

Quantum Nonlinear Optical Pulse Propagation

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[Outline]

1. How quantum solitons are formed?
2. How quantum soliton squeezing is produced?
3. How quantum soliton entanglement is produced?
4. Different types of quantum solitons.

How classical solitons are formed?

Nonlinear Schroedinger equation (NLSE):

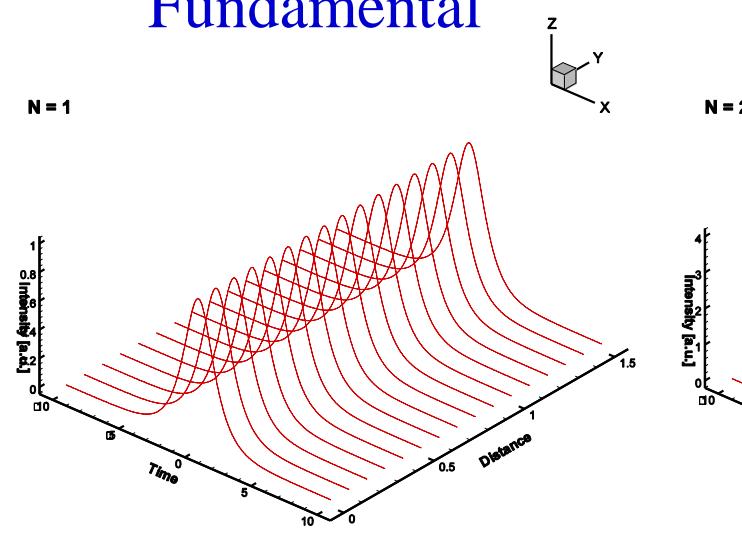
$$i \frac{\partial}{\partial z} U(z,t) = -\frac{1}{2} \frac{\partial^2}{\partial t^2} U(z,t) - |U(z,t)|^2 U(z,t)$$

Fundamental solitons:

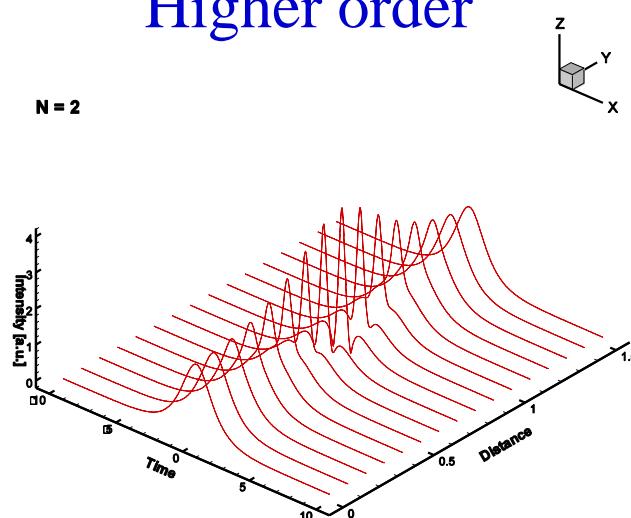
$$U(z,t) = \frac{n_0}{2} \exp[i \frac{n_0^2}{8} z + i \theta_0] \operatorname{sech}[\frac{n_0}{2} t]$$

A. Hasegawa and F. Tappert, Appl. Phys. Lett. Vol. 23, Issue 3, pp. 142-144, (1973).

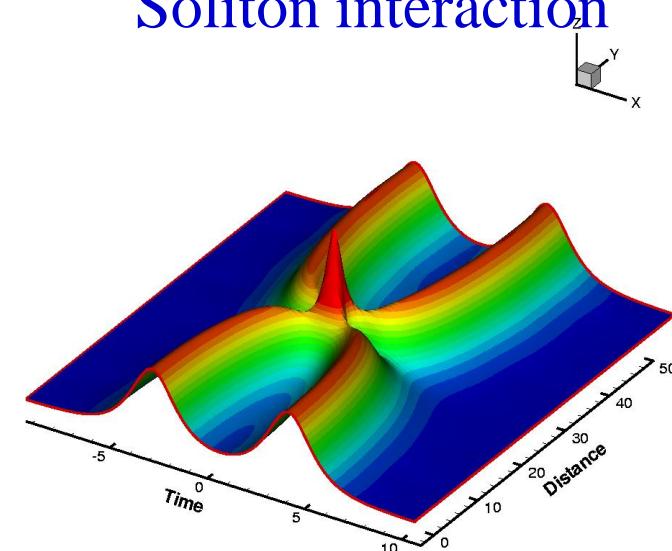
Fundamental



Higher order



Soliton interaction



Quantum description of optical pulses

$$\hat{a}(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{a}(\Delta\omega) e^{-i\Delta\omega\tau} d\Delta\omega$$

$$|\Psi(z)\rangle = \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f_n(z, \tau_1, \dots, \tau_n) \hat{a}^\dagger(\tau_1) \cdots \hat{a}^\dagger(\tau_n) d\tau_1 \cdots d\tau_n |0\rangle$$

Quantum Nonlinear Schrodinger Equation

$$\frac{\partial}{\partial z} \hat{a}(z, \tau) = i \frac{1}{2} \frac{\partial^2}{\partial \tau^2} \hat{a}(z, \tau) + i \hat{a}^\dagger(z, \tau) \hat{a}(z, \tau) \hat{a}(z, \tau)$$

$$\frac{\partial}{\partial z} f_n(z, \tau_1, \dots, \tau_n) = i \left(\frac{1}{2} \sum_{j=1}^n \frac{\partial^2}{\partial \tau_j^2} + \sum_{1 \leq j < k \leq n} \delta(\tau_j - \tau_k) \right) f_n(z, \tau_1, \dots, \tau_n)$$

Bethe's ansatz method

Microscopic quantum state solution

For $\tau_1 < \tau_2 < \dots < \tau_n$

No nonlinearity: $f_n(z, \tau_1, \dots, \tau_n) \propto e^{ik_1\tau_1 + ik_2\tau_2 + \dots + ik_n\tau_n}$

With nonlinearity:

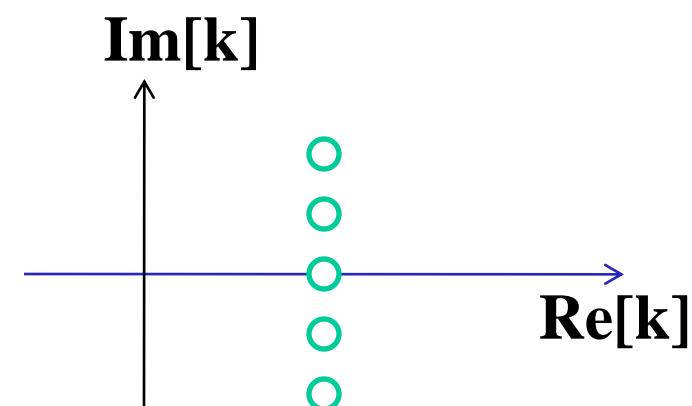
$$f_n(z, \tau_1, \dots, \tau_n) = A_{\{1,2,3,\dots,n\}} e^{ik_1\tau_1 + ik_2\tau_2 + \dots + ik_n\tau_n} + A_{\{2,1,3,\dots,n\}} e^{ik_2\tau_1 + ik_1\tau_2 + \dots + ik_n\tau_n} + \text{other permutations}$$

$$A_{\{2,1,3,\dots,n\}} = \frac{k_2 - k_1 - i}{k_2 - k_1 + i} A_{\{1,2,3,\dots,n\}}$$

Bound eigenstates:

$$\text{If } k_j = p - \frac{i}{2}(n - 2j + 1)$$

then only the first term is non-zero.



How quantum solitons are formed?

Bound eigenstates

$$| n, p, z \rangle = N_c \text{Exp}[-i K(n, p) z] \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \text{Exp}\left[i p \sum_{k=1}^n \tau_k - \frac{1}{2} \sum_{1 \leq i < j \leq n} |\tau_j - \tau_i|\right] d\tau_1 \cdots d\tau_n |0\rangle$$

$$K(n, p) = \frac{1}{2} n p^2 - \frac{1}{24} n (n^2 - 1)$$

Soliton pulses

$$| \Psi(z) \rangle = \sum_n \int g_n(p) | n, p, z \rangle d p$$

$$\theta(n, p) = \{K(n+1, p) - K(n, p)\} z \approx \frac{z}{2} p^2 - \frac{z}{8} n^2$$

$$\frac{\partial}{\partial p} \{K(n+1, p) - K(n, p)\} = p$$

Quantum soliton pulses are changing:
phase spreading and position spreading effects

Quantum soliton perturbation theory

NLSE

$$i \frac{\partial}{\partial z} U(z,t) = -\frac{1}{2} \frac{\partial^2}{\partial t^2} U(z,t) - |U(z,t)|^2 U(z,t)$$

Soliton pulses

$$U(z,t) = \frac{n_0}{2} \exp \left[i p_0 t + i \frac{n_0^2}{8} z - i \frac{p_0^2}{2} z + i \theta_0 \right] \operatorname{sech} \left[\frac{n_0}{2} (t - T_0 - p_0 z) \right]$$

Phase spreading

$$\Delta \hat{n}(z) = \Delta \hat{n}(0)$$

$$\Delta \hat{\theta}(z) = \Delta \hat{\theta}(0) + \frac{n_0}{4} z \Delta \hat{n}(0)$$

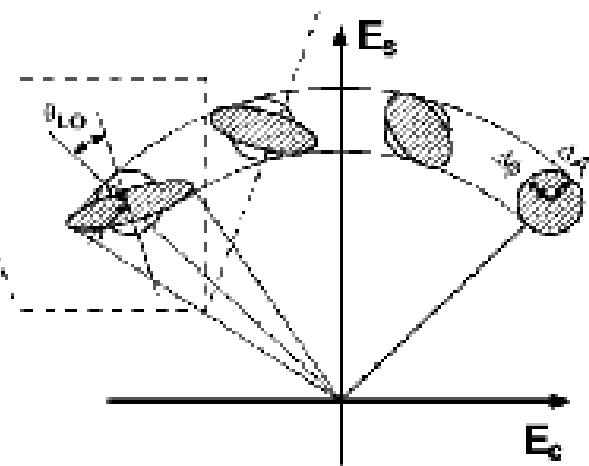
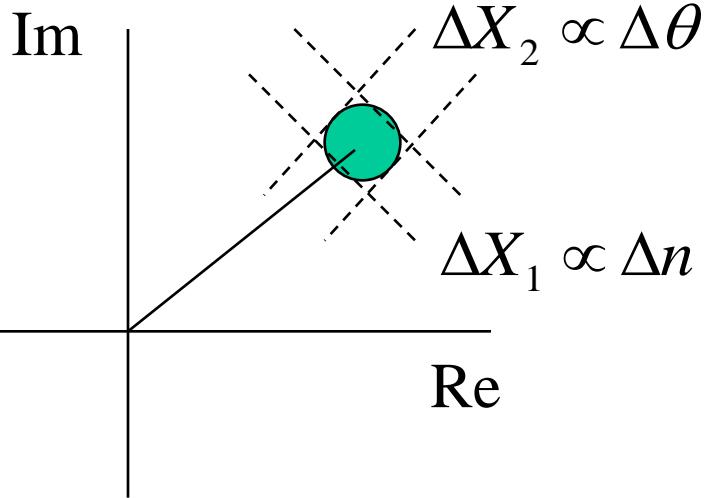
Assuming $p_0 = 0$

Position spreading

$$\Delta \hat{p}(z) = \Delta \hat{p}(0)$$

$$\Delta \hat{T}(z) = \Delta \hat{T}(0) + z \Delta \hat{p}(0)$$

Phase spreading and quadrature squeezing

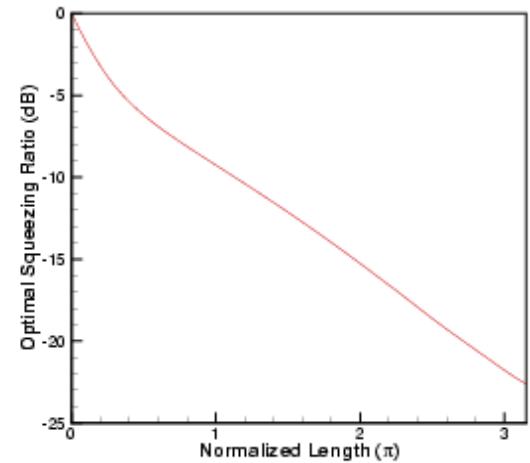


$$\Delta \hat{n}(z) = \Delta \hat{n}(0)$$

$$\Delta \hat{\theta}(z) = \Delta \hat{\theta}(0) + \frac{n_0}{4} z \Delta \hat{n}(0)$$

$$\Delta \hat{X}_\theta(z) = \alpha_1 \Delta \hat{n}(z) + \alpha_2 \Delta \hat{\theta}(z)$$

$$R = \min \frac{\text{Var}[\Delta \hat{X}_\theta(z)]}{\text{Var}[\Delta \hat{X}_\theta(0)]}$$



How to detect?

$$\hat{u}(z,t) = \Delta\hat{n}(z)u_n(t) + \Delta\hat{\theta}(z)u_\theta(t) + \Delta\hat{p}(z)u_p(t) + \Delta\hat{T}(z)u_T(t) + continuum$$

Projection

$$\Delta\hat{n}(z) = \langle f_n(t) | \hat{u}(z,t) \rangle$$

$$\Delta\hat{\theta}(z) = \langle f_\theta(t) | \hat{u}(z,t) \rangle$$

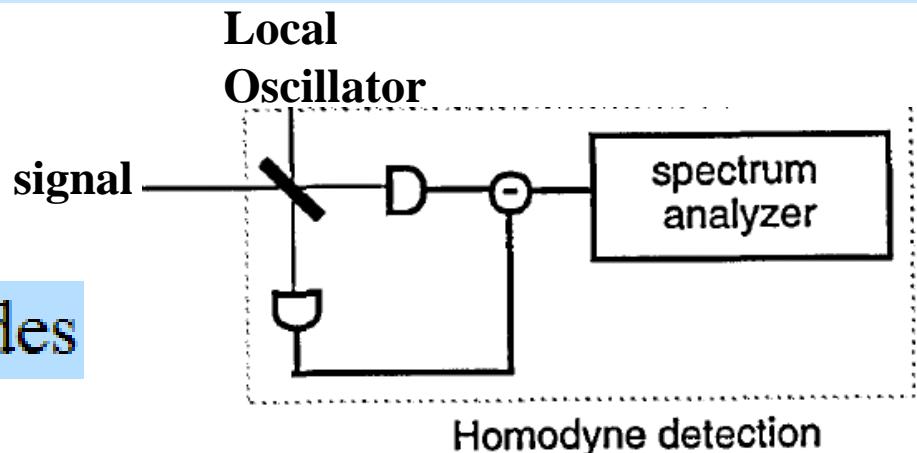
$$\Delta\hat{p}(z) = \langle f_p(t) | \hat{u}(z,t) \rangle$$

$$\Delta\hat{T}(z) = \langle f_T(t) | \hat{u}(z,t) \rangle$$

Optically, such projection can be implemented by homodyne detection.

Projection interpretation of homodyne detection

$$\hat{M}(z) = \langle f_{\text{LO}}(t) | \hat{u}(z, t) \rangle = \int \frac{1}{2} [f_{\text{LO}}^*(t) \hat{u}(z, t) + h.c.] dt$$



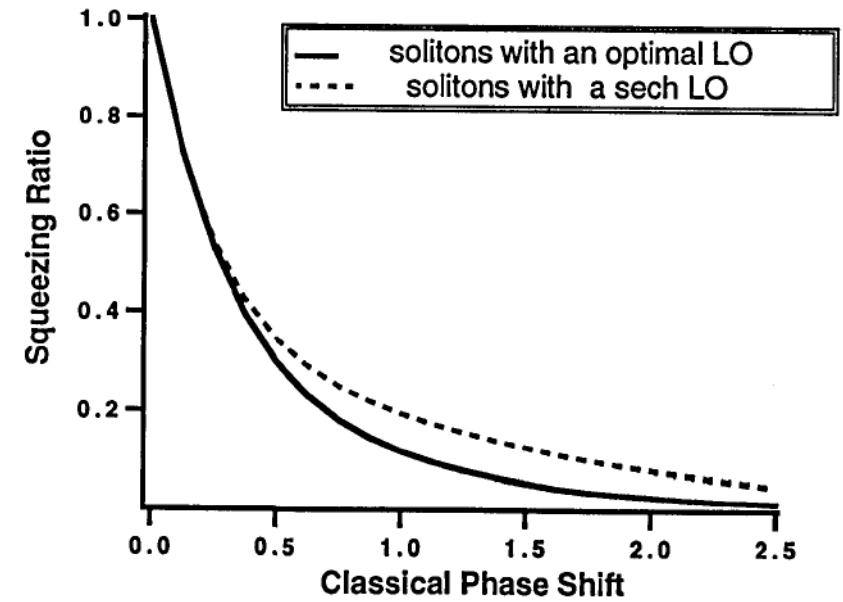
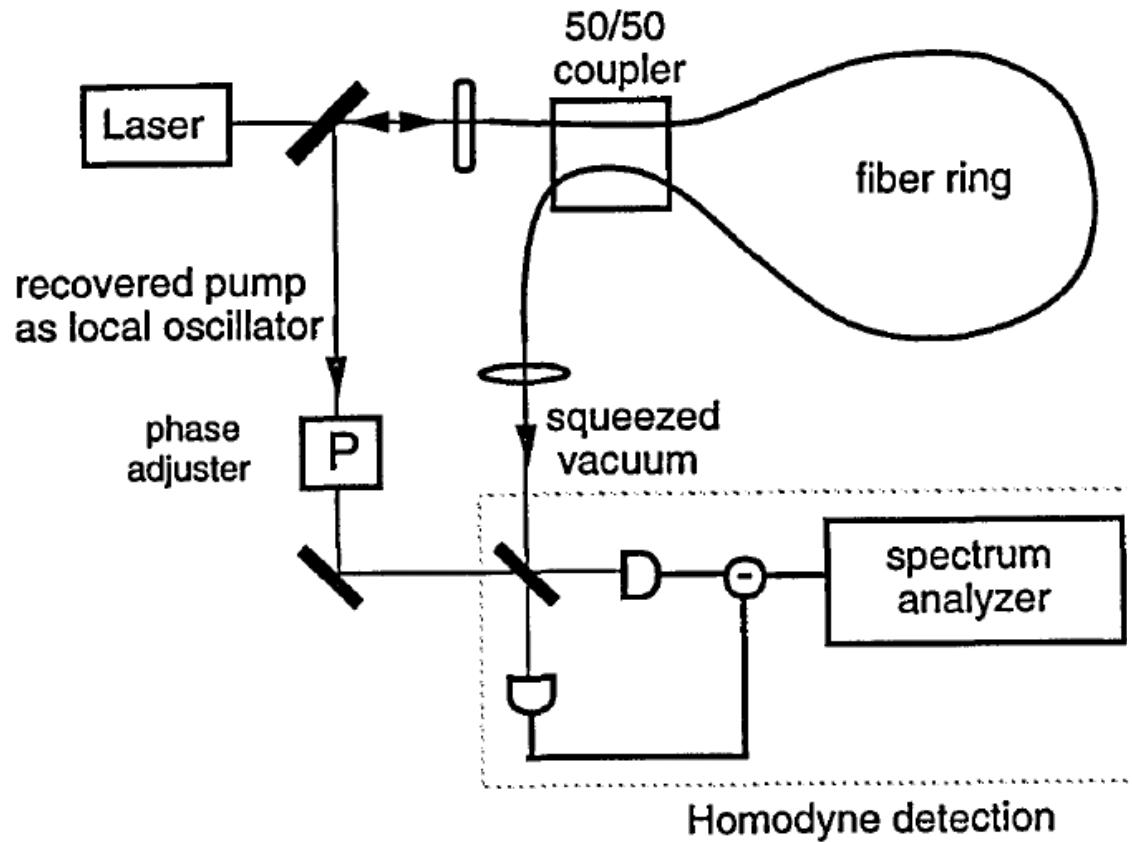
$$\hat{u}(t) = \hat{a} u_m(t) + \text{other modes}$$

$$\hat{a} = \hat{q} + i \hat{p}$$

$$\hat{q} = \langle u_m(t) | \hat{u}(t) \rangle = \frac{1}{2} \int [u_m^*(t) \hat{u}(t) + u_m(t) \hat{u}^\dagger(t)] dt$$

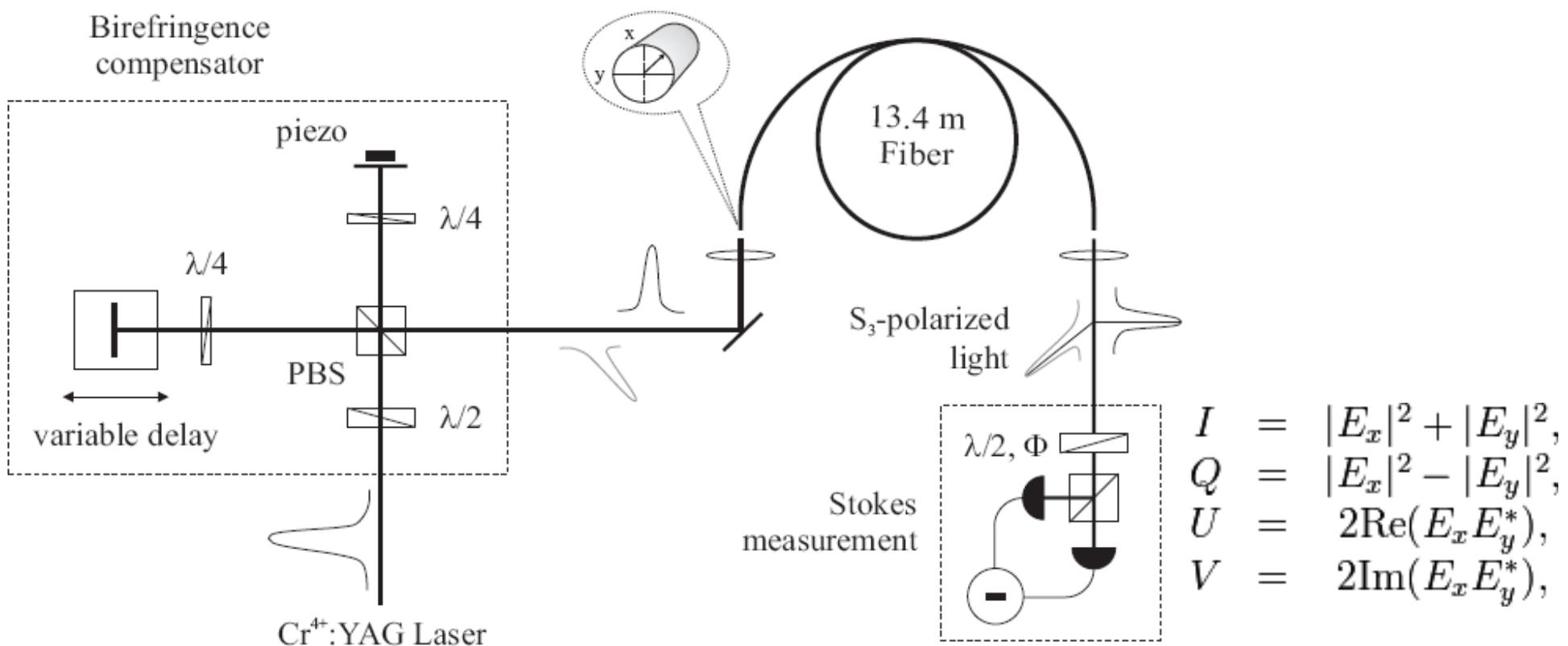
$$\hat{p} = \langle i u_m(t) | \hat{u}(t) \rangle = \frac{i}{2} \int [-u_m^*(t) \hat{u}(t) + u_m(t) \hat{u}^\dagger(t)] dt$$

Squeezed vacuum state generation and detection



First Experiment: M. Rosenbluh and R. M. Shelby, Phys. Rev. Lett. 66, 153(1991).

Polarization squeezing

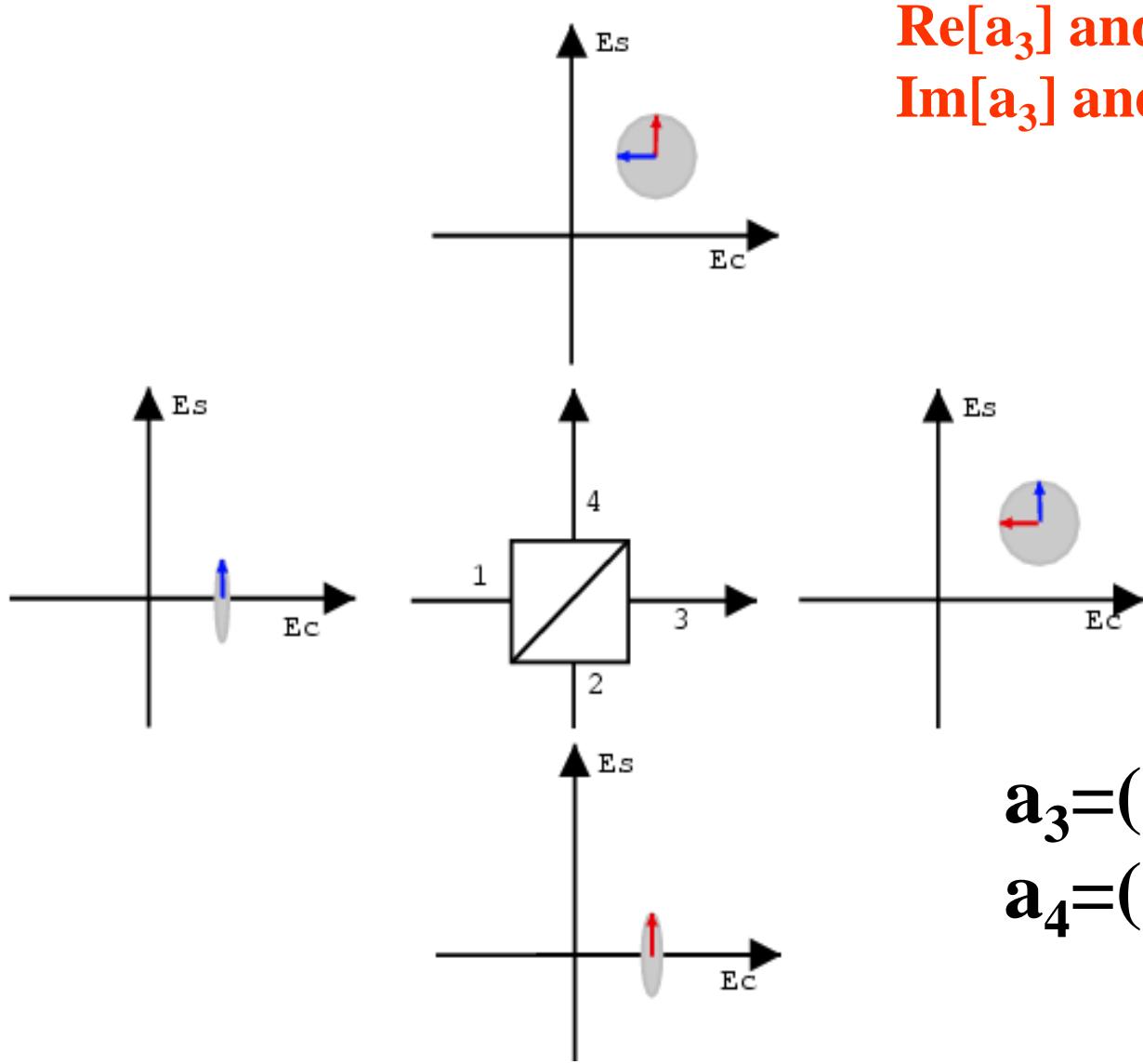


$$\hat{S}_\theta = \cos(\theta) \hat{S}_1 + \sin(\theta) \hat{S}_2$$

**Observed noise reduction
of -6.8 ± 0.3 dB**

1. J. F. Corney, P. Drummond, J. Heersink, V. Josse, G. Leuchs, and U.L. Andersen, Phys. Rev. Lett. 97, 023606 (2006).
2. R.-F. Dong, J. Heersink, J. F. Corney, P. D. Drummond, U. L. Andersen, and G. Leuchs, Opt. Lett. 33, 116 (2008).

Generation of EPR entangled beams with two independent squeezed states



Re[a₃] and Im[a₄] are correlated
Im[a₃] and Re[a₄] are correlated

$$a_3 = (a_1 + i a_2)/2^{0.5}$$
$$a_4 = (i a_1 + a_2)/2^{0.5}$$

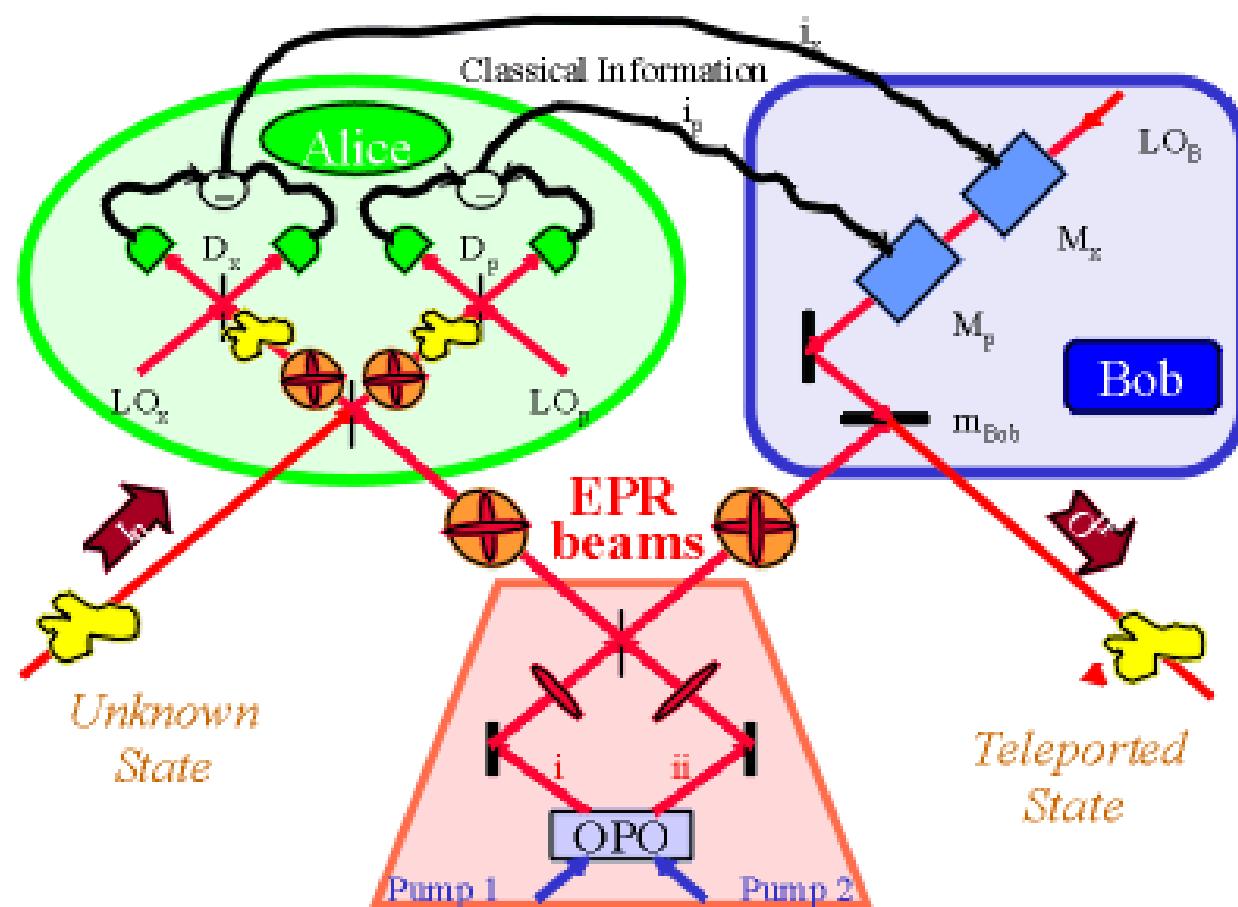
Teleportation of Continuous Quantum Variables

Samuel L. Braunstein

SEECS, University of Wales, Bangor LL57 1UT, United Kingdom

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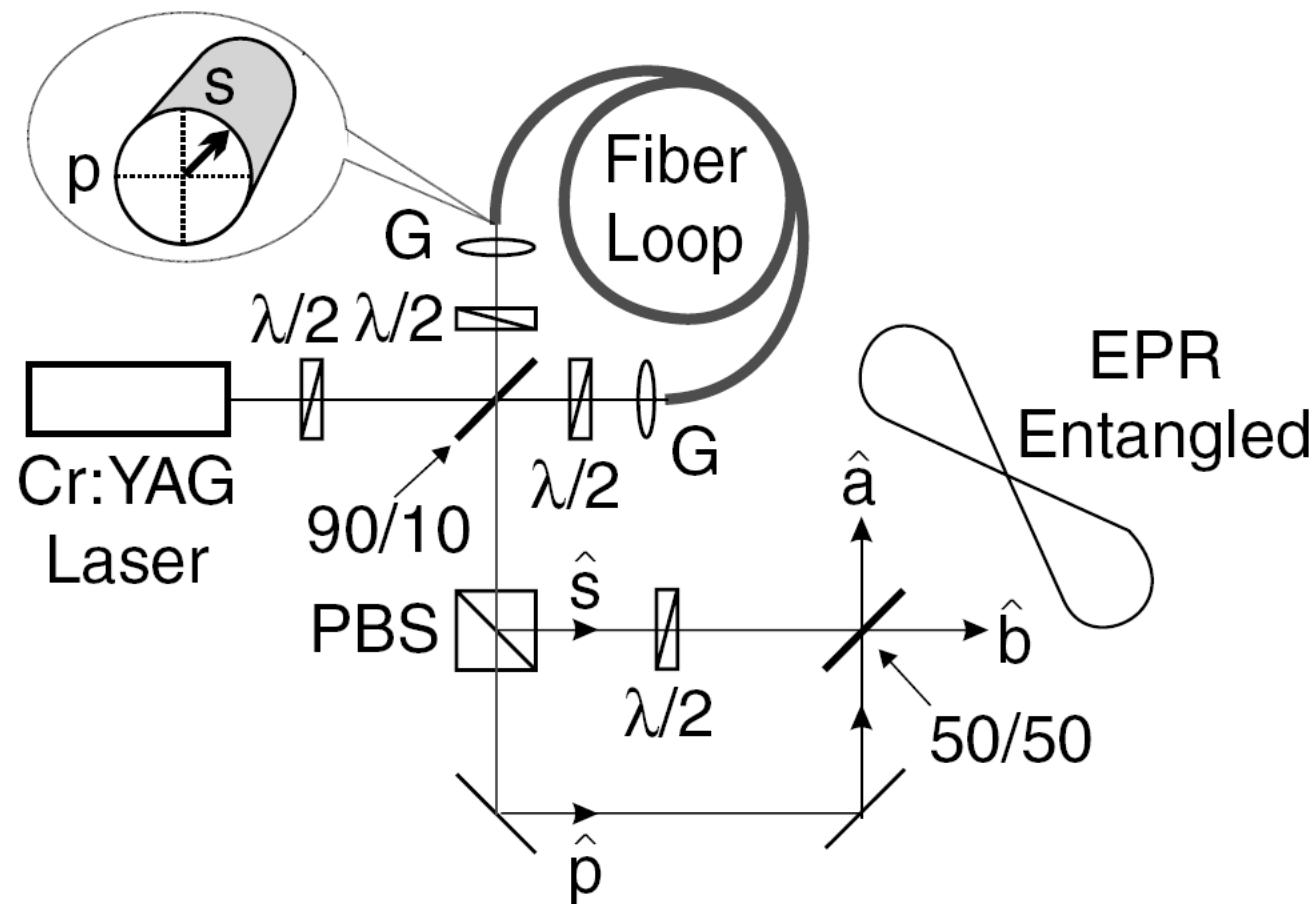


Generation of Continuous Variable Einstein-Podolsky-Rosen Entanglement via the Kerr Nonlinearity in an Optical Fiber

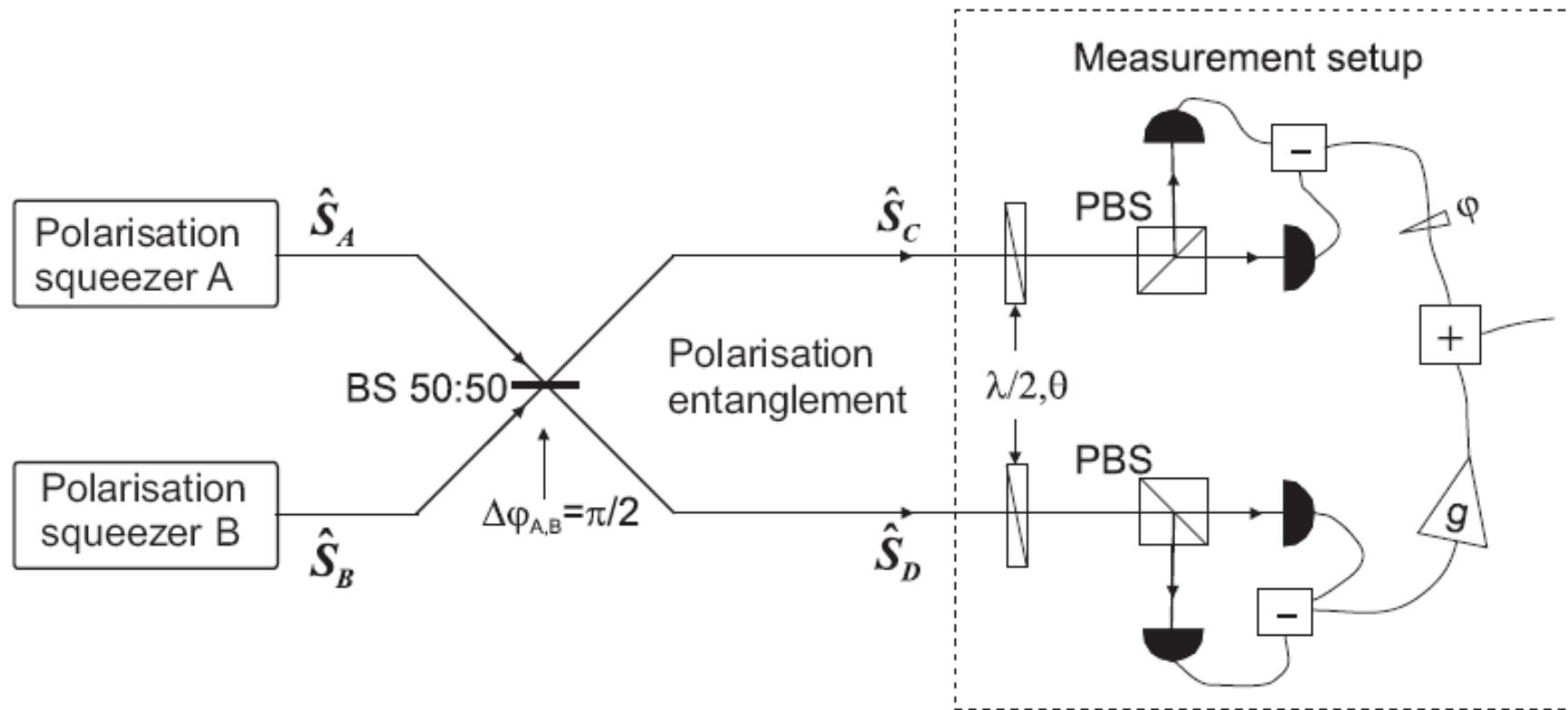
Ch. Silberhorn,¹ P. K. Lam,^{1,2} O. Weiß,¹ F. König,¹ N. Korolkova,¹ and G. Leuchs¹

¹Zentrum für Moderne Optik, Universität Erlangen-Nürnberg, Staudtstraße 7/B2, D-91058 Erlangen, Germany

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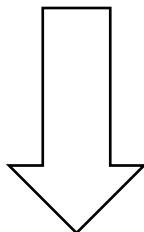
Polarization entanglement states



The quantum correlations along the squeezed and the anti-squeezed Stokes parameters were observed to be -4.1 ± 0.3 dB and -2.6 ± 0.3 dB below the shot noise level respectively

What if

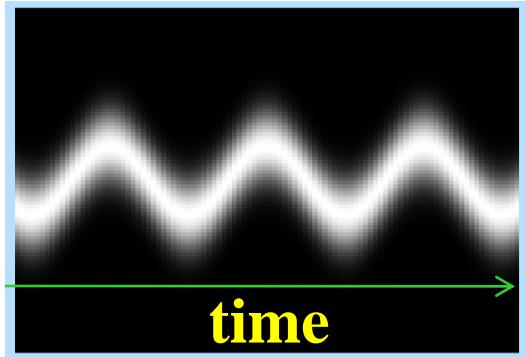
- 1. Non-soliton cases?**
- 2. With loss?**
- 3. With higher order dispersion?**
- 4. With self-Raman?**
- 5. Different local oscillators?**



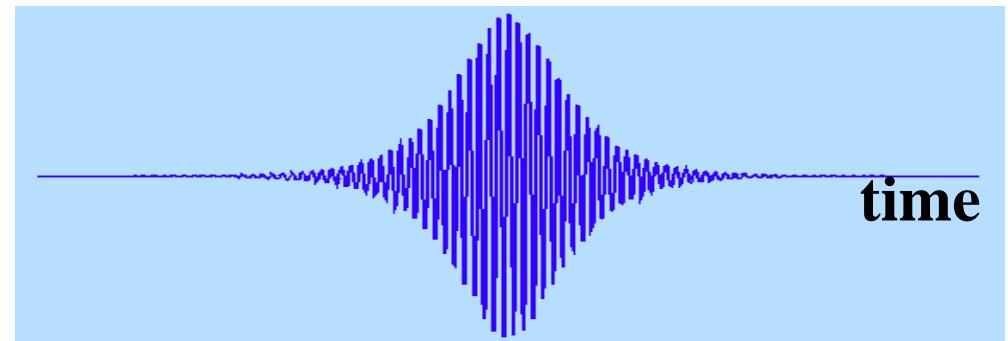
**General numerical methods for
quantum nonlinear optical pulse propagation**

Quantum effects of optical pulses

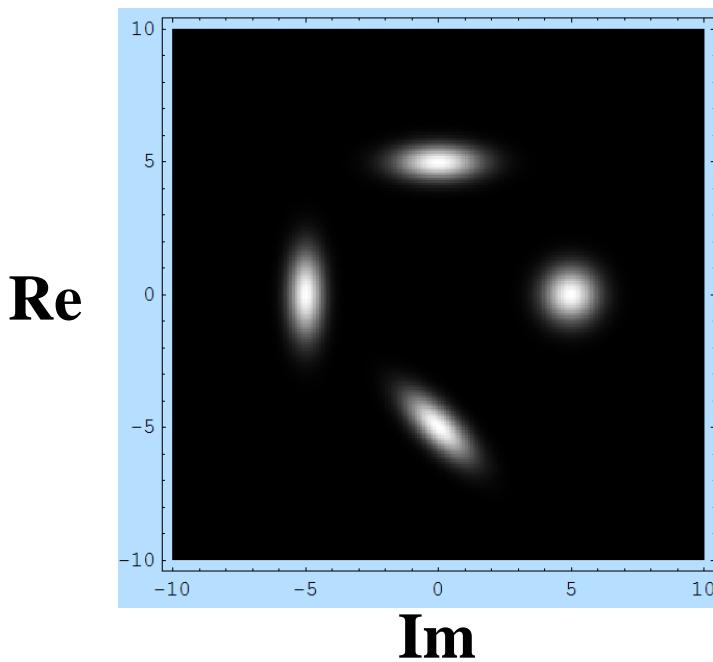
Single light mode



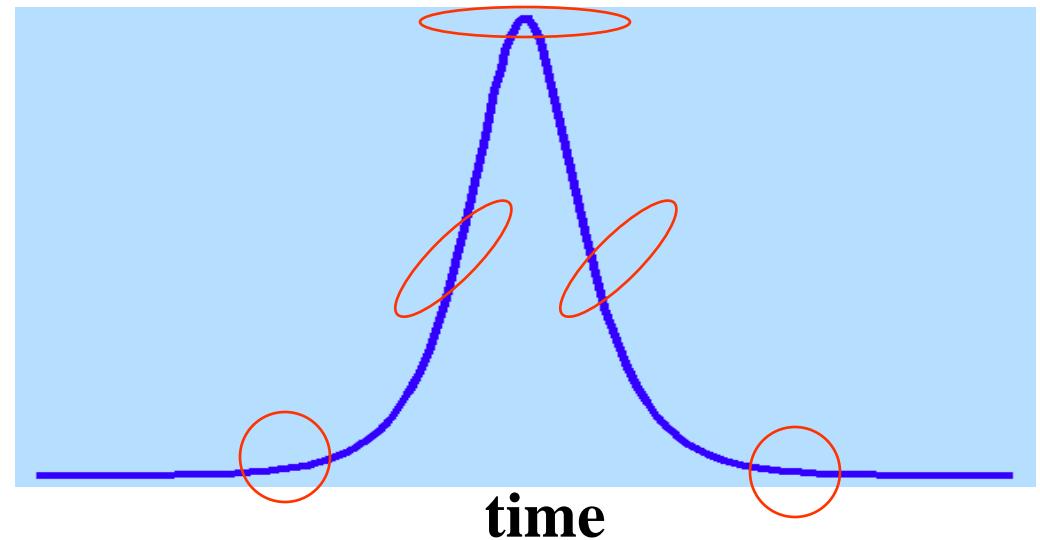
Optical pulse (multi-mode)



Phasor diagram of single mode



Time-sliced phasor diagram



Backpropagation method

$$\frac{\partial}{\partial z} \hat{U} = F(\hat{U}, \hat{U}^\dagger)$$

$$\frac{\partial}{\partial z} \hat{u} = P \bullet \hat{u}$$

$$\frac{\partial}{\partial z} u^A = P^A \bullet u^A$$

$$\frac{\partial}{\partial z} \langle u^A | u \rangle = 0$$

$$\hat{M}(z) = \langle u^A(z, t) | u(z, t) \rangle = \langle u^A(0, t) | u(0, t) \rangle$$

$$\text{Var}[\hat{M}(z)] = \frac{1}{4} \int |u^A(0, t)|^2 dt$$

Linearization approximation

Relate the output operators to the input operators.

1. Quantum correlation also can be calculated.
2. Additional noise terms also can be included.

Quantum noise calculation

$$\frac{\partial}{\partial z} \hat{U} = F(\hat{U}, \hat{U}^\dagger)$$

$$\frac{\partial}{\partial z} \hat{u} = P \bullet \hat{u}$$

$$\frac{\partial}{\partial z} u^A = P^A \bullet u^A$$

$$\frac{\partial}{\partial z} \langle u^A | u \rangle = 0$$

$$\hat{M}(z) = \langle u^A(z, t) | u(z, t) \rangle = \langle u^A(0, t) | u(0, t) \rangle$$

$$\text{Var}[\hat{M}(z)] = \frac{1}{4} \int |u^A(0, t)|^2 dt$$

Linearization approximation

Relate the output operators to the input operators.

1. Quantum correlation also can be calculated.
2. Additional noise terms also can be included.

Optimized projection function

$$u^A(z, t) = f_L(t)$$

$$u^A(0, t) = A_0 \leftarrow_z \bullet f_L(t)$$

$$\hat{u}(z, t) = L_z \leftarrow_0 \bullet \hat{u}(0, t)$$

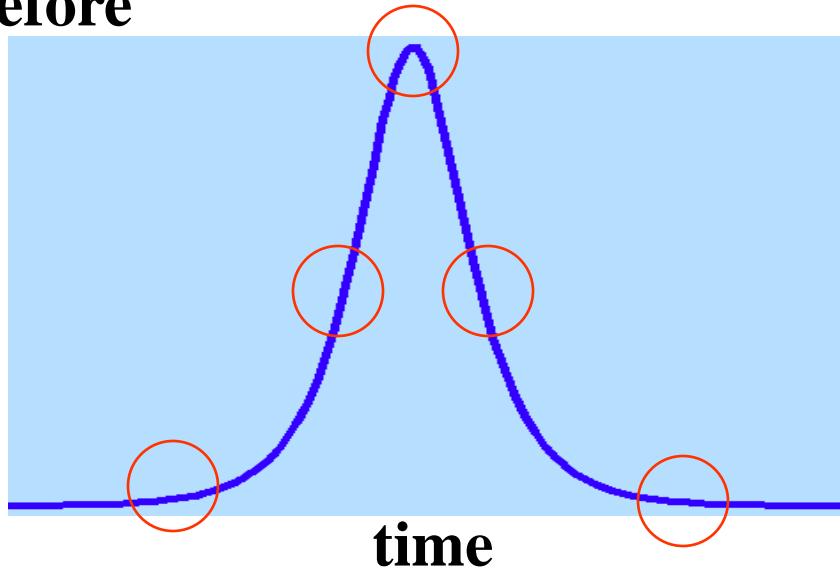
$$R = \frac{\text{Var}[\hat{M}(L)]}{\text{Var}[\hat{M}(0)]} = \frac{\text{Var}[\langle f_L(t) | \hat{u}(z, t) \rangle]}{\text{Var}[\langle f_L(t) | \hat{u}(0, t) \rangle]} = \frac{\langle A_0 \leftarrow_z \bullet f_L(t) | A_0 \leftarrow_z \bullet f_L(t) \rangle}{\langle f_L(t) | f_L(t) \rangle}$$

$$\delta R = 0 \rightarrow L_z \leftarrow_0 A_0 \leftarrow_z \bullet f_L(t) = \lambda f_L(t)$$

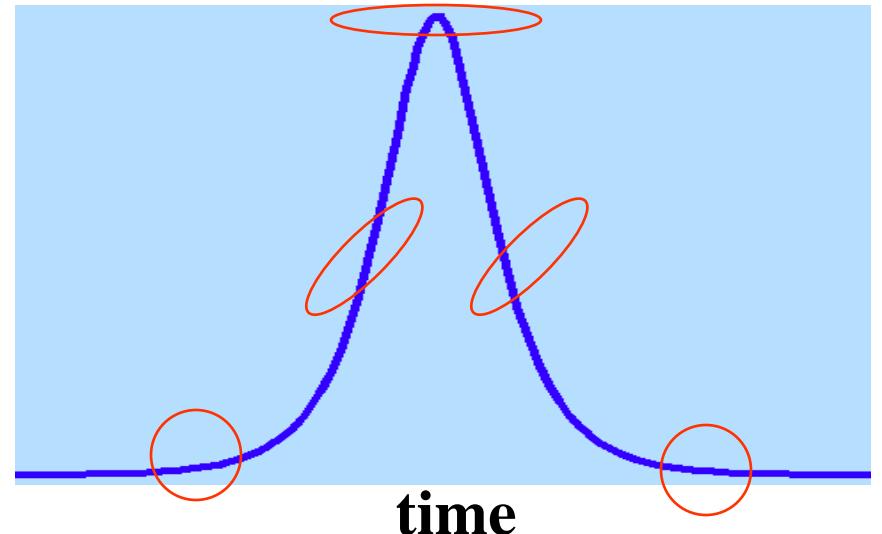
$$R_{\text{opt}} = \lambda$$

How squeezing is produced?

Before



After



$\hat{u}(z, t)$ is related to $\hat{u}(0, t)$ and $\hat{u}^\dagger(0, t)$ by a linear transform

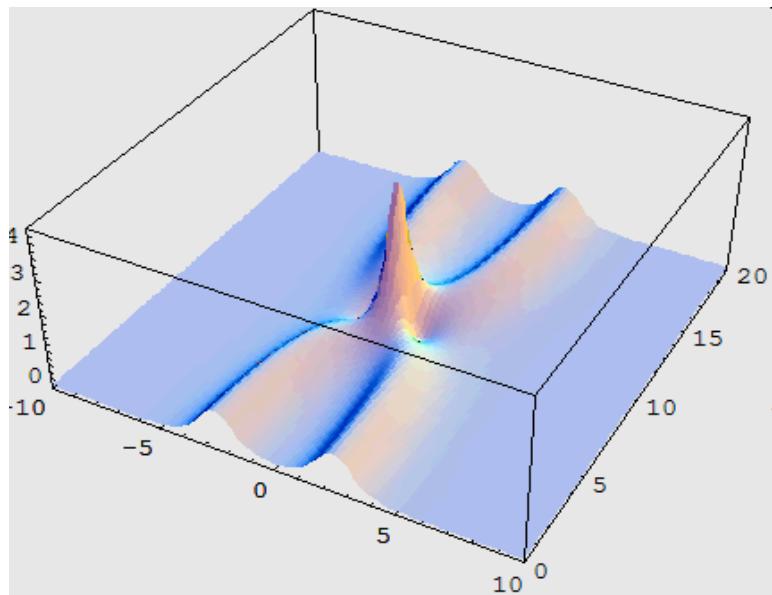
The original quantum state are multi – mode independent coherent states.

The new quantum state are multi – mode entangled gaussian states.

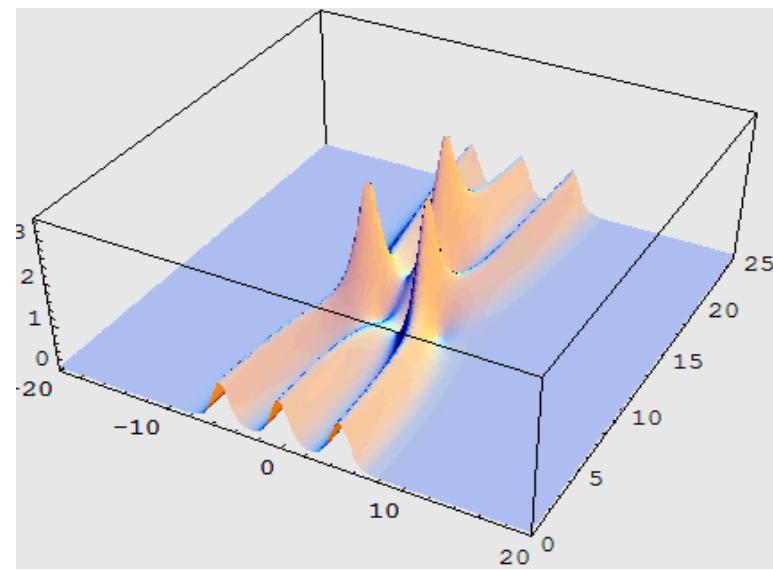
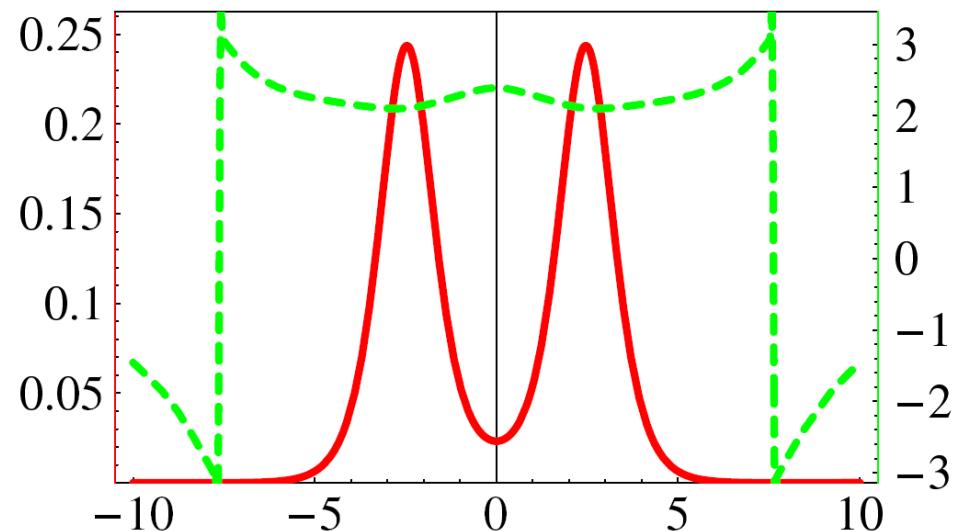
$$\hat{M} = \langle f_L(t) | u(z, t) \rangle = \langle F_L(t) | u(0, t) \rangle$$

$$\text{Var}[\hat{M}(z)] = \frac{1}{4} \int |F_L(t)|^2 dt$$

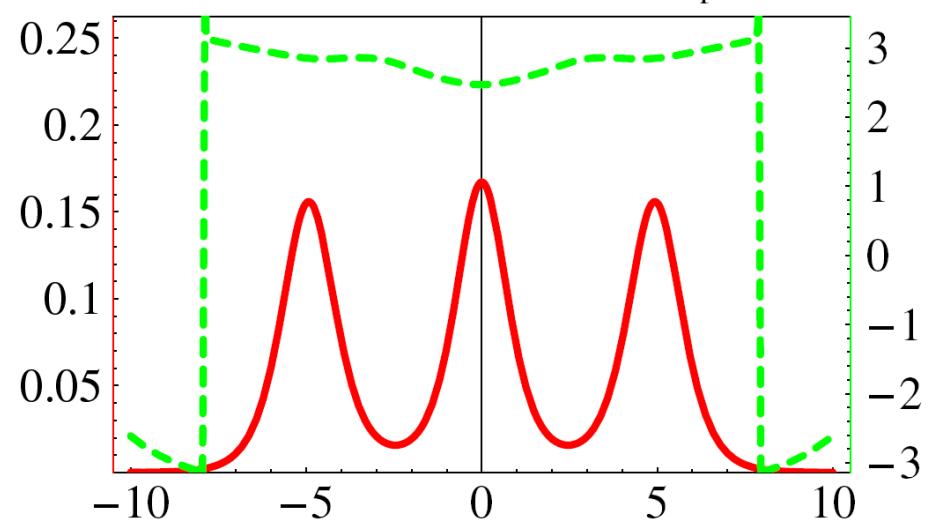
Multi-partite solitons



Intensity and phase of f_{opt}



Intensity and phase of f_{opt}



How squeezing leads to entanglement

$$f_1(t) \propto f_{\text{opt}}(t) \text{ for } t > 0$$

$$f_2(t) \propto f_{\text{opt}}(t) \text{ for } t < 0$$

$$\hat{q}_1 = \langle f_1 | \hat{u} \rangle, \hat{p}_1 = \langle i f_1 | \hat{u} \rangle, \hat{q}_2 = \langle f_2 | \hat{u} \rangle, \hat{p}_2 = \langle i f_2 | \hat{u} \rangle.$$

$f_1(t) + f_2(t)$: **the optimum squeezing/anti-squeezing mode.**

$f_1(t) - f_2(t)$: **orthogonal to the optimum mode.**

$$\text{Squeezing ratio of } \frac{\text{Var}[\hat{q}_1 + \hat{q}_2]}{\text{Var}[\hat{p}_1 - \hat{p}_2]} \leq \frac{\lambda_{\text{opt}}}{\lambda_{\text{snd}}} < 1$$

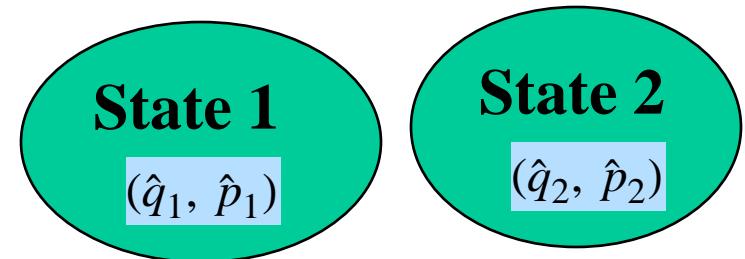
$$\lambda_{\text{opt}}$$

$$\frac{1}{\lambda_{\text{snd}}}$$

Inseparability criterion of two-partite quantum states

Definition of separable quantum states:

$$\rho = \sum_i P_i \rho_{i,1} \otimes \rho_{i,2}$$



Sufficient criterion for inseparability:

If \hat{q}_1 is correlated with \hat{q}_2 and \hat{p}_1 is correlated with \hat{p}_2 , and

$\text{Var}[\hat{q}_1|\hat{q}_2] * \text{Var}[\hat{p}_1|\hat{p}_2] <$ Heisenberg uncertainty product

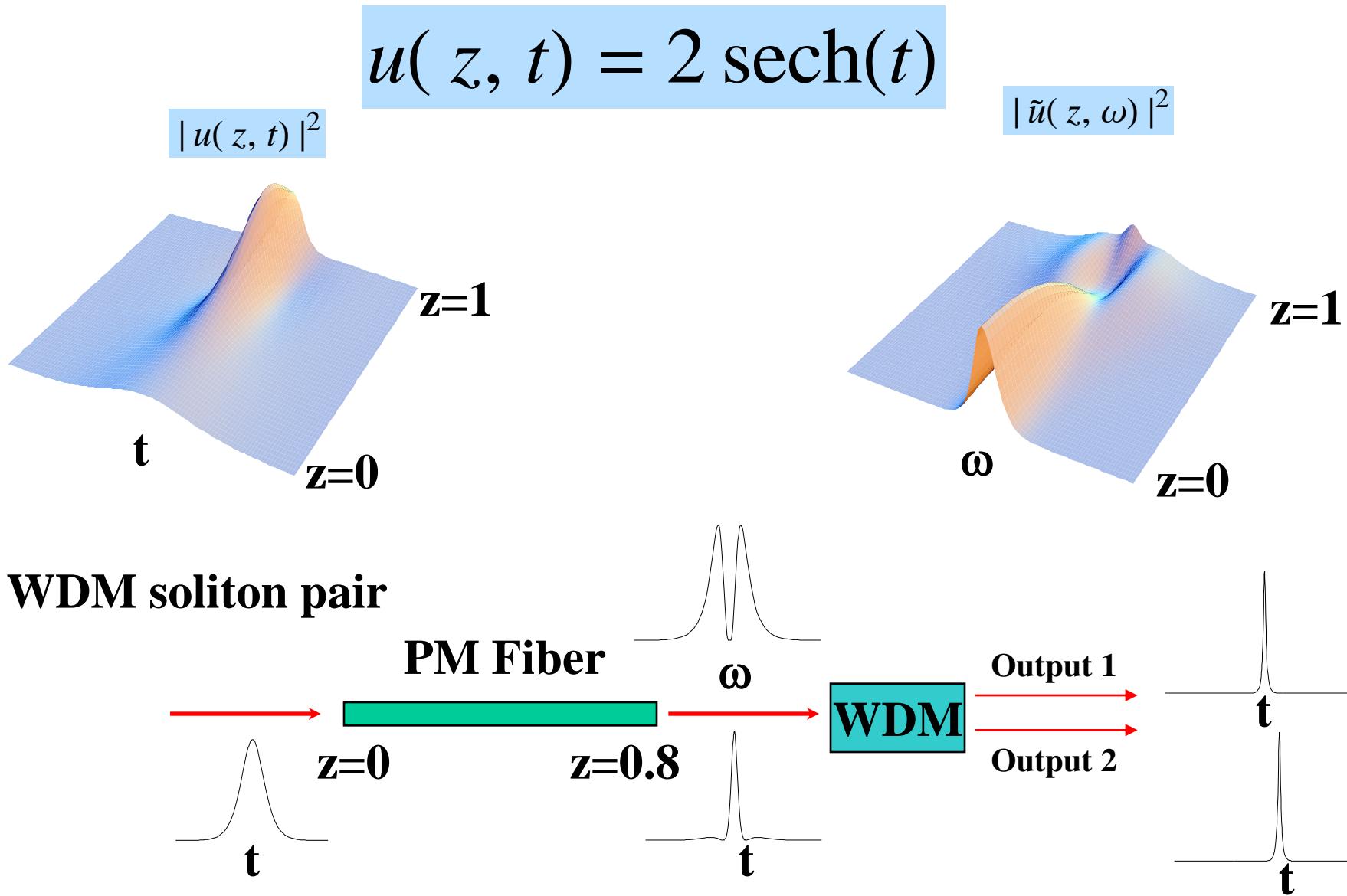
Quantum correlation \neq Quantum entanglement (inseparability)

For example,

$$\rho = \int P(\alpha) |\alpha\rangle_1 |\alpha\rangle_2 \langle \alpha|_1 \langle \alpha|_2 d\alpha$$

Correlated but not entangled.

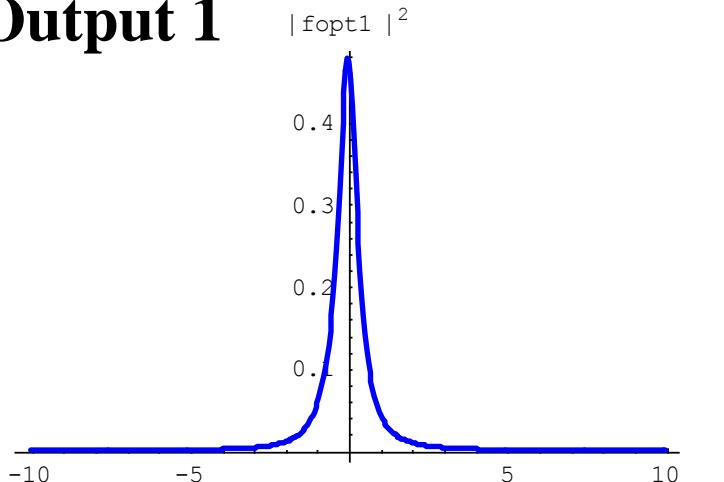
Entangled frequency multiplexed quantum solitons (I)



Entangled frequency multiplexed quantum solitons (II)

Projection mode functions

Output 1

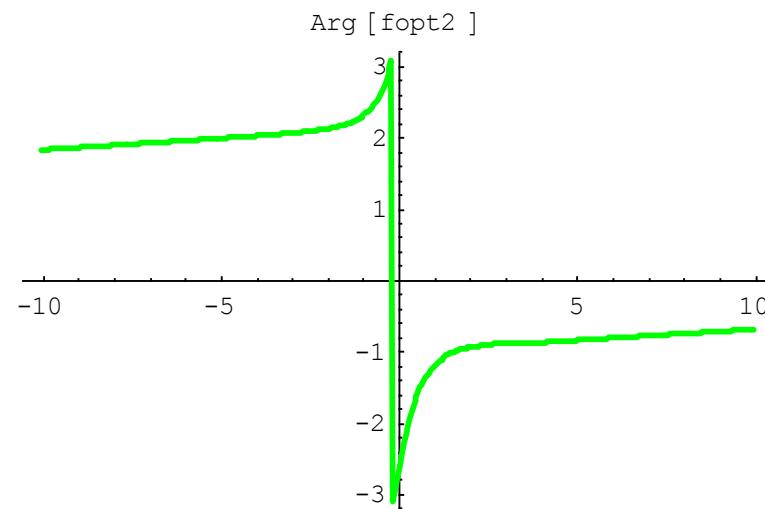
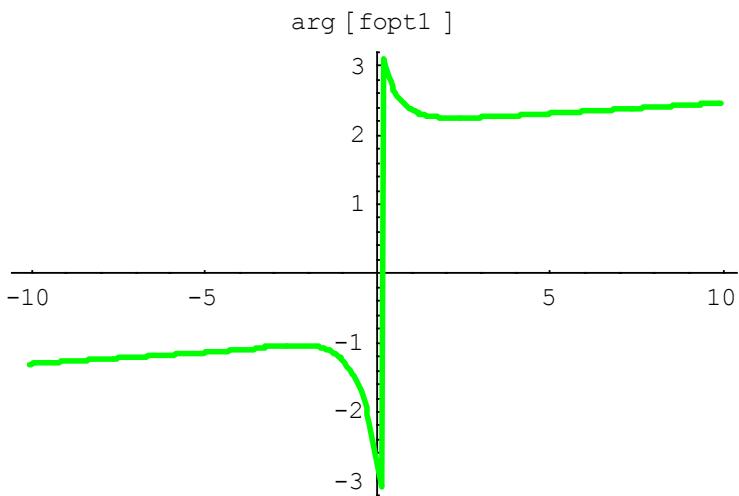
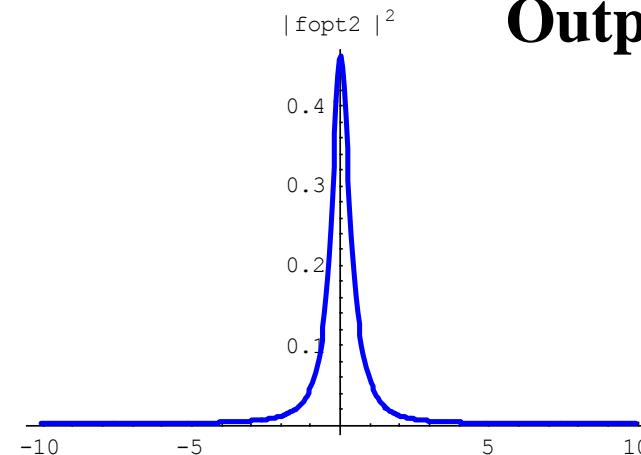


$$\lambda_{\text{opt}} = -24 \text{ dB}$$

$$\lambda_{\text{snd}} = -17 \text{ dB}$$

Squeezing ratio of $\text{Var}[\hat{q}_1 + \hat{q}_2] / \text{Var}[\hat{p}_1 - \hat{p}_2] < -7 \text{ dB}$

Output 2

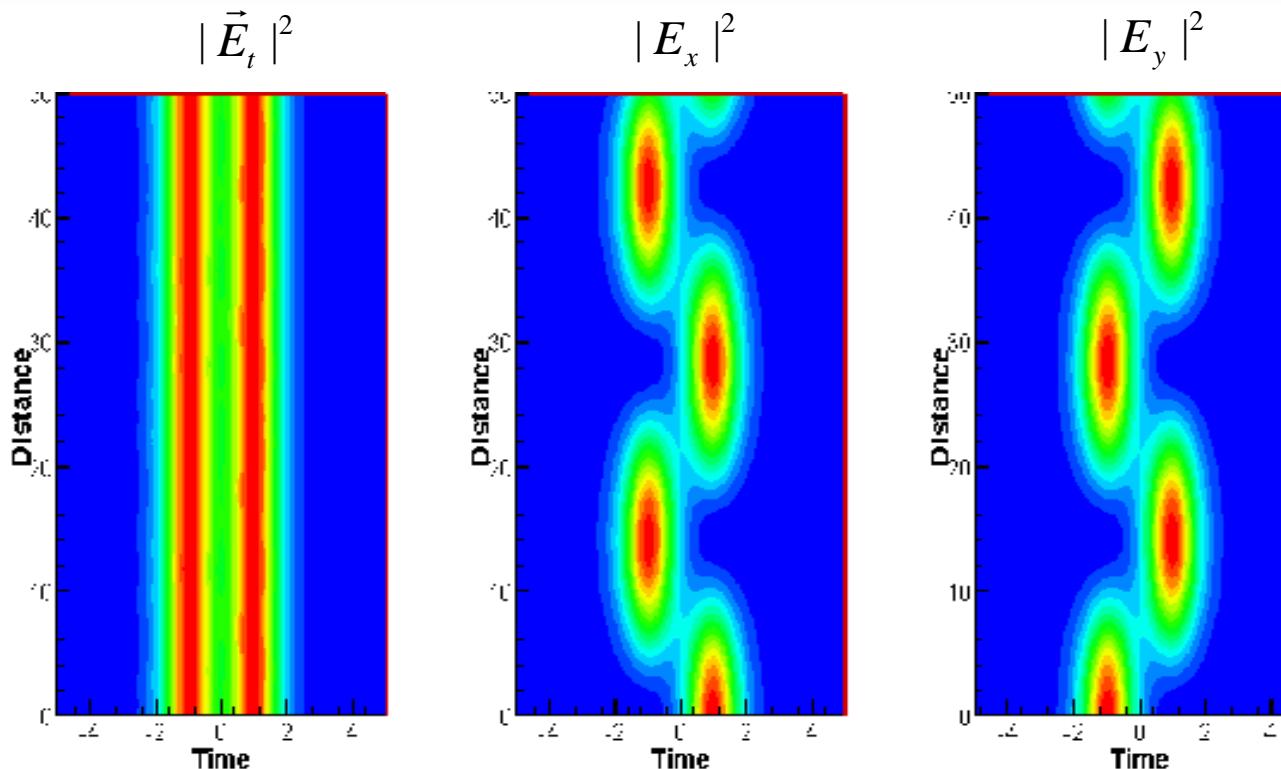


PDM bound solitons

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2U + B|V|^2U = 0$$
$$i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2V + B|U|^2V = 0$$

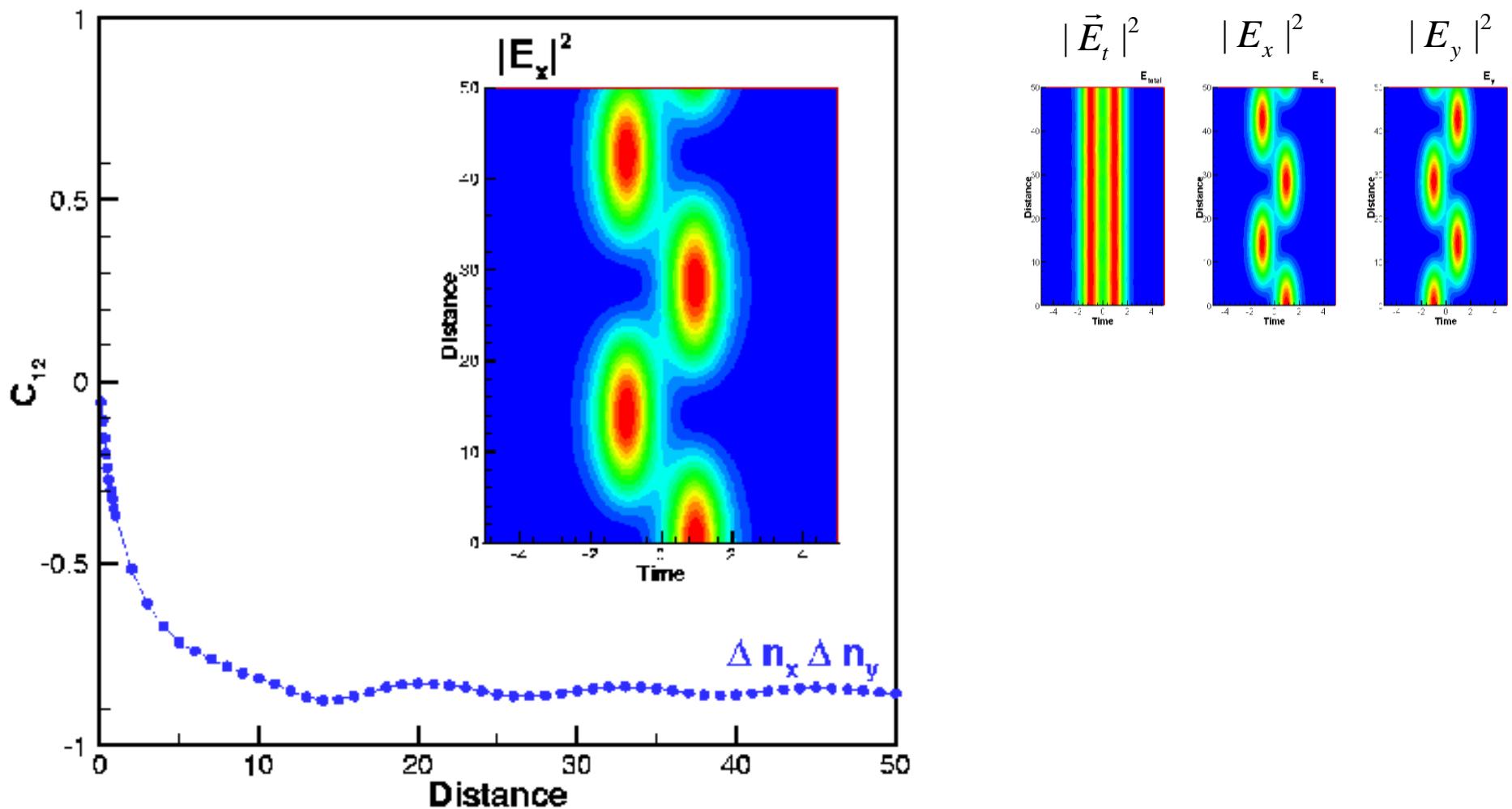
U, V : Fields
in circular
polarizations

and $A = 1/3$, $B = 2/3$



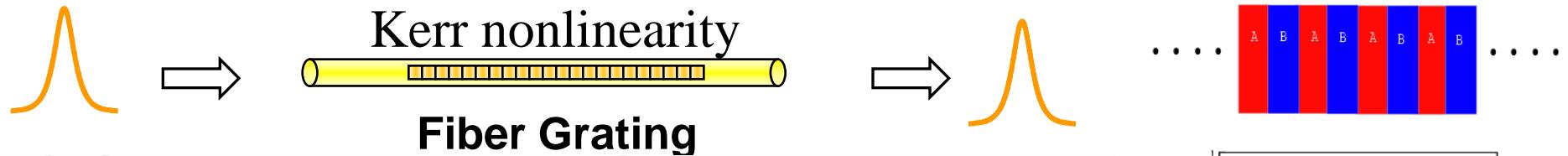
M. Haelterman et al.,
Optics Letters 18,
1406 (1993).

Quantum correlation of PDM soliton pairs



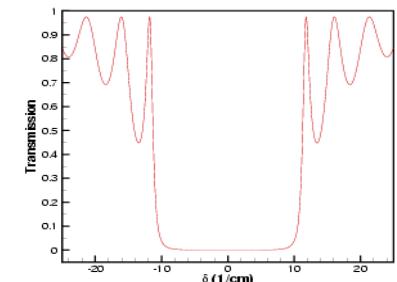
Fiber Bragg Grating Solitons

Solitons in 1D nonlinear photonic crystals



$$\frac{1}{v_g} \frac{\partial}{\partial t} U_a(z, t) + \frac{\partial}{\partial z} U_a = i\delta U_a + i\kappa U_b + i\Gamma |U_a|^2 U_a + 2i\Gamma |U_b|^2 U_a$$

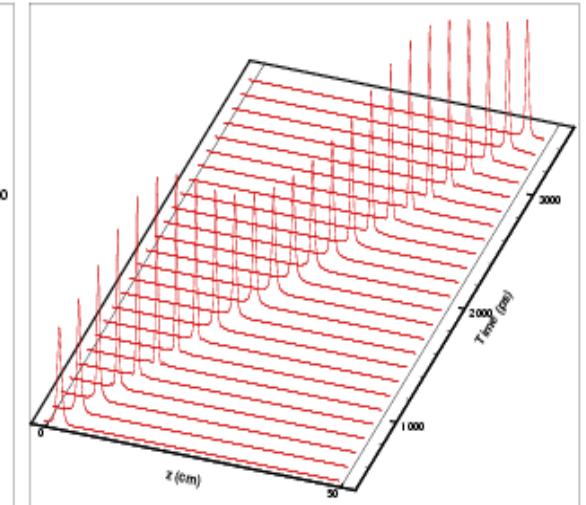
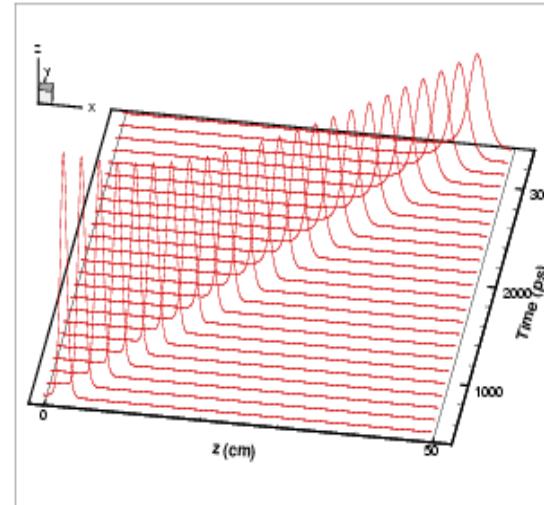
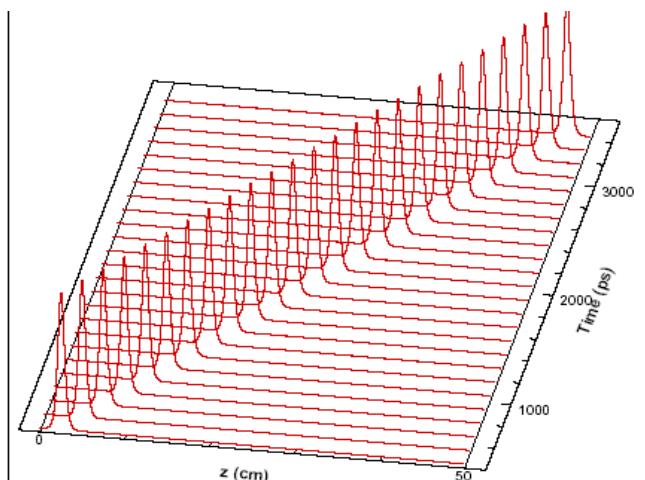
$$\frac{1}{v_g} \frac{\partial}{\partial t} U_b(z, t) - \frac{\partial}{\partial z} U_b = i\delta U_b + i\kappa U_a + i\Gamma |U_b|^2 U_b + 2i\Gamma |U_a|^2 U_b$$



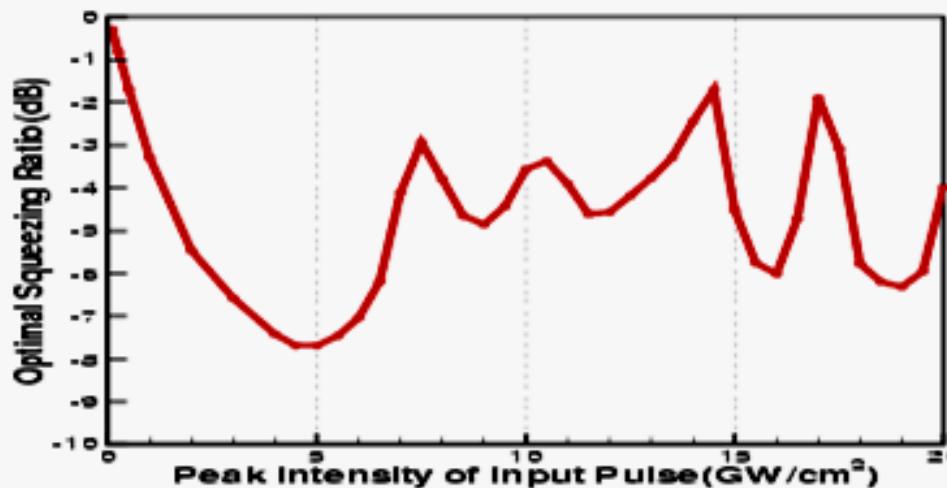
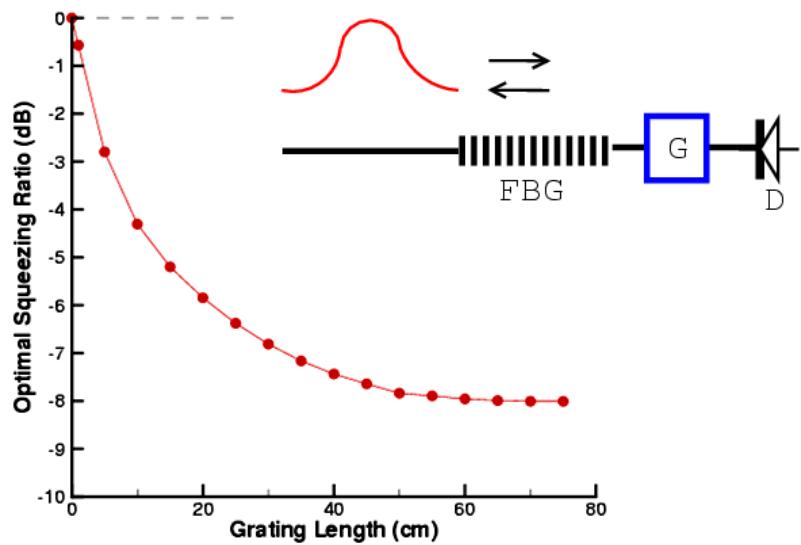
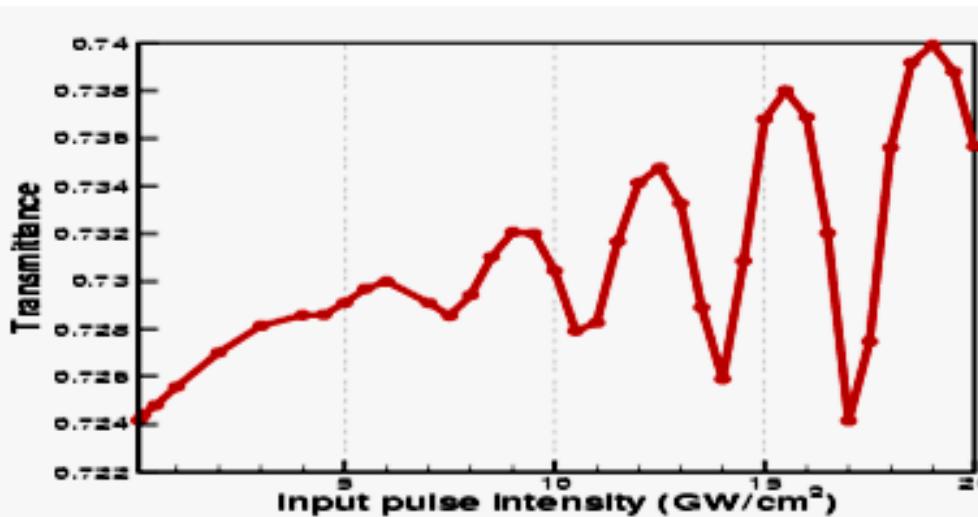
stationary

below

above



Amplitude squeezing of fiber Bragg grating solitons



$$\hat{M} = M + \Delta\hat{M}$$

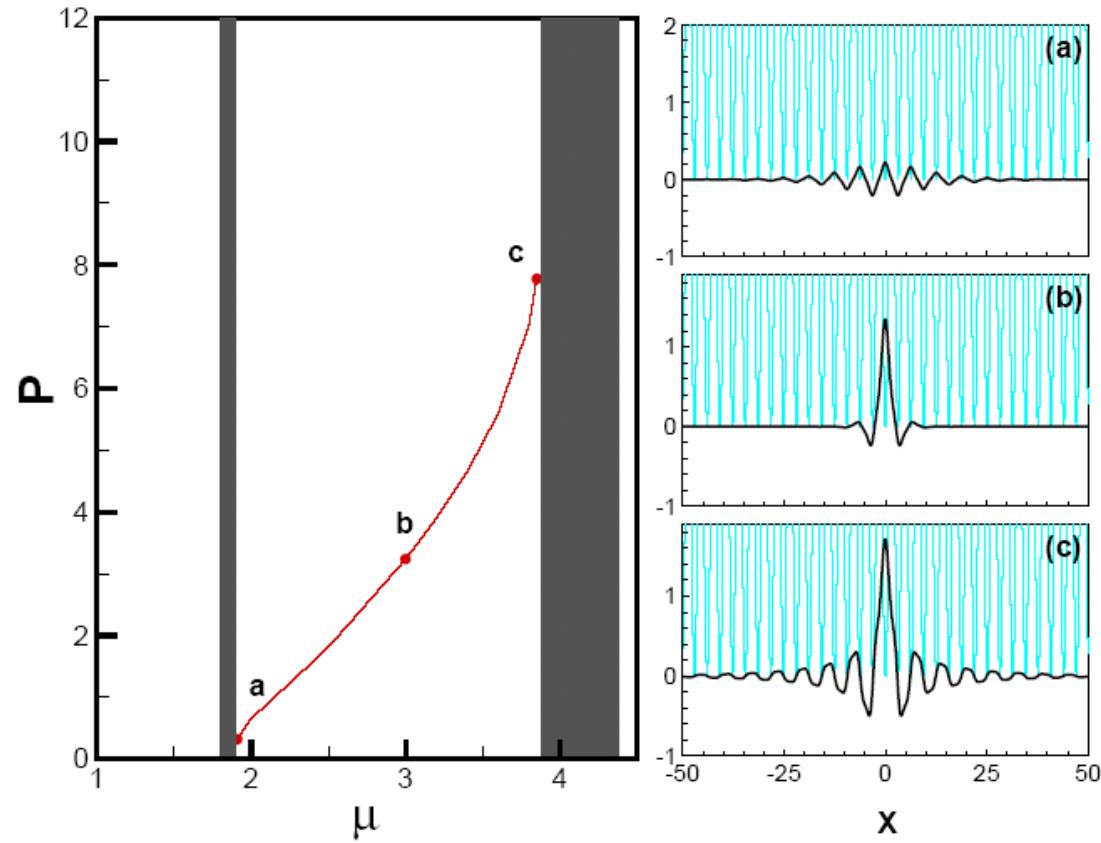
$$R = \frac{\langle \Delta\hat{M}^2 \rangle}{\langle \Delta\hat{M}^2 \rangle_{coherent\ state}}$$

R<1 : Squeezing

R>1 : Anti-squeezing

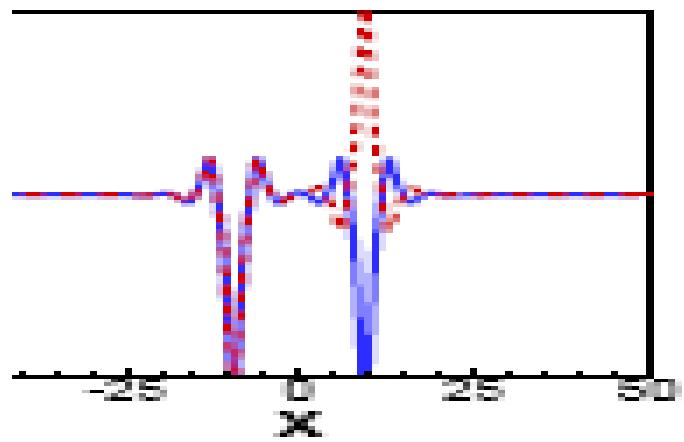
BEC matter-wave gap solitons

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{1}{2} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi + g_{1D} |\Psi|^2 \Psi,$$

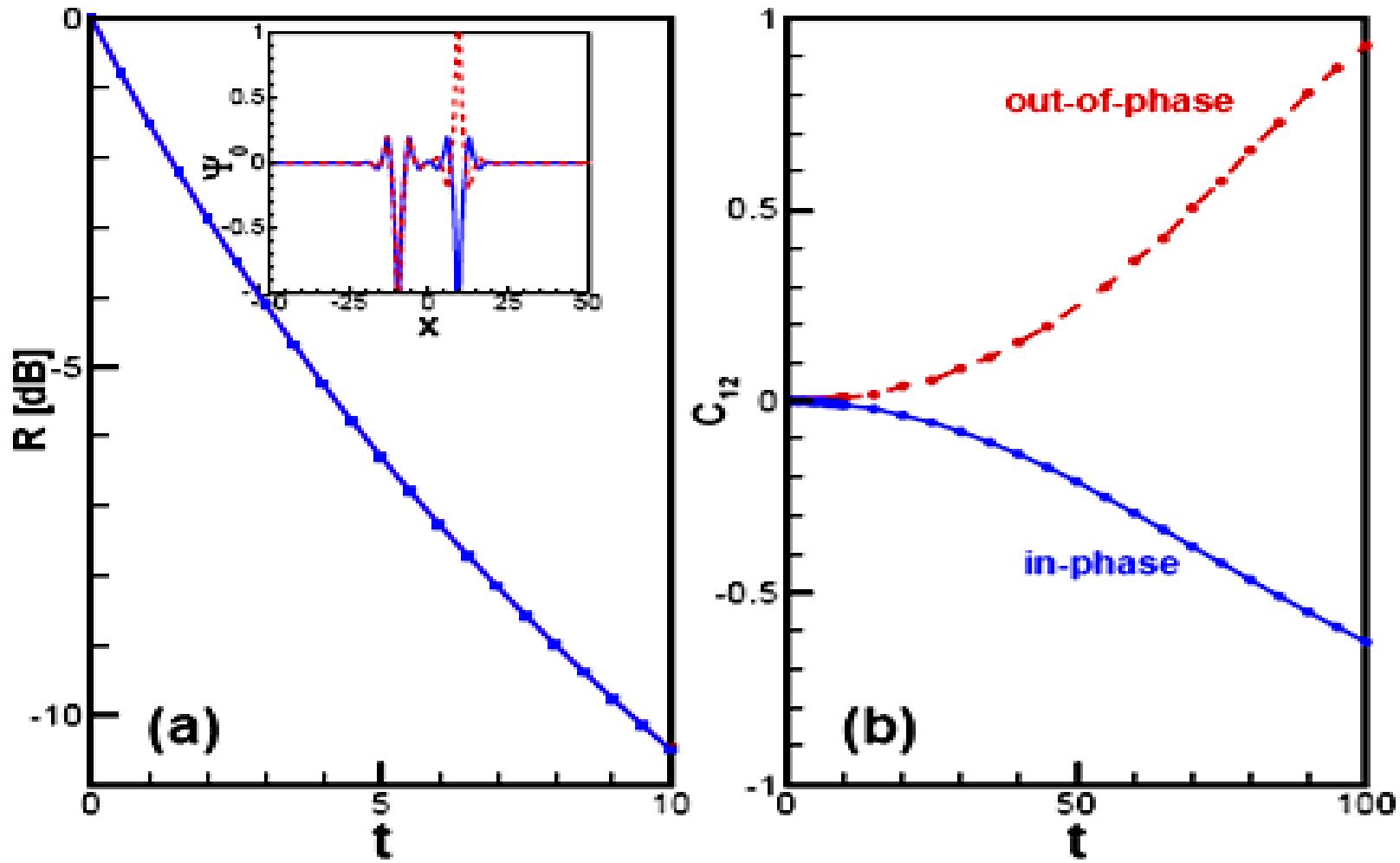


$$V(x) = V_0 \sin^2(x)$$

$$\Psi(t, x) = \psi(x) \exp(-i\mu t)$$



Quantum noises of matter-wave gap solitons



Quantum properties of SIT solitons

Squeezing through self induced transparency in a microstructured hollow core fibre

Ch. Marquardt, U.L. Andersen and G. Leuchs

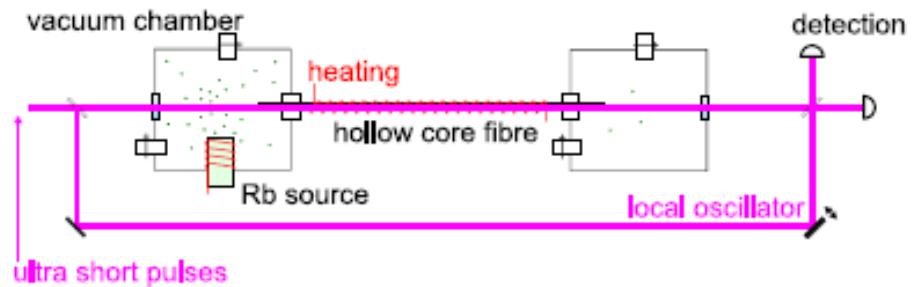
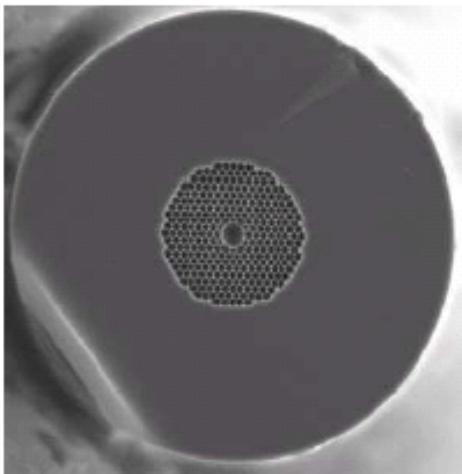


Figure 2: Experimental setup of Rb filling chambers and homodyne detection of light pulses propagating through a Rb vapour filled hollow core fibre.

Quantum equations of SIT solitons

$$\int_z^{z+\Delta z} \hat{P}(z, t) dz = \sum_{z \leq z_j \leq z + \Delta z} \hat{p}_j(t)$$

$$\int_z^{z+\Delta z} \hat{N}(z, t) dz = \sum_{z \leq z_j \leq z + \Delta z} \hat{n}_j(t)$$

$$\frac{\partial \hat{U}(z, t)}{\partial t} = -c \frac{\partial \hat{U}(z, t)}{\partial z} + K \hat{P}(z, t)$$

$$\frac{\partial \hat{P}(z, t)}{\partial t} = K \hat{N}(z, t) \hat{U}(z, t)$$

$$\frac{\partial \hat{N}(z, t)}{\partial t} = -2 K \left\{ \hat{P}^\dagger(z, t) \hat{U}(z, t) + \hat{U}^\dagger(z, t) \hat{P}(z, t) \right\}$$

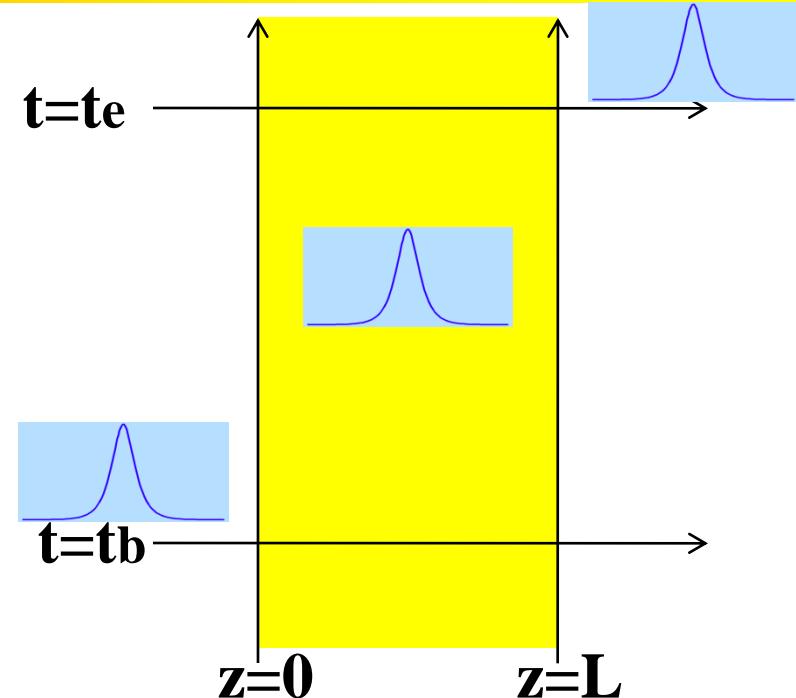
For the Bethe's ansatz approach, see S. John and V. I. Rupasov, *Europhys. Lett.* 46, p.326, 1999.

Linearized equations of SIT solitons

$$\frac{\partial}{\partial t} \hat{u} = -\frac{\partial}{\partial z} \hat{u} + \frac{r}{2} \hat{p},$$

$$\frac{\partial}{\partial t} \hat{p} = \frac{1}{2} (U_0 \hat{n} + N_0 \hat{u})$$

$$\frac{\partial}{\partial t} \hat{n} = -\left(P_0^* \hat{u} + U_0 \hat{p}^\dagger + U_0^* \hat{p} + P_0 \hat{u}^\dagger \right),$$



$$\hat{M}(t_e) = \int [f_L^*(z) \hat{u}(z, t_e) + f_L(z) \hat{u}^\dagger(z, t_e)] dz$$

$$\hat{M}(t_e) = \int dz \left[u^{A*}(z, t_b) \hat{u}(z, t_b) + u^A(z, t_b) \hat{u}^\dagger(z, t_b) + p^{A*}(z, t_b) \hat{p}(z, t_b) + p^A(z, t_b) \hat{p}^\dagger(z, t_b) + n^A(z, t_b) \hat{n}(z, t_b) \right]$$

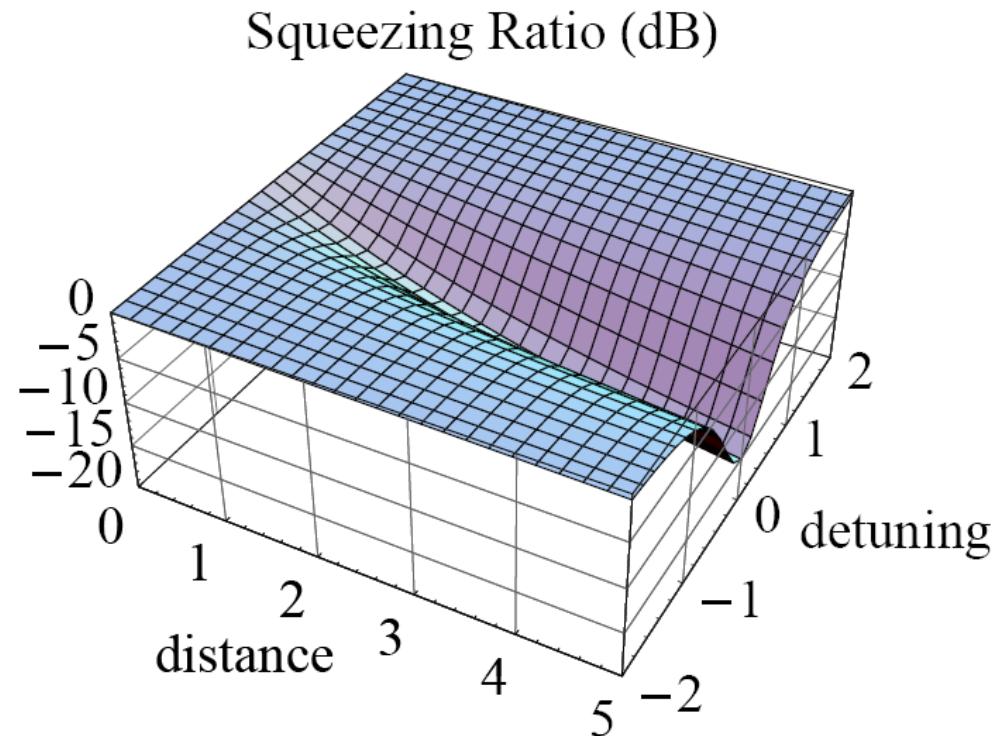
Squeezing of SIT solitons

$$\Delta n(t) = \Delta n_0 ,$$

$$\Delta\theta(t) = \Delta\theta_0 + \frac{\partial\omega}{\partial n_0} \Delta n_0 t + \frac{\partial\omega}{\partial p_0} \Delta p_0 t$$

$$\Delta p(t) = \Delta p_0 ,$$

$$\Delta x(t) = \Delta x_0 + \frac{\partial v_g}{\partial n_0} \Delta n_0 t + \frac{\partial v_g}{\partial p_0} \Delta p_0 t$$



At resonance, the squeezing is actually through the coupling of the photon number and the pulse position operators.

Quantum squeezing and correlation of self-induced transparency solitons

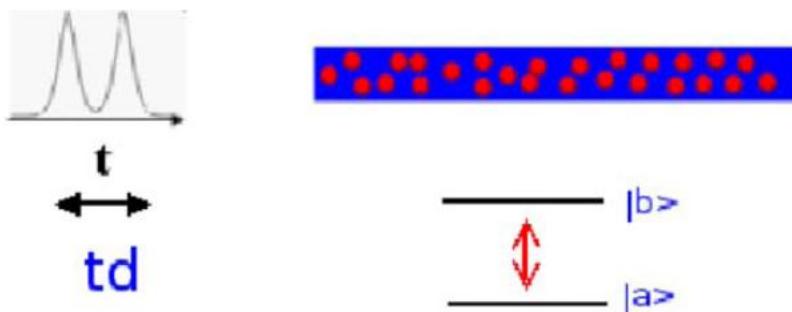
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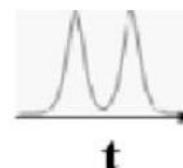
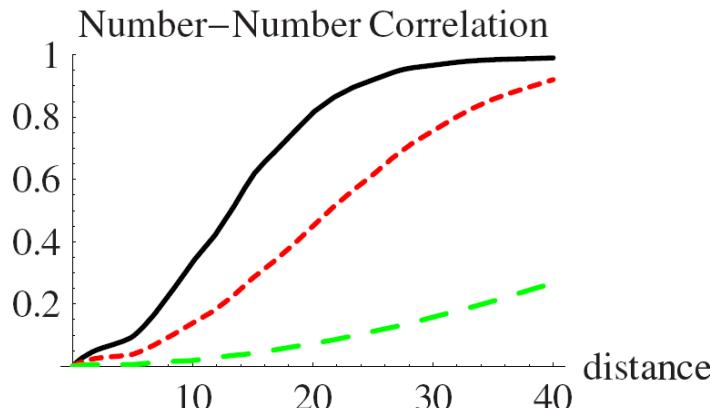
²*Department of Photonics, National Chiao-Tung University, Hsinchu, Taiwan 300, Republic of China*

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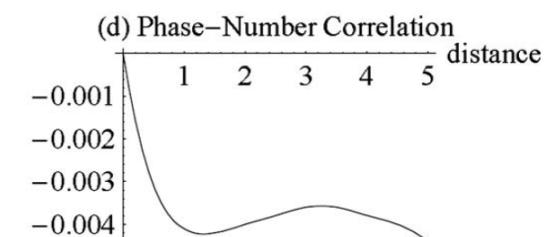
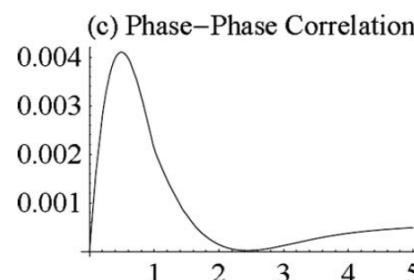
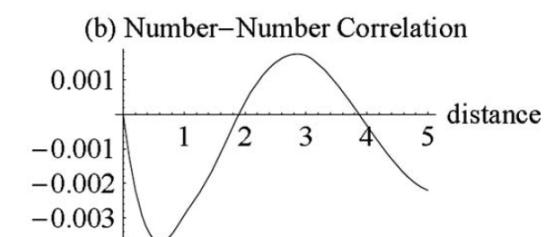
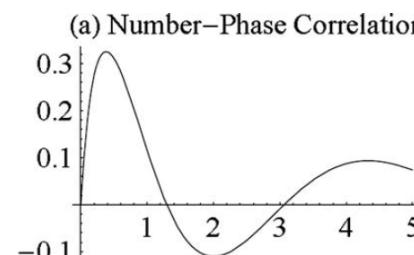
(Received 6 February 2009; published 25 September 2009)



In-phase soliton pairs



Out-of-phase soliton pairs



Conclusions

- Theories of quantum nonlinear optical propagation are reviewed.
- Quantum soliton, squeezing, correlation, and entanglement are explained.
- How squeezing leads to entanglement is clarified.
- Entangled quantum solitons can be generated through nonlinear interaction.
- Time-, polarization-, or frequency-multiplexed schemes are analyzed.
- Different optical soliton platforms are investigated.