



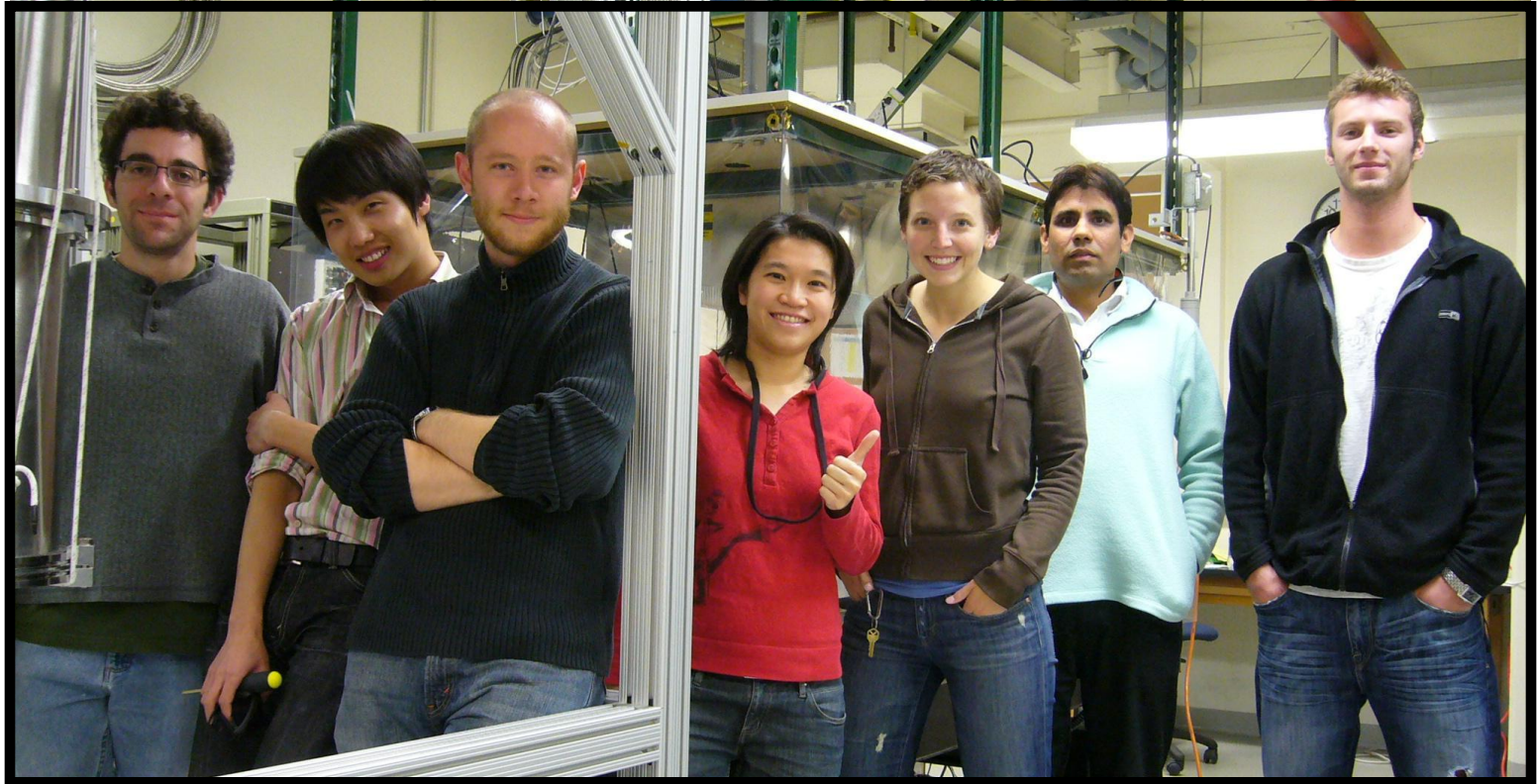
University of Nevada, Reno

**A study of anisotropic inelastic collisions
and slow/stopped light with nuclear spin**

**Mei-Ju Lu
2011Jun03**

Group photo

<http://www.physics.unr.edu/xap/>



From the left:

Dr. Jonathan Weinstein (PI), Tian Li (G), Ryan Baker (UG), Mei-Ju Lu (G),
Aja Ellis (UG), Vijay Singh (G) and Kyle Hardman (G)

Outline

- **Motivation**
- **Apparatus**
Buffer-gas cooling
- **Measurements**
Collisional-induced relaxation
Atom-light coherence
Light storage and retrieval
- **Conclusions**

Collisions

- **Elastic collisions:**

Colliding particles remain in the same internal energy states during the collisional events.

- **Inelastic collisions:**

Colliding particles change their internal states, with their chemical constitutions remaining the same.

- **Chemical reaction:**

Resulting products after collisions are different from the colliding particles.

Inelastic collisions

- Colliding particles change their internal states, with their chemical constitutions remaining the same.

Zeeman relaxation (m -changing)

Fine-structure changing collisions (J -changing)

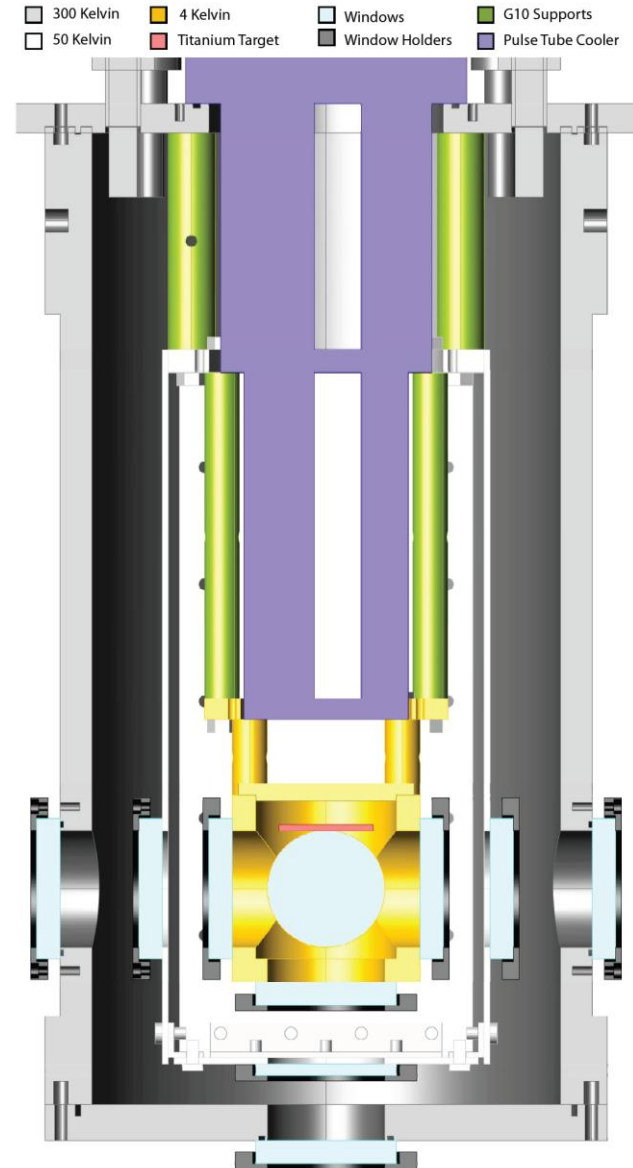
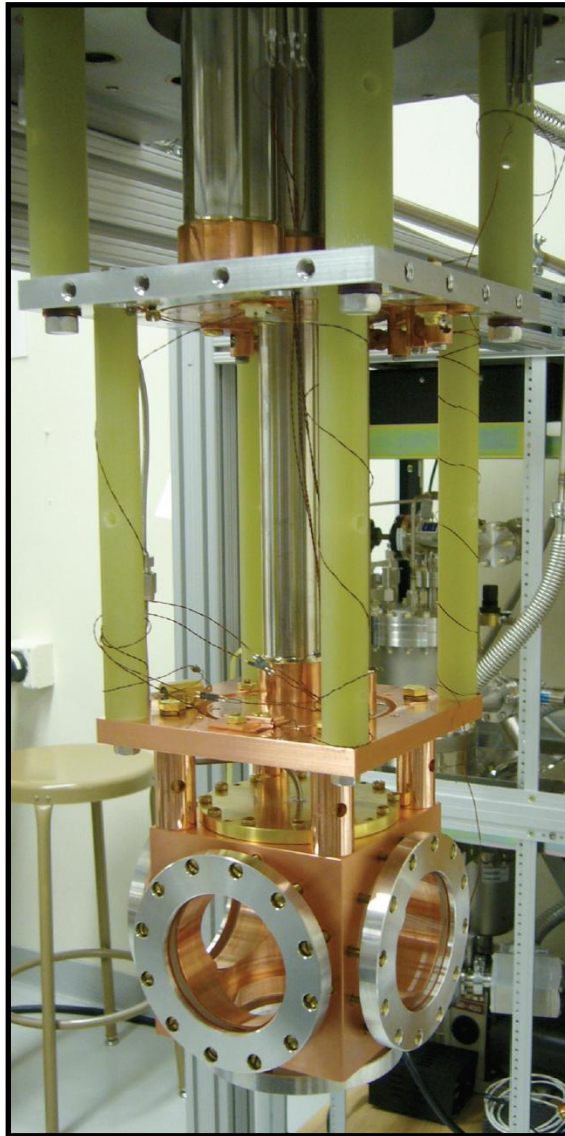
Hyperfine-structure changing collisions (F -changing)

"Bad" collisions!~

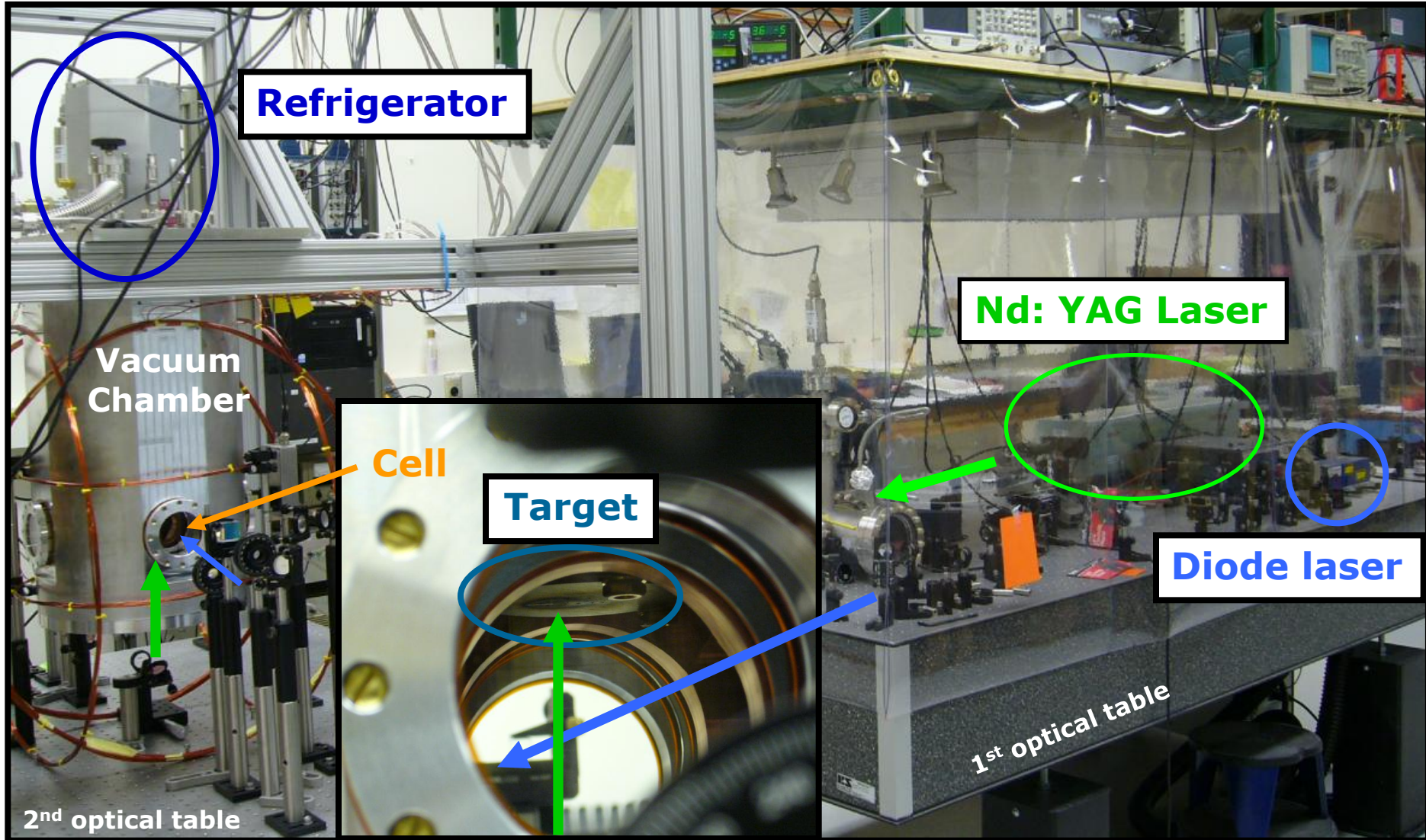
Inelastic collisions cause a loss in the magnetic trap.

It is difficult to prepare atoms or molecules in a specific state.

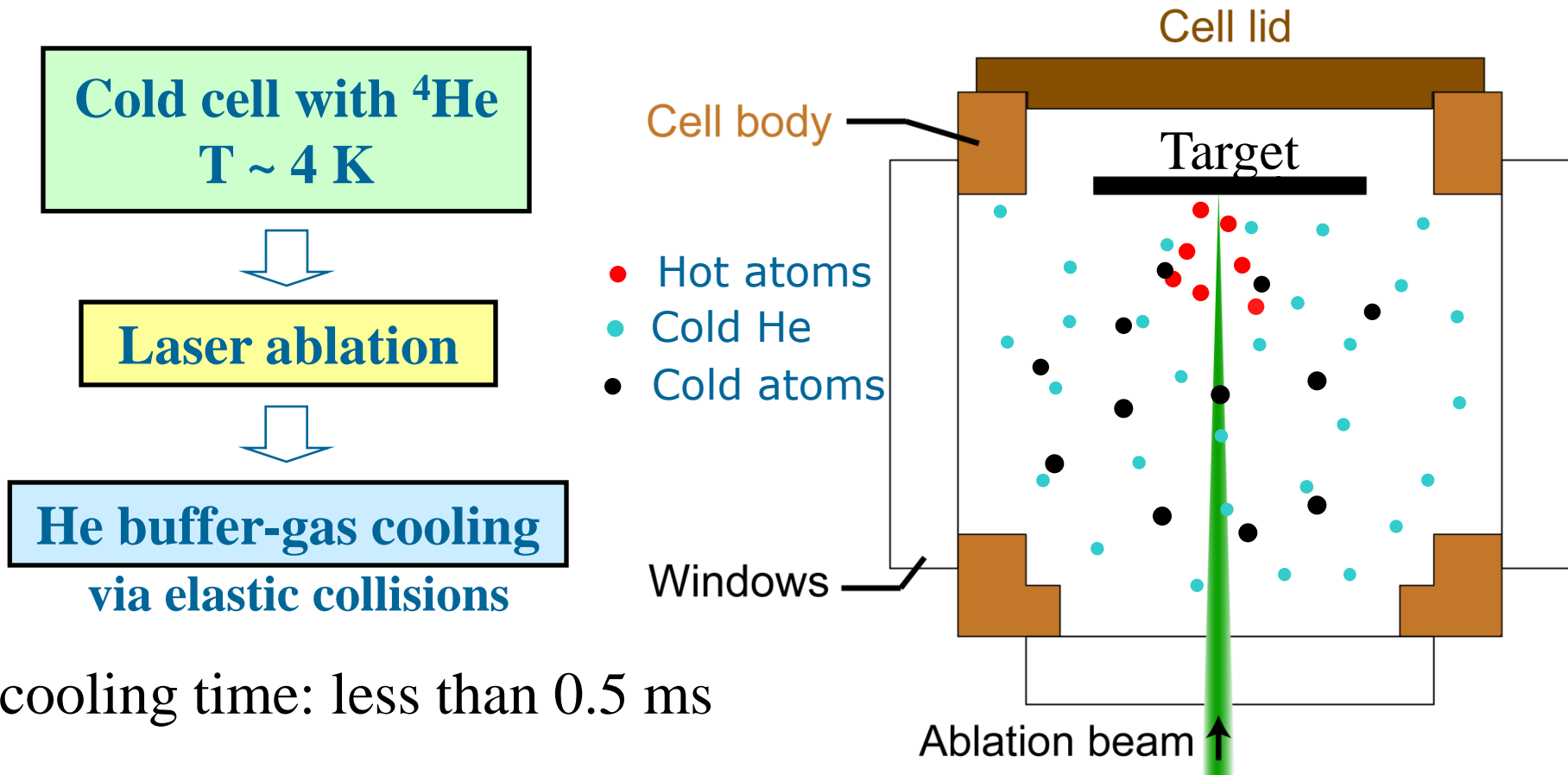
Cryogenic system



Apparatus

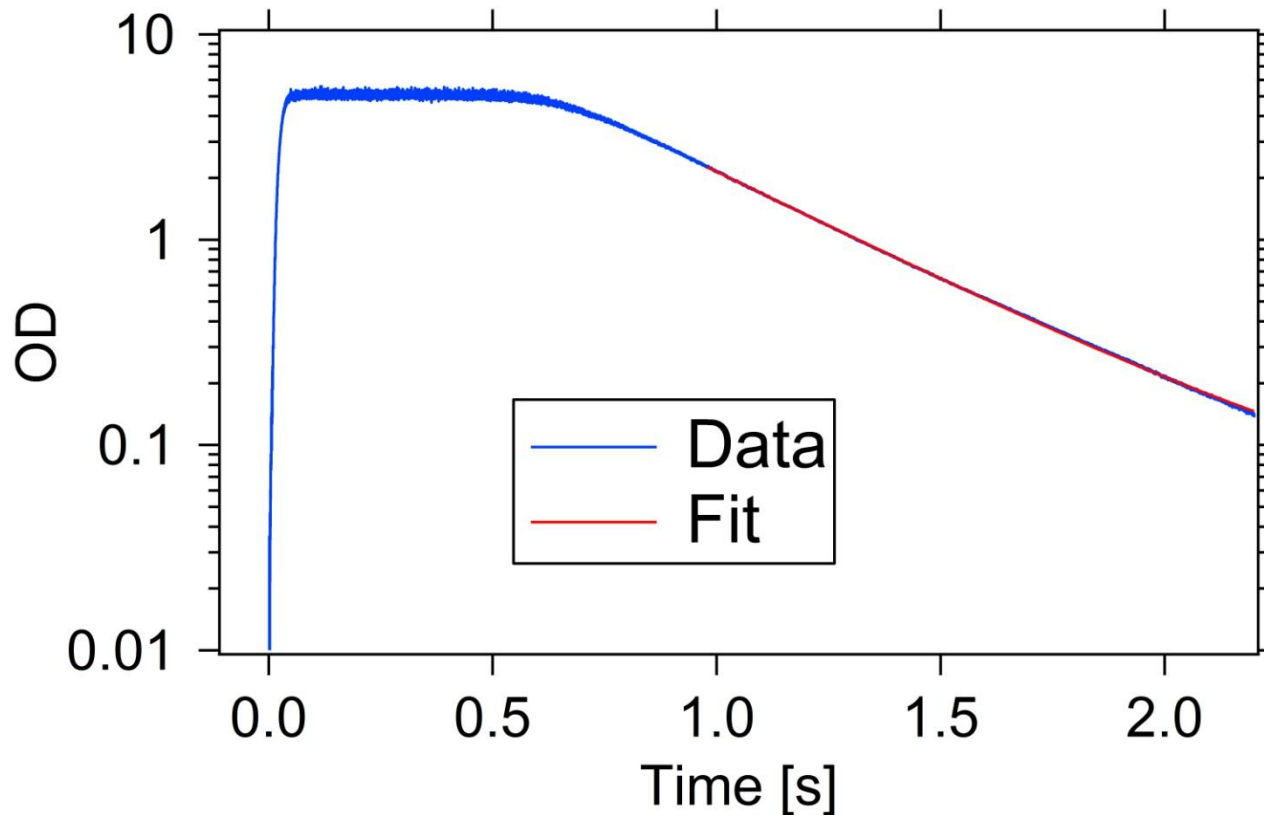


Buffer-gas-cooled atoms



Diffusion lifetimes

1. Optical Density (OD) = atomic density \times absorption cross section \times path length
2. Transmission = $e^{-\text{OD}}$; Transmission = 1 - Absorption



3. The lowest diffusion mode: $n(t) = n_0 \exp\left[-\frac{t}{\tau_D}\right]$

Cryogenically-cooled atomic vapor cell

Buffer-gas cooling is general to atoms or molecules
(eg: We have observed Ti, Ga, In, Yb, Li, TiO & CaH at 5 K)

Note:

Not as cold as laser cooling (μK)

BGC is limited by helium vapor pressure (0.3K - 300K)

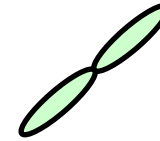
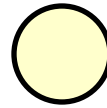
For our purpose, BGC is sufficient.

High optical density (eg: ^{173}Yb , OD ~ 80)

Large atomic numbers (eg: ^{173}Yb , $N \sim 10^{13}$ atoms)

Large atomic density (eg: ^{173}Yb , $n \sim 10^{10} \text{ cm}^{-3}$)

In general:



	S-state atoms	Non-S-state atoms
Eg:	Yb[1S_0] Rb[$^2S_{1/2}$] Cr[7S_3]	C[3P_0] Si[2P_0] O[3P_2]
Orbital angular momentum	Zero	Non-zero
Electronic shell structure	Spherical	Aspherical
Interaction potential	Isotropic	Anisotropic

The electronic interaction anisotropy usually causes large inelastic collisions.

Derevianko et al., PRL 90(6), 063002 (2003);
R. V. Krems et al, J. Phys. Chem. A 108, 8941 (2004)

Prior Work

	S-state atoms	Non-S-state atoms
Events	Yb[¹ S ₀]-He Rb[² S _{1/2}]-He	C[³ P ₀]-He Si[² P ₀]-He Al[² P _{1/2}]-Ar O[³ P ₂]-H C[³ P ₀]-H
Rate coefficient [cm ³ s ⁻¹]	<10 ⁻¹⁷	>10 ⁻¹²

Lu et al, Opt. Lett. 35, 622 (2010) ; Walker et al, PRA 56,2090 (1997); Picard et al, JCP 108, 10319 (1998);
Picard et al, JCP 117, 10109 (2002); Abrahamsson et al, Astrophysics, J. 654, 1171 (2007)

$$k = \frac{1}{\tau n}$$

k : rate coefficient

$1/\tau$: inelastic collision rate

n : density

Exceptions

- Measured Ti-He Zeeman relaxation rate coefficient: $k_m \sim 10^{-14} \text{ cm}^3\text{s}^{-1}$.

Hancox et al, PRL 94, 013201(2005); Krems et al, PRL 94, 013202 (2005)

- Submerged-shell structure: **Titanium** [$3d^2 4s^2 \ ^3F_J$], ^{48}Ti ($I=0$)
a suppression of the anisotropic interaction potential
-

- Similar suppression for the fine-structure changing collisions?
- Fine-structure changing collisions is an important cooling mechanism in cold molecular clouds (~ 10 K).

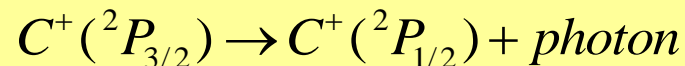
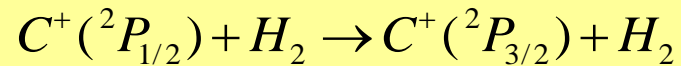
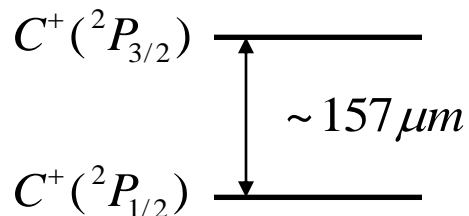
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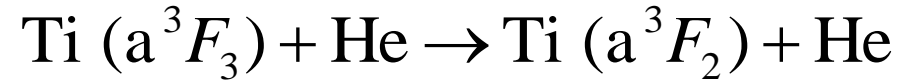


Predicted rate coefficient $k \sim 10^{-10} \text{ cm}^3 \text{ s}^{-1}$

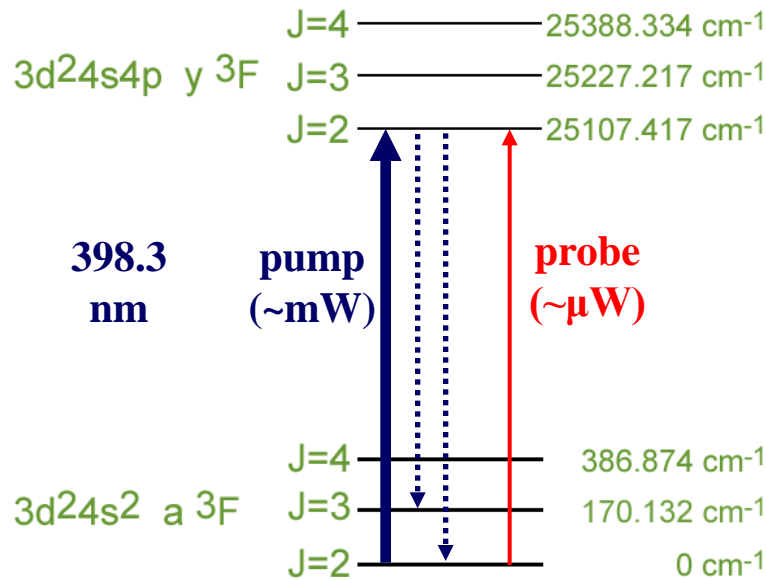
Flower et al J. Phys. B. At. Mol. Opt. Phys. 21 L451 (1988)

^{48}Ti -He fine-structure changing collisions

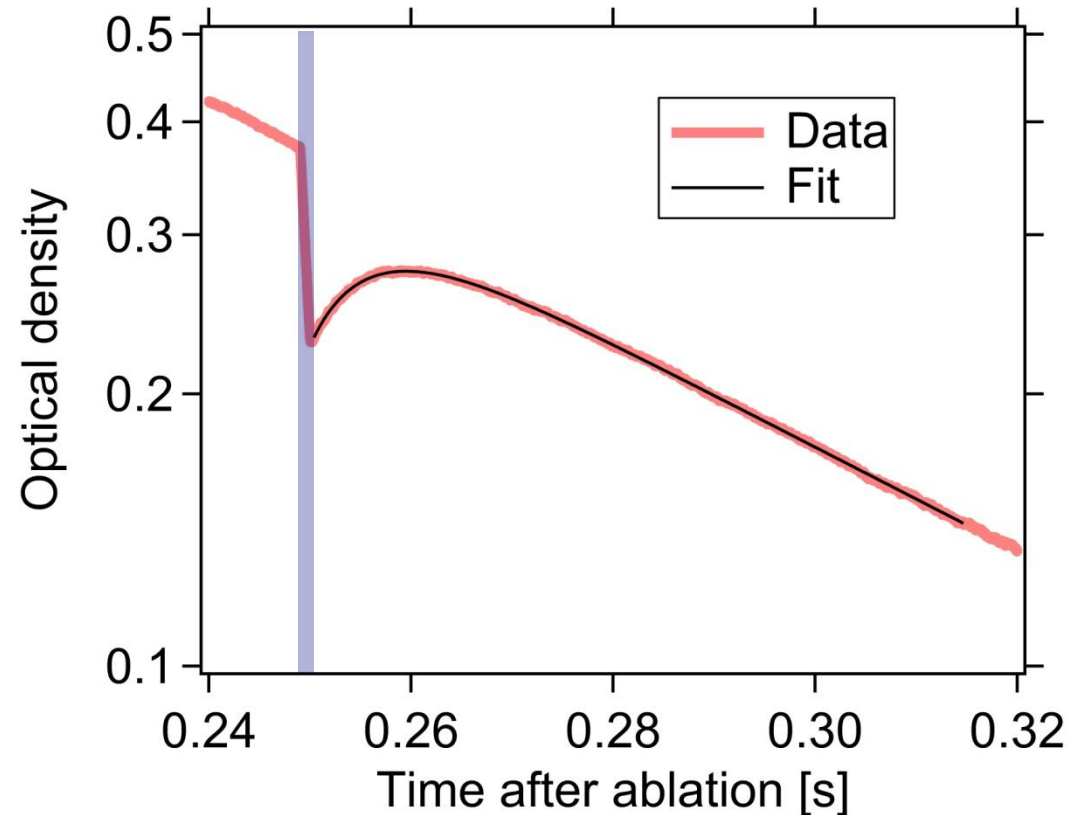
Optical pumping to perturb J population



^{48}Ti level structure

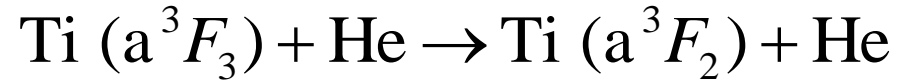


NIST Atomic Spectra Database

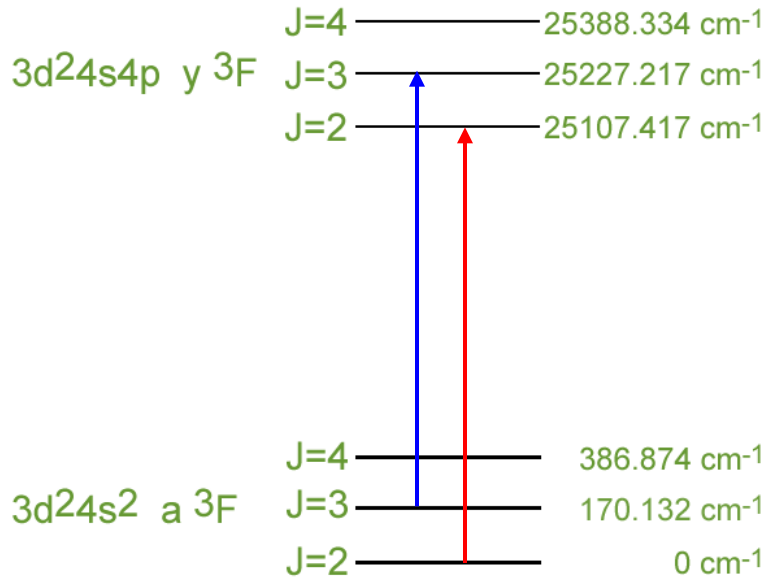


^{48}Ti -He fine-structure changing collisions

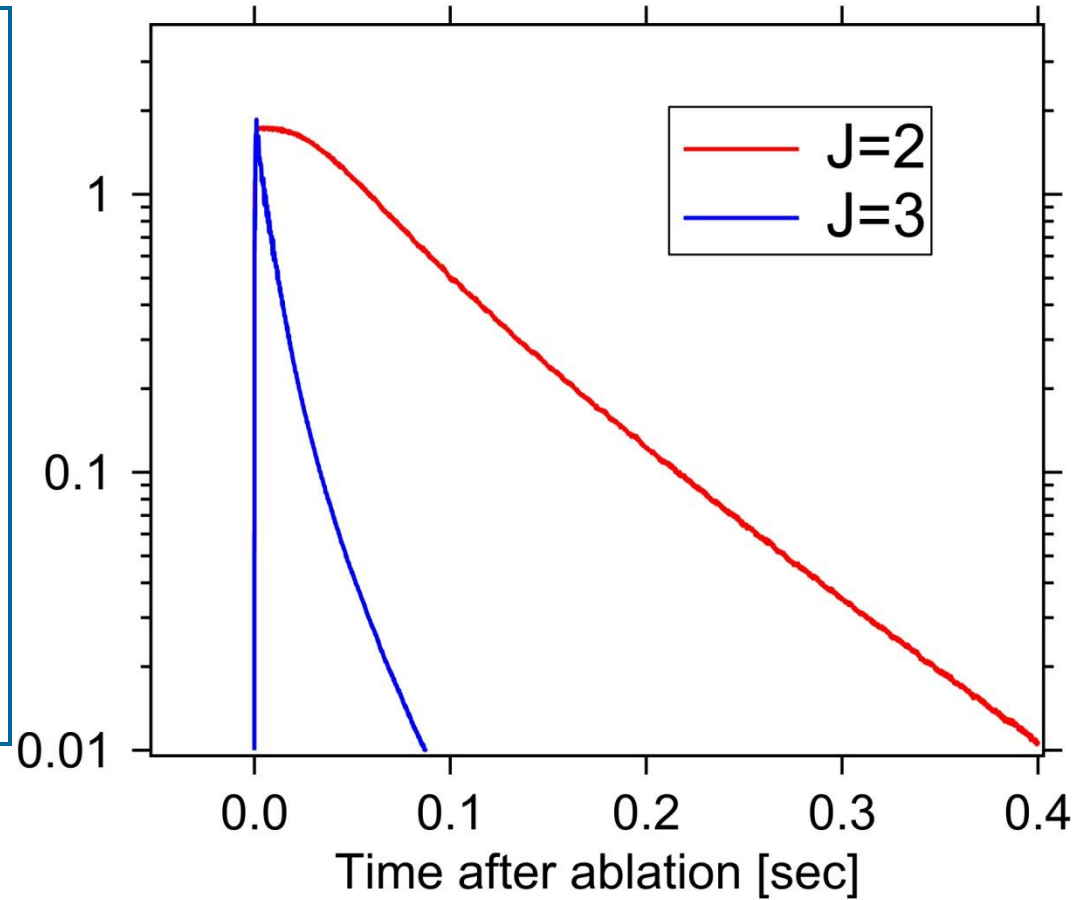
Monitoring the J population



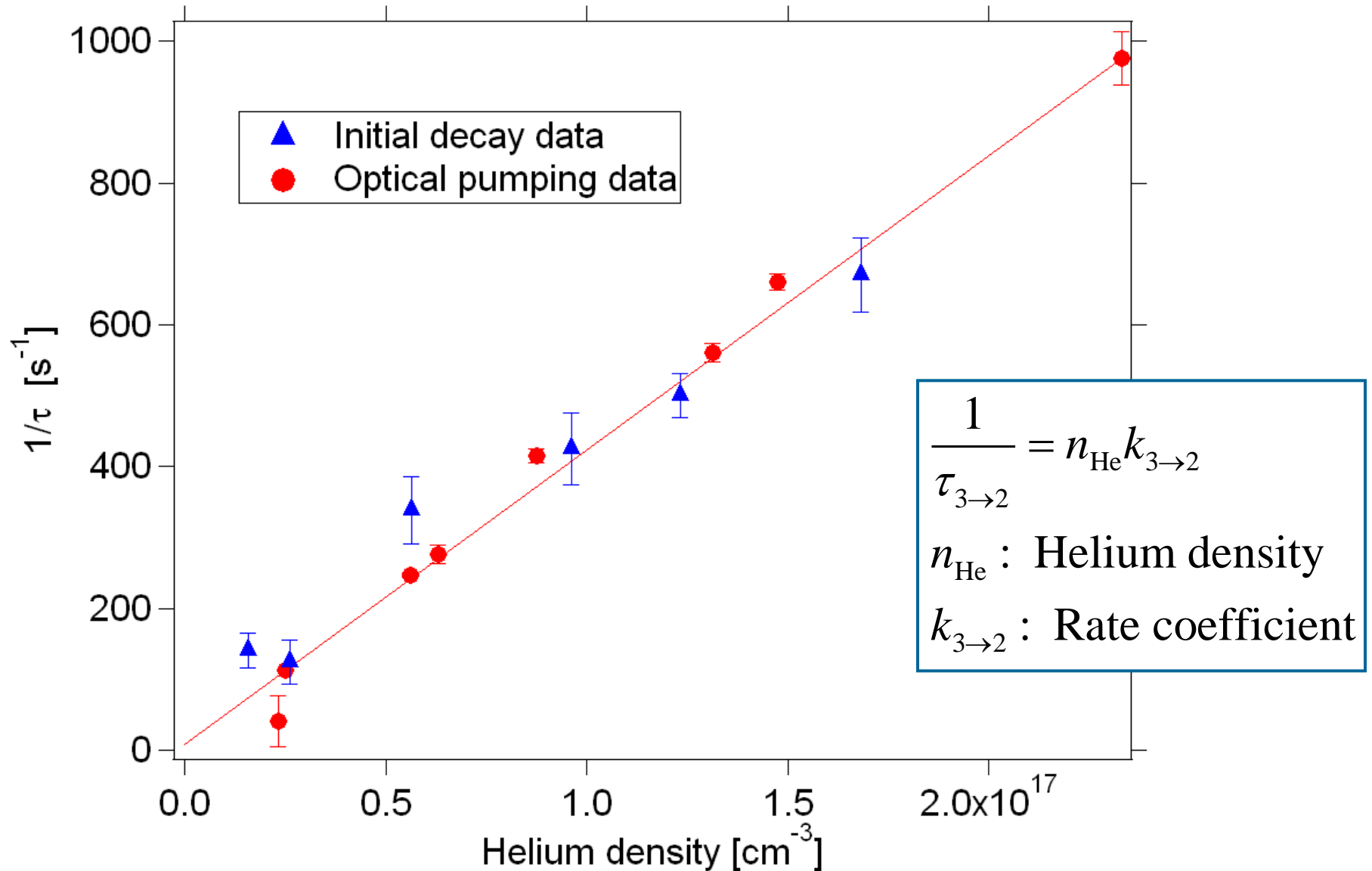
Ti level structure



NIST Atomic Spectra Database



^{48}Ti -He fine-structure changing collisions



$^{48}\text{Ti-He}$ fine-structure changing collisions

- **Submerged-shell structure:** Titanium [$3d^2 4s^2 {}^3F_J$]

$^{48}\text{Ti-He}$	
T [K]	$k_J [\text{cm}^3\text{s}^{-1}]$
5.2	$(4.4 \pm 0.7) \times 10^{-15}$
9.9	$(5.3 \pm 0.8) \times 10^{-15}$
15.6	$(7.7 \pm 1.2) \times 10^{-15}$
19.9	$(9.8 \pm 1.5) \times 10^{-15}$

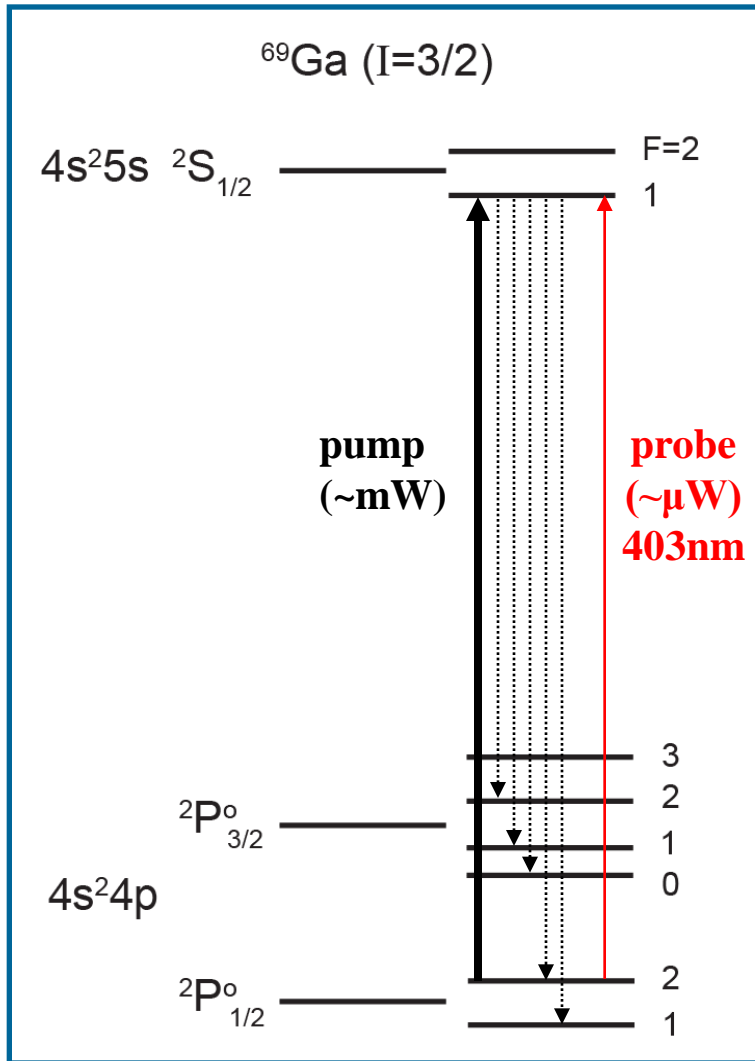
Lu et al, PRA 77, 060701(R) (2008)

1. A similar suppression in fine-structure changing collisions as that in Zeeman relaxation.
2. The result of Ti-He fine-structure measurements has been applied to a theoretical calculation.

Zygelman et al, PRA 78, 012795 (2008)

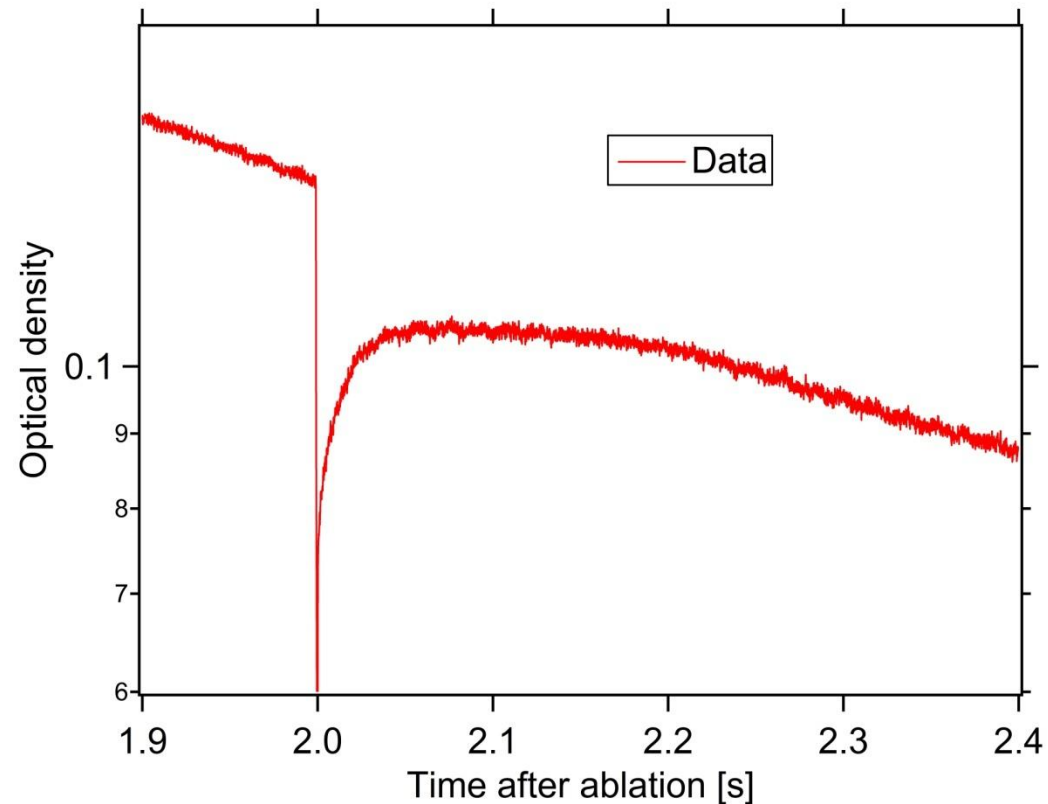
Open-shell atoms

- $^2P_{1/2}$ -state atoms: Gallium [$4s^24p\ ^2P_{1/2}$] and Indium [$5s^25p\ ^2P_{1/2}$]

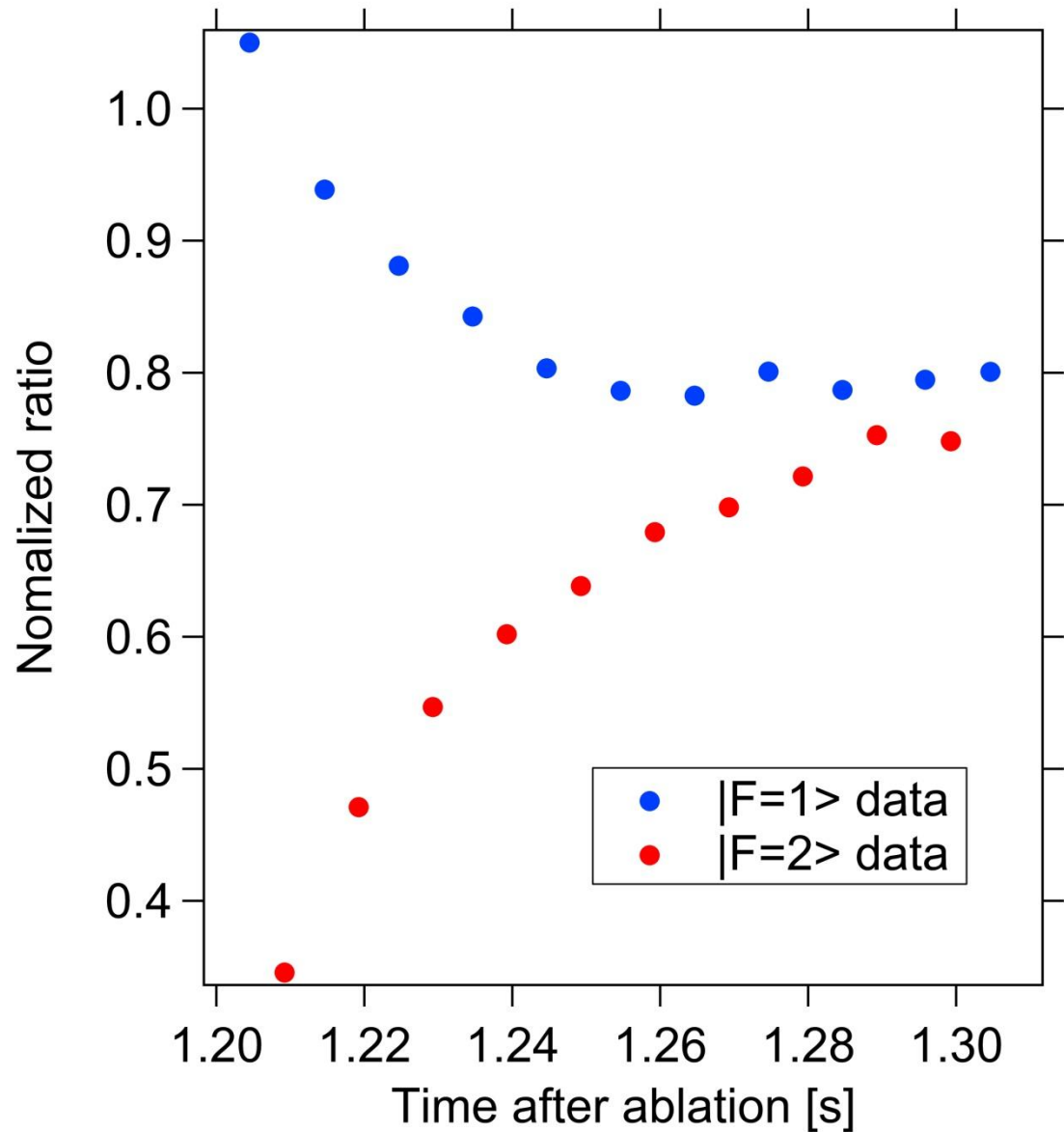
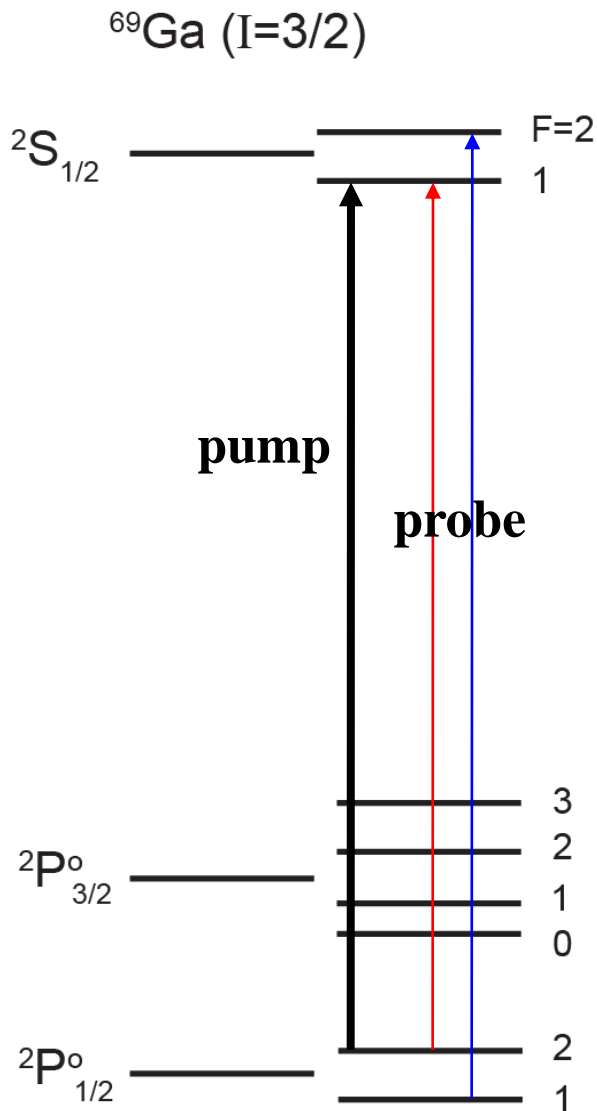


Probe ground state population

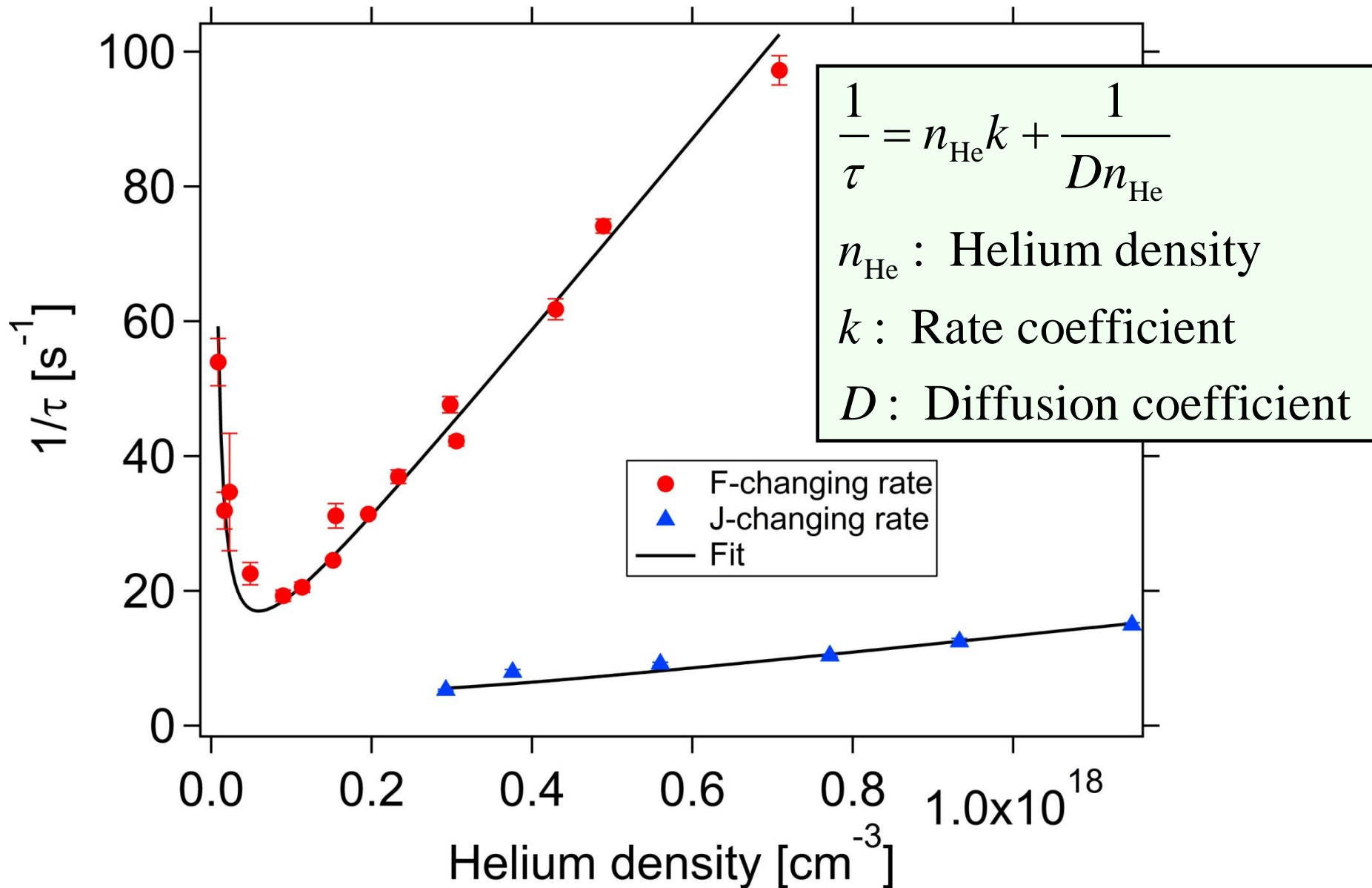
Optical pumping to perturb population



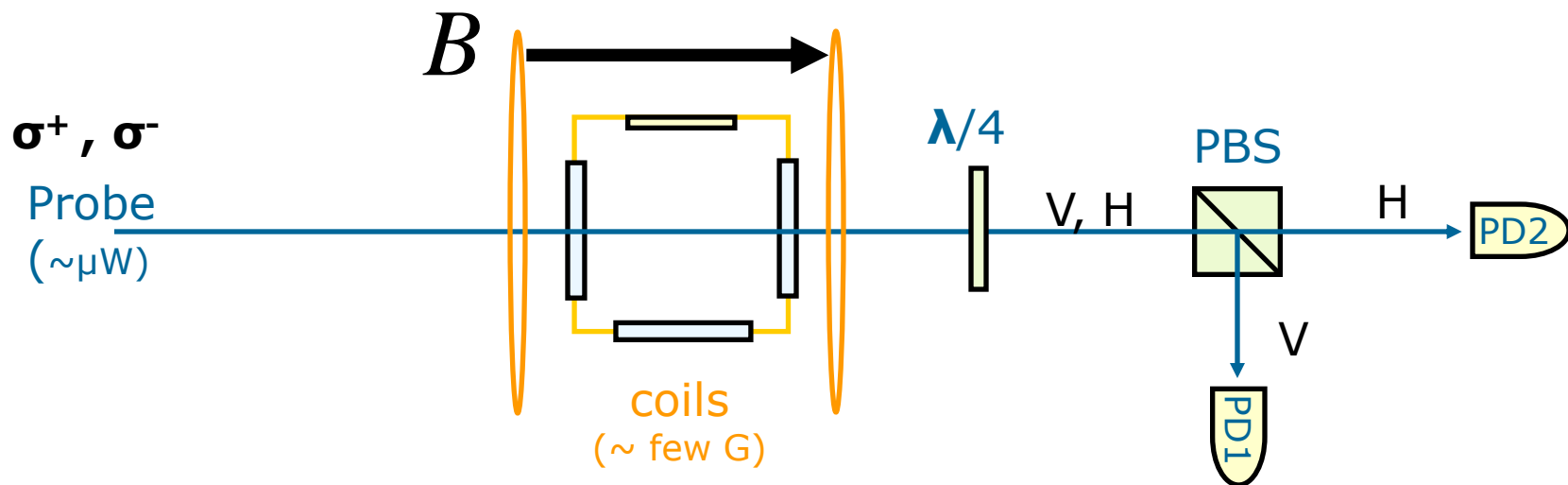
Ga-He F -changing collisions



Ga-He F and J -changing rates

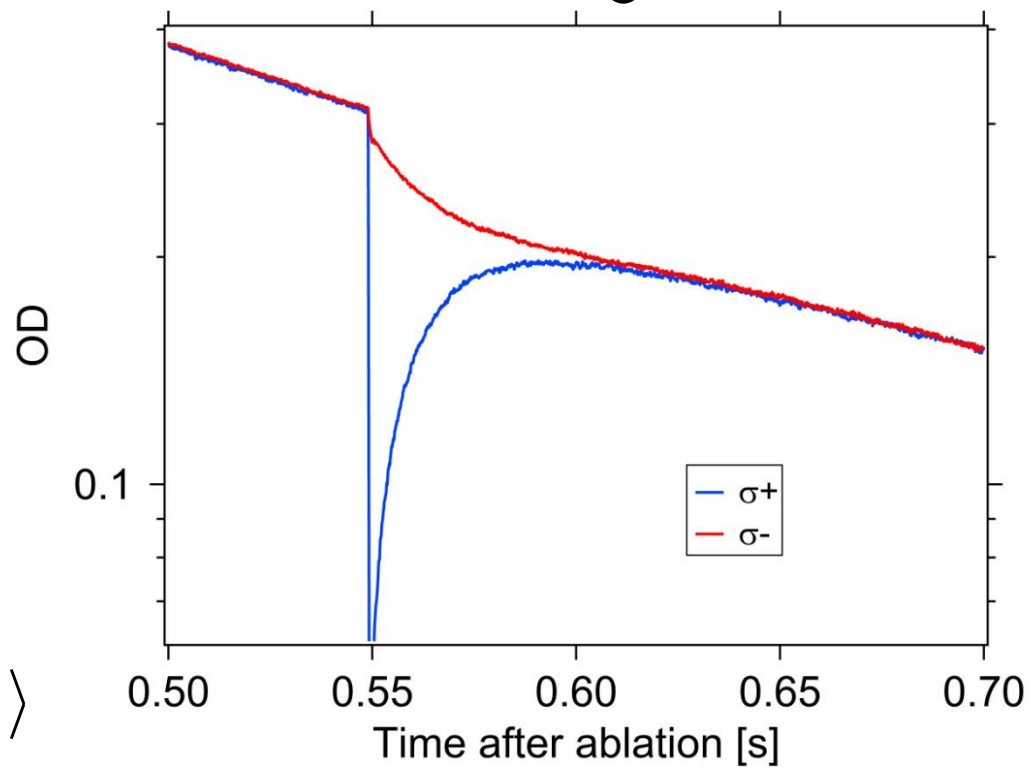


Ga-He Zeeman relaxation

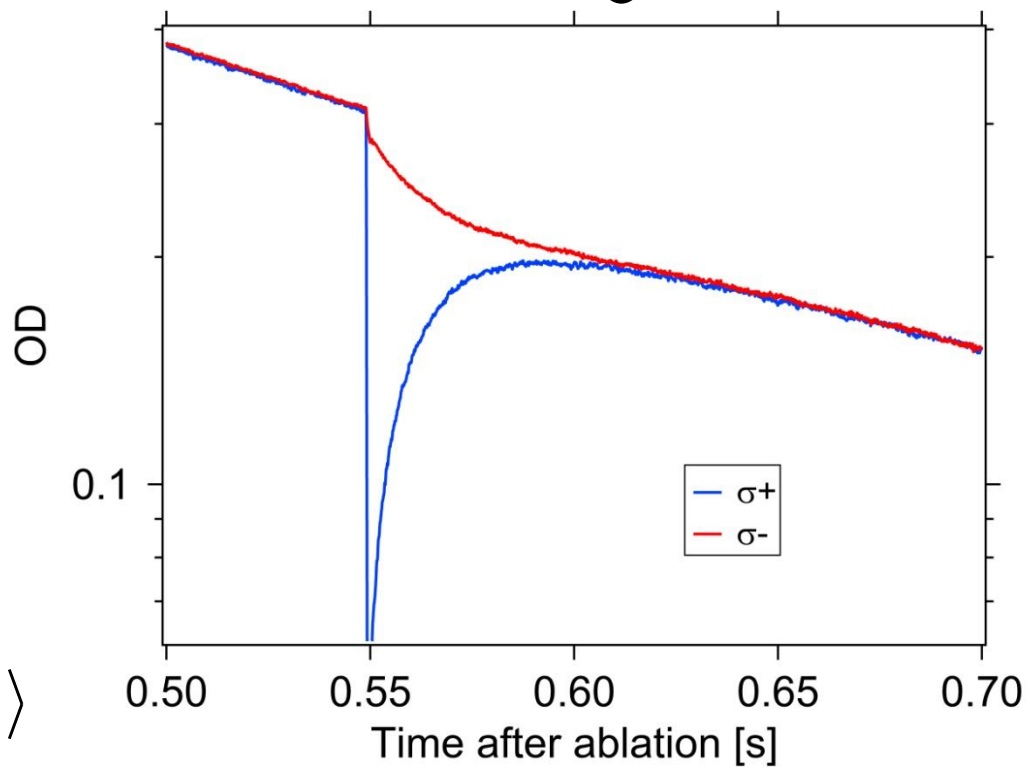
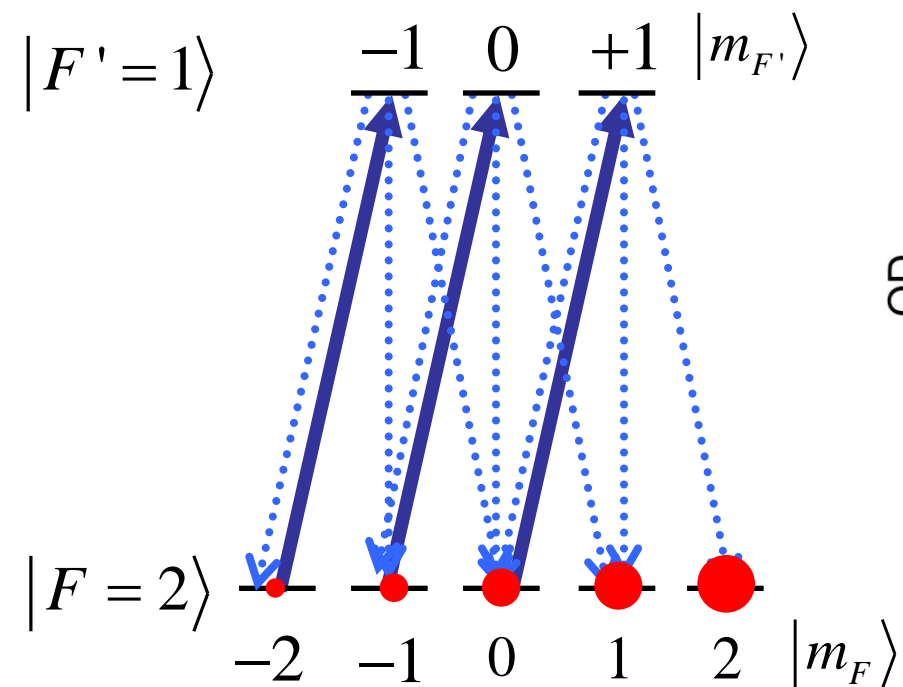
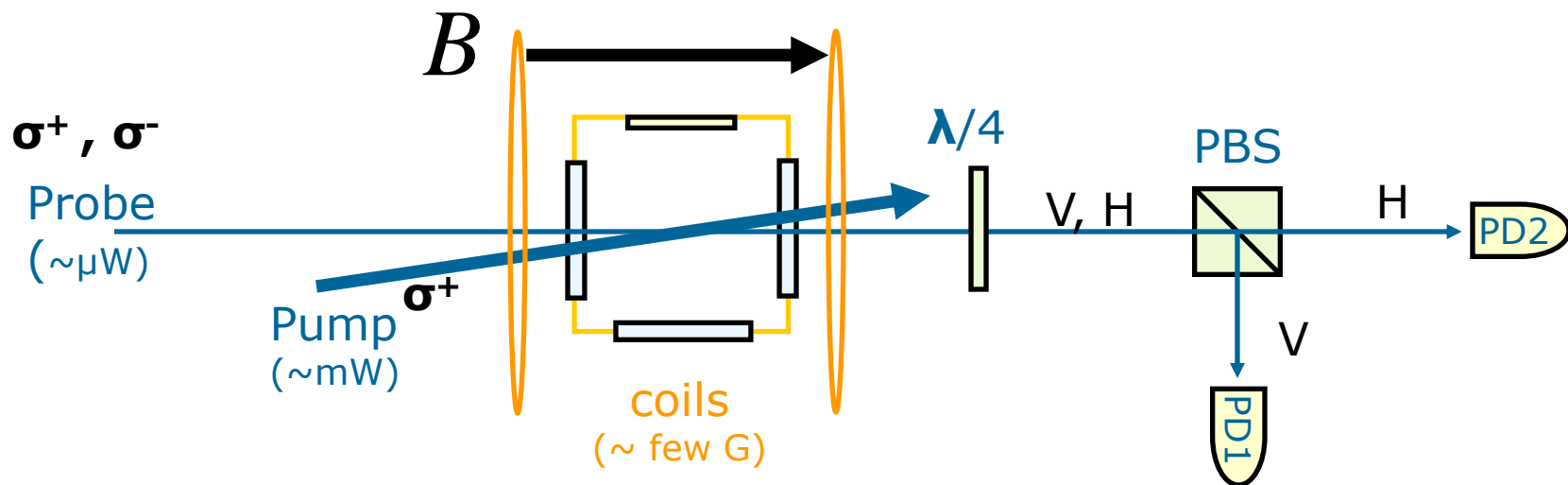


$|F' = 1\rangle$ -1 0 +1 $|m_{F'}\rangle$

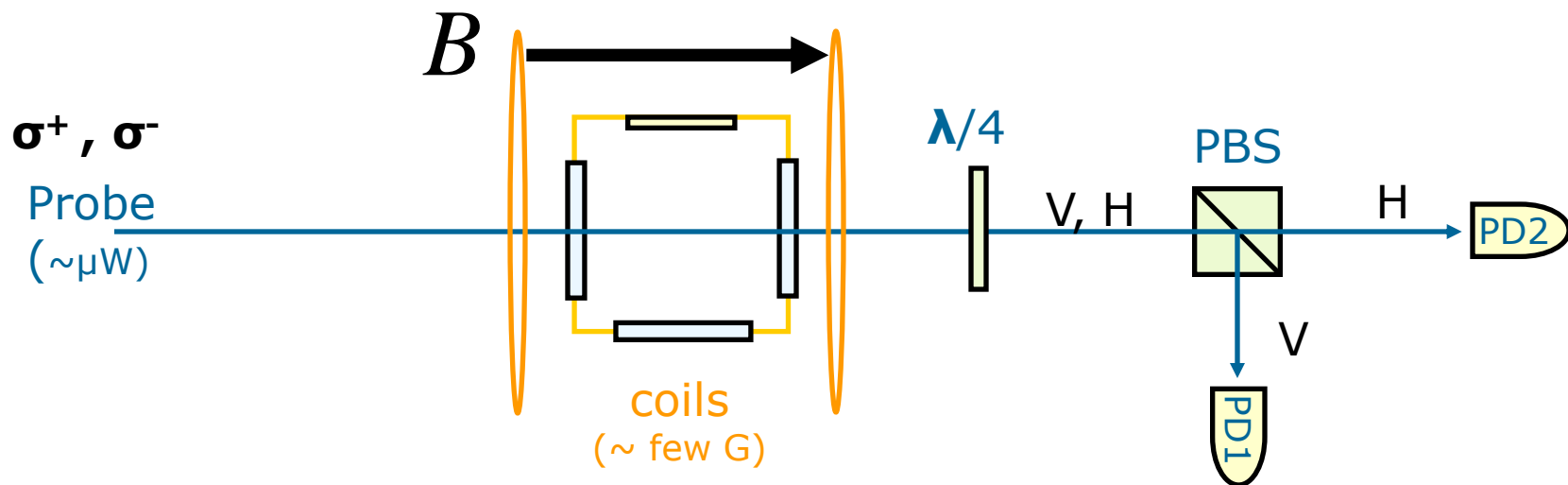
$|F = 2\rangle$ ● ● ● ● ●
 -2 -1 0 1 2 $|m_F\rangle$



Ga-He Zeeman relaxation

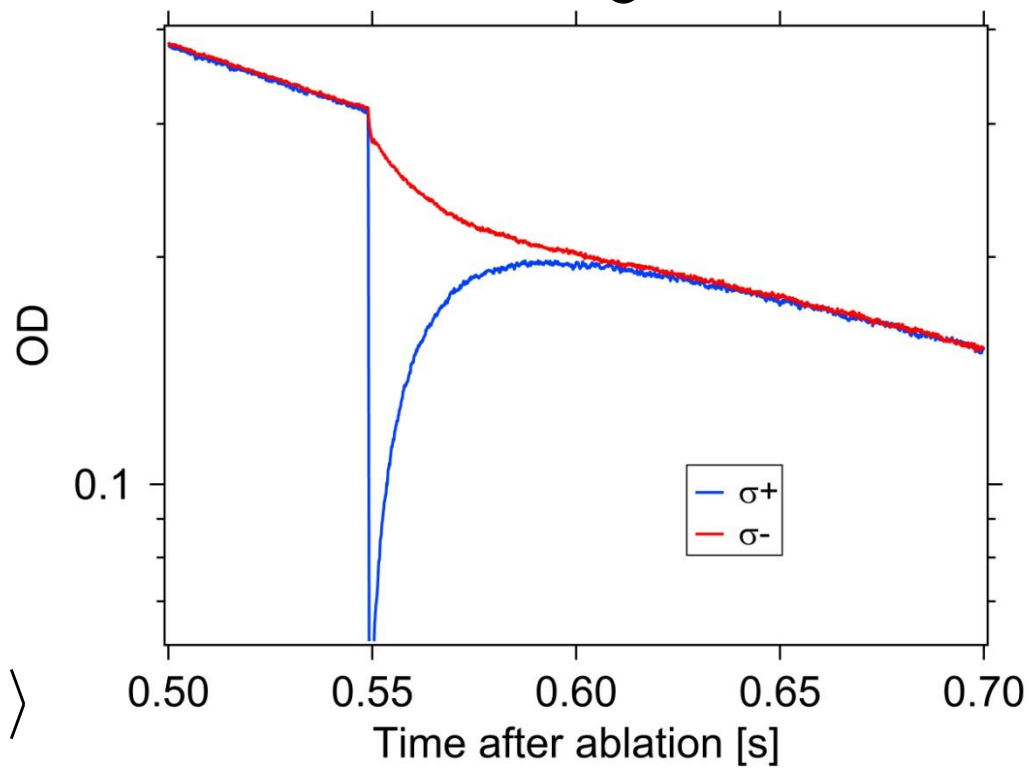


Ga-He Zeeman relaxation



$|F' = 1\rangle$ -1 0 +1 $|m_{F'}\rangle$

$|F = 2\rangle$ \bullet \bullet \bullet \bullet \bullet
-2 -1 0 1 2 $|m_F\rangle$



Events	$^{69}\text{Ga}[^2\text{P}_{1/2}]\text{-He}$		$^{115}\text{In}[^2\text{P}_{1/2}]\text{-He}$	
	Experiment	Theory	Experiment	Theory
k_J	(1.0+-0.3)	1.3	< 8	0.0004
k_F	(5.3+-1.3)	6.6	< (2.3+-1.4)	0.1
k_m	< 300	2.3	< 50	3.8

(in the unit of $10^{-17} \text{ cm}^3\text{s}^{-1}$)

Tscherbul et al., PRA 80, 040701(R) (2009)

- Prior work:
 - a. $\text{C}[^3\text{P}_0]$, $\text{Si}[^2\text{P}_0]\text{-He}$; $\text{Al}[^2\text{P}_{1/2}]\text{-Ar}$; $\text{O}[^3\text{P}_2]$, $\text{C}[^3\text{P}_0]\text{-H}$: $k > 10^{-12} \text{ cm}^3\text{s}^{-1}$
 - b. $\text{Ti}[^3\text{F}_2]\text{-He}$: $k_J \sim 10^{-15} \text{ cm}^3\text{s}^{-1}$
- The electron-density distribution of atoms in $^2\text{P}_{1/2}$ electronic states is spherically symmetric and that of $^2\text{P}_{3/2}$ atoms is not.
- Ground-state fine-structure splitting
 - Ga: 826 cm^{-1}
 - In: 2213 cm^{-1}

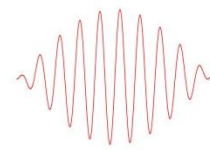
Summary

- The combined technique of laser ablation, buffer-gas cooling and optical pumping is applicable to measure inelastic collisions.
- We have measured a suppression in Ti-He fine-structure changing collisions due to titanium's submerged shell structure.
- The measured inelastic collisions rates of Ga-He and In-He are about five orders smaller than other open-shelled atoms and about one order smaller than Ti-He.
- $^2P_{1/2}$ -state atoms may be good for evaporative cooling in a magnetic trap.

Light storage and retrieval

- Quantum communication:
Carrier: photons (fastest, polarization)
Storage: atoms (spin states, long coherence time)
- A direct transmission of quantum states for a long distance is impossible!
In an optical fiber: Transmission = $10^{-0.2L/10}$
L=5 km, T=0.8; L=100 km, T= 10^{-2} ; L=500 km, T= 10^{-10}
- Quantum repeater Briegel et al., PRL 81(26), 5932 (1998); DLCZ Nature 414, 413 (2001)
Successful operations are probabilistic.
Need quantum memory
- Classical light storage and retrieval
Slow/stopped light

Slow light



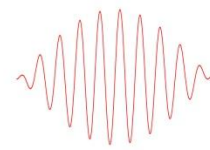
For a light pulse traveling in a dispersive medium:

$$v_{\text{phase}} = \frac{c}{n}; \quad v_{\text{group}} = \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}}$$

i) $\frac{dn(\omega)}{d\omega} \rightarrow \text{large}, v_{\text{group}} \rightarrow \text{slow}$

ii) no absorption of the light pulse

Slow light



For a light pulse traveling in a dispersive medium:

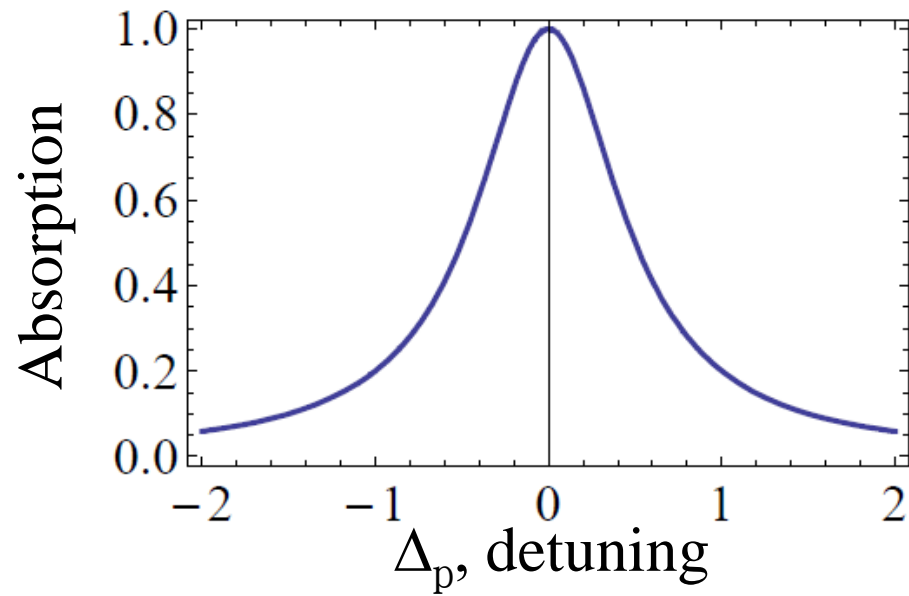
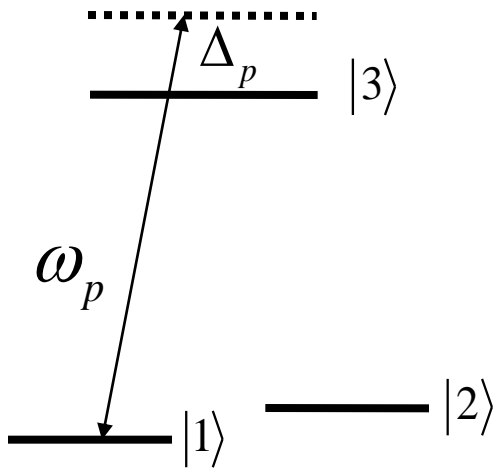
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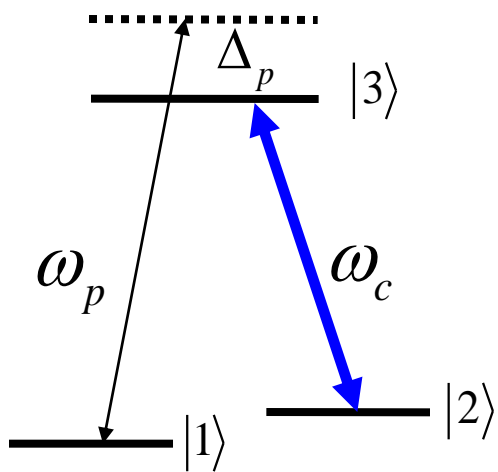
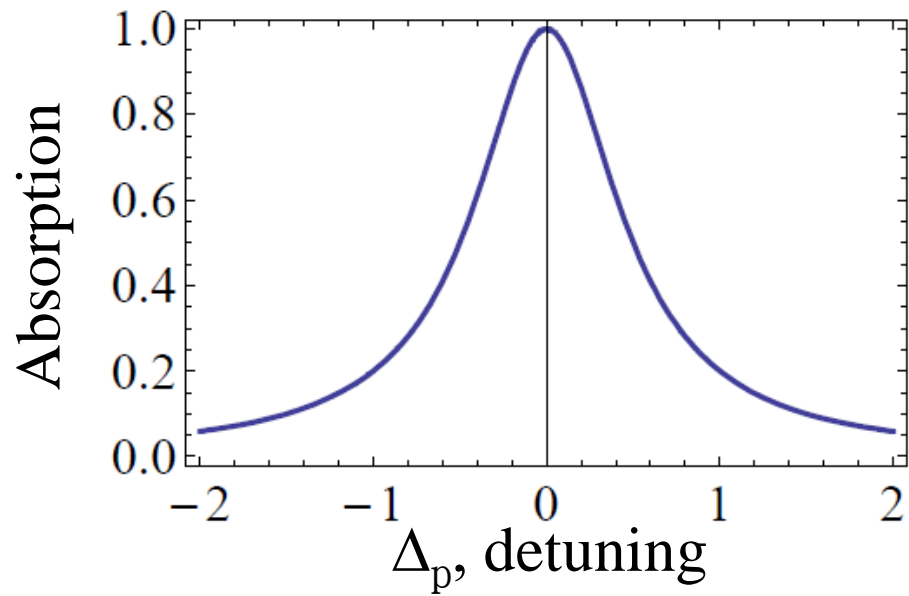
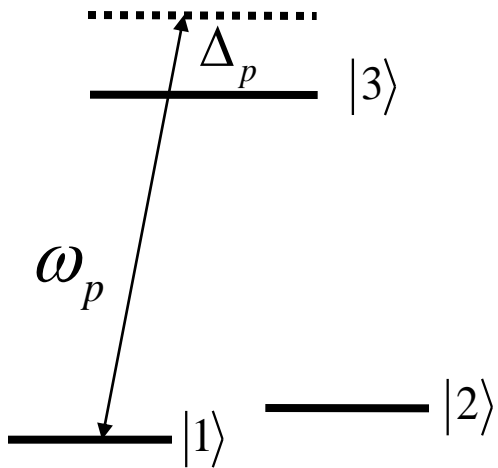
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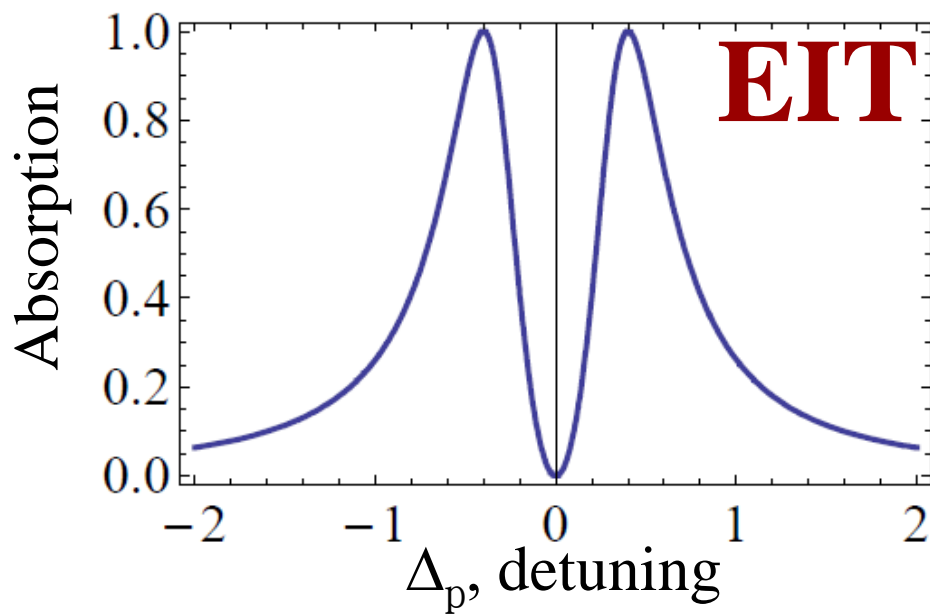
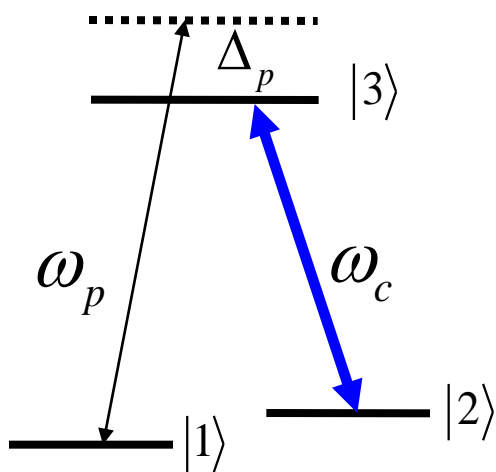
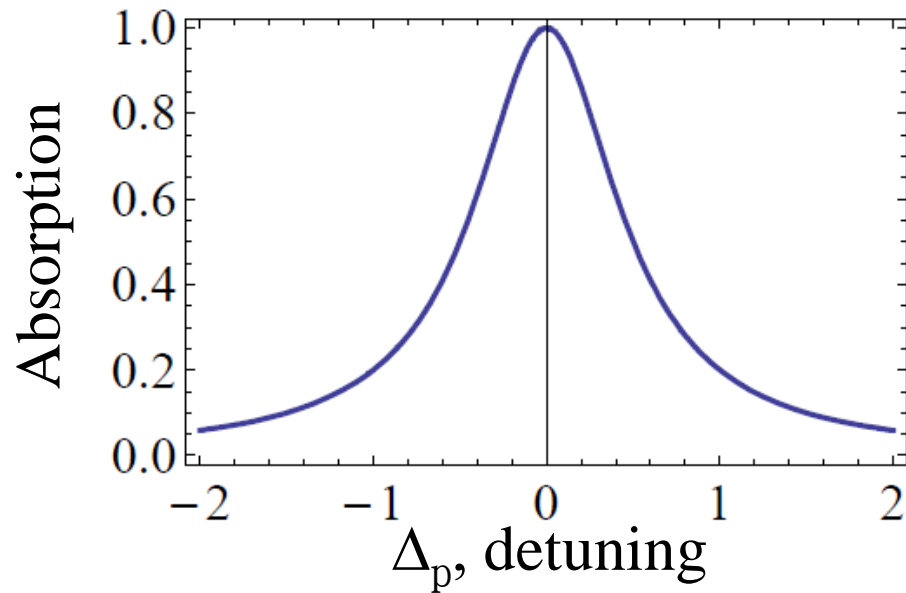
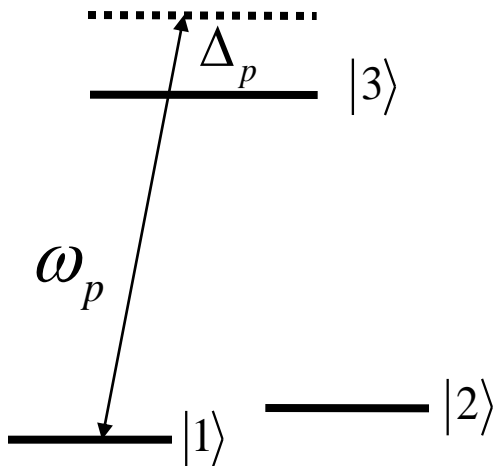
ii) no absorption of the light pulse

-
- For a linear absorptive medium, the derivative is small.
 - Light pulse can be slow down by using electromagnetically-induced transparency (EIT).

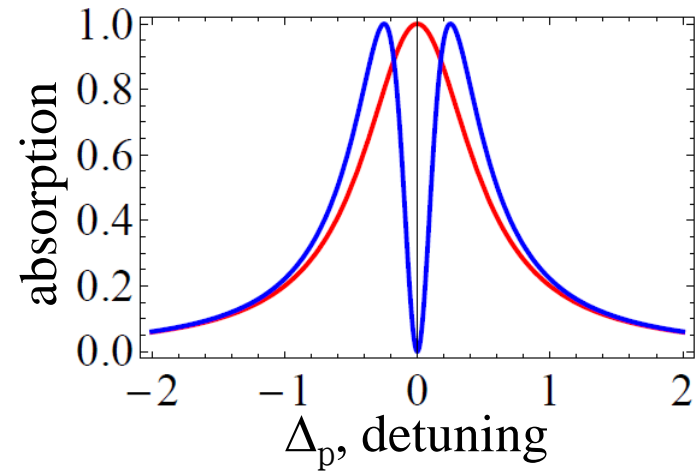
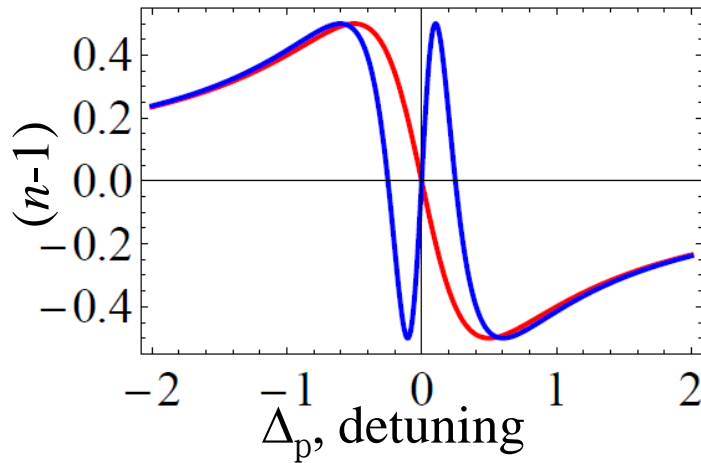
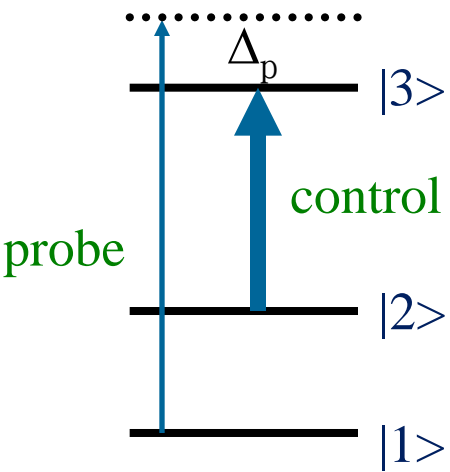
Harris et al., PRA 46(1), R29 (1992)



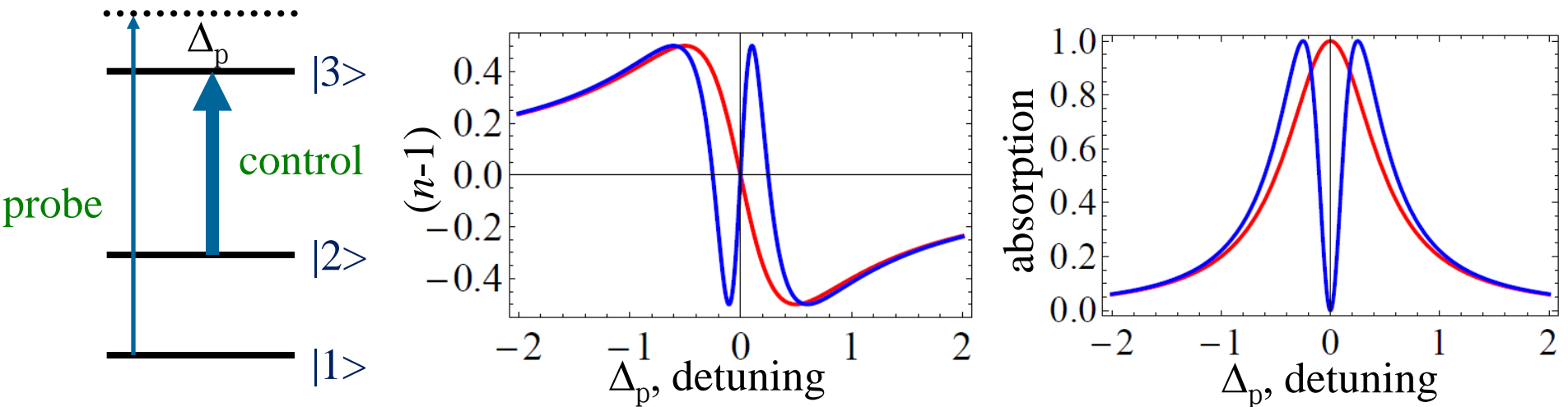




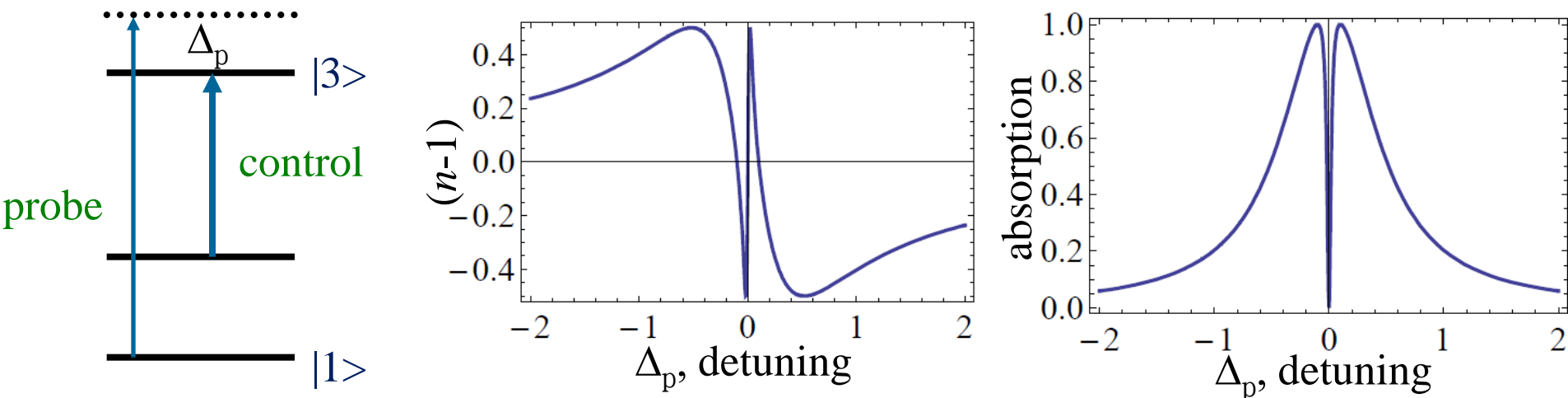
Electromagnetically-induced transparency (EIT)



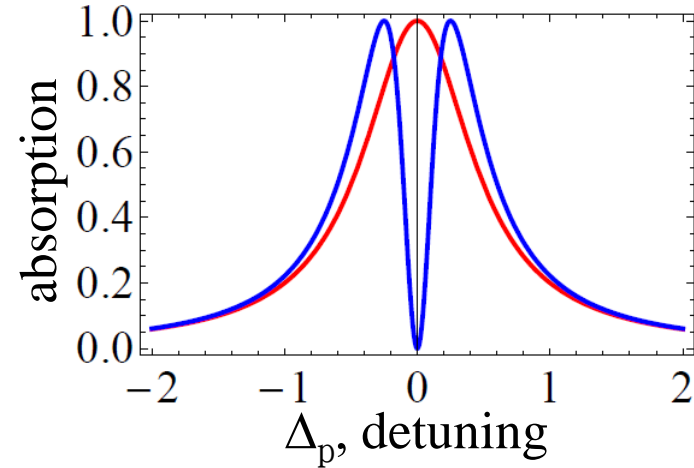
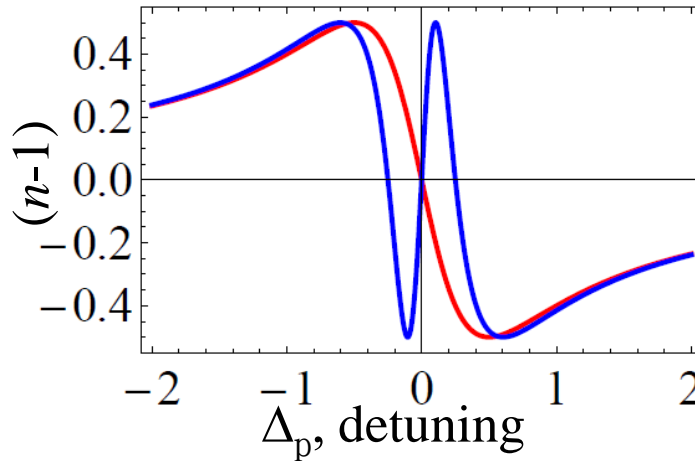
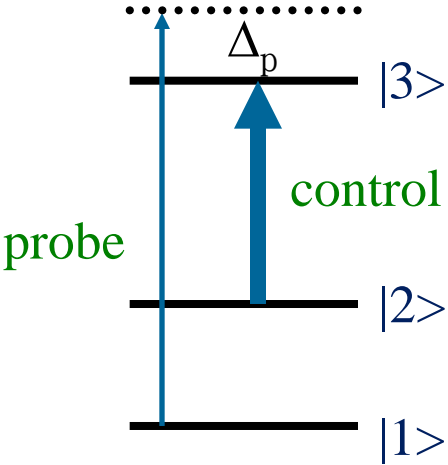
Electromagnetically-induced transparency (EIT)



The nonlinear effect depends on the control beam power.



Electromagnetically-induced transparency (EIT)



$$V_{\text{group}} = \frac{c}{n(\omega) + \omega \frac{dn(\omega)}{d\omega}}$$

- i) $\frac{dn(\omega)}{d\omega} > 0$; $\frac{dn(\omega)}{d\omega}$ depends on the control beam power
- ii) No absorption when on resonance.

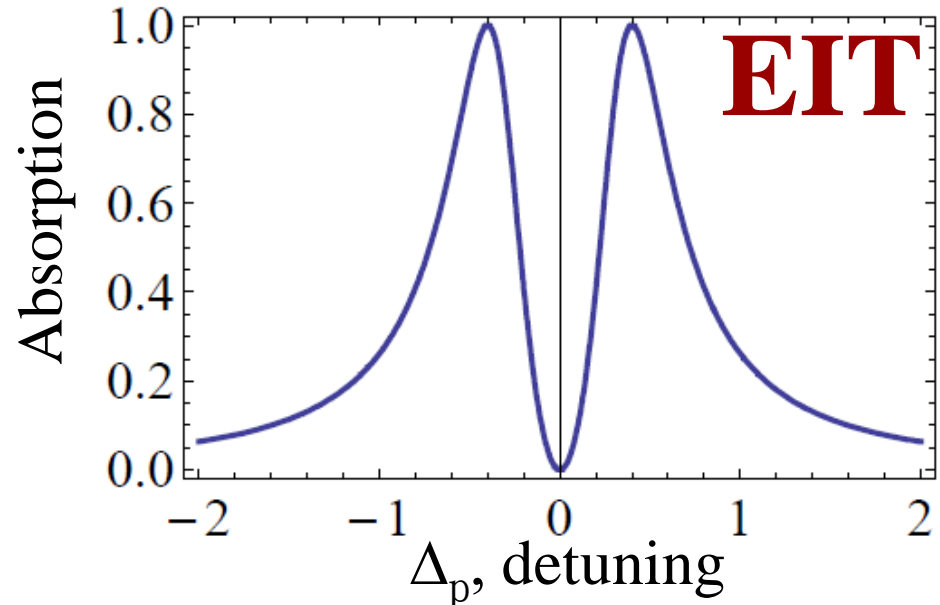
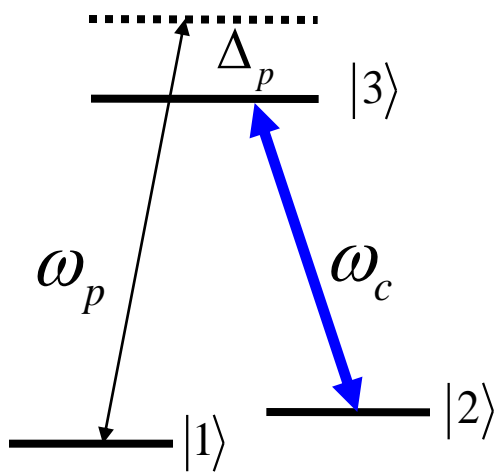
Get the condition to Slow Light!~

$$\text{Dark state } |a^0\rangle = \cos\theta|1\rangle - \sin\theta|2\rangle$$

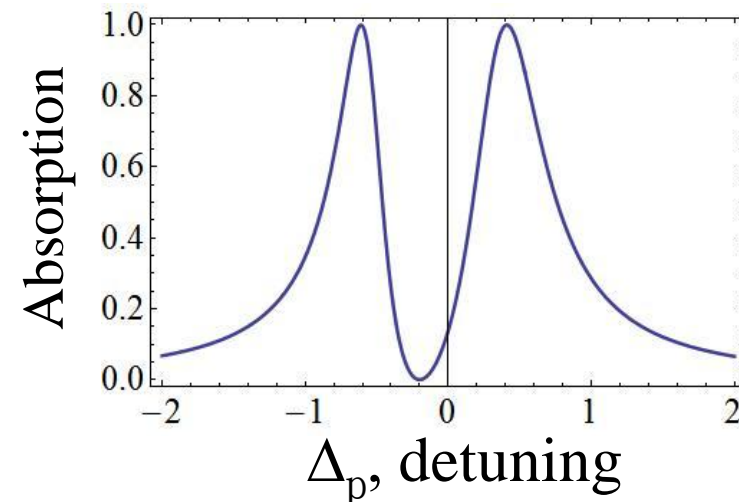
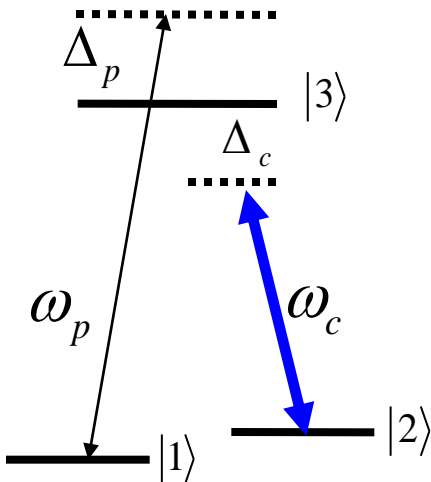
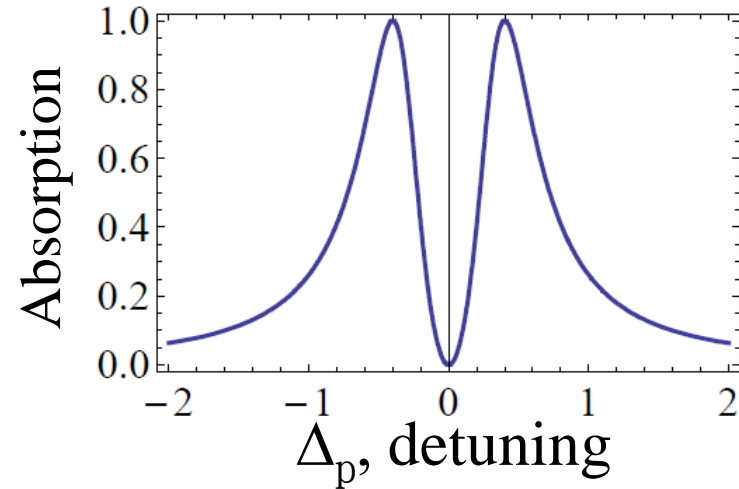
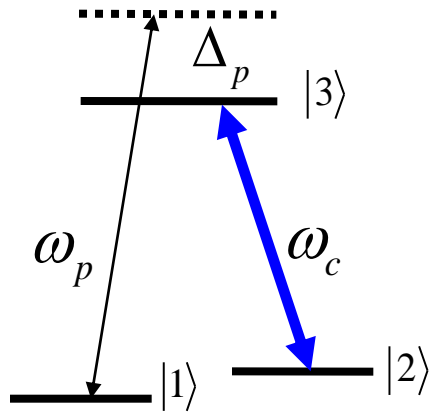
$$\text{Bright state } |a^+\rangle = \sin\theta\sin\phi|1\rangle + \cos\theta\sin\phi|2\rangle + \cos\phi|3\rangle$$

$$\text{Bright state } |a^-\rangle = \sin\theta\cos\phi|1\rangle + \cos\theta\cos\phi|2\rangle - \sin\phi|3\rangle$$

- Angles are associated to the intensities of both fields
- **No excited state population: no absorption**



Two-photon resonance: ($\delta = \Delta_p - \Delta_c = 0$) $\hbar(\omega_p - \omega_c) = E_2 - E_1$
 EIT happens as long as the frequency difference between two optical fields matches with the energy difference of two lower energy levels.



EIT-based atomic ensemble

1. Strong atom-photon coupling

Need: **large atomic density**

2. Good isolation from environment

Need: **long coherence times**

often disturbed by

inelastic collisions, stray magnetic field,
and thermal diffusion/ atomic motion

Other system: electronic spin of alkali gas or solids

UNR: use a pure nuclear spin system to accomplish those needs

Atom choices

- **Ground state 1S_0 atoms ($J=0$, $L=0$ and $S=0$)**
no coupling between electronic and nuclear spins.
nuclear spin has much better collisional behaviors than
electronic spin.

- **Isotopes with nuclear spin ($I \neq 0$)** $\frac{\mu_N}{\mu_e} \sim \frac{1}{1837}$
less effects from stray magnetic fields.

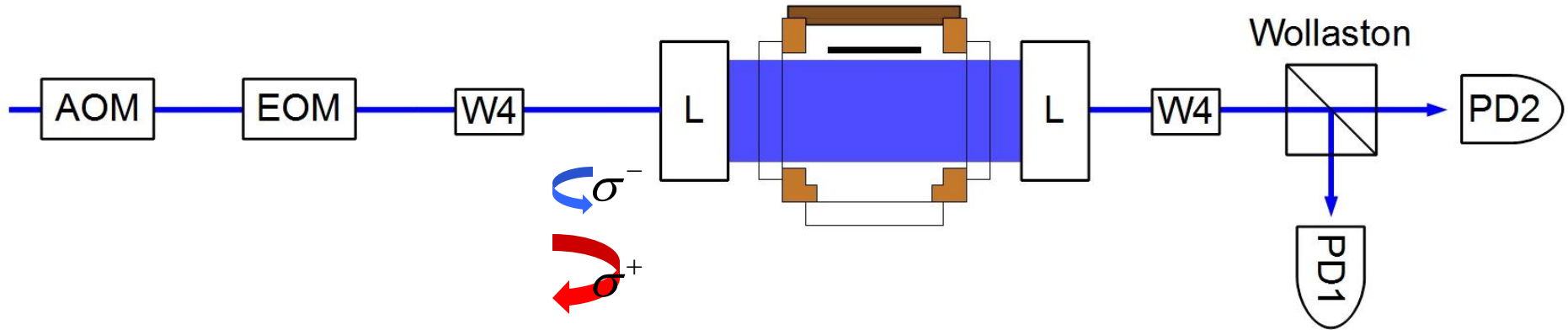
- **Atomic Ytterbium** (optical transition ~ 400 nm)

^{168}Yb , ^{170}Yb , ^{172}Yb , ^{174}Yb , ^{176}Yb

^{171}Yb ($I=1/2$), and ^{173}Yb ($I=5/2$) structure for EIT

good natural abundance

Setup



AOM: acousto-optic modulator, **controlling beam power**

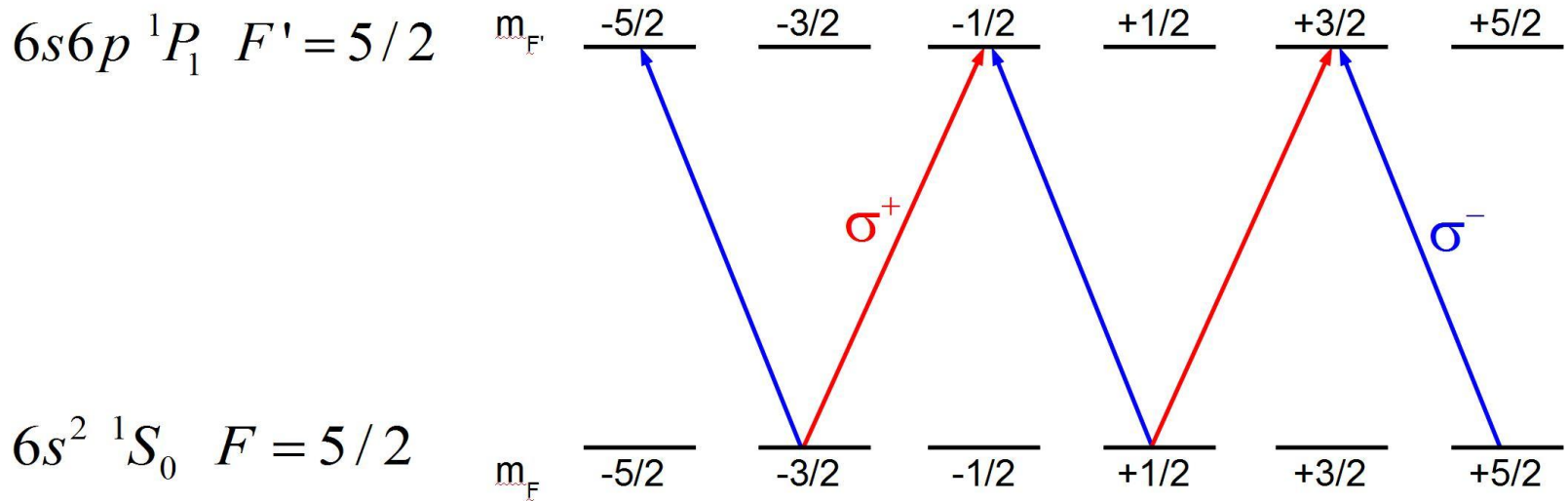
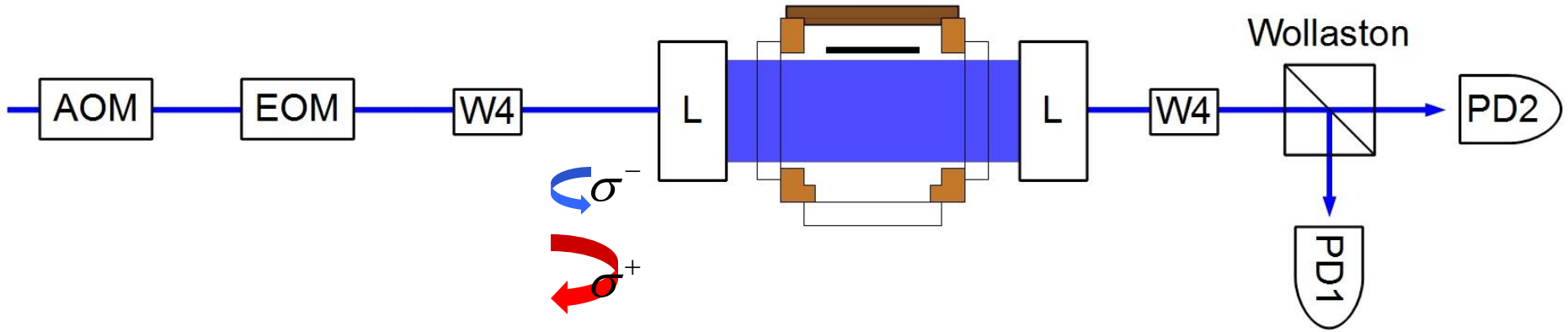
EOM: electro-optic modulator, **rotating beam polarization**

W4: quarter-wave plate, **turning linear-pol. to circular-pol.**

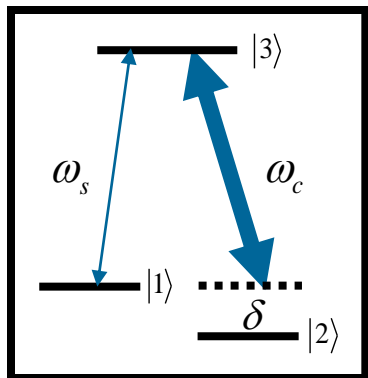
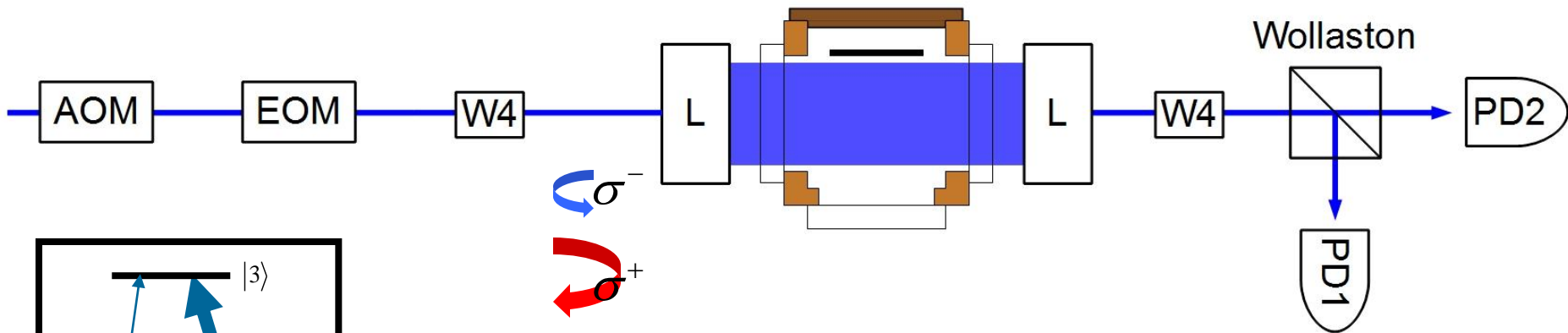
L: lens combination, **expanding the beam**

Detection device = W4 + Wollaston Prism + PD1 and PD2

Yb EIT

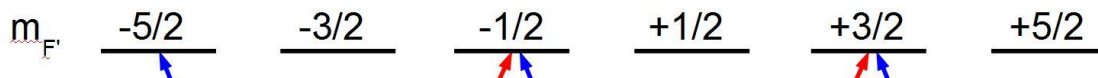


Yb EIT

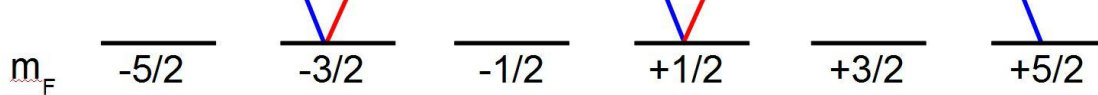


Applying a magnetic field to induce two-photon detuning δ

$$6s6p \ ^1P_1 \ F' = 5/2$$

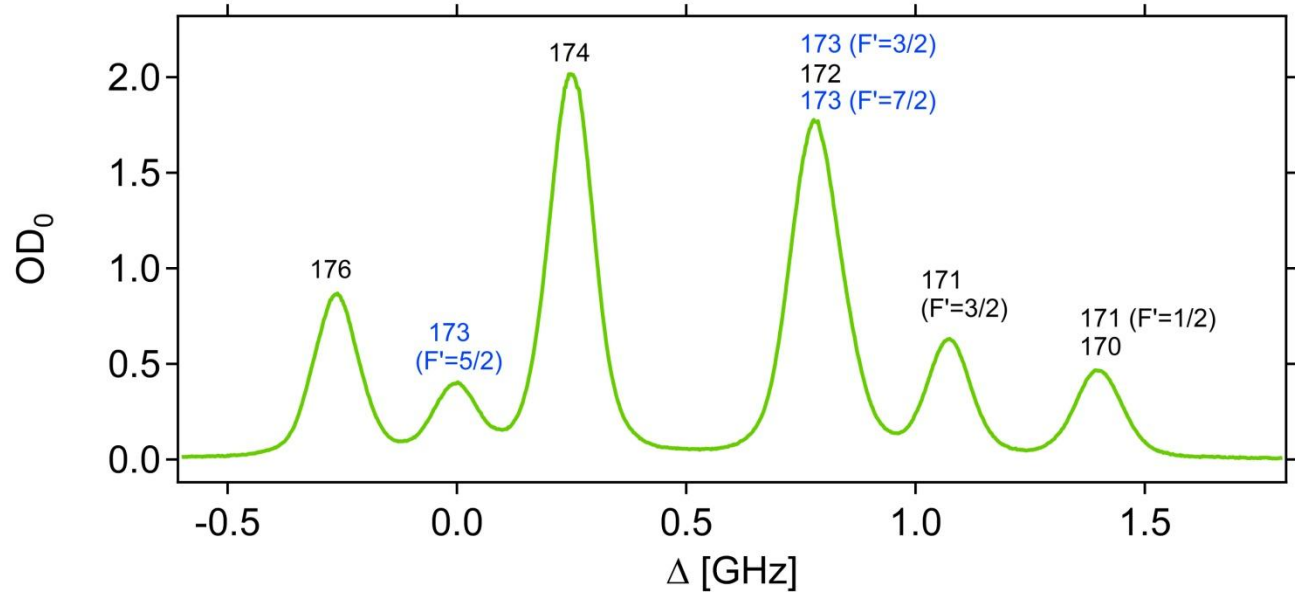
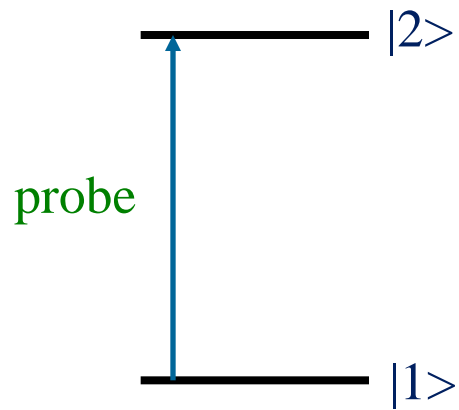


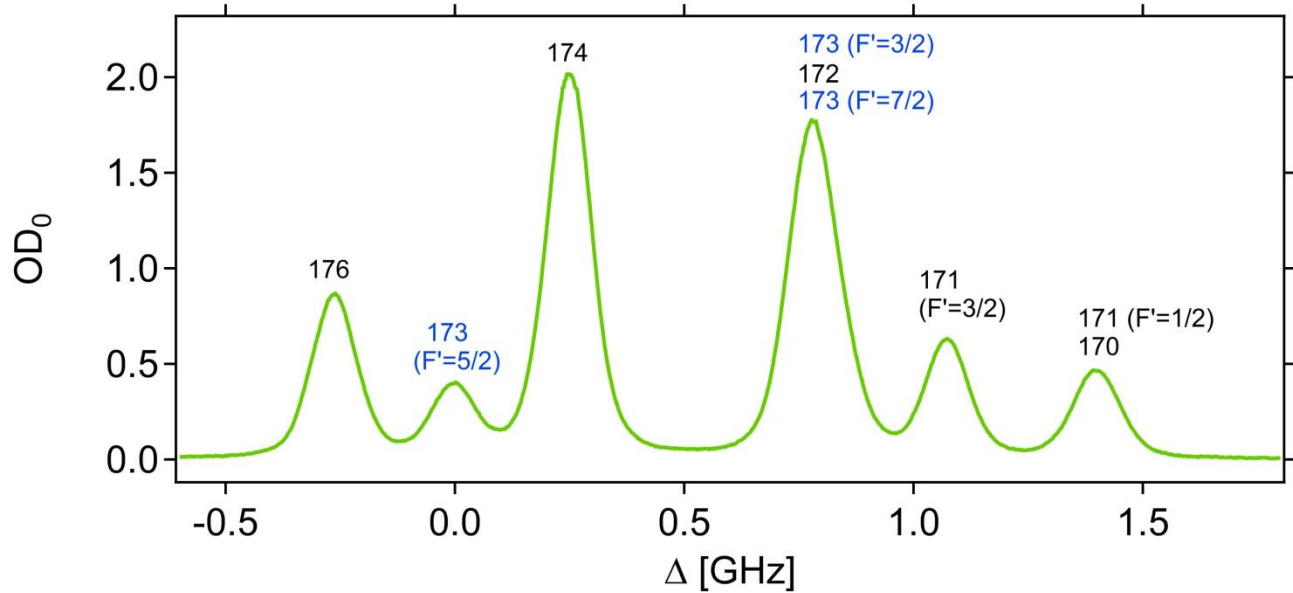
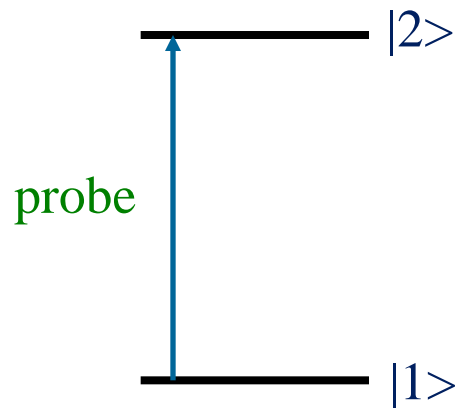
$$6s^2 \ ^1S_0 \ F = 5/2$$



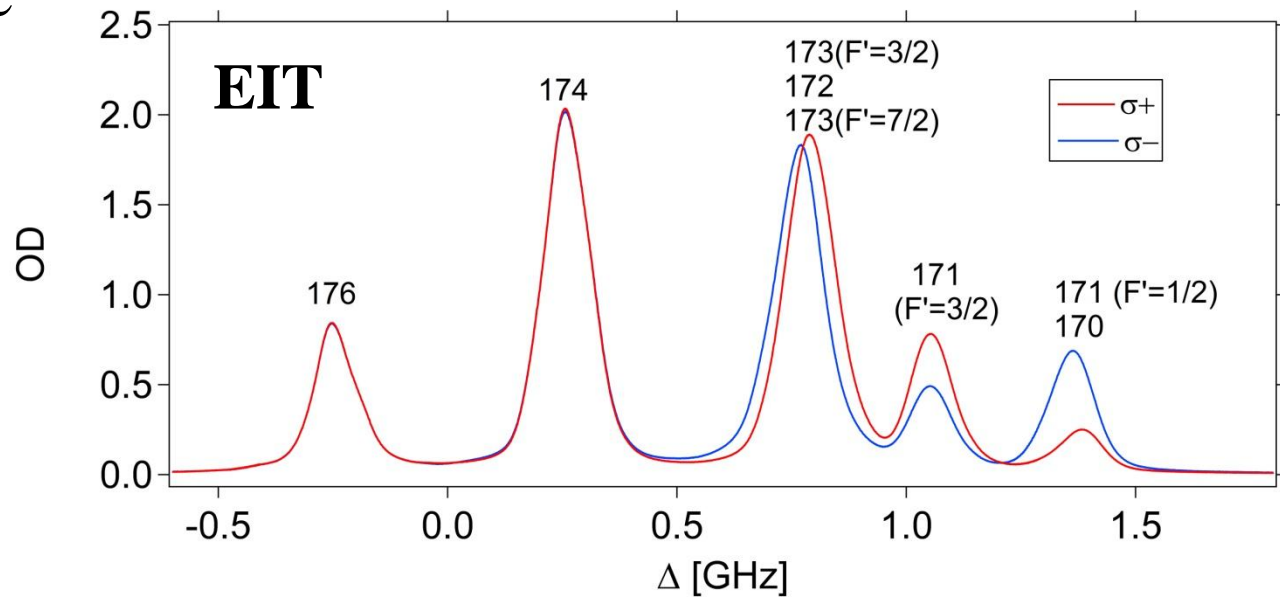
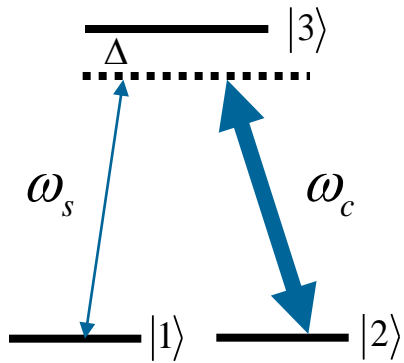
σ^+

σ^-





Two-photon resonance
($\delta=0$)



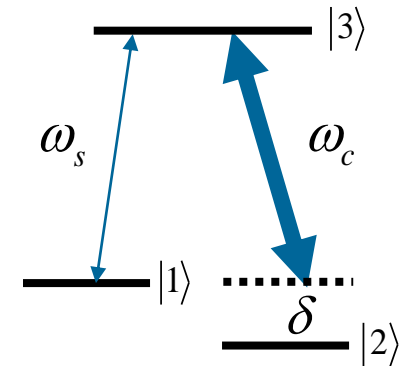
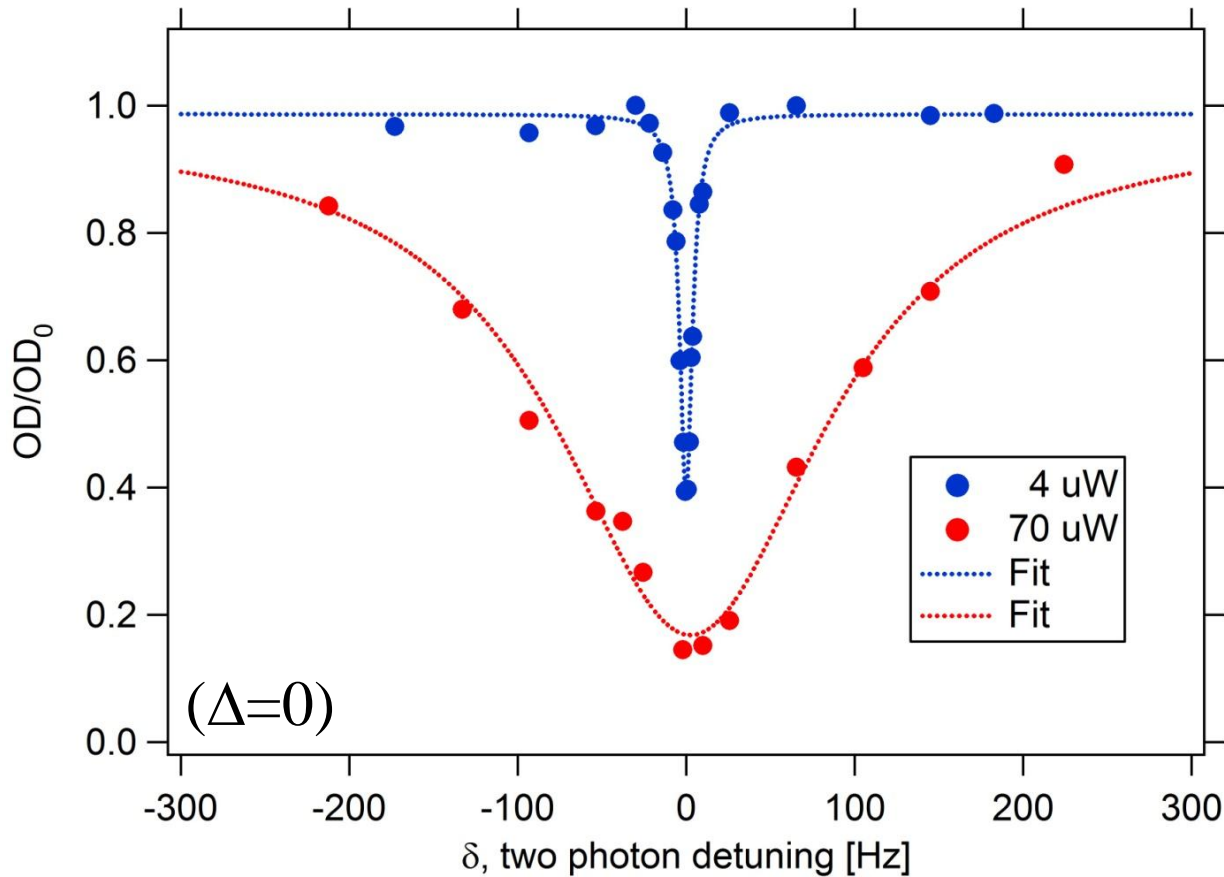
For a Doppler-broaden thermal gas,

EPL 82, 54002 (2008)

$$\text{Dip FWHM: } \Gamma_{\text{EIT}} = 2\gamma_g + \frac{\Omega_c^2}{W_D + \gamma}$$

Rabi frequency

$$\Omega_c \propto \sqrt{I_c}$$



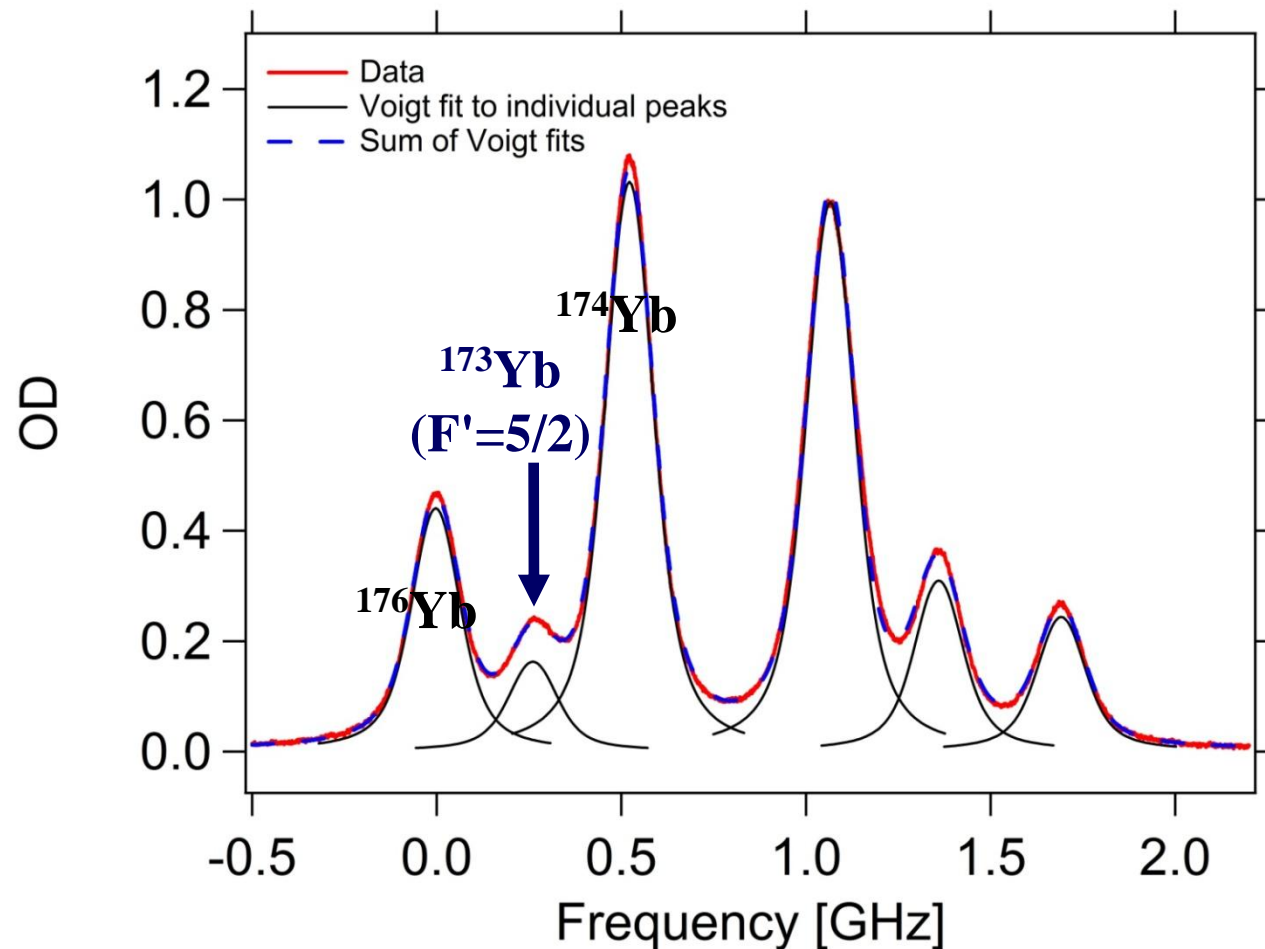
$$\Gamma_{\text{EIT}} = 10 \text{ Hz}$$

$$\Gamma_{\text{EIT}} = 200 \text{ Hz}$$

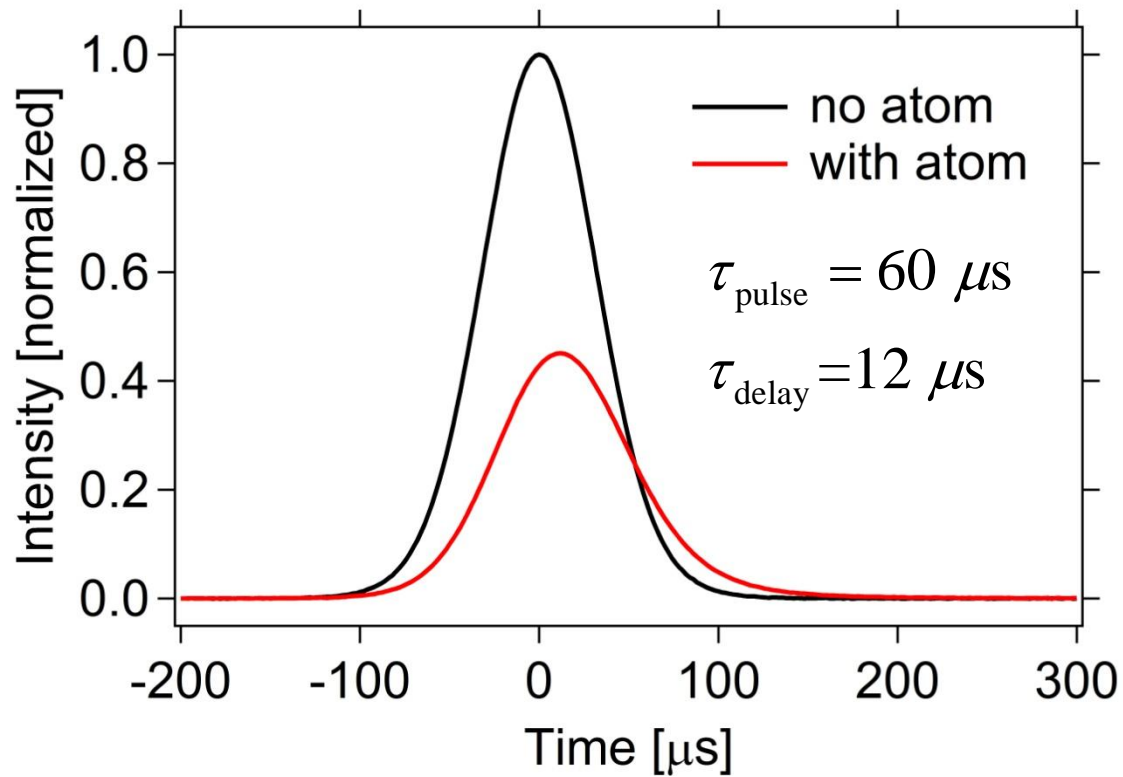
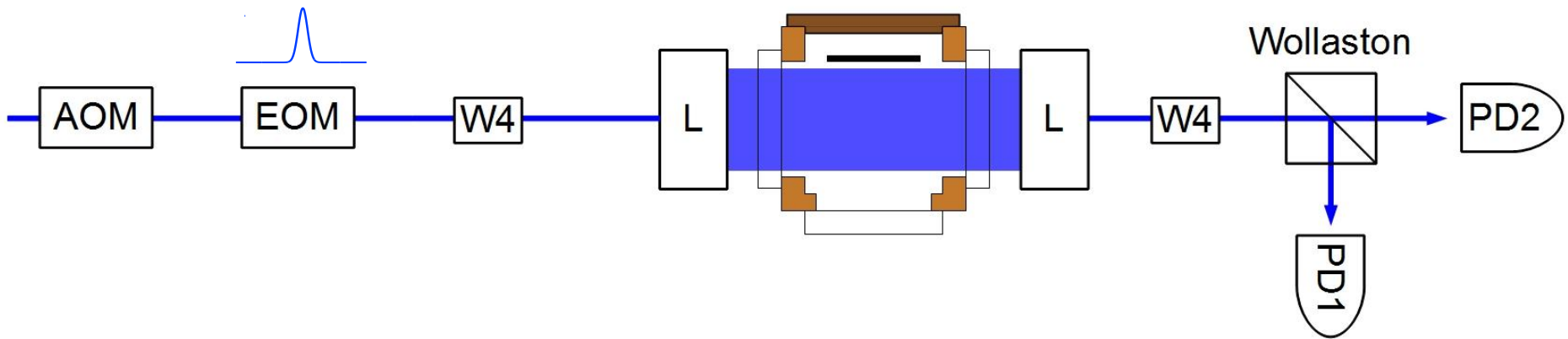
Why isn't the transparency better?

- Off-resonance absorption of light by the other isotopes

Solution: switching to an isotopically-enriched ^{173}Yb target

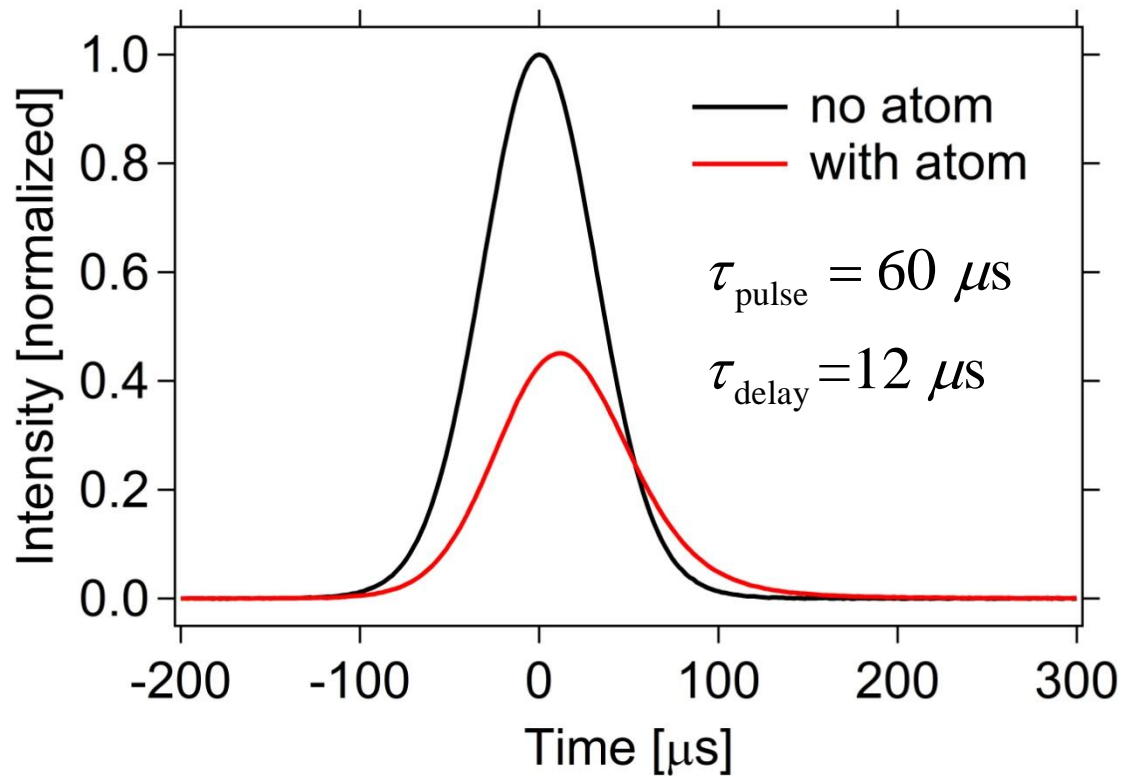
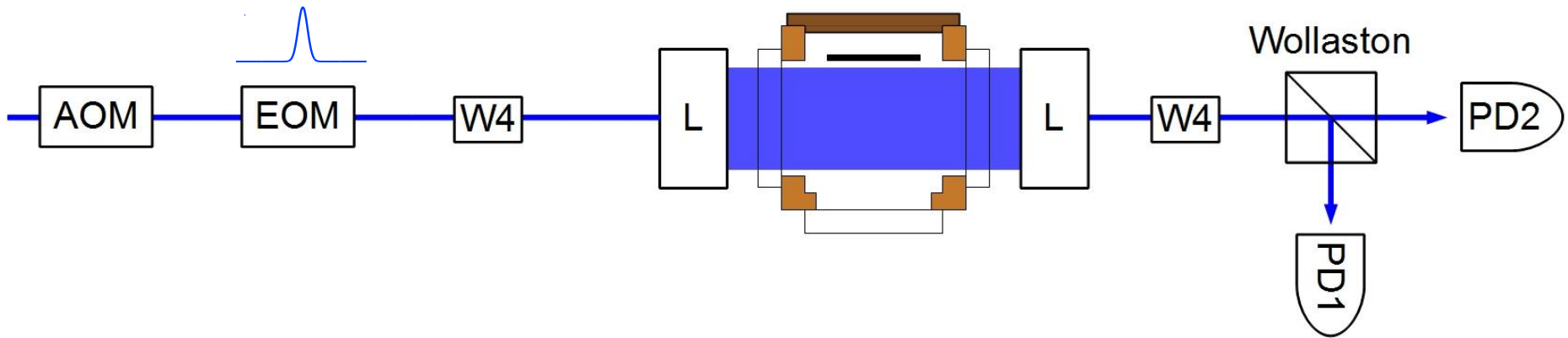


Slow light



$$\tau_{\text{delay}} = \frac{L}{v_{\text{sg}}} - \frac{L}{c};$$

Slow light



$$\tau_{\text{delay}} = \frac{L}{v_g} - \frac{L}{c};$$

τ_{delay} up to 1 ms

$v_g \sim 100 \text{ m/s}$

$v_g \sim 3 \times 10^{-7} c$

Delay-bandwidth product (DBW)

$$\tau_{\text{delay}} = \text{OD}_0 \frac{(W_D + \gamma)\Omega_c^2}{(2\gamma_g(W_D + \gamma) + \Omega_c^2)^2}; \Gamma_{\text{EIT}} = 2\gamma_g + \frac{\Omega_c^2}{W_D + \gamma}$$

Goldfarb et. al. *EPL* **82** 54002 (2008)

i) For long τ_{delay} , need large OD_0 ; small Ω_c , γ_g

(want $v_g \rightarrow 0$, $\Omega_c \rightarrow 0$)

ii) For no absorption, require $\frac{1}{\tau_{\text{pulse}}} < \Gamma_{\text{EIT}}$, \therefore large Ω_c

Delay-bandwidth product (DBW)

$$\tau_{\text{delay}} = \text{OD}_0 \frac{(W_D + \gamma)\Omega_c^2}{(2\gamma_g(W_D + \gamma) + \Omega_c^2)^2}; \Gamma_{\text{EIT}} = 2\gamma_g + \frac{\Omega_c^2}{W_D + \gamma}$$

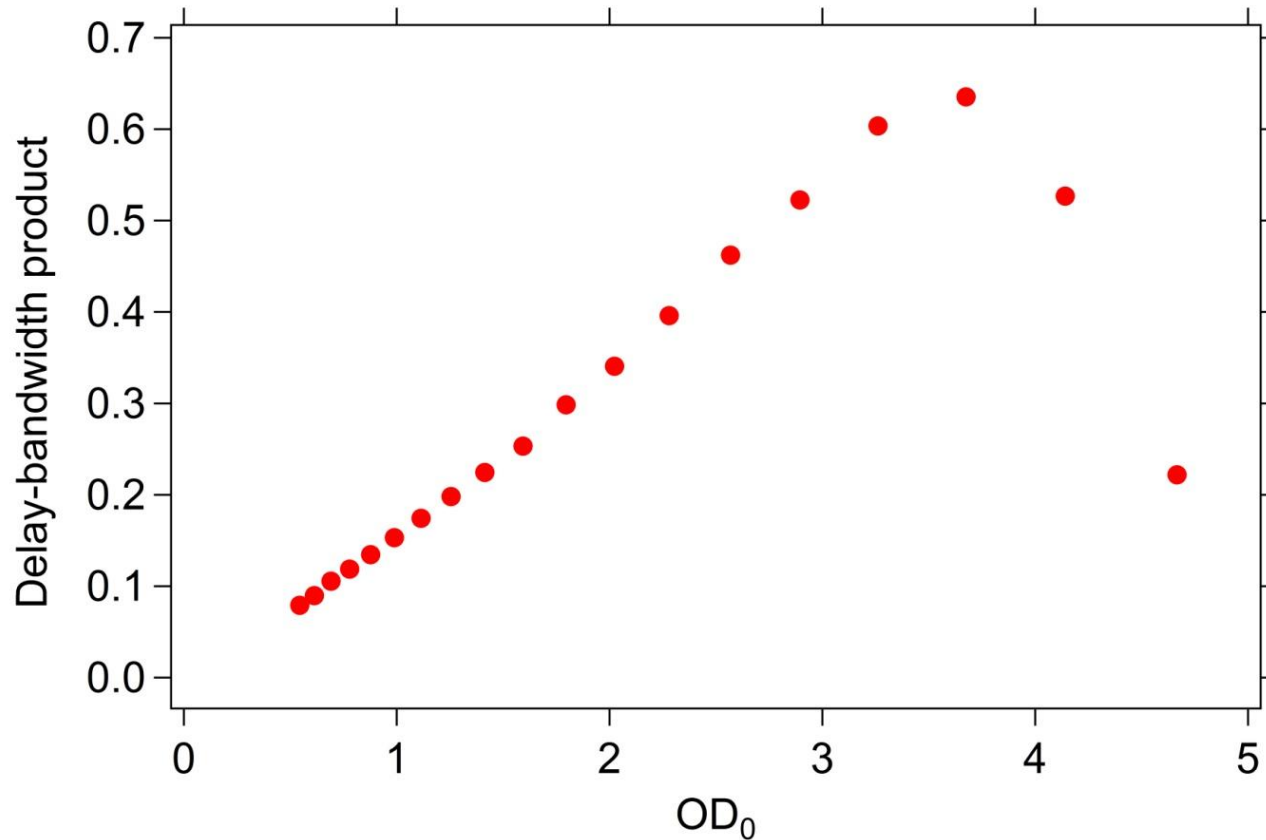
Goldfarb et. al. *EPL* **82** 54002 (2008)

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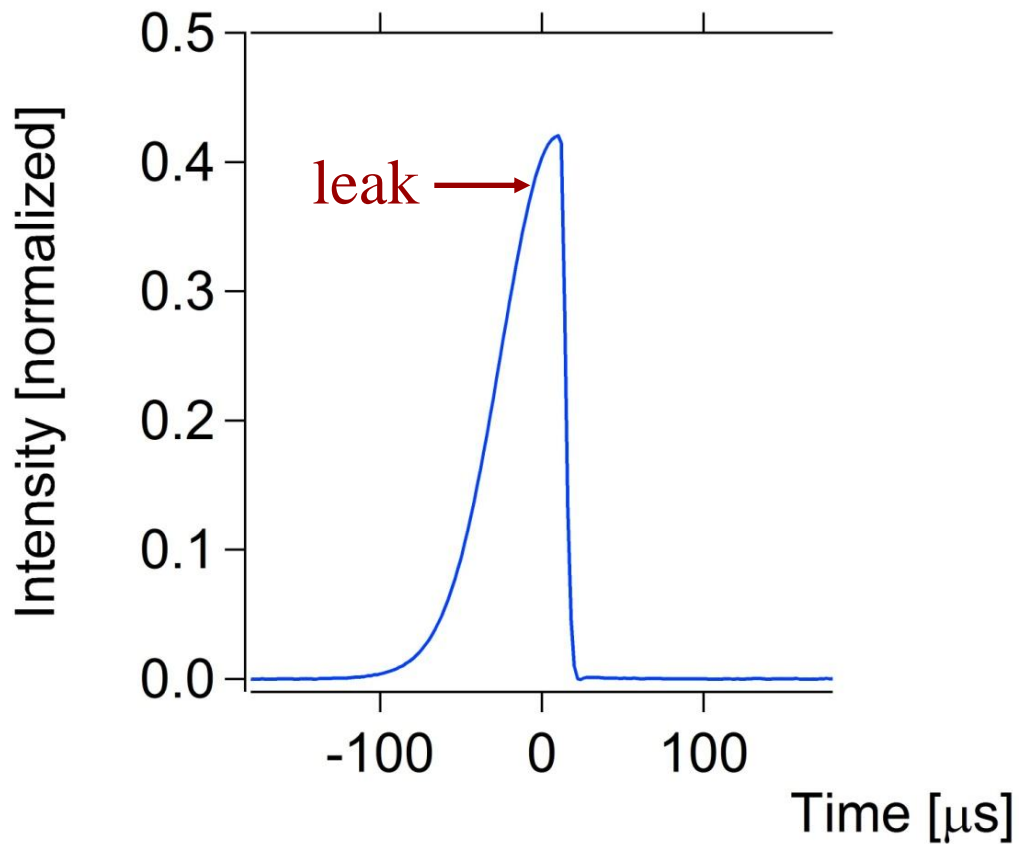
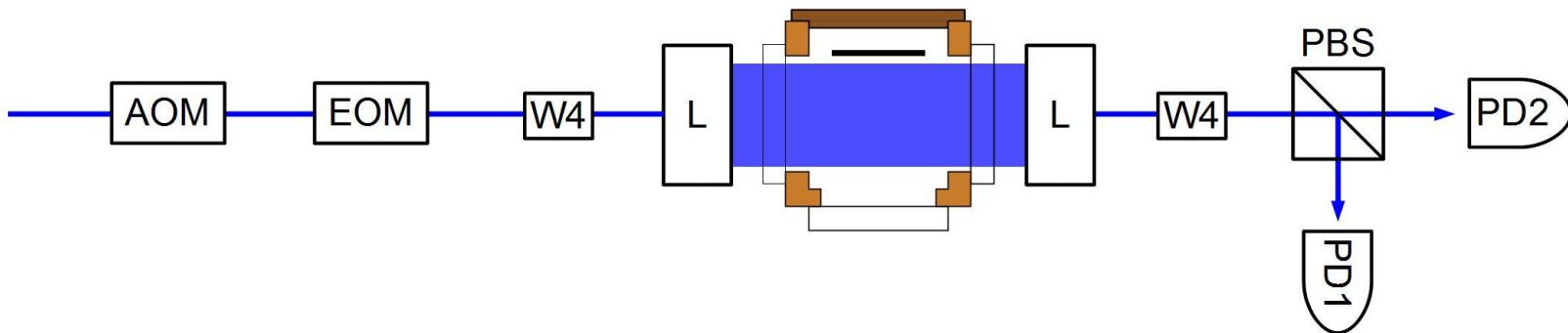
$$1 \ll \text{DBW} = \tau_{\text{delay}} \cdot \frac{1}{\tau_{\text{pulse}}} < \text{OD}_0$$



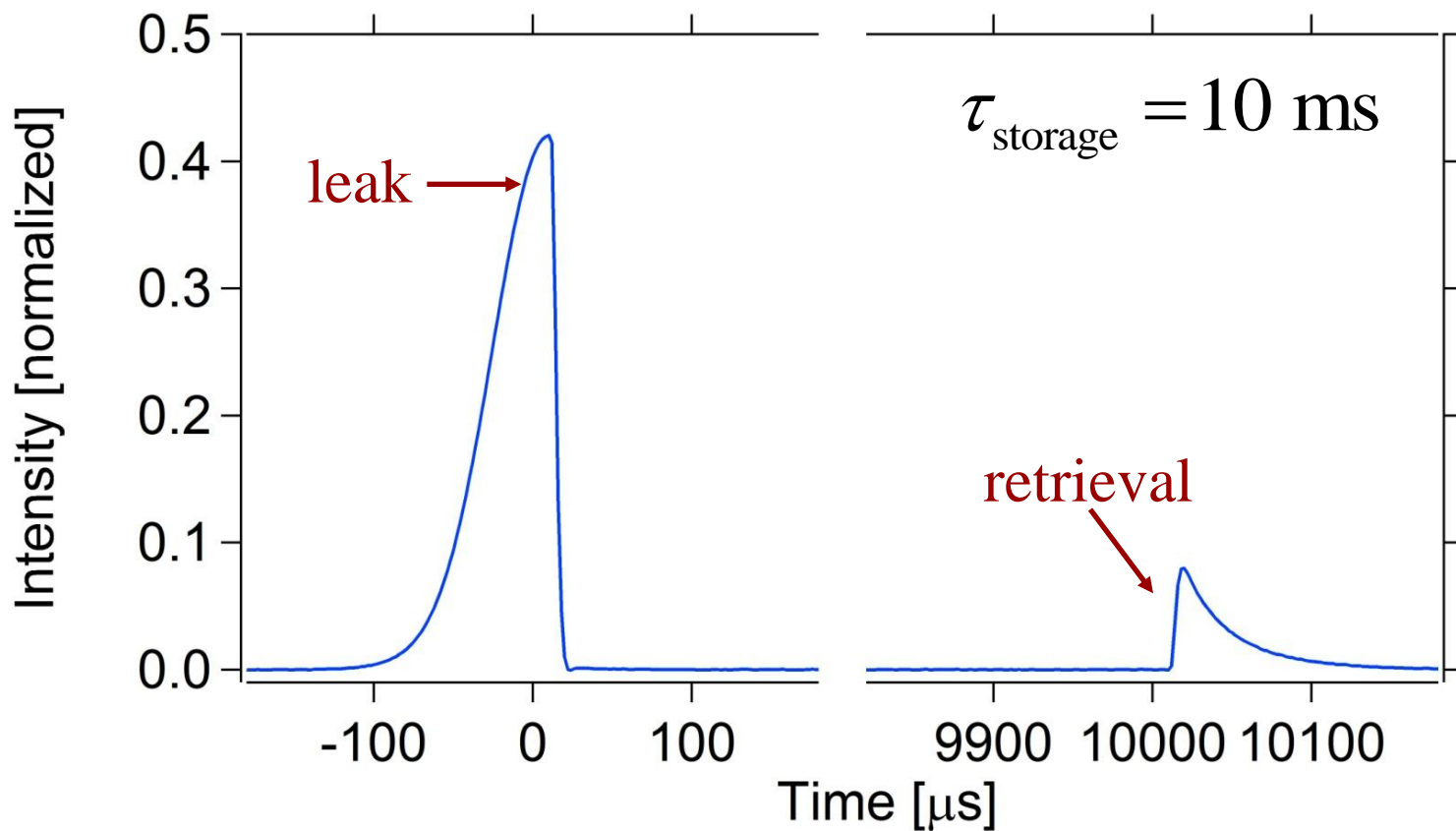
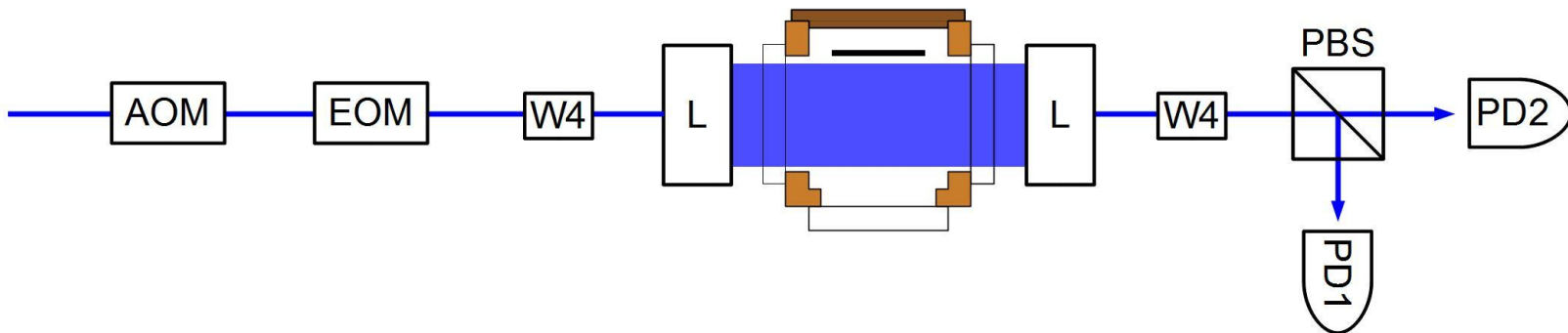
- **Radiation trapping**

Re-emitted photons w/ random polarization mess up atom-light coherence.

Stopped light

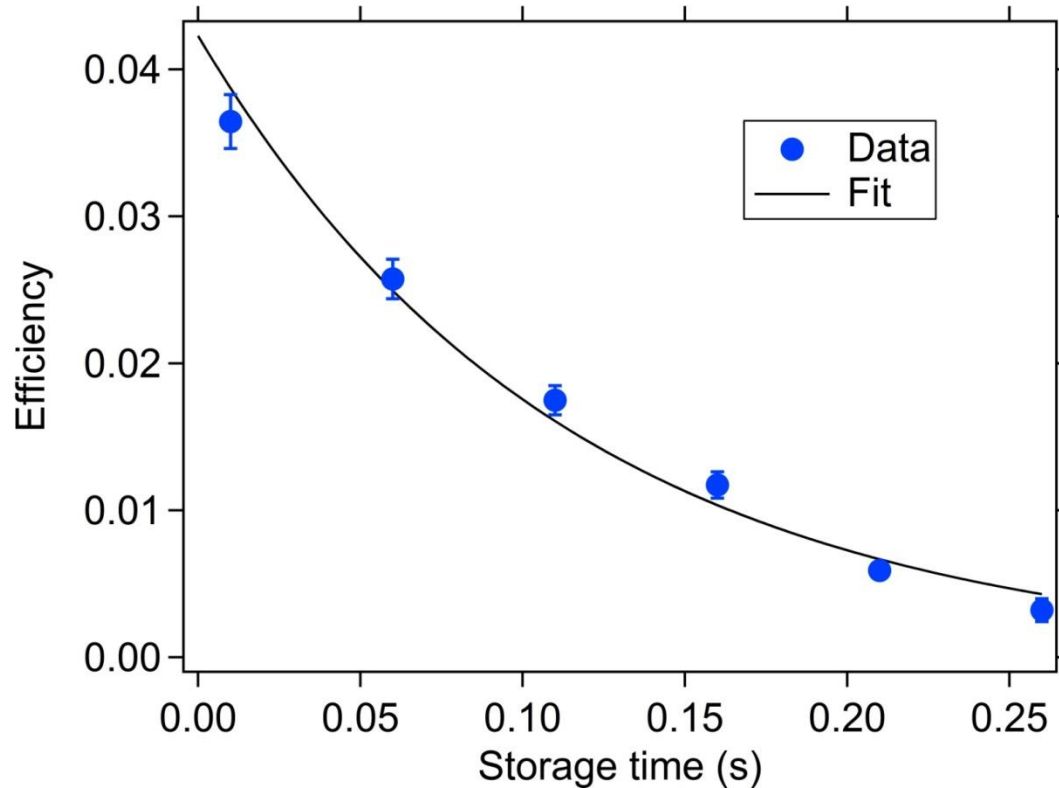


Stopped light



Stopped light

$$\text{Efficiency} = \frac{\text{the energy of the retrieved pulse}}{\text{the energy of the input pulse}}$$

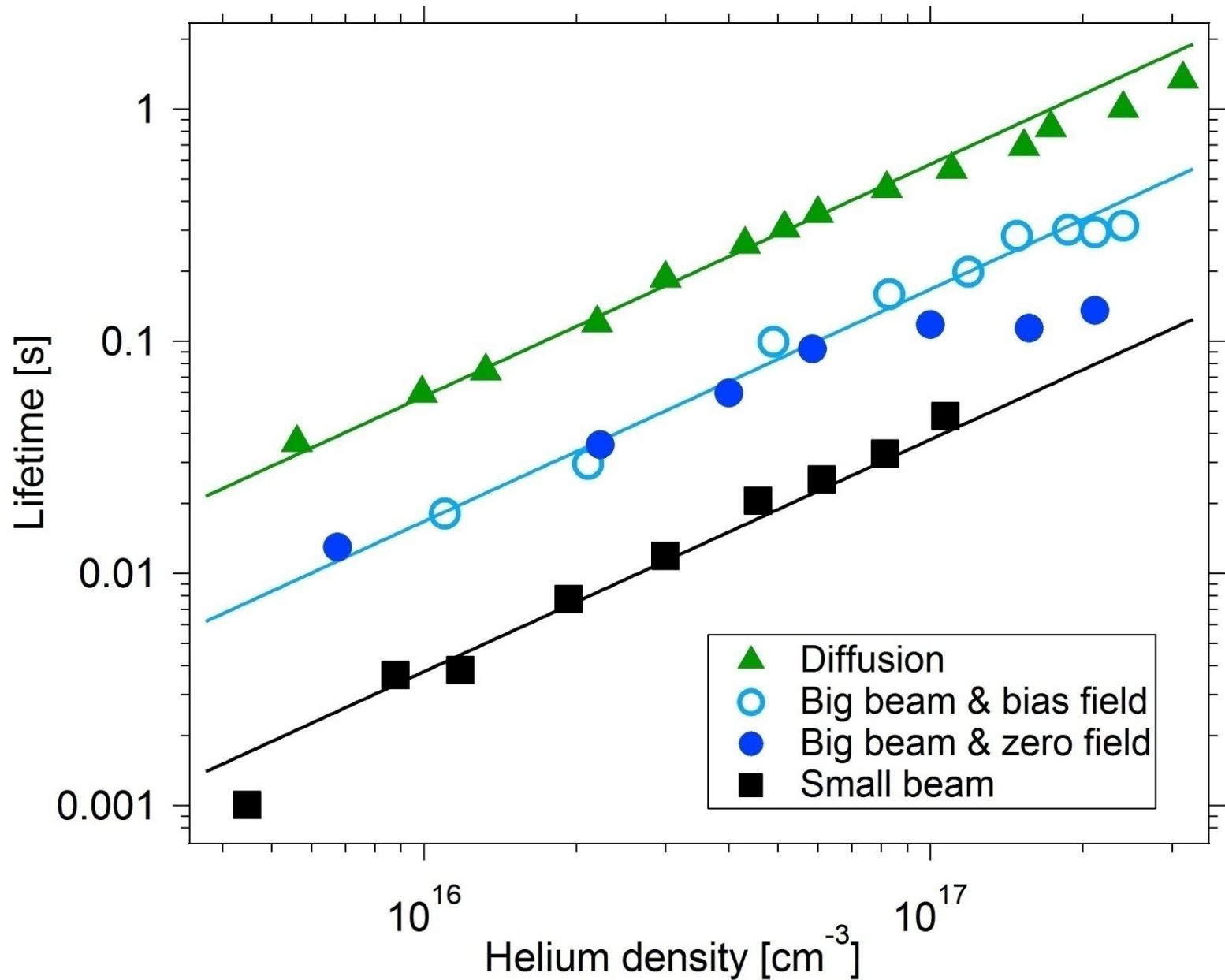


Decoherence due to diffusion:

an exponential fit gives a storage lifetime of 0.11s.

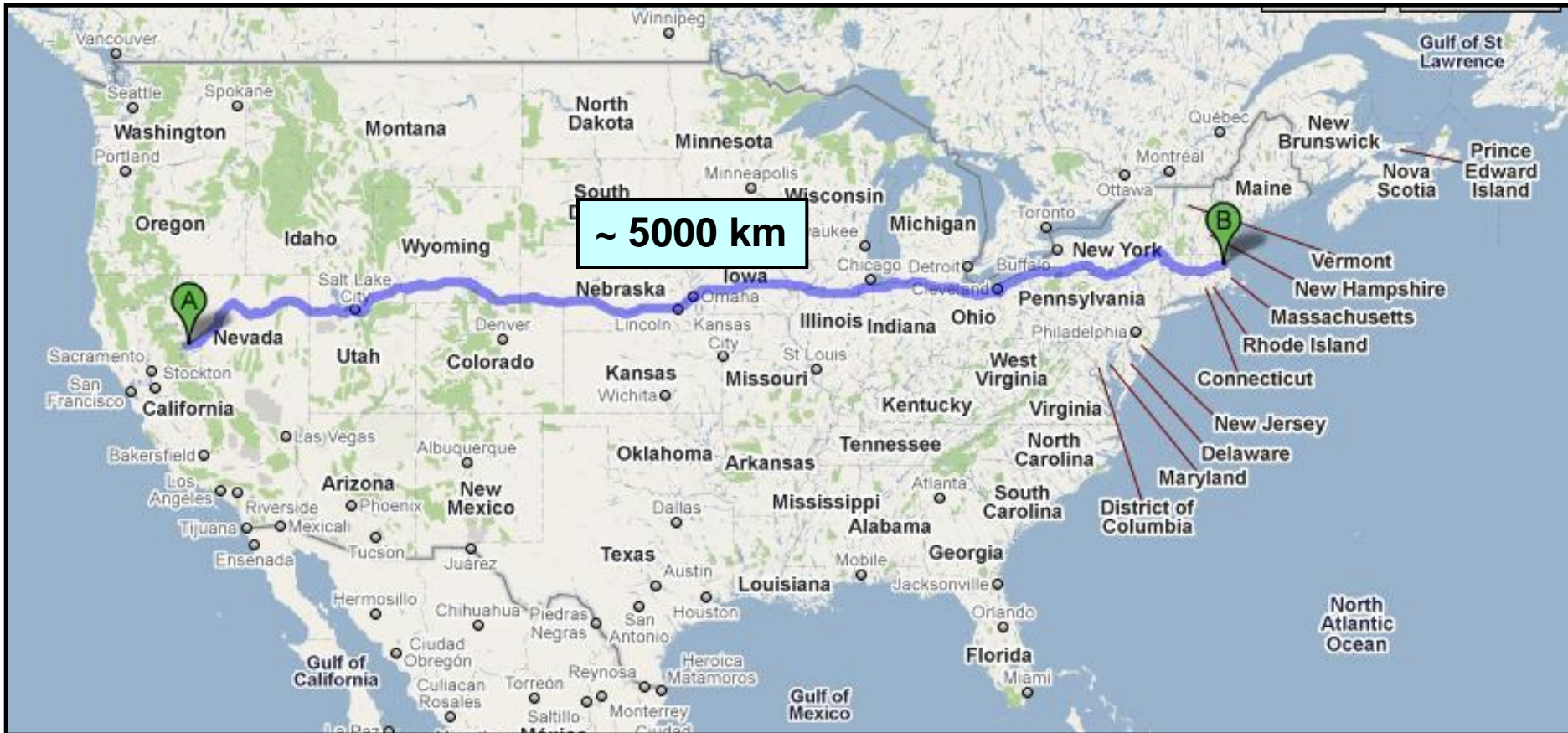
Stopped light

A storage lifetime is up to 0.3 s.

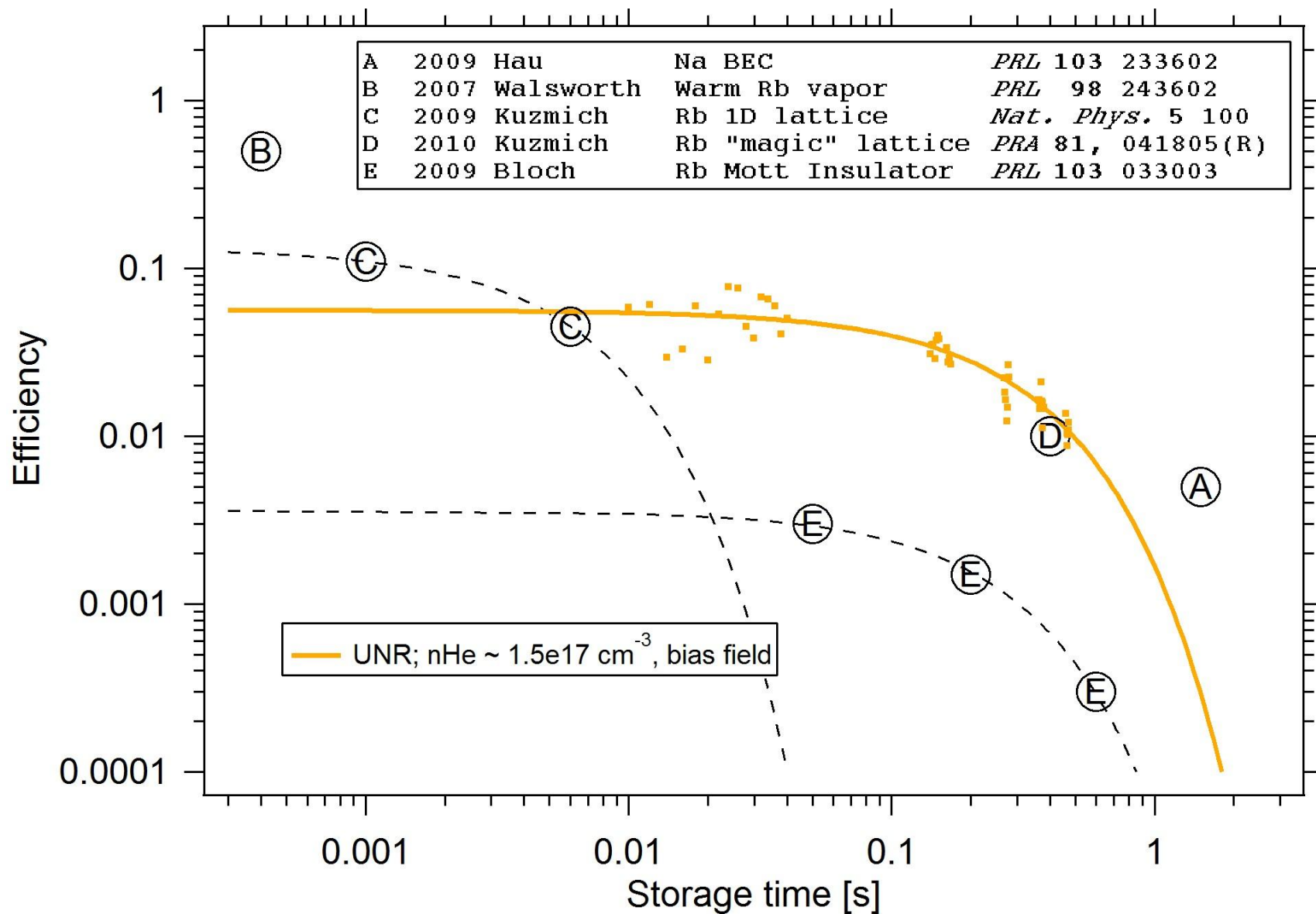


Quantum communication

It takes about 17 ms for a light pulse to travel across the north America.



Yb data v.s. *state of the art*



Conclusions

- We use cryogenically-cooled 1S_0 atomic ensembles to have high optical densities and long spin coherence times.
- By applying EIT techniques in a nuclear spin system, we have observed an atom-light coherence time better than 100 ms.
- We have a storage lifetime up to hundreds of milliseconds and show a competitive performance than other system.

$${}^2P_{1/2} : s = 1/2, l = 1, j = 1/2$$

$$|j = 1/2, m_j = 1/2\rangle = \sqrt{\frac{2}{3}}Y_1^1 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}}Y_1^0 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$$

$$|\psi|^2 \propto \frac{2}{3}|Y_1^1|^2 + \frac{1}{3}|Y_1^0|^2 = 1$$

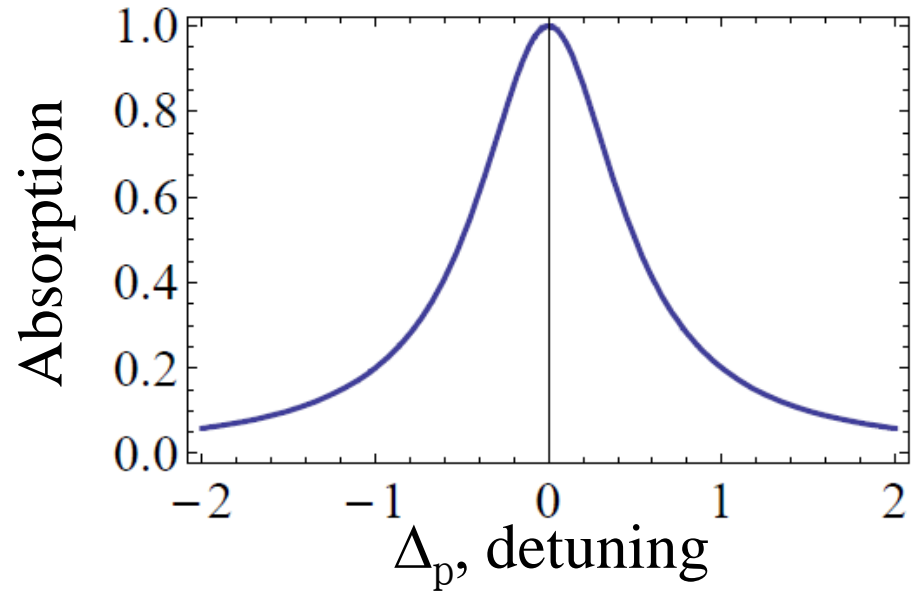
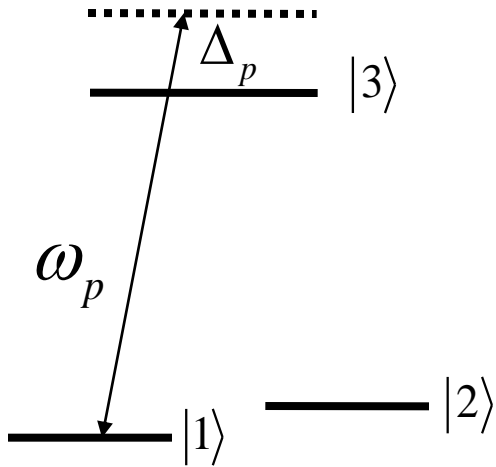
$${}^2P_{3/2} : s = 1/2, l = 1, j = 3/2$$

$$|j = 3/2, m_j = 3/2\rangle = Y_1^1 \left| \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$|j = 3/2, m_j = 1/2\rangle = \sqrt{\frac{1}{3}}Y_1^1 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \sqrt{\frac{2}{3}}Y_1^0 \left| \frac{1}{2}, \frac{1}{2} \right\rangle$$

$$|\psi_{3/2,3/2}|^2 \propto |Y_1^1|^2 = \frac{3}{8\pi} \sin^2 \theta$$

$$|\psi_{3/2,1/2}|^2 \propto \frac{1}{3}|Y_1^1|^2 + \frac{2}{3}|Y_1^0|^2 = \frac{1}{8\pi} (1 + \cos^2 \theta)$$



Kramers-Kronig Relation

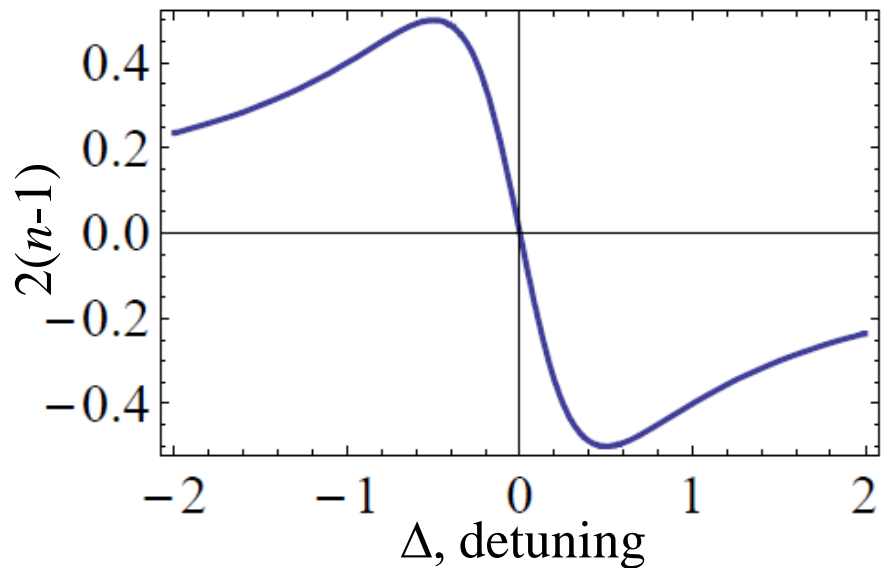
Classical Electrodynamics (1998)

J. D. Jackson

χ : susceptibility

$\text{Im}[\chi]$: absorption

$1 + \frac{1}{2} \text{Re}[\chi]$: index of refraction



Citations to Previously Published Work

- Fine-structure changing collisions in atomic titanium

Mei-Ju Lu, Kyle S. Hardman, Jonathan D. Weinstein, and Bernard Zygelman
Physical Review A 77, 060701(R) (2008)

- Inelastic titanium-titanium collisions

Mei-Ju Lu, Vijay Singh, and Jonathan D. Weinstein
Physical Review A 79, 050702(R) (2009)

- Cold TiO-He collisions

Mei-Ju Lu and Jonathan D. Weinstein
New Journal of Physics 11, 055015 (2009)

- Suppression of Zeeman relaxation in cold collisions of $^2P_{1/2}$ atoms

T. V. Tscherbul, A. A. Buchachenko, A. Dalgarno, M.-J. Lu, and J. D. Weinstein
Physical Review A 80, 040701(R) (2009)

- Electromagnetically induced transparency with nuclear spin

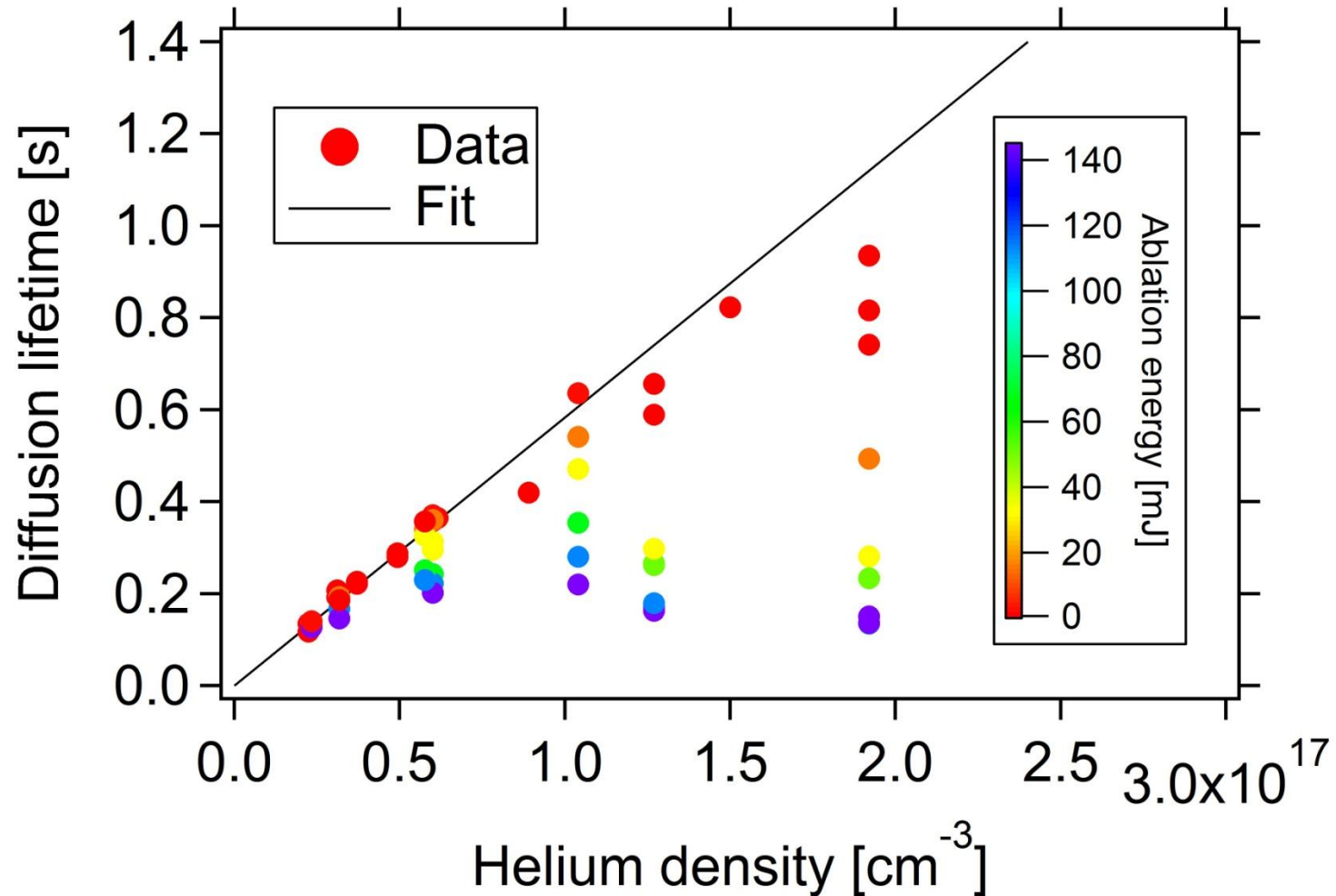
Mei-Ju Lu and Jonathan D. Weinstein
Optics Letters 35, 622 (2010)

- Stopped light with a cryogenic ensemble of ^{173}Yb atoms

Mei-Ju Lu, Franklin Jose, and Jonathan D. Weinstein
Physical Review A 82, 061802(R) (2010)

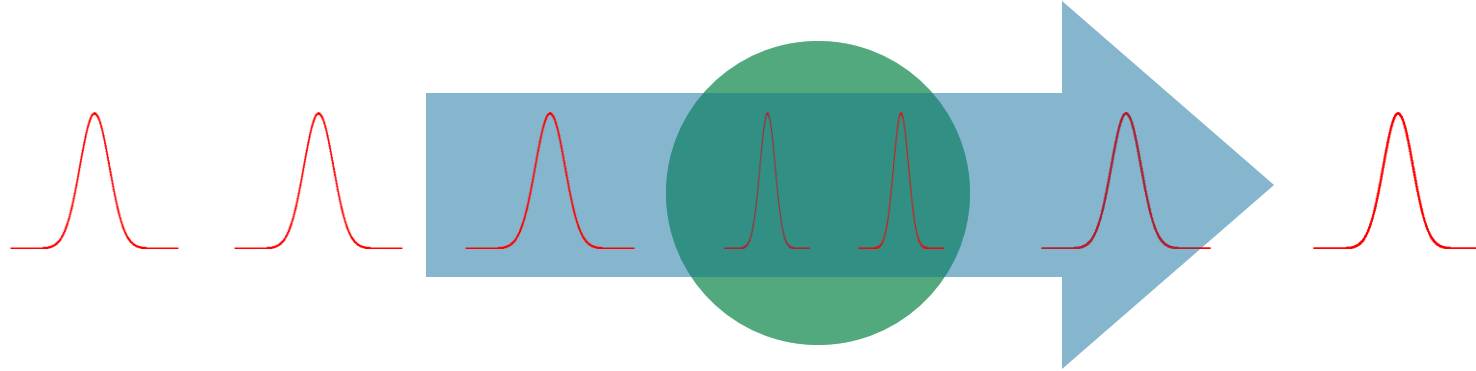
Diffusion lifetimes

- i) At low n_{He} , τ_{D} is linearly increasing with n_{He} .
- ii) At high n_{He} , diffusion is not the only atom loss mechanism.



Slow/Stop light

$$1 \ll \text{DBW} = \tau_{\text{delay}} \cdot \frac{1}{\tau_{\text{pulse}}} < \text{OD}_0$$



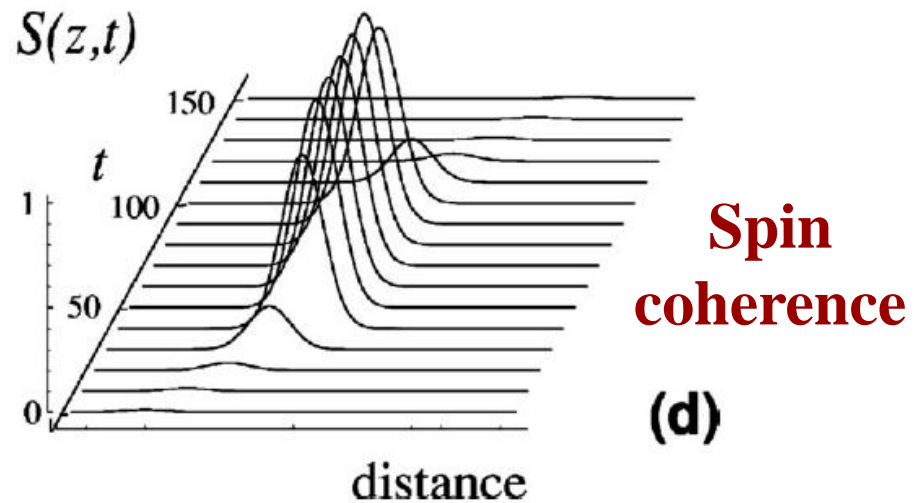
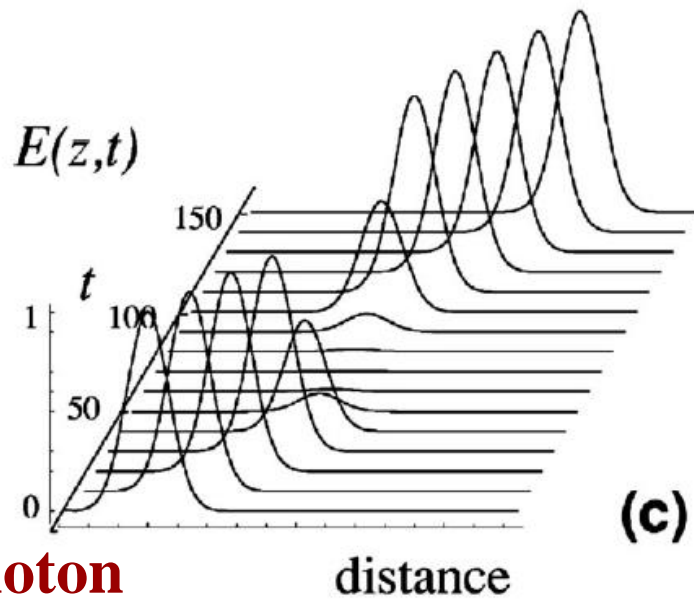
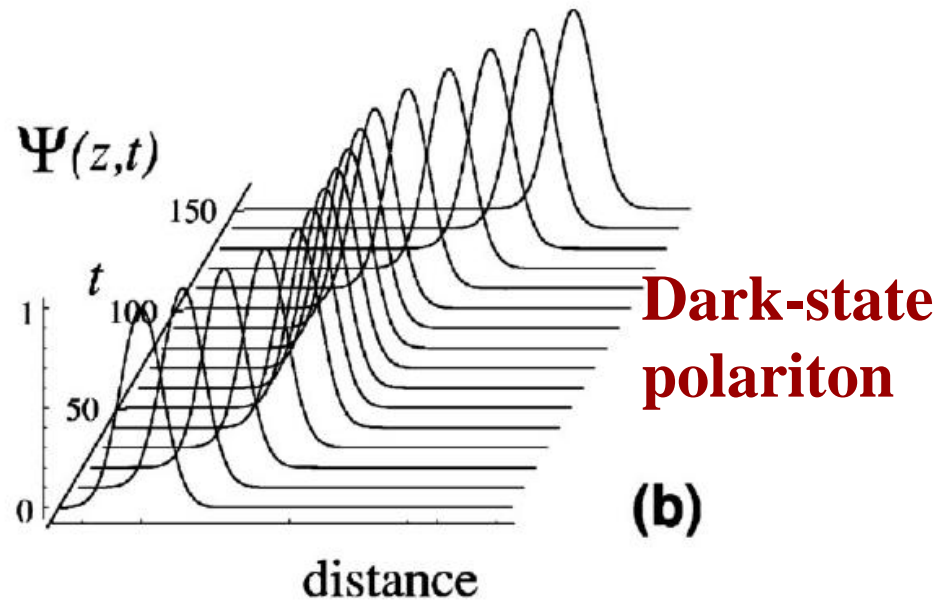
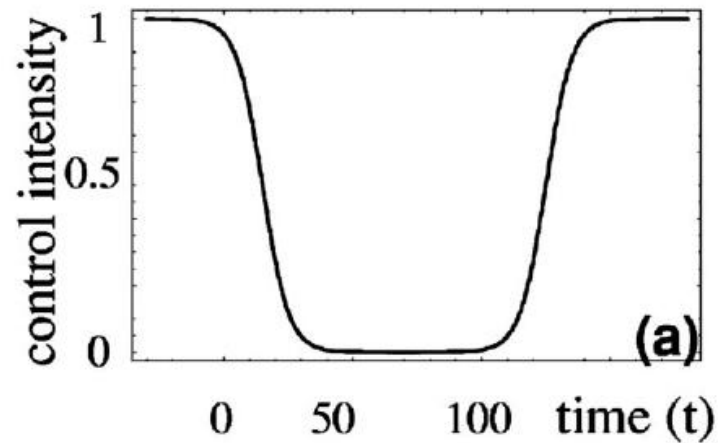
Spatial compression: c / v_g

For a $60 \mu\text{s}$ long pulse (18000 m long in space),

$$\text{if } \tau_{\text{delay}} = 12 \mu\text{s}, v_g = \frac{10 \text{ cm}}{12 \mu\text{s}} = 8.3 \times 10^5 \frac{\text{cm}}{\text{s}}.$$

$$\text{Spatial compression} = \frac{c}{v_g} = 3.6 \times 10^4 = \frac{18000 \text{ m}}{0.5 \text{ m}}$$

The length of the atomic cloud is 10 cm



Decoherence

- **Inelastic collisions**

coated cell, buffer gas, $J = 0$ atoms, Mott insulator

- **Inhomogeneous magnetic fields**

magnetic shielding,

nuclear spins,

using magnetically insensitive states (clock transitions)

- **Thermal diffusion/ atomic motion**

Cryogenic He buffer gas, optical lattice/ Mott insulator,

Bose-Einstein Condensation,

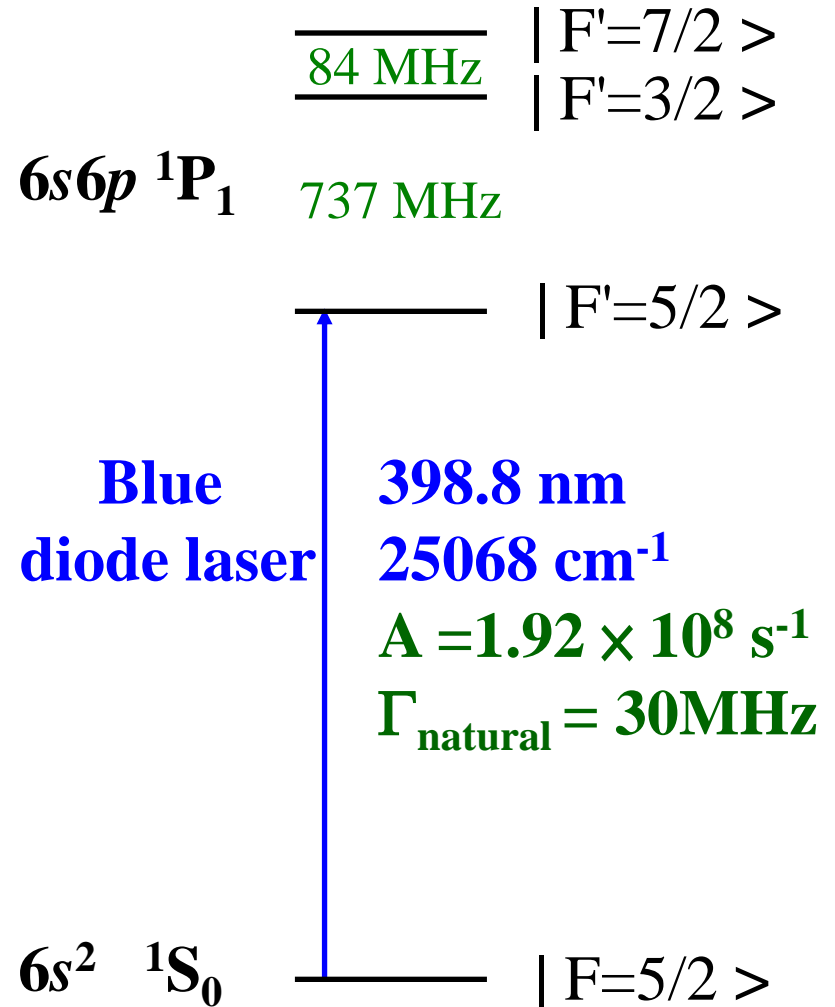
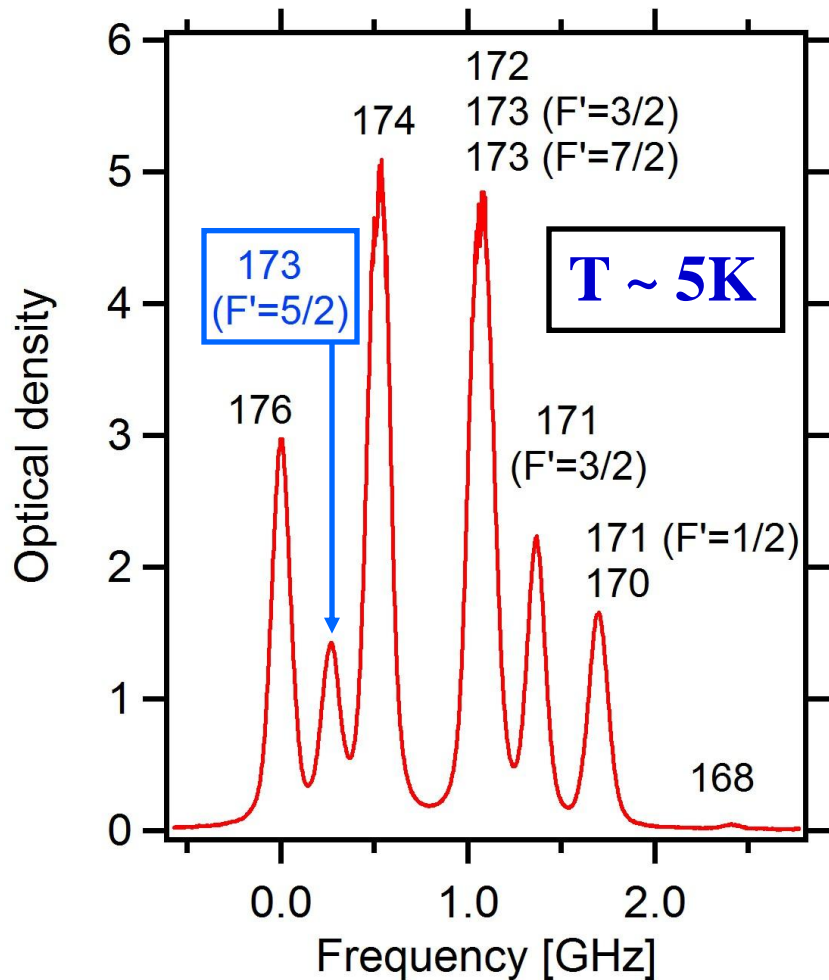
solids/crystal

Yb spectrum

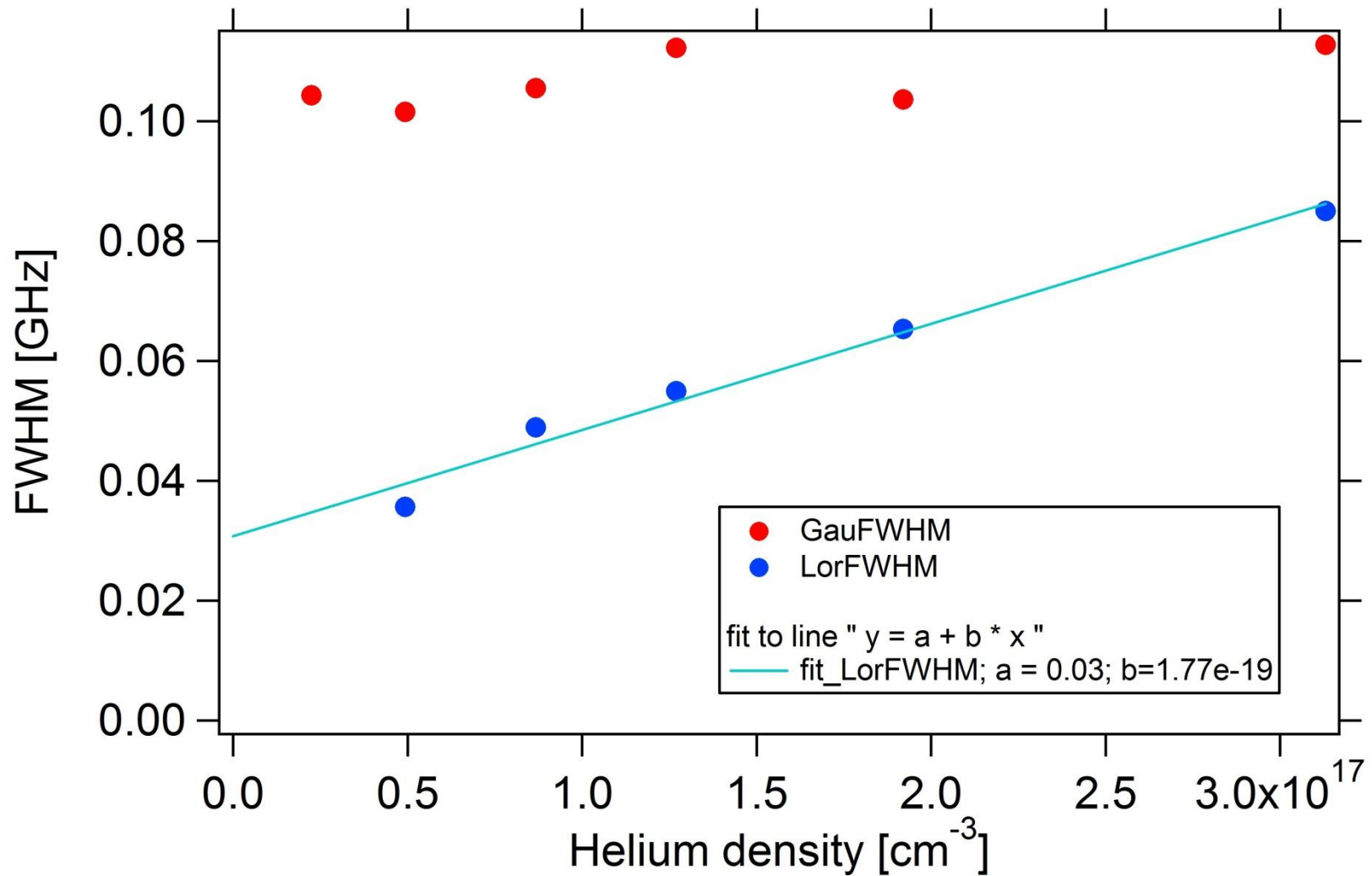
Isotope	Nuclear Spin	Abundance	Gryomagnetic ratio γ [Hz / Gauss]
^{173}Yb	5/2	16.12 %	- 206.5

Transmission = $e^{-\text{OD}}$

Optical Density (OD) = atom density \times absorption cross section \times path length



Yb Pressure broadening



Prior Work

$$\frac{1}{\tau} = k \cdot n$$

$$k = \sigma \cdot \bar{v}_r$$

$\frac{1}{\tau}$: rate

k : rate coefficient

n : atomic density

σ : cross-section

\bar{v}_r : relative velocity

Events	Rate coefficient [cm ³ s ⁻¹]
¹⁷³ Yb[¹ S ₀]-He	< 9 × 10 ⁻¹⁸
Rb[² S _{1/2}]-He	~10 ⁻¹⁹
Al[² P _{1/2}]-Ar C[³ P ₀], Si[² P ₀]-He O[³ P ₂], C[³ P ₀]-H	10 ⁻¹² ~10 ⁻¹⁰
Ca*-Ca* [4s4p ³ P ₂]	3 × 10 ⁻¹⁰
Yb*-Yb* [6s6p ³ P ₂]	1.0(3) × 10 ⁻¹¹
Sr*-Sr* [5s5p ³ P ₀]	(5 ± 3) × 10 ⁻¹²

Lu et al, Opt. Lett. 35, 622 (2010)

Walker et al, PRA 56,2090 (1997)

Picard et al, JCP 108, 10319 (1998)

Picard et al, JCP 117, 10109 (2002)

Abrahamsson et al, Astrophysics, J. 654, 1171 (2007)

Hemmerich et al, PRL 96, 073003 (2006)

Yamaguchi et al, PRL 101, 233002 (2008)

Traverso et al, PRA 79, 060702 (2009)