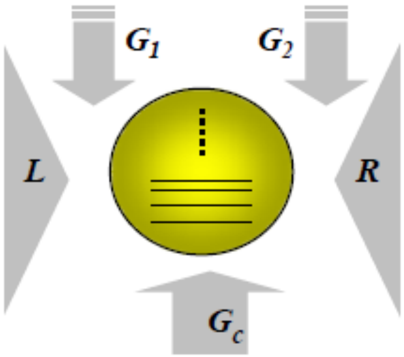
- The super-operators are exactly derived:

$$\mathcal{L}_\alpha^-(t)\rho(t) = \sum_{ij}\left\{\lambda_{\alpha ij}(t)[a_j\rho(t)a_i^\dagger + \rho(t)a_j a_i^\dagger] + \kappa_{\alpha ij}(t)a_j\rho(t)a_i^\dagger + \text{H.c.}\right\}$$

$$\mathcal{L}_\alpha^+(t)\rho(t) = -\sum_{ij}\left\{\lambda_{\alpha ij}(t)[a_i^\dagger a_j\rho(t) + a_i^\dagger\rho(t)a_j] + \kappa_{\alpha ij}(t)a_i^\dagger a_j\rho(t) + \text{H.c.}\right\}$$

- Transient current:

$$I_\alpha(t) = \frac{e}{\hbar}\text{tr}_s[\mathcal{L}_\alpha^+(t)\rho(t)] = -\frac{e}{\hbar}\text{tr}_s[\mathcal{L}_\alpha^-(t)\rho(t)]$$



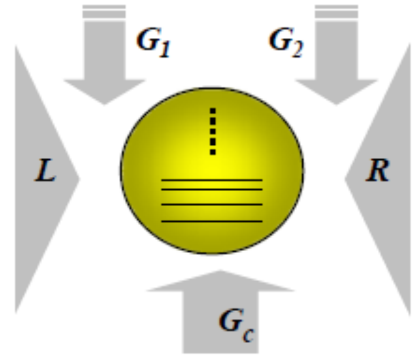
where

$$\boldsymbol{\kappa}_\alpha(t) = \int_{t_0}^t d\tau\, \boldsymbol{g}_\alpha(t,\tau)\boldsymbol{u}(\tau)[\boldsymbol{u}(t)]^{-1}$$

$$\boldsymbol{\lambda}_\alpha(t) = \int_{t_0}^t d\tau\{\boldsymbol{g}_\alpha(t,\tau)\boldsymbol{v}(\tau) - \widetilde{\boldsymbol{g}}_\alpha(t,\tau)\bar{\boldsymbol{u}}(\tau)\} - \boldsymbol{\kappa}_\alpha(t)\boldsymbol{v}(t)$$

retarded and lesser Green functions

Equations of Motion for *u(t)* and *v(t)*



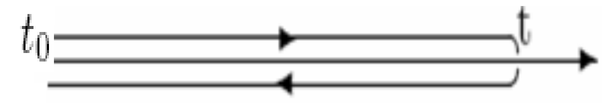
Non-perturbation equations

$$\dot{\boldsymbol{u}}(\tau) + \mathrm{i}\boldsymbol{\epsilon}(\tau)\boldsymbol{u}(\tau) + \sum_{\alpha}\int_{t_0}^{\tau} \mathrm{d}\tau' \boldsymbol{g}_{\alpha}(\tau, \tau')\boldsymbol{u}(\tau') = 0$$

device energy levels

$$\dot{\boldsymbol{v}}(\tau) + \mathrm{i}\boldsymbol{\epsilon}(\tau)\boldsymbol{v}(\tau) + \sum_{\alpha}\int_{t_0}^{\tau} \mathrm{d}\tau' \boldsymbol{g}_{\alpha}(\tau, \tau')\boldsymbol{v}(\tau') = \sum_{\alpha}\int_{t_0}^{t} \mathrm{d}\tau' \widetilde{\boldsymbol{g}}_{\alpha}(\tau, \tau')\bar{\boldsymbol{u}}(\tau')$$

Non-Markovian memory



where

system-reservoir couplings

$$\boldsymbol{g}_{\alpha ij}(\tau, \tau') = \sum_{k} V_{i\alpha k}(\tau) V^{*}_{j\alpha k}(\tau')\mathrm{e}^{-\mathrm{i}\int_{\tau'}^{\tau} \mathrm{d}\tau_1 \epsilon_{\alpha k}(\tau_1)}$$

reservoir's spectra

$$\widetilde{\boldsymbol{g}}_{\alpha ij}(\tau, \tau') = \sum_{k} V_{i\alpha k}(\tau) V^{*}_{j\alpha k}(\tau') f_{\alpha}(\epsilon_{\alpha k})\mathrm{e}^{-\mathrm{i}\int_{\tau'}^{\tau} \mathrm{d}\tau_1 \epsilon_{\alpha k}(\tau_1)}$$

initial particle distribution in reservoirs

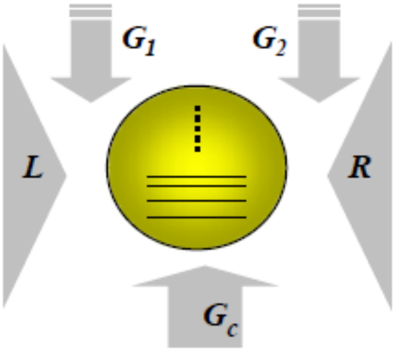
Dissipation-fluctuation theorem

Tu & WMZ, PRB78, 235311 (2008)

Reproduce NEGF:

Jin, Tu, WMZ & Yan, NJP 12, 183013 (2010)

- We reproduce and further generalize the transient current:



$$I_\alpha(t) = -\frac{2e}{\hbar}\mathrm{Re}\int_{t_0}^{t} d\tau\, \mathrm{Tr}\Big\{g_\alpha(t,\tau)v(\tau) - \tilde{g}_\alpha(t,\tau)\bar{u}(\tau) + g_\alpha(t,\tau)u(\tau)\rho^{(1)}(t_0)u^\dagger(t)\Big\}$$

$$= -\frac{2e}{\hbar}\mathrm{Re}\int_{t_0}^{t} d\tau\, \mathrm{Tr}\Big\{\Sigma_\alpha^r(t,\tau)G^<(\tau,t) + \Sigma_\alpha^<(t,\tau)G^a(\tau,t)\Big\}.$$

Wingreen, Jauho & Meir, PRB48, 8487 (1993)

where

$$G^<(\tau,t) = i[u(\tau)\rho^{(1)}(t_0)u^\dagger(t) + v(\tau)]$$
$$= G^r(\tau,t_0)G^<(t_0,t_0)G^a(t_0,t)$$
$$+ \int_{t_0}^{\tau} d\tau_1 \int_{t_0}^{t} d\tau_2\, G^r(\tau,\tau_1)\Sigma^<(\tau_1,\tau_2)G^a(\tau_2,t).$$

As a result of the exact transport theory based on master equation

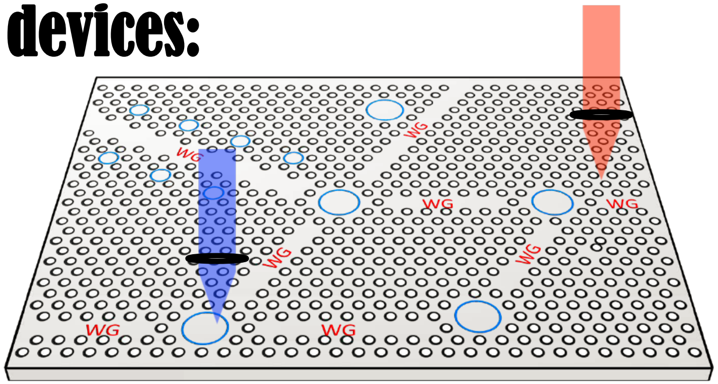
- full nonequilibrium dynamics can be described with the exact master equation.
- quantum decoherence in transport dynamics can be explicitly addressed from the time-evolution of the reduced density matrix.
- the initial state dependence is included so that the non-Markovian memory structure in various transport processes and quantum measurement can be explored explicitly.
- the theory can be used to study various transport phenomena, including energy transfer and heat transfer, etc.

- It may also be used to develop the theory for quantum feedback controlling???

Outline

- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Nonequilibrium dynamics of nanophotonic devices:



$$H = H_S + H_E + H_I$$

$$i\partial\rho_{\text{tot}}(t)/\partial t = [H, \rho_{\text{tot}}(t)]$$

$$\Rightarrow \quad \rho_{\text{tot}}(t) = e^{-iH(t-t_0)}\rho_{\text{tot}}(t_0)e^{iH(t-t_0)}$$

$$\rho_{\text{tot}}(t_0) = \rho(t_0) \otimes \rho_E(t_0)$$

Tu & WMZ, Phys. Rev. B 78, 235311 (2008)

$\Rightarrow$

The System

$$\langle\alpha_f|\rho(t)|\alpha'_f\rangle = \int d\mu(\alpha_0)d\mu\alpha'_0\langle\alpha_0|\rho(t_0)|\alpha'_0\rangle \times \mathcal{J}(\alpha^*_f, \alpha'_f, t|\alpha_0, \alpha'^*_0, t_0)$$

In the coherent state representation

The Environment

WMZ et al., Rev. Mod. Phys. 62, 867 (1990)

Transport theory for photonic network

Lei & WMZ, arXiv:1011.1475 (2010)

Exact master equation:

$$\frac{d\rho(t)}{dt} = -i[H_S(t),\rho(t)] + \sum_\alpha [\mathcal{L}^+_\alpha(t) + \mathcal{L}^-_\alpha(t)]\rho(t) - i\sum_\alpha [f_{\alpha i}(t)a_i^\dagger + f^*_{\alpha i}(t)a_i, \rho(t)]$$

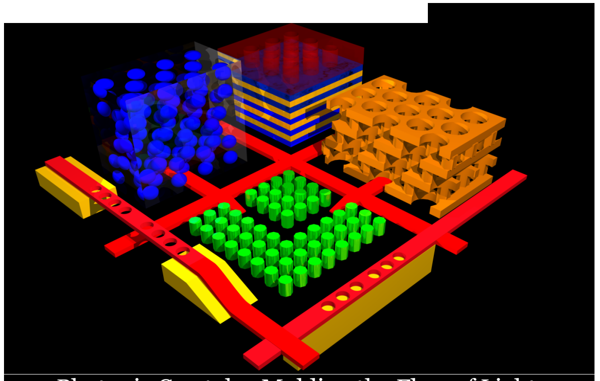
- The super-operators are exactly derived:

$$\mathcal{L}^+_\alpha(t)\rho(t) = \sum_{ij}\{\lambda_{\alpha ij}(t)[a_j\rho(t)a_i^\dagger - \rho(t)a_j a_i^\dagger] - \kappa_{\alpha ij}(t)a_i^\dagger a_j\rho(t) + \text{H.c.}\}$$

$$\mathcal{L}^-_\alpha(t)\rho(t) = \sum_{ij}\{\lambda_{\alpha ij}(t)[a_i^\dagger\rho(t)a_j - a_i^\dagger a_j\rho(t)] + \kappa_{\alpha ij}(t)a_j\rho(t)a_i^\dagger + \text{H.c.}\}$$

- Transient photocurrent:

$$I_\alpha(t) = \text{tr}_\text{s}[\mathcal{L}^+_\alpha(t)\rho(t)] = -\text{tr}_\text{s}[\mathcal{L}^-_\alpha(t)\rho(t)]$$


Photonic Crystals : Molding the Flow of Light, By Johns D. Joannopoulos

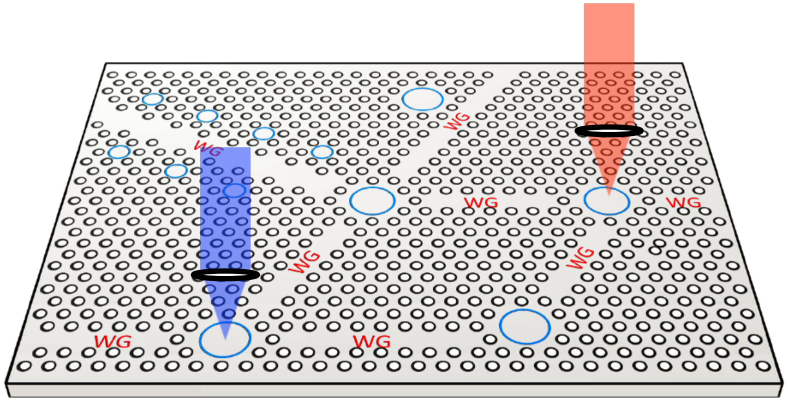
where

$$\kappa_\alpha(t) = \int_{t_0}^t d\tau \mathrm{g}_\alpha(t,\tau)u(\tau,t_0)u^{-1}(t,t_0) ,$$

$$\lambda_\alpha(t) = \int_{t_0}^t d\tau[\mathrm{g}_\alpha(t,\tau)v(\tau,t) - \widetilde{\mathrm{g}}_\alpha(t,\tau)\bar{u}(\tau,t)] - \kappa_\alpha(t)v(t,t) ,$$

$$f_\alpha(t) = i\kappa_\alpha(t)y(t) - i\int_{t_0}^t d\tau \mathrm{g}_\alpha(t,\tau)y(\tau) \qquad y(\tau) = -i\int_{t_0}^\tau u(\tau,\tau')f(\tau')d\tau' ,$$

Non-Markovian dynamics



Non-perturbation equations:

$$\frac{d\boldsymbol{u}(\tau,t_0)}{d\tau}+i\boldsymbol{\omega}\boldsymbol{u}(\tau,t_0)+\int_{t_0}^{\tau}\mathrm{g}(\tau,\tau')\boldsymbol{u}(\tau',t_0)d\tau'=0\ ,$$

$$\frac{d\bar{\boldsymbol{u}}(\tau,t)}{d\tau}+i\boldsymbol{\omega}\bar{\boldsymbol{u}}(\tau,t)-\int_{\tau}^{t}\mathrm{g}(\tau,\tau')\bar{\boldsymbol{u}}(\tau',t)d\tau'=0\ ,$$

$$\frac{d\boldsymbol{v}(\tau,t)}{d\tau}+i\boldsymbol{\omega}\boldsymbol{v}(\tau,t)+\int_{t_0}^{\tau}\mathrm{g}(\tau,\tau')\boldsymbol{v}(\tau',t)d\tau'=\int_{t_0}^{t}\widetilde{\mathrm{g}}(\tau,\tau')\bar{\boldsymbol{u}}(\tau',t)d\tau'$$

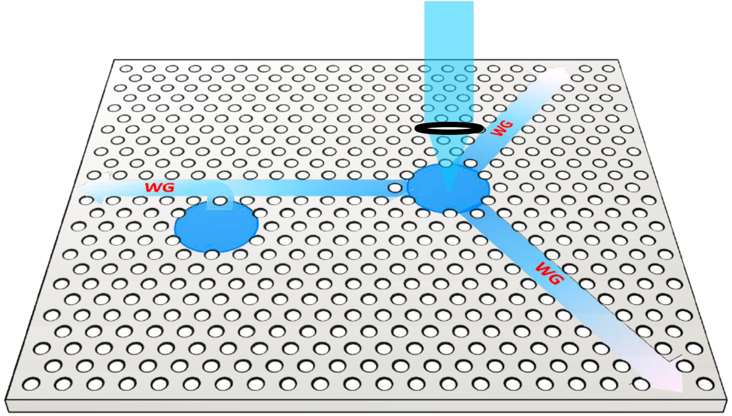
$$\frac{d\boldsymbol{y}(\tau)}{d\tau}+i\boldsymbol{\omega}\boldsymbol{y}(\tau)+\int_{t_0}^{\tau}\mathrm{g}(\tau,\tau')\boldsymbol{y}(\tau')d\tau'=-i\boldsymbol{f}(\tau)$$

$$\bar{\boldsymbol{u}}(\tau,t)=\boldsymbol{u}^{\dagger}(t,\tau)\ ,$$

$$\boldsymbol{y}(\tau)=-i\int_{t_0}^{\tau}\boldsymbol{u}(\tau,\tau')\boldsymbol{f}(\tau')d\tau'\ ,$$

$$\boldsymbol{v}(\tau,t)=\int_{t_0}^{\tau}d\tau'\int_{t_0}^{t}d\tau''\boldsymbol{u}(\tau,\tau')\widetilde{\mathrm{g}}(\tau',\tau'')\bar{\boldsymbol{u}}(\tau'',t)$$

Generalize the NEGF theory:



- Non-equilibrium GFs

  Jin, Tu, WMZ & Yan, NJP (2010)

$$u_{ij}(t_1,t_2)=\theta(t_1-t_2)\langle[a_i(t_1),a_j^\dagger(t_2)]\rangle\equiv iG^r_{ij}(t_1,t_2),$$

$$\bar{u}_{ij}(t_1,t_2)=\theta(t_2-t_1)\langle[a_i(t_1),a_j^\dagger(t_2)]\rangle\equiv -iG^a_{ij}(t_1,t_2),$$

$$\rho^{(1)}_{ij}(t_1,t_2)=\langle a_j^\dagger(t_2)a_i(t_1)\rangle\equiv -iG^<_{ij}(t_1,t_2).$$

- Explicit and complete solution:

  Lei & WMZ, arXiv:1011.1475 (2010)

$$\Big\{i\frac{d}{d\tau}-\omega\Big\}G^r(\tau,t_0)=\delta(\tau-t_0)+\int_{t_0}^{\tau}\Sigma^r(\tau,\tau')G^r(\tau',t_0)d\tau'$$

$$G^<(\tau,t)=iy(\tau)y^\dagger(t)-G^r(\tau,t_0)\langle a^\dagger(t_0)\rangle y^\dagger(t)+y(\tau)\langle a(t_0)\rangle G^a(t_0,t)$$
$$+G^r(\tau,t_0)G^<(t_0,t_0)G^a(t_0,t)+\int_{t_0}^{\tau}d\tau_1\int_{t_0}^{t}d\tau_2\; G^r(\tau,\tau_1)\Sigma^<(\tau_1,\tau_2)G^a(\tau_2,t)$$

Photonic quantum transport theory

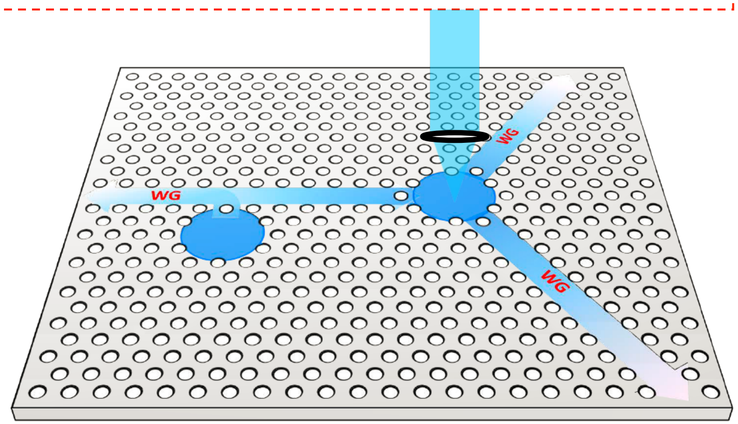
Lei & WMZ, arXiv:1011.1475 (2010)

Generalized quantum kinetic equation

$$G^<(\tau,t) = iy(\tau)y^\dagger(t) - G^r(\tau,t_0)\langle a^\dagger(t_0)\rangle y^\dagger(t) + y(\tau)\langle a(t_0)\rangle G^a(t_0,t) + G^r(\tau,t_0)G^<(t_0,t_0)G^a(t_0,t) + \int_{t_0}^{\tau} d\tau_1 \int_{t_0}^{t} d\tau_2\, G^r(\tau,\tau_1)\Sigma^<(\tau_1,\tau_2)G^a(\tau_2,t)$$

$$n_i(t) = \mathrm{tr}_s[a_i^\dagger a_i \rho(t)] = iG_{ii}^<(t,t)$$

Transient photocurrent



$$I_\alpha(t) = 2\mathrm{Re}\int_{t_0}^{t} d\tau \mathrm{Tr}[\mathrm{g}_\alpha(t,\tau)\rho^{(1)}(\tau,t) - \tilde{\mathrm{g}}_\alpha(t,\tau)\bar{u}(\tau,t)]$$

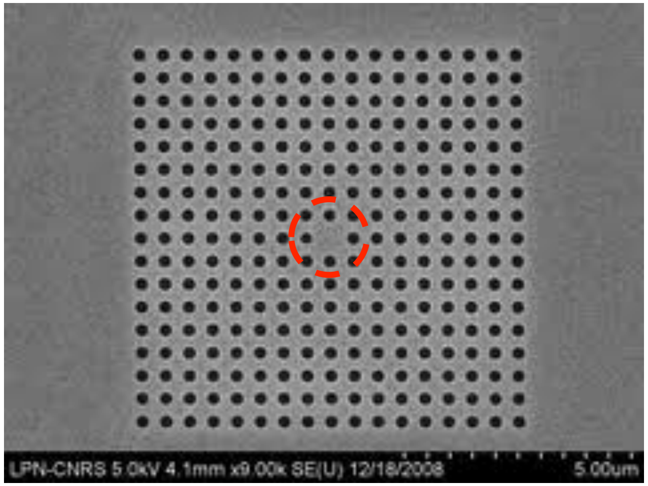
$$= 2\mathrm{Re}\int_{t_0}^{t} d\tau \mathrm{Tr}[\Sigma_\alpha^r(t,\tau)G^<(\tau,t) + \Sigma_\alpha^<(t,\tau)G^a(\tau,t)]$$

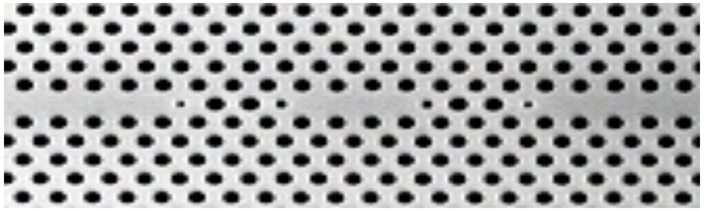
Outline

- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Quantum devices with micro/nano cavities

High-Q photonic nanocavity





A typical quantum device: with micro/nano-cavity build on photonic crystals coupled to waveguides, which has the potential application for light propagating and for storage.

Let us start with such a cavity coupled to a general reservoir

$$H_{\mathrm{tot}} = \omega_C a^\dagger a + \sum_k \omega_k b_k^\dagger b_k + \sum_k (V_k a^\dagger b_k + V_k^* b_k^\dagger a)$$

Exact master equation:

Xiong, WMZ, Wang, Wu, PRA 82, 012105 (2010)

$$\dot{\rho}(t) = -i\omega_0'(t)[a^\dagger a, \rho(t)] + \kappa(t)\{2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\} + \widetilde{\kappa}(t)\{a^\dagger\rho(t)a + a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)aa^\dagger\},$$

where

$$\omega_0'(t) = -\mathrm{Im}[\dot{u}(t)u^{-1}(t)],$$
$$\kappa(t) = -\mathrm{Re}[\dot{u}(t)u^{-1}(t)],$$
$$\widetilde{\kappa}(t) = \dot{v}(t) - 2v(t)\mathrm{Re}[\dot{u}(t)u^{-1}(t)],$$

and

$$\dot{u}(\tau) + i\omega_0 u(\tau) + \int^{\tau} d\tau' g(\tau - \tau')u(\tau') = 0,$$
$$v(t) = \int_{t_0}^{t} d\tau_1 \int_{t_0}^{t} d\tau_2\, \overline{u}(\tau_1)\widetilde{g}(\tau_1 - \tau_2)\overline{u}^*(\tau_2).$$

$$g(\tau - \tau') = \int_0^\infty \frac{d\omega}{2\pi} J(\omega)e^{-i\omega(\tau-\tau')}, \quad \widetilde{g}(\tau - \tau') = \int_0^\infty \frac{d\omega}{2\pi} J(\omega)\overline{n}(\omega, T)e^{-i\omega(\tau-\tau')},$$

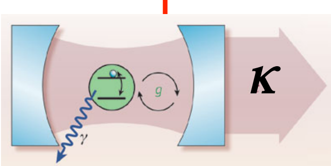
$$J(\omega) = 2\pi g(\omega)|V(\omega)|^2$$

Born-Markov approximation:

Born-Markov approx.: taking the coefficients in the master equation up to the second order of the coupling between the cavity field and the thermal field

Markov limit: $t \gg \tau_\varepsilon$ (the character time of the thermal field) , or equivalently, taking $t \to \infty$:

Carmichael's textbook



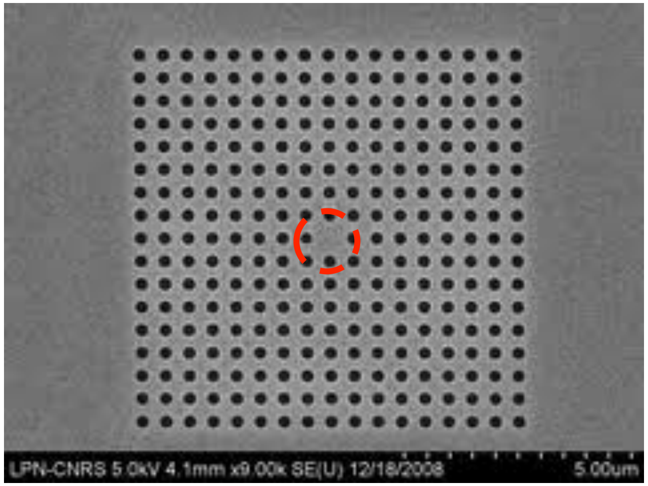
$$\dot{\rho}(t) = -\frac{i}{\hbar}(\omega_C + \Delta)[a^\dagger a, \rho(t)] + \kappa\left[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\right] + 2\kappa\bar{n}(\omega_C, T)\left[a^\dagger\rho(t)a + a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)aa^\dagger\right],$$

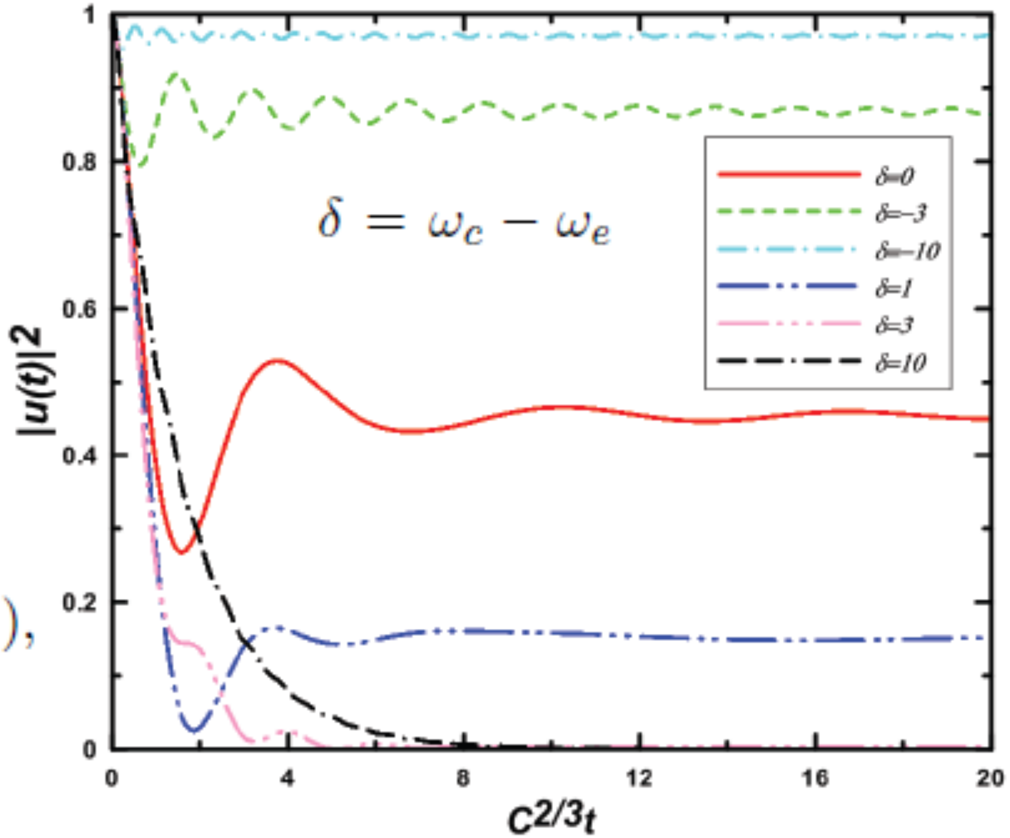
with

$$\Delta = \mathcal{P}\int_0^\infty d\omega \frac{g(\omega)|V(\omega)|^2}{\omega - \omega_0}, \quad \kappa = \pi g(\omega_C)|V(\omega_C)|^2,$$

Photon confinement in photonic crystals

a nanocavity (defect) in photonic crystals: Nonmarkovian dynamics and Photon confinement!





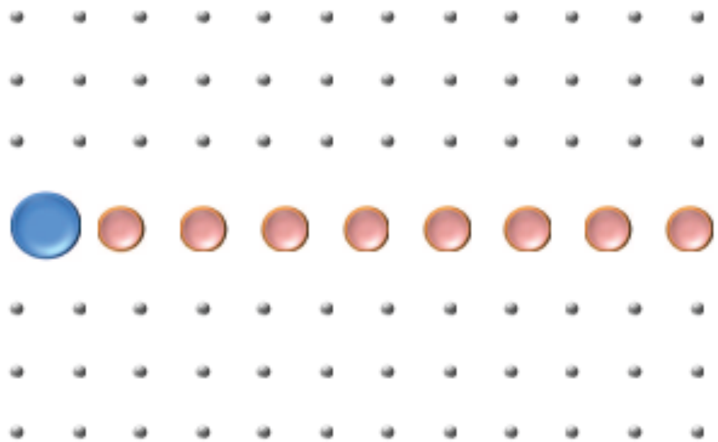
$$J(\omega) = \frac{C}{\pi} \frac{\sqrt{\omega - \omega_e}}{\omega - \omega_e + \epsilon} \theta(\omega - \omega_e),$$

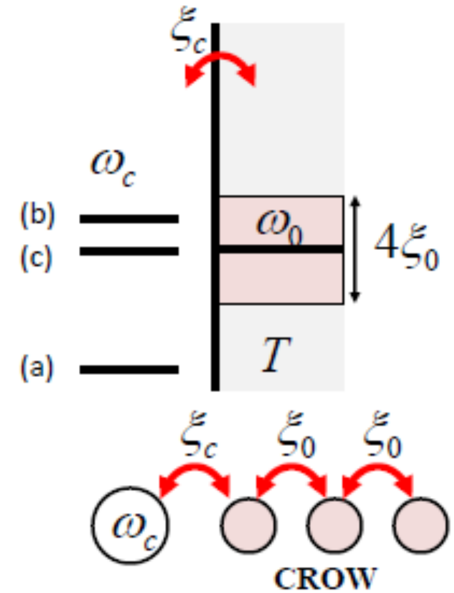
S. John, PRL58, 2486 (1990)

Wu & WMZ, (2011)

photonic crystals are lossless materials

Nanocavity coupled to a waveguide





$$H=\omega_c a^\dagger a+\sum_n \omega_0 a_n^\dagger a_n-\sum_{n=1}^{N}\xi_0(a_n^\dagger a_{n+1}+\text{H.c.})+\xi(a^\dagger a_1+\text{H.c.})$$

- Taking the waveguide as a reservoir

$$\omega_k=\omega_0-2\xi_0\cos(k),\ V_k=\sqrt{2/\pi}\,\xi\sin(k),$$

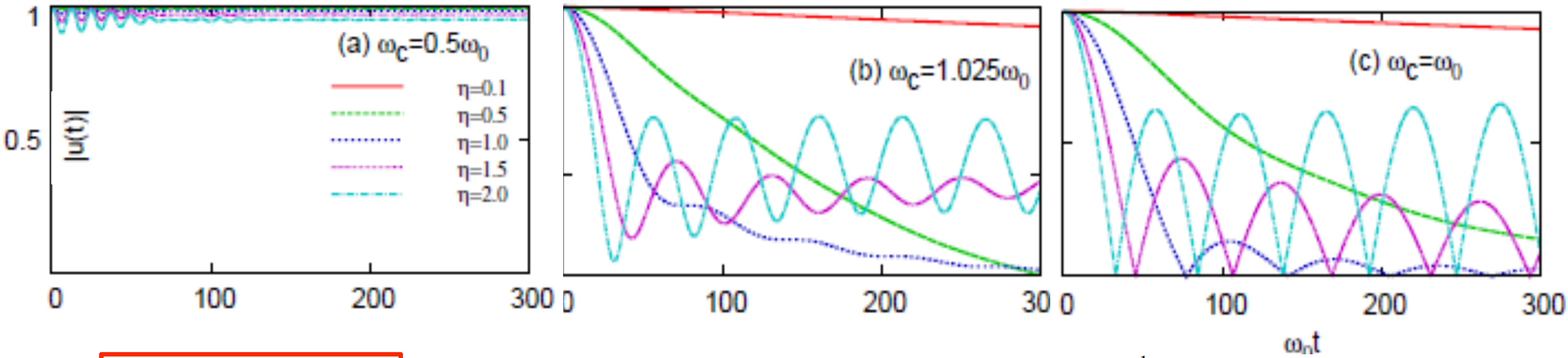
- Spectral density:

$$J(\omega)=\left(\frac{\xi}{\xi_0}\right)^2\sqrt{4\xi_0^2-(\omega-\omega_0)^2}\qquad \omega_0-2\xi_0<\omega<\omega_0+2\xi_0$$

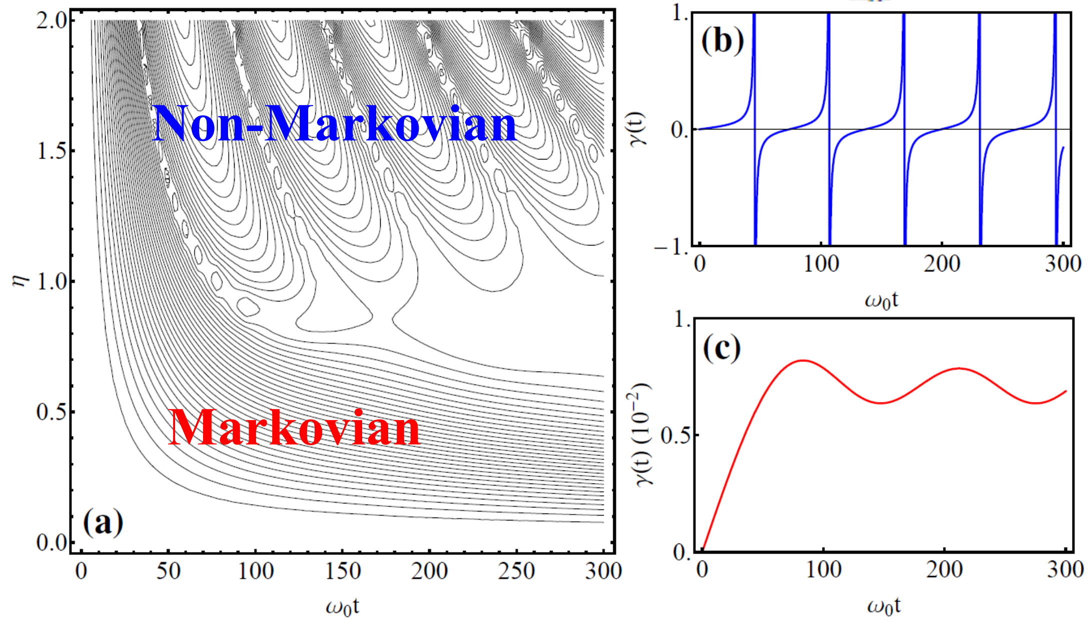
Wu, Lei, WMZ & Xiong, Opt. Express, 18, 18407 (2010)

Non-Markovian dynamics

Wu, Lei, WMZ & Xiong, Opt. Express, 18, 18407 (2010)

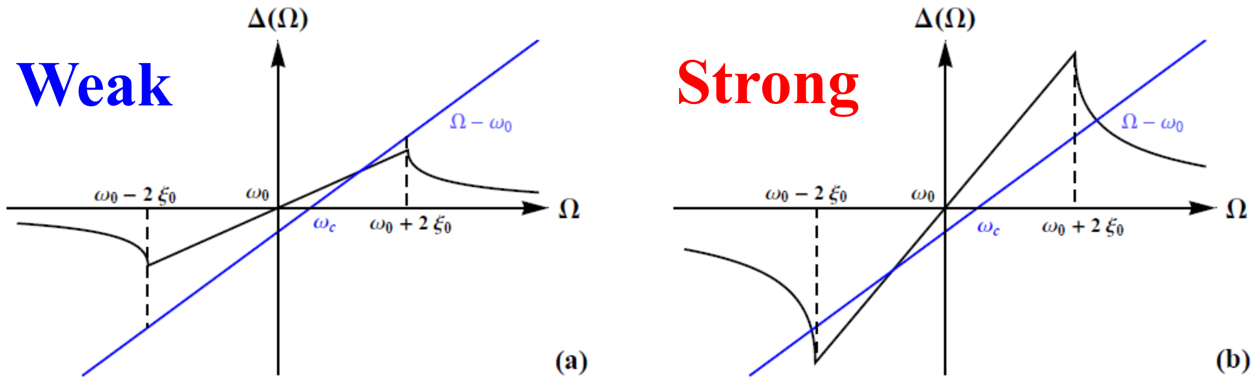


$$\langle a(t)\rangle = u(t)\langle a(t_0)\rangle$$



A phase transition from Markovian to Non-Markovian dynamics

Analysis – Strong Interaction



Bound modes

$$\Omega - \omega_c = \Delta(\Omega)$$

Critical Coupling

$$\eta_c = \sqrt{2 - \frac{|\omega_c - \omega_0|}{\xi_0}}$$

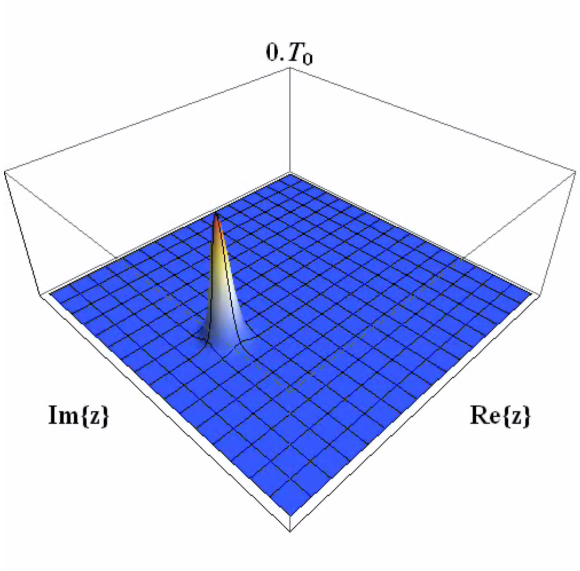
Non-dissipative Steady state solution

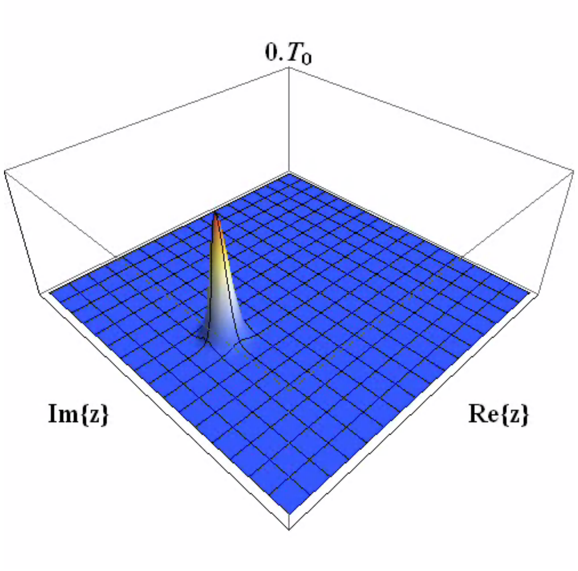
$$\omega_c = \omega_0$$

$$u_{\rm st}(t) = \frac{\eta^2 - 2}{\eta^2 - 1} e^{-i\omega_0 t} \cos\left(\frac{\eta^2}{\sqrt{\eta^2 - 1}} \xi_0 t\right)$$

Time Evolution of Wigner function

Initial Coherent State



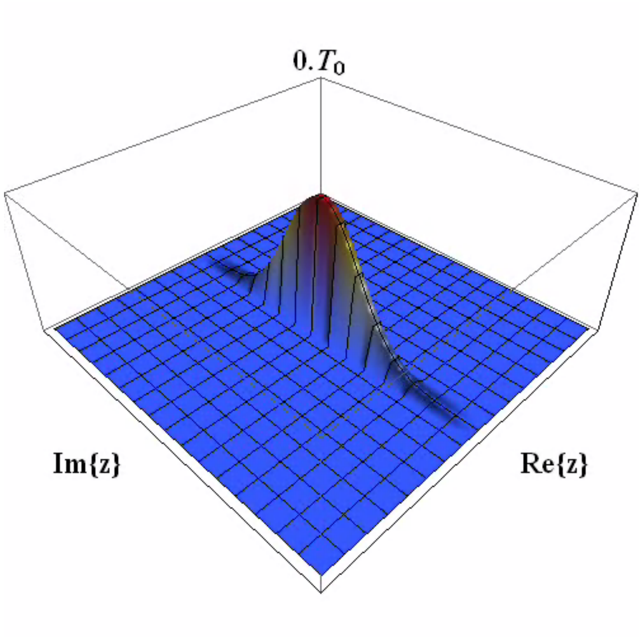


η=0.15 T=0.5K

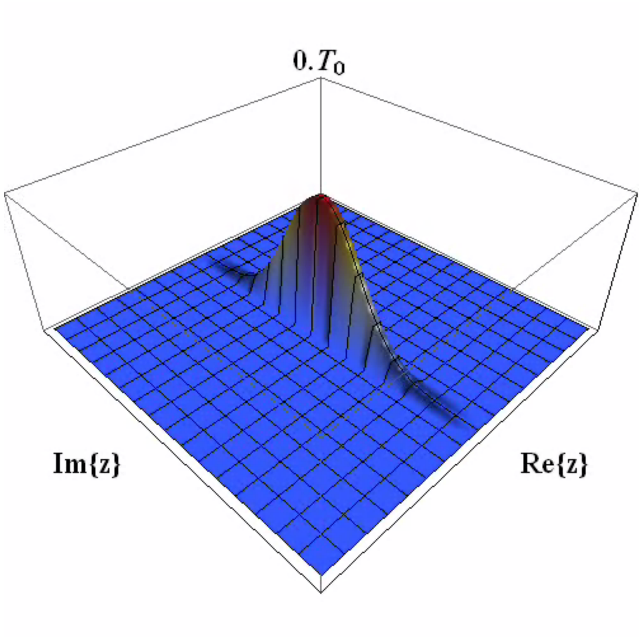
η=2.0 T=0.05K

Time Evolution of Wigner function

Initial Squeezed State



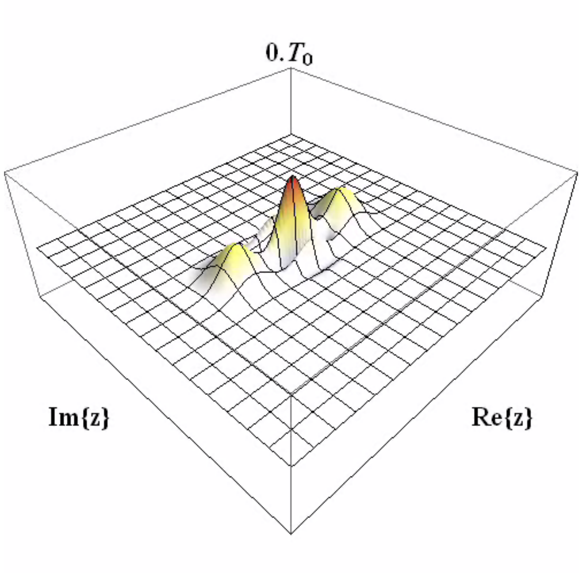
η=0.15 T=0.5K

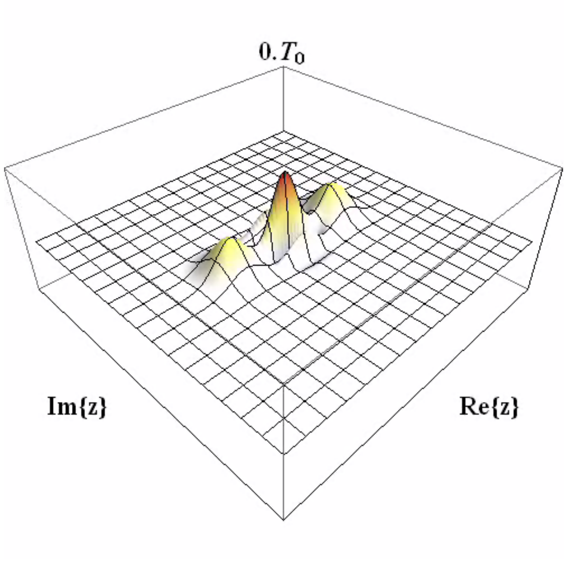


η=2.0 T=0.05K

Time Evolution of Wigner function

Initial "Schrodinger-like" Cat State

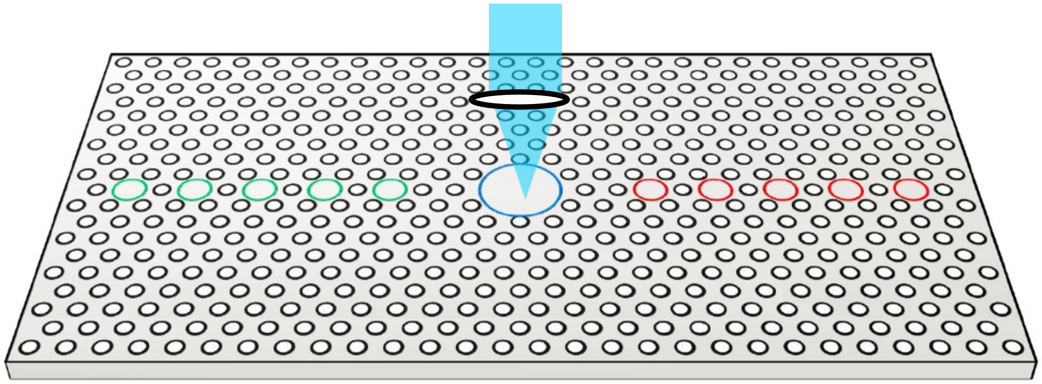


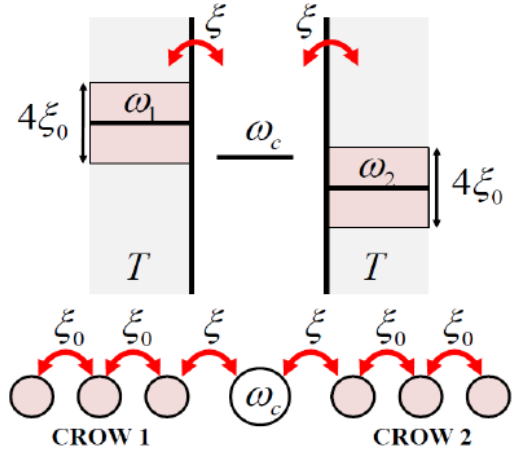


η=0.15 T=0.5K   *η=2.0 T=0.05K*

- **Quantum Coherence Protection via Strong Coupling !**

Driven nanocavity





➢Hamiltonian

$$H = \omega_c a^\dagger a + (E_0 e^{-i\omega_d t} a^\dagger + E_0 e^{i\omega_d t} a)$$
$$+ \sum_{\alpha=1}^{2} \sum_k \omega_{\alpha k} c^\dagger_{\alpha k} c_{\alpha k} + \sum_{\alpha=1}^{2} \sum_k (V_{\alpha k} a^\dagger c_{\alpha k} + V^*_{\alpha k} a c^\dagger_{\alpha k})$$

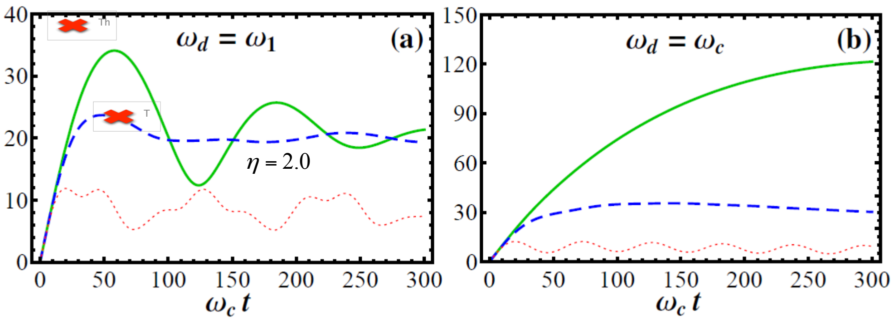
Lei &WMZ, arXiv: 1011.1475 (2010).

Photonic coherence controlled by external driving field

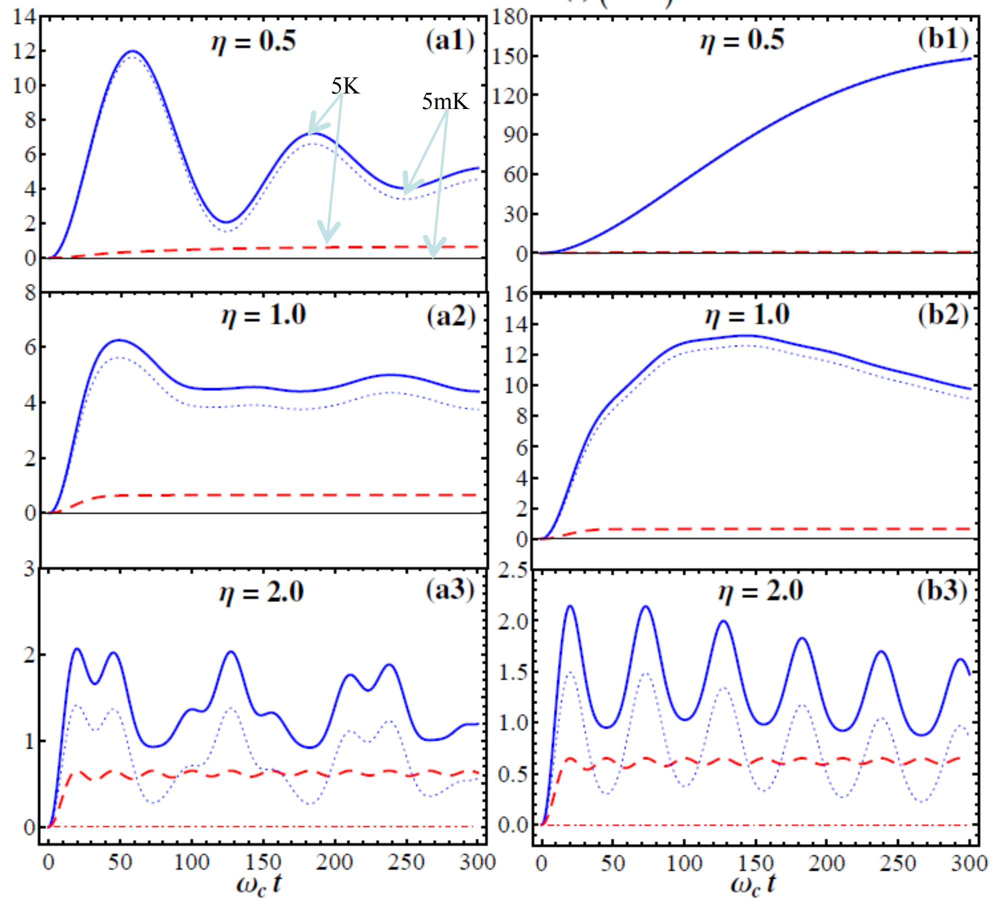
$$\langle a(t)\rangle = y(t) = -i\int_{t_0}^{\tau} u(\tau,\tau')f(\tau')d\tau'$$

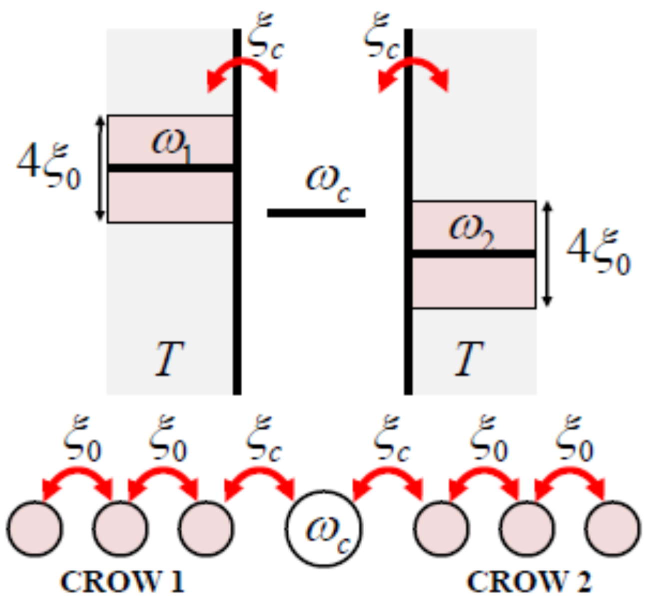
$$n(t) = \langle a^{\dagger}(t)a(t)\rangle = v(t,t) + |y(t)|^2$$

Field Amplitude |<a(t)>|

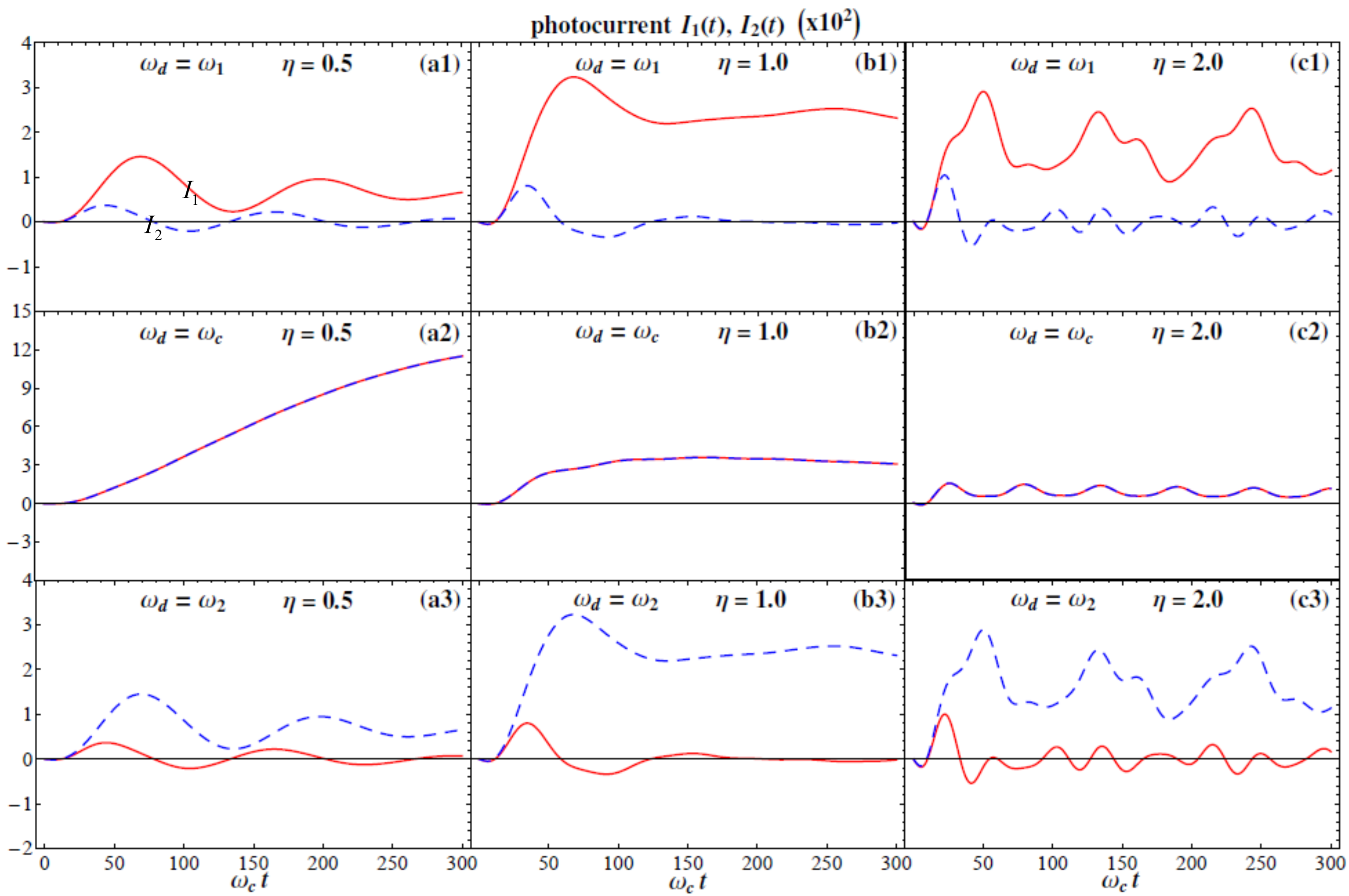


Photon Number $n(t)$ (x10^2)

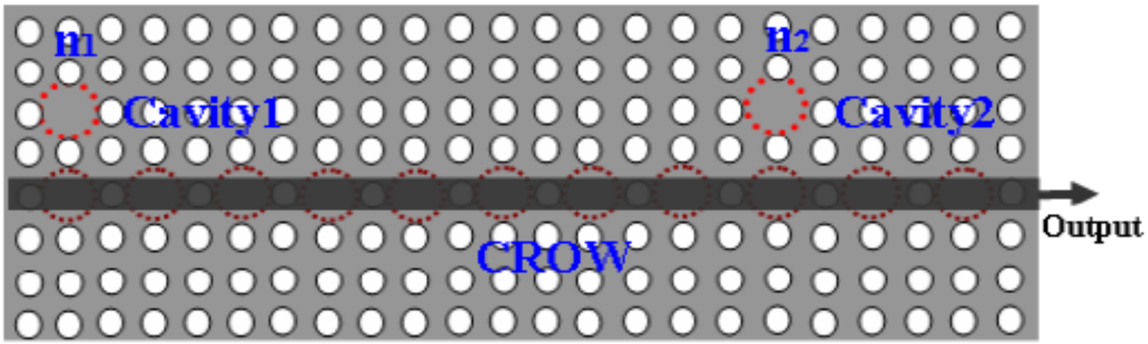




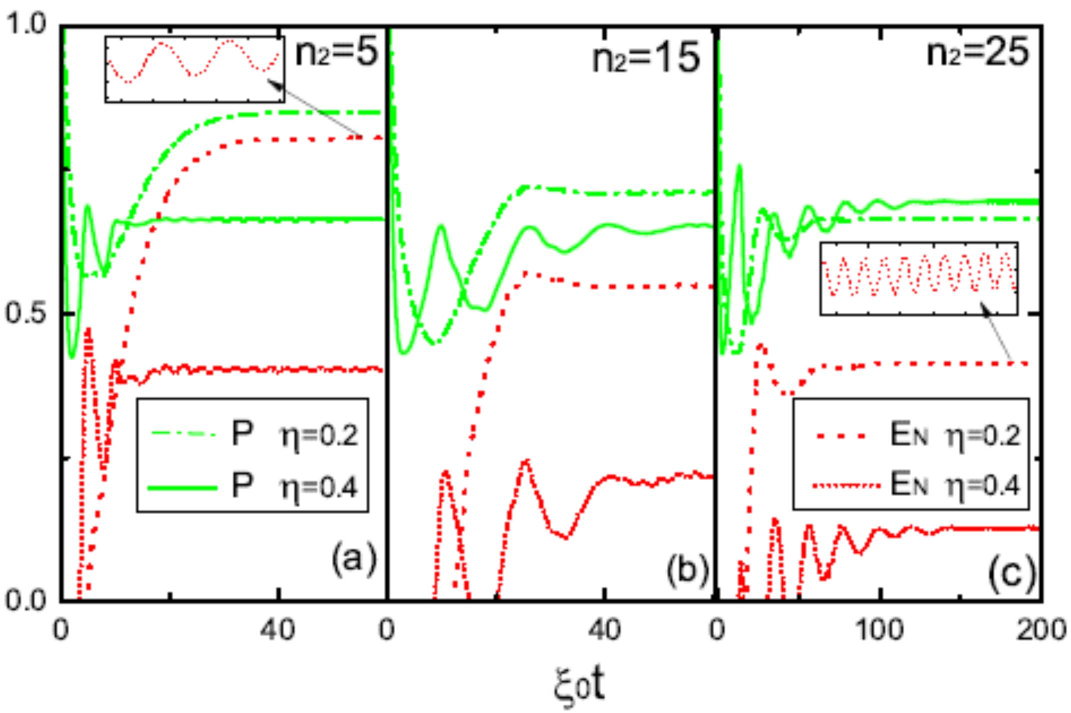
Photonic transport controlled by external driving field

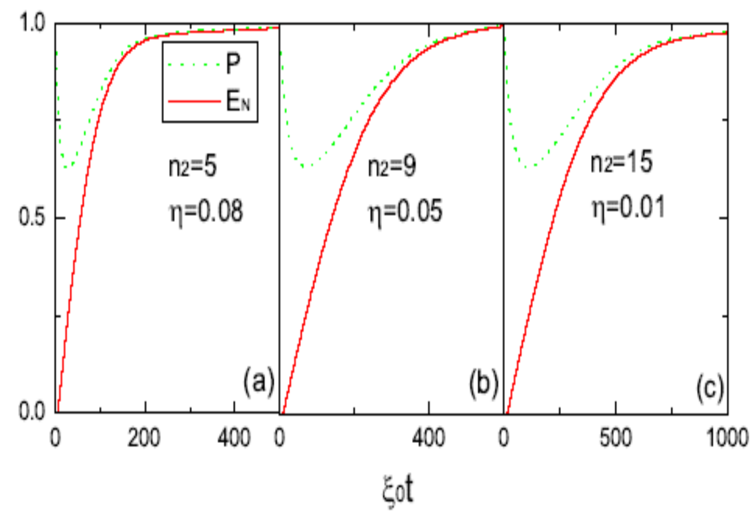


- Entanglement generation between two spatially-separated nanocavities through a waveguide



Tan, WMZ & Li,
PRA83, 062310 (2011)





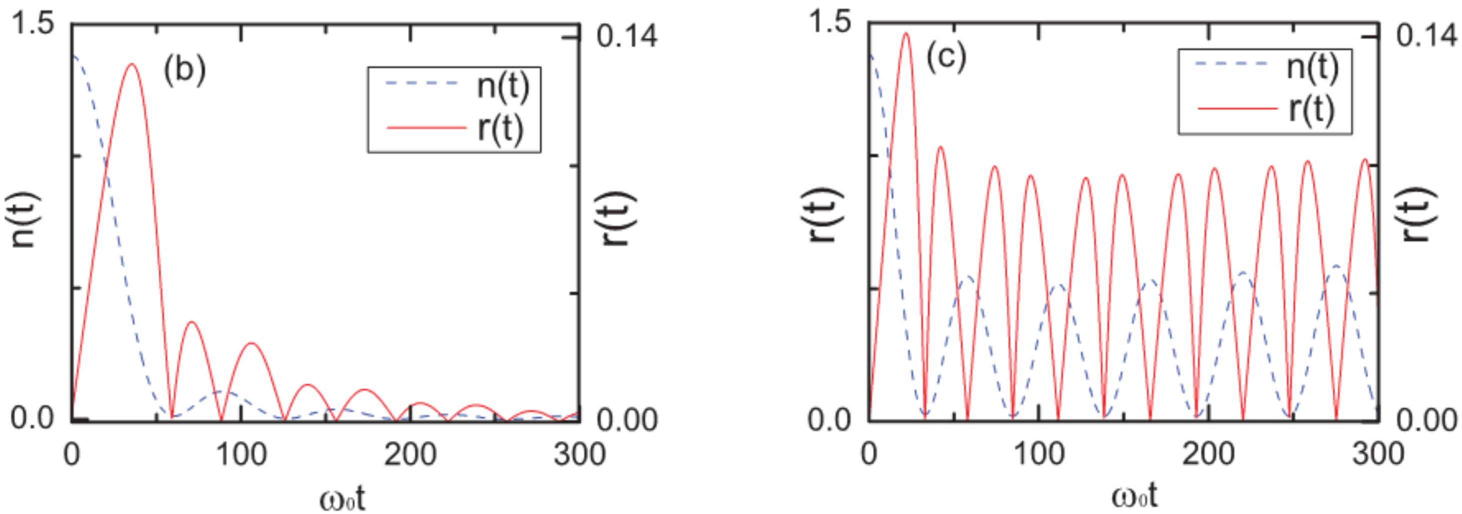
Entangled squeezed state:
Logarithmic negativity

Other interesting development

- Exact master equation with initial system-reservoir correlations

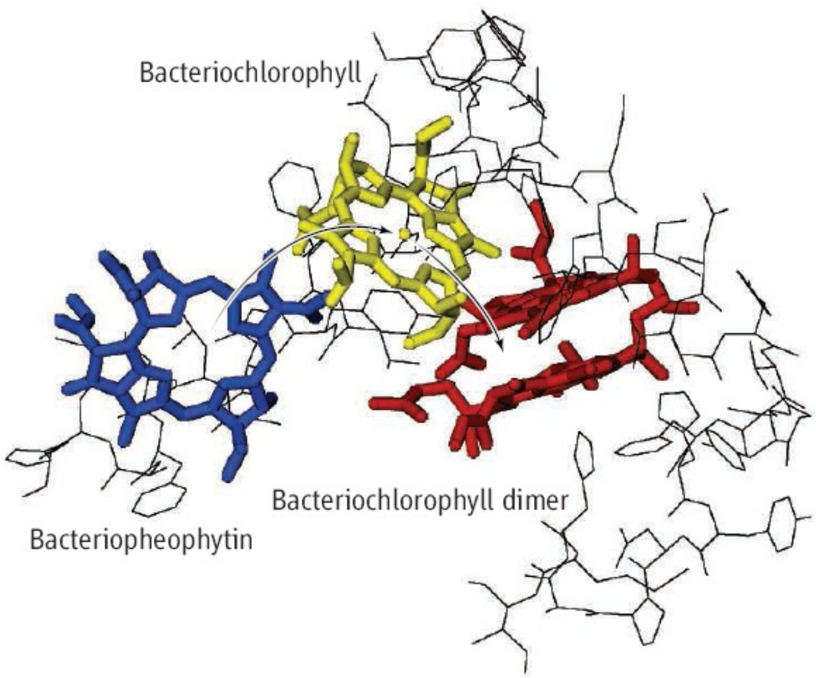
Tan & WMZ, Phys. Rev. A83, 032102 (2011)

$$\begin{aligned}\dot{\rho}(t) = & -i\Delta(t)[a^\dagger a, \rho] \\ & + \gamma_1(t)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) \\ & + \gamma_2(t)(a\rho a^\dagger + a^\dagger \rho a - a^\dagger a\rho - \rho a a^\dagger) \\ & + \gamma_3^*(t)(2a\rho a - aa\rho - \rho aa) \\ & + \gamma_3(t)(2a^\dagger \rho a^\dagger - a^\dagger a^\dagger \rho - \rho a^\dagger a^\dagger),\end{aligned}$$



Energy transfer in photosynthesis:

1. Recent experiments show that the energy transfer in photosynthesis may involve quantum coherence channel. H. Lee, Y.-C. Cheng and G. R. Fleming, Science, 316, 1462 (2007)



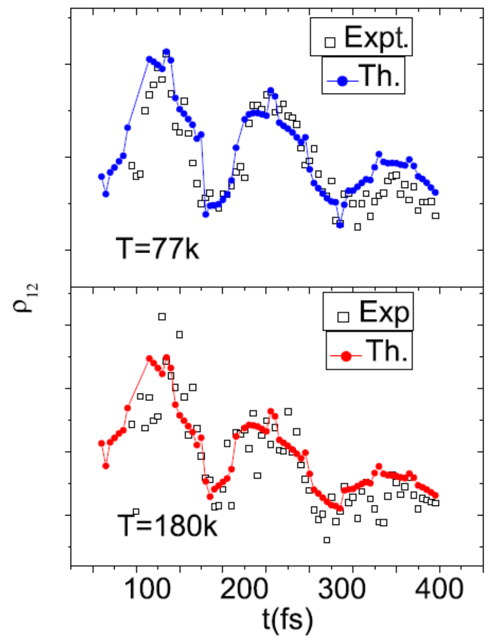


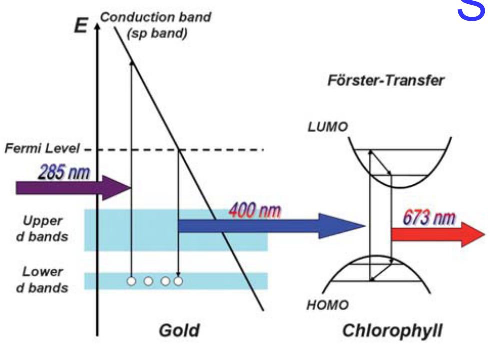
FIG. 2. (Color online) The evolutions of the off-diagonal coherent terms of the reduced density matrix for the two-level subsystem with the Ohmic bath at temperature (a) 77 and (b) 180 K.

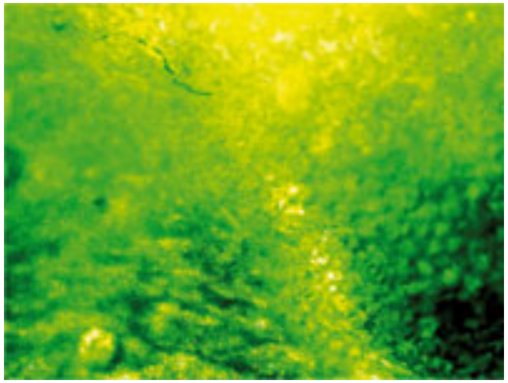
Liang, WMZ & Zhuo, PRE81, 011906 (2010)

Nanoparticles make leaves glow:

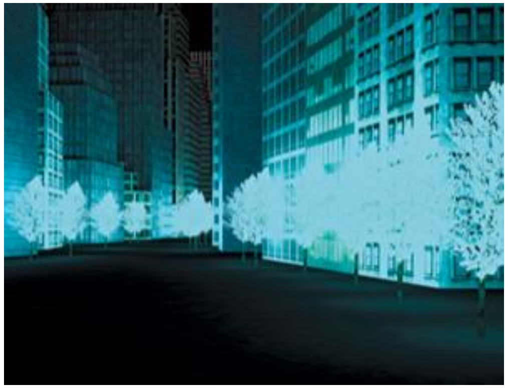
A new idea on Bio-LED

Su, Tu, Tseng, Chang & WMZ, Nanoscale 2, 2639 (2010).





- Highlight in Chemistry World
- Interviews by NewScientist and Reuters
- Selected as Cutting Edge Chemistry in 2010
- Reported by Discovery News and other over hundred medias over the world

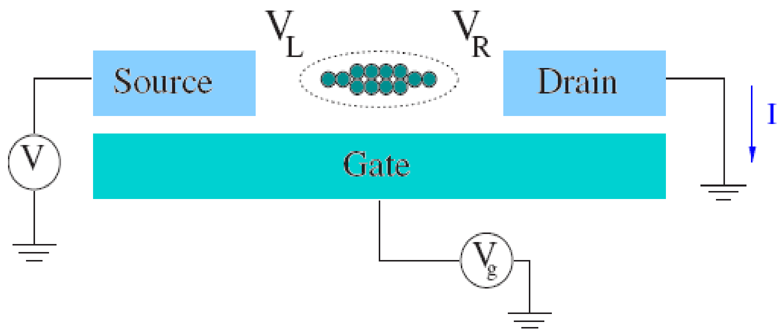


Outline

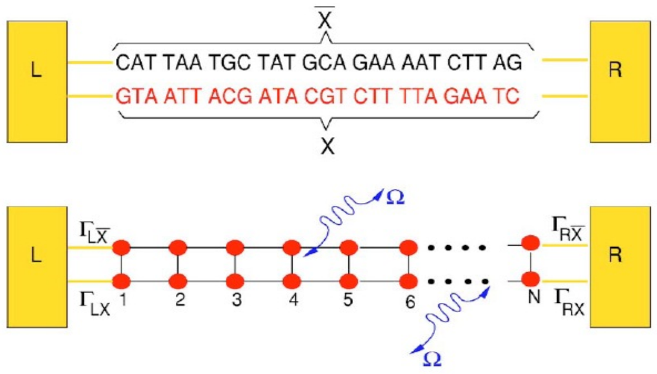
- Introduction of dissipative transport dynamics
- General theory for electronic quantum transport
- Development of photonic quantum transport theory
- Applications to various nanophotonic devices
- Prospective and further development

Bio-junctions: further development

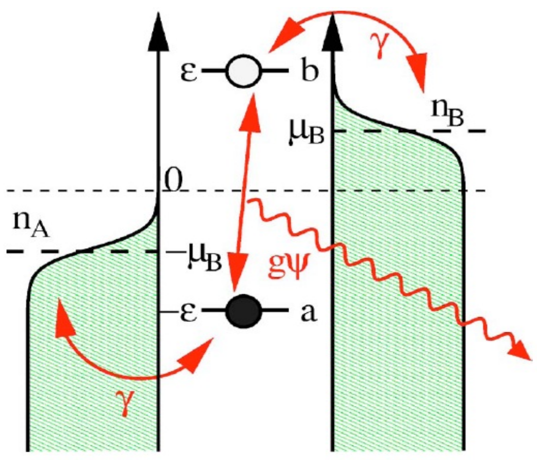
1. Molecular electronics



2. DNA junctions

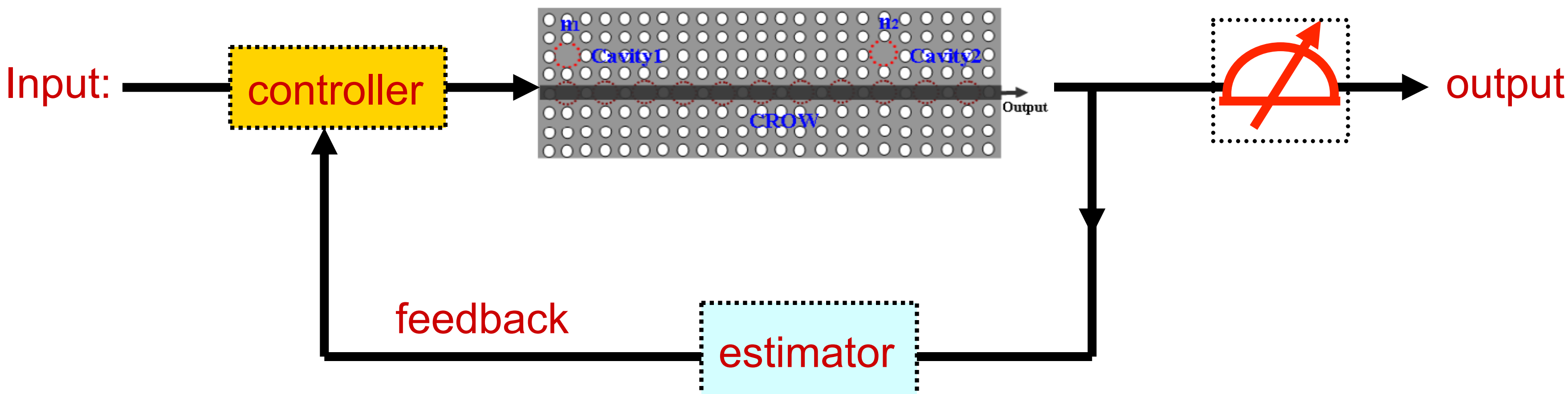


3. Photonic electronics



Theory for quantum feedback controls ??

$$\begin{cases} \dfrac{d\rho(t)}{dt} = -i[H_S(t), \rho(t)] + \sum_\alpha [\mathcal{L}_\alpha^+(t) + \mathcal{L}_\alpha^-(t)]\rho(t), & \text{State evolution} \\ I_\alpha(t) = \mathrm{tr_s}[\mathcal{L}_\alpha^+(t)\rho(t)] = -\mathrm{tr_s}[\mathcal{L}_\alpha^-(t)\rho(t)] & \text{Measurement} \end{cases}$$



- How to make Feedback controls ???

Summary:

- the exact master equation is first developed for studying the time-evolution of the entangled squeezed state and entangled coherent state at zero-temperature.
  An & WMZ, PRA76, 042127 (2007)
  An, Feng & WMZ, QIC 9, 317 (2009)

- then we developed the exact master equation for studying the non-Markovian decoherence dynamics of various nanoelectronic devices at an arbitrary temperature.
  Tu & WMZ, PRB78, 235311 (2008)
  Tu, Lee & WMZ, QIP 8, 631 (2009)

- the exact quantum transport theory is further developed from the exact master equation for studying the transient electronic transport phenomena in mesoscopic systems, which generalizes the transport theory of Keldysh's non-equilibrium GF technique. Jin, Tu, WMZ & Yan, NJP 12, 183013 (2010)

- the exact master equation including explicitly the initial system-reservoir is also obtained.
  Tan & WMZ, PRA83, 032102 (2011)

- in this work, we extend the exact master equation and transport theory with explicitly external fields applied to the system and also the reservoirs
  Lei & WMZ, arXiv: 1011.1475 (2010)

Applications:

- phase localization and decoherence dynamics in double-dot AB interferometer.
  Tu, WMZ & Jin, PRB83, 115318 (2011)
  Tu, WMZ, Jin, Entin-Wohlman & Aharony (in preparation)

- precision control of qubit coherence through cross-correlations.
  Jin, WMZ, Tu & Wang, arXiv:1103. 5099 (2011)

- non-Markovian dynamics in nanocavity systems.
  Xiong, WMZ, Wang & Wu, PRA 82, 012105 (2010)
  Wu, Lei, WMZ & Xiong, Opt. Express. 18, 18407 (2010)

- entanglement generation through nanostructure wave-guide
  Tan & WMZ, PRA83, 062310 (2011)

- single-electron turnstile pumping mechanism
  Lin & WMZ, APL 99, 072105 (2011)

- noise spectrum and full-counting statistics
  Jin *et al.*, arXiv: 1105.0136 (2011)

Conclusions:

- It has been attempted for many decades without a very satisfactory answer to find the exact master equation for an arbitrary open quantum system since Pauli first proposed the phenomenological master equation in 1928.

- We utilized the coherent state path integral approach to reformulate Feynman-Vernon influence functional approach and derived an exact master equation for a large class of nanoelectronic devices (electronic nanostructures coupled with multiple electrodes for control and measurement) and various nanophotonic devices.

- We believe that the new master equation is a crucial step toward establishing the nonequilibrium quantum theory for arbitrary open systems, one of the most difficulty problems that has been struggled for many decades without a significant achievement during the 20th century.

  - Hopefully, such theory can also be extended to the study of bio-systems and other more complex open systems in nature

Prospective: Physics in 21th Century

- **Closed systems:**

  most of problems have been solved with well-developed perturbation theories except for some strongly correlated systems that need a nonperturbation treatment which has not been developed yet

- **Open systems:**

  most of problems have been over-looked in 20th century and also no well-developed theory has be established to address many unsolved issues

Acknowledgement:

Chun U Lei	(NCKU, Caltech.)
Matisse W. Y. Tu	(NCKU)
Jinshuang Jin	(Hongzhou Normal Univ.)
Jun Hong An	(Lazhou University, NUS)
Ming-Tsung Lee	(RCAS, Academic Sinica)
Menh-Hsiu Wu	(NCKU)
Hua-Tang Tan	(Huachung Normal Univ.)
Heng-Na Xiong	(Zhejiang Univ.)
Chuan-Yu Lin	(NCKU)
Nien-An Wang	(NCKU)

Historical development of the master equation for open systems

- Pauli Master equation (W. Pauli, *Festschrift zum 60. Geburtstage A. Sommerfelds* (Hirzel, Leipzig, p.30, 1928)

$$\dot{P}_k = -\gamma_k P_k + \sum_{k'} (W_{kk'} P_{k'} - W_{k'k} P_k).$$

- Generalized master equation (S. Nakajima, Prog. Theo. Phys. **20**, 948, 1958; R. Zwanzig, J. Chem. Phys. **33**, 1338, 1960):

$$\rho_{\text{tot}} = \wp\rho_{\text{tot}} + (1-\wp)\rho_{\text{tot}}$$

$$\begin{aligned}\frac{d}{dt}\wp\rho_{\text{tot}}(t) =& \wp L\wp\rho_{\text{tot}}(t) + \wp L e^{(1-\wp)Lt}(1-\wp)\rho_{\text{tot}}(0) \\ &+ \int_0^t d\tau \wp L e^{(1-\wp)Lt}(1-\wp)L\wp\rho_{\text{tot}}(t-\tau)\end{aligned}$$

- Master equation under Born Approximation (e.g. F. Haake, Z. Phys. **223**, 364, 1969):

$$\rho_{\text{tot}}(0) = \rho(0) \otimes \rho_{\text{r}}^{\text{Eq}}$$

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H_S, \rho(t)] - \frac{1}{\hbar^2}\int_{t_0}^t d\tau \langle [\widetilde{H}_I(t), [\widetilde{H}_I(\tau), \widetilde{\rho}(\tau)]]^I \rangle_E$$

GME has been mainly used for investigating non-Markovian processes due to the fact that the master equation involves an explicit time integration

Markovian processes

- Since 1970's, one made further approximation: taking the perturbation up to the second order ➔ Born-Markov (or Redfield or Lindblad) master equation, for example:

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H_s, \rho(t)] + \frac{\gamma(t)}{2}(2\sigma^-\rho(t)\sigma^+ - \sigma^+\sigma^-\rho(t) - \rho(t)\sigma^+\sigma^-)$$
$$+\tilde{\gamma}(t)(\sigma^+\rho(t)\sigma^- + \sigma^-\rho(t)\sigma^+ - \sigma^-\sigma^+\rho(t) - \rho(t)\sigma^+\sigma^-)$$

PHYSICAL REVIEW VOLUME 138, NUMBER 4B 24 MAY 1965

Master Equations and Markov Processes

IRWIN OPPENHEIM*
Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts

AND

KURT E. SHULER
National Bureau of Standards, Washington, D. C.

(Received 13 November 1964)

The processes described by generalized master equations (GME), derived from the Liouville equation on the basis of various physical and dynamic arguments, have been termed Markovian or non-Markovian depending upon whether the GME did not or did involve an explicit time integration. We show that these designations are not in accord with the (very specific) mathematical definitions of Markovian and non-Markovian processes. We demonstrate that the GME does not contain sufficient information to determine whether or not the stochastic process described by it is Markovian or non-Markovian.