Efficiency of light-frequency conversion in an atomic ensemble

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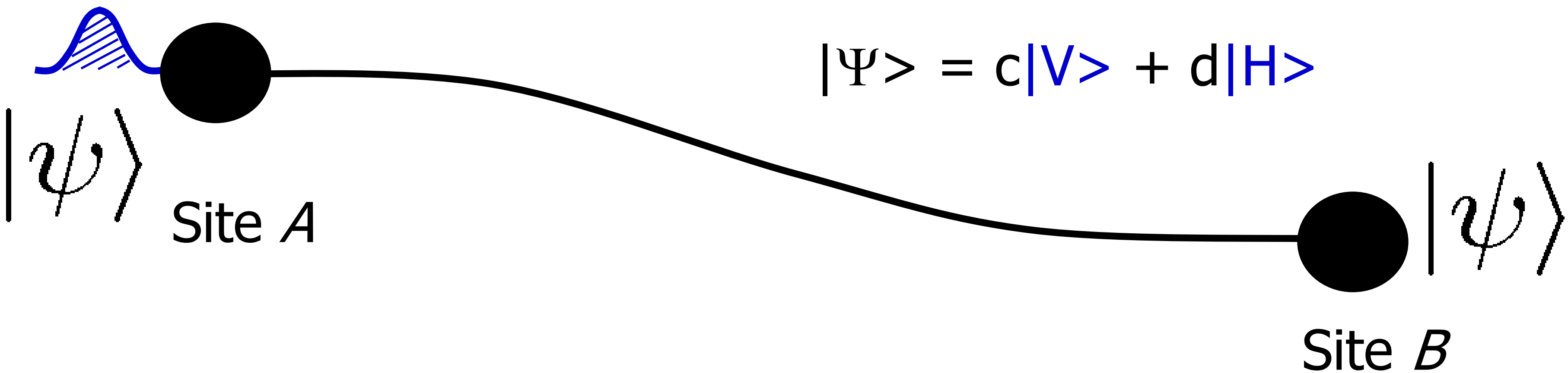
Outline

- Quantum repeater: a protocol for long distance quantum communication
- Cascade emission
- Telecom wavelength conversion using a diamond configuration
- Conclusion

Quantum repeater

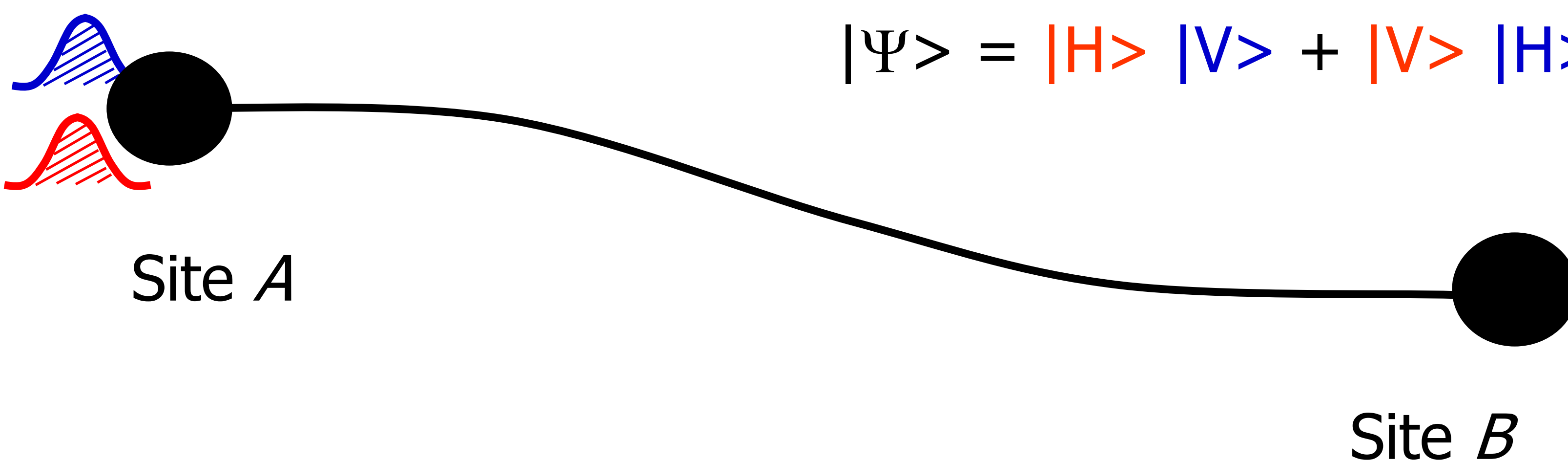
Quantum state transmission

$|\Psi\rangle = c|V\rangle + d|H\rangle$

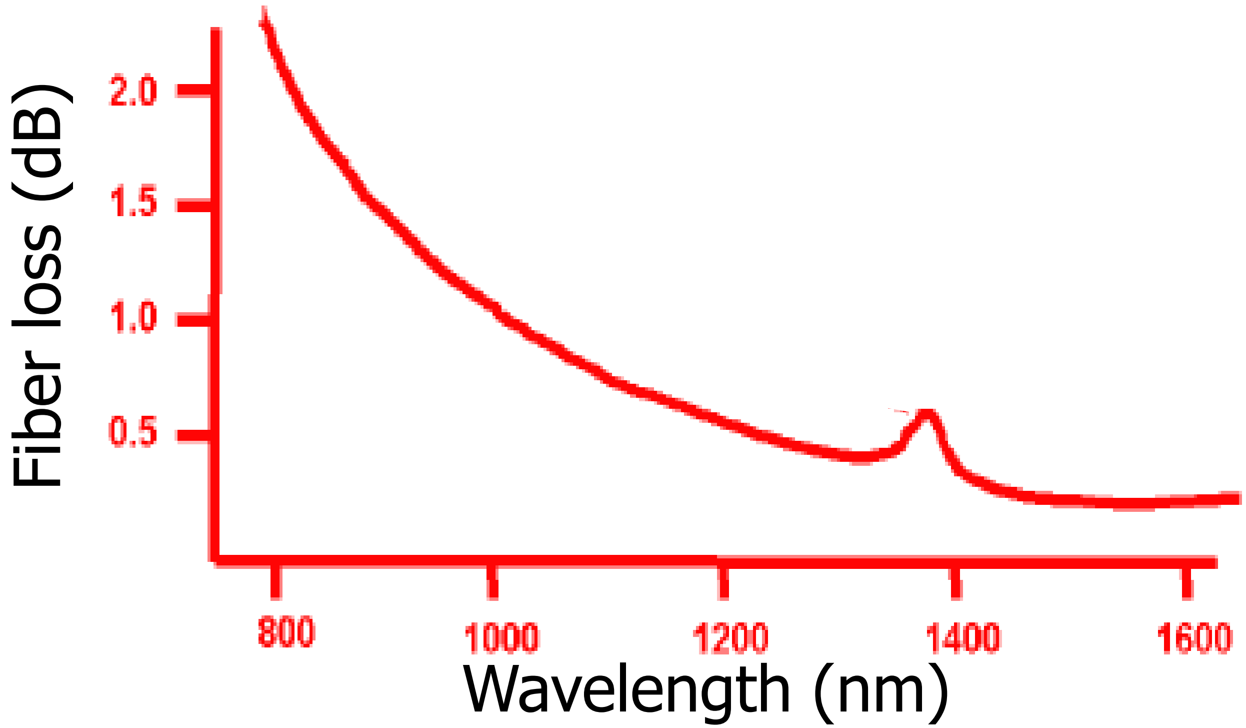


Entanglement distribution

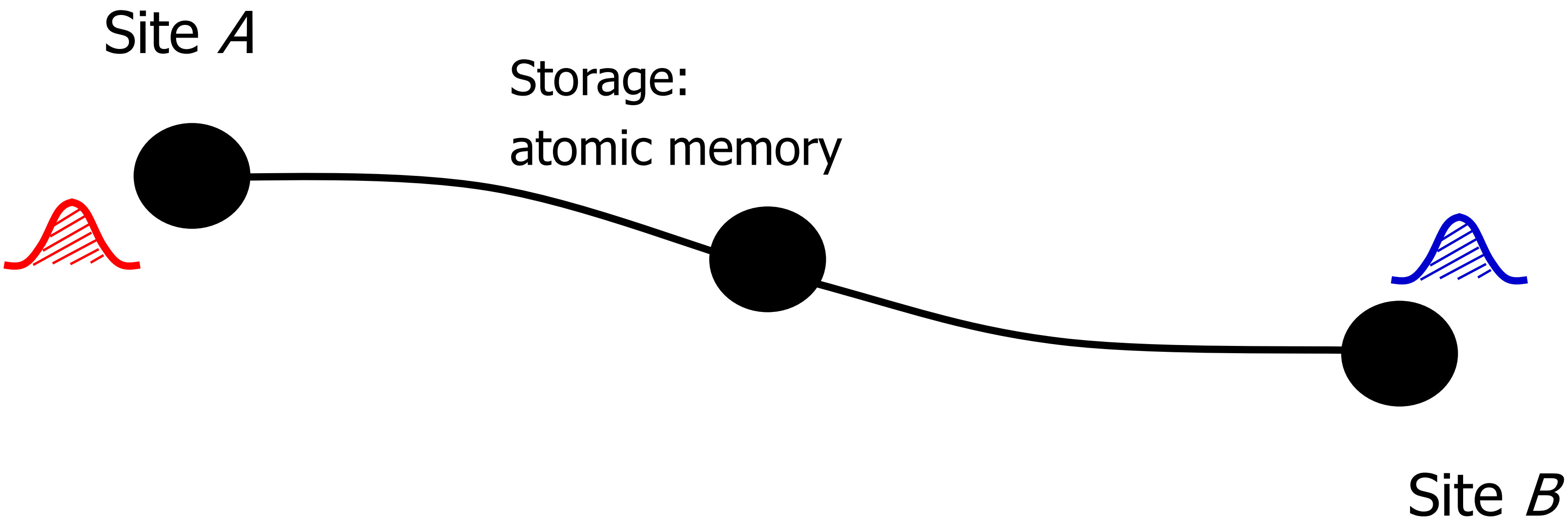
$|\Psi\rangle = |H\rangle\,|V\rangle + |V\rangle\,|H\rangle$



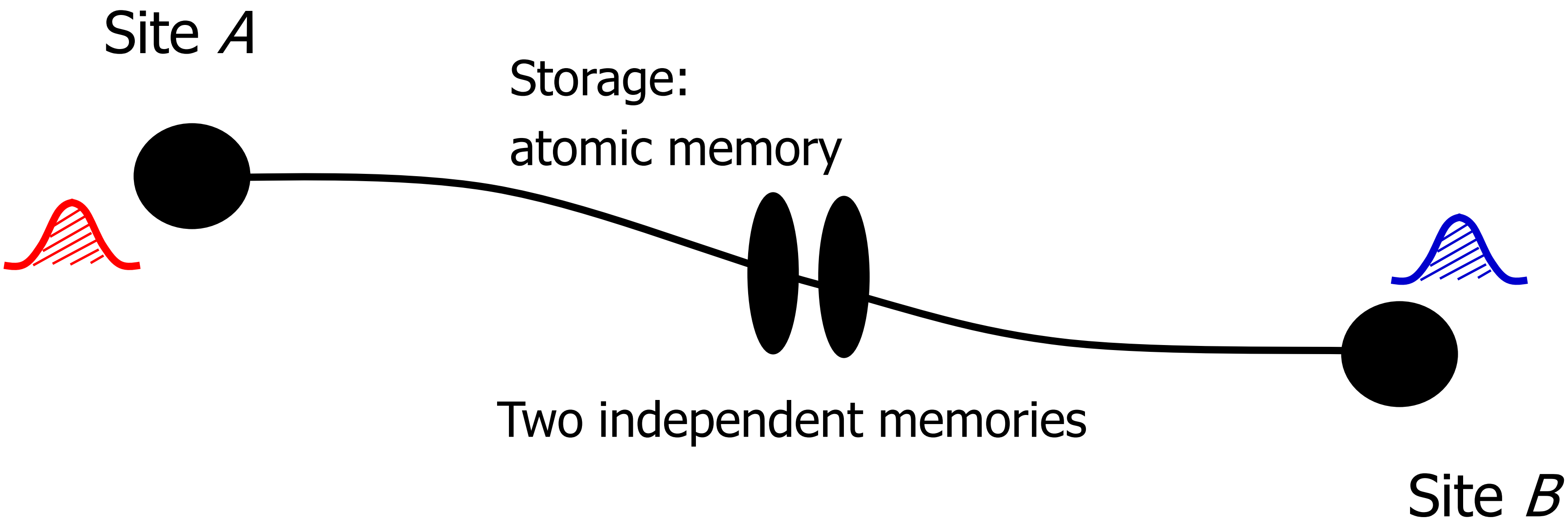
Attenuation for fiber transmission



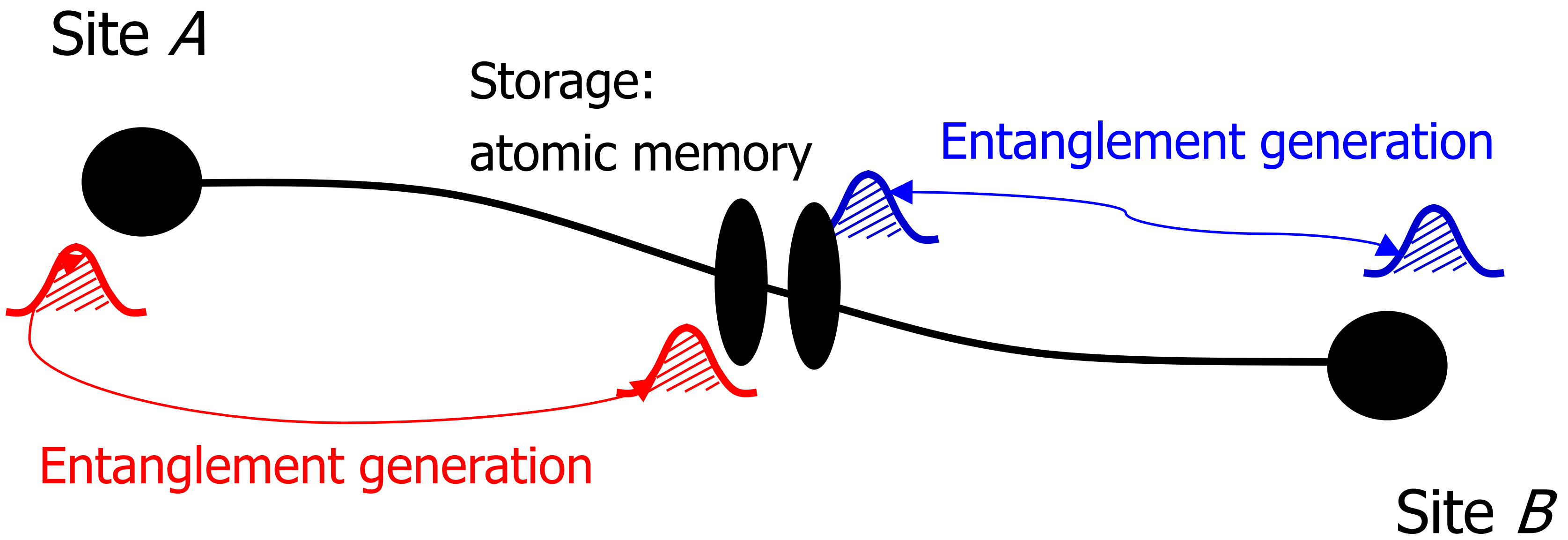
Quantum repeater



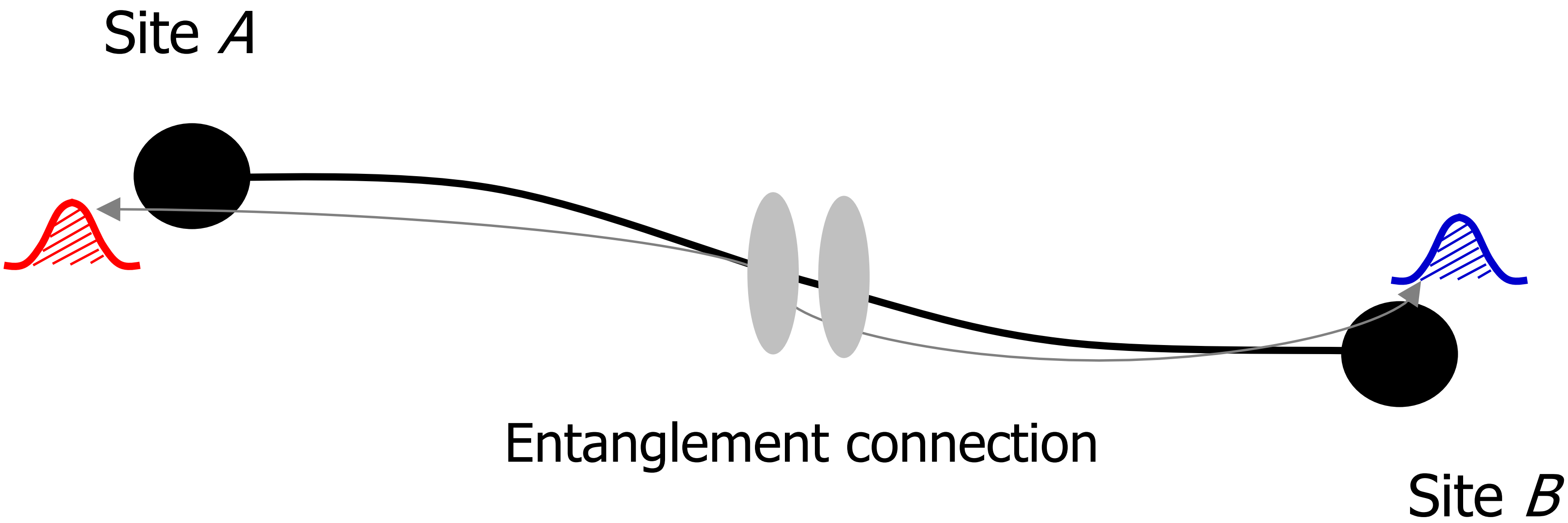
Quantum repeater



Quantum repeater

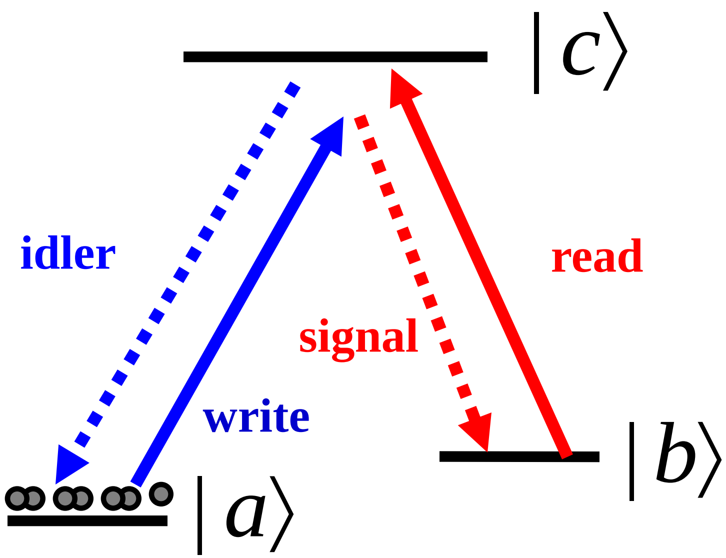


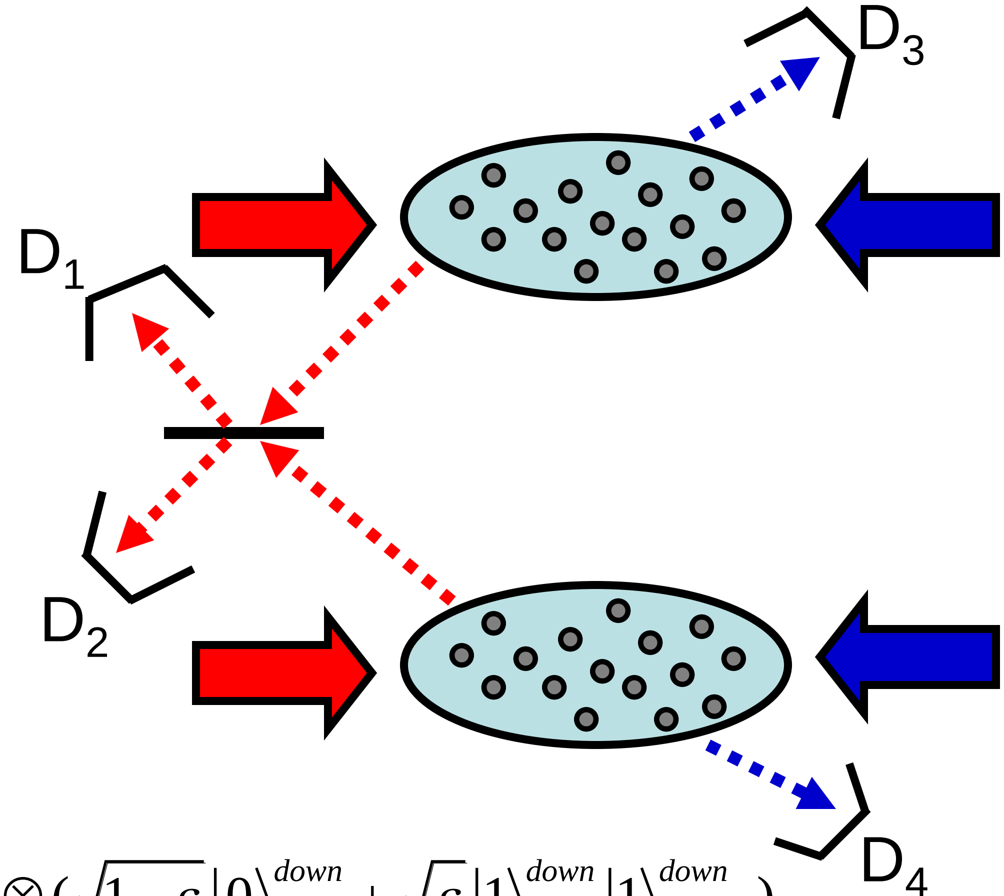
Quantum repeater



Duan-Lukin-Cirac-Zoller quantum repeater

Raman scheme



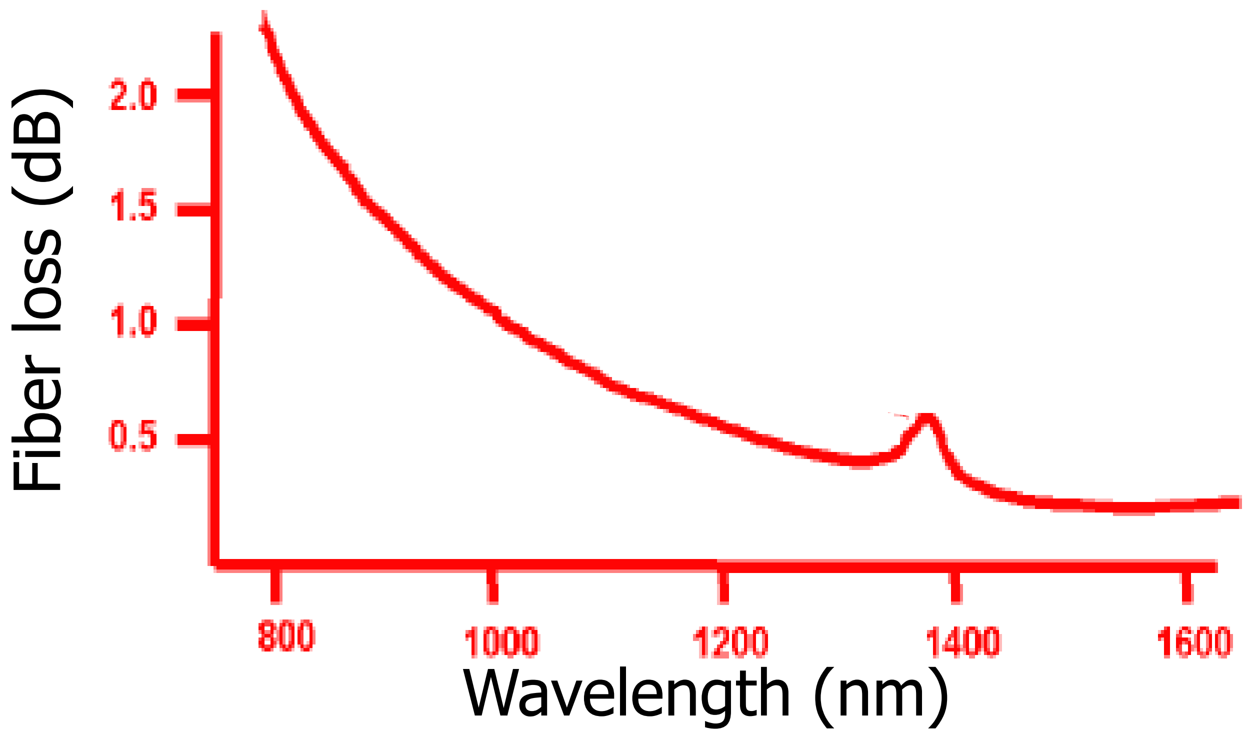


$$|\Psi\rangle = (\sqrt{1-\varepsilon}\,|0\rangle^{up} + \sqrt{\varepsilon}\,|1\rangle^{up}_{atom}|1\rangle^{up}_{photon}) \otimes (\sqrt{1-\varepsilon}\,|0\rangle^{down} + \sqrt{\varepsilon}\,|1\rangle^{down}_{atom}|1\rangle^{down}_{photon})$$

$$|\Psi\rangle_{\text{single click}} = \sqrt{\varepsilon}\sqrt{1-\varepsilon}(|1\rangle^{up}_{atom}|0\rangle^{down}_{atom} + |0\rangle^{up}_{atom}|1\rangle^{down}_{atom}) + O(\varepsilon)$$

L.-M. Duan et al, *Nature* **414**, 413 (2001)

Attenuation for fiber transmission



Atomic Cascade Transition

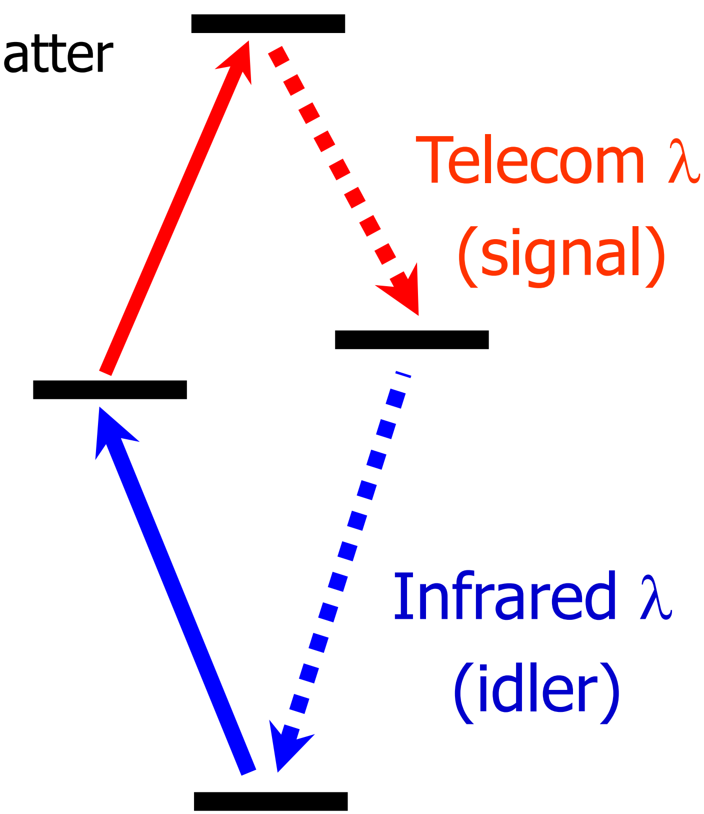
Quantum telecommunication using atomic cascade transitions

(1) Two-photon excitation of alkalis gives telecom λ,

(2) Two photons are polarization entangled,

(3) Storage of the idler then gives telecom-matter entanglement.

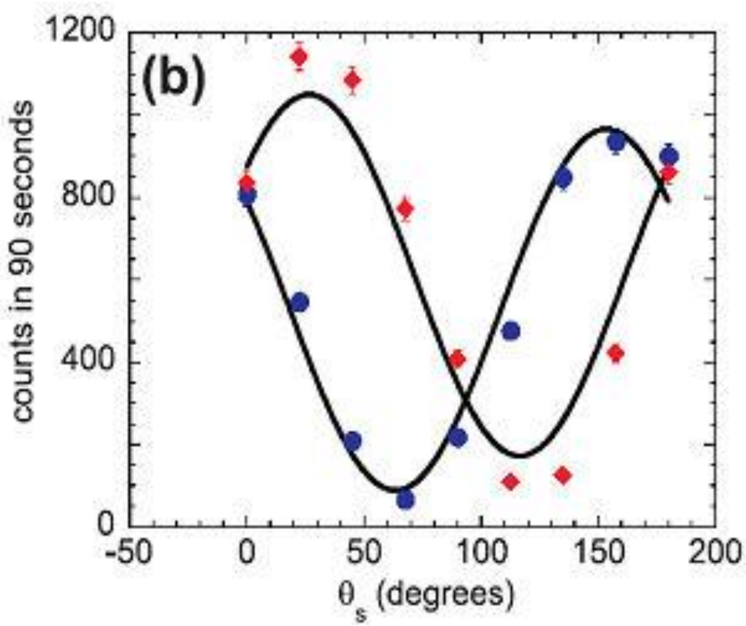
1.32 mm	$6\ ^2S_{1/2}$ to $5\ ^2P_{1/2}$
1.37 mm	$6\ ^2S_{1/2}$ to $5\ ^2P_{3/2}$
1.48 mm	$4\ ^2D_{3/2}$ to $5\ ^2P_{1/2}$
1.53 mm	$4\ ^2D_{5/2}$ to $5\ ^2P_{3/2}$

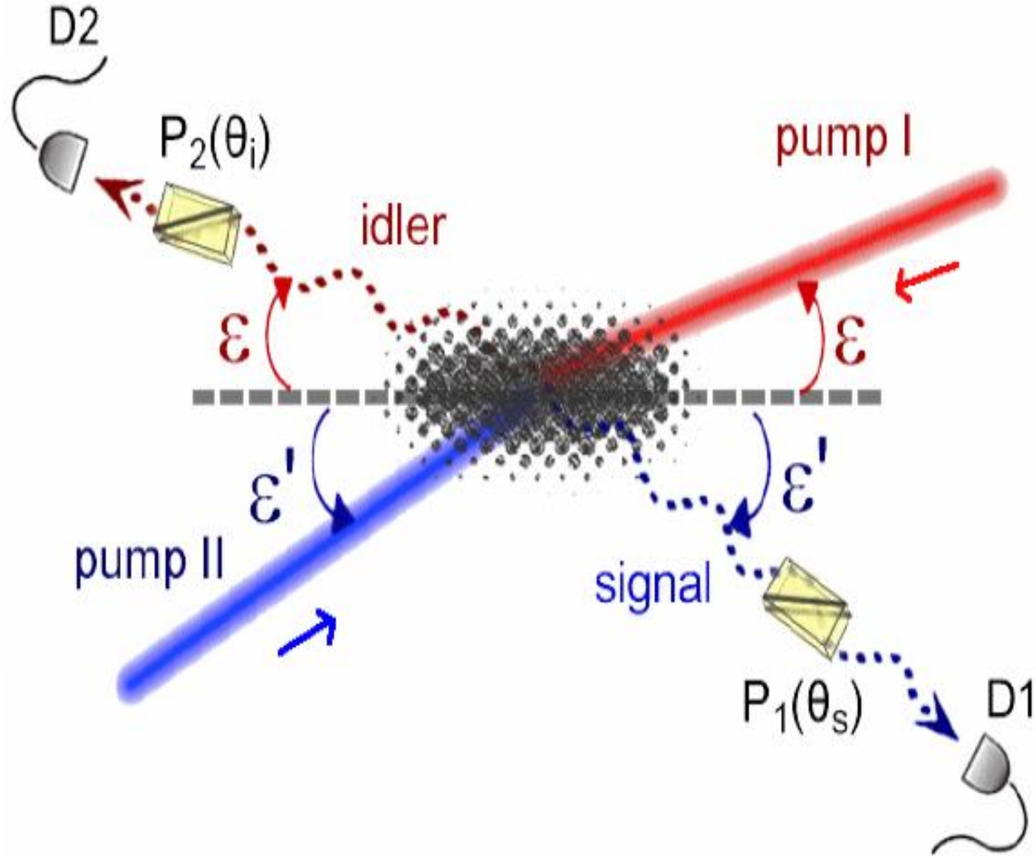


T. Chanelière, et al., *PRL* **96**, 093604 (2006)

Quantum telecommunication using atomic cascade transitions

- Polarization correlations





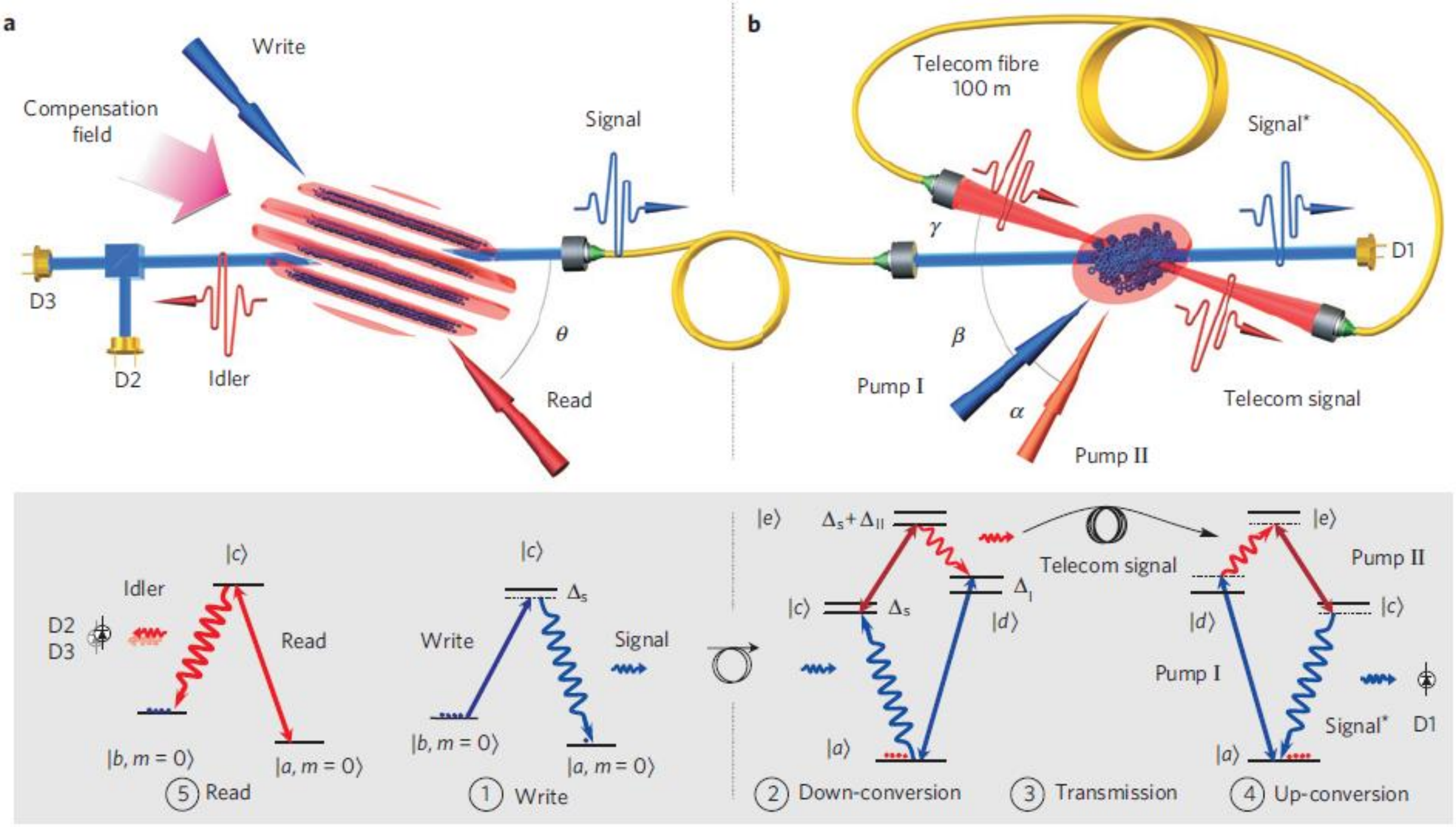
Bell inequality violation:

$$S_{\exp} = 2.132 \pm 0.036 \not\leq 2$$

T. Chanelière, et al., *PRL* **96**, 093604 (2006)

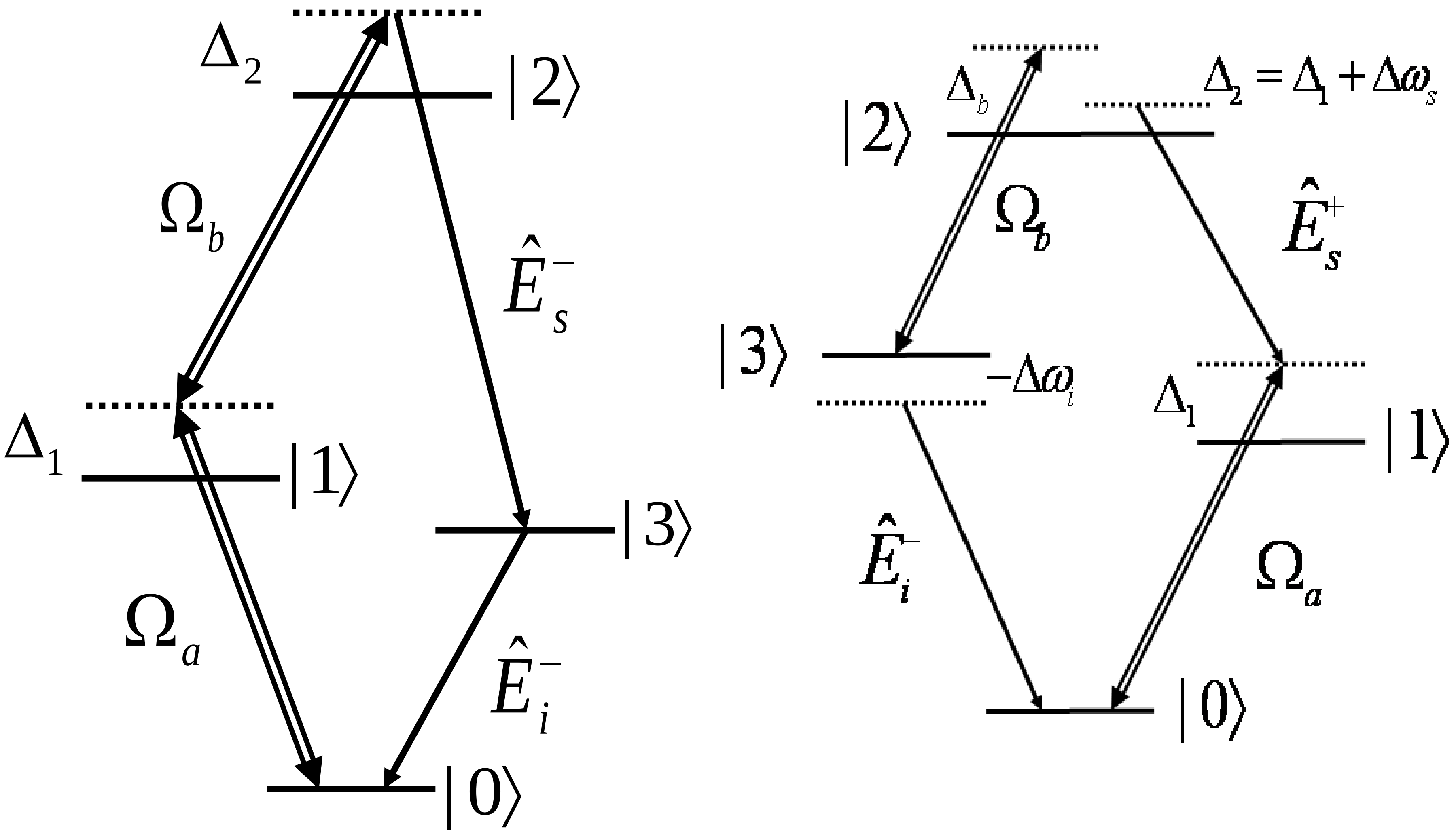
Telecom wavelength conversion

Quantum memory with telecom-wavelength conversion



A. G. Radnaev, et al., *Nature Physics* 6, 894 (2010)

Cascade: noise-initiated emission Vs Diamond: input-output



Hamiltonian and field quantization

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \quad \hat{H}_I = -\vec{d} \cdot \vec{E},$$

$$\begin{aligned}\hat{H}_0 \ = \ & \sum_{i=1}^{3} \sum_{l=-M}^{M} \hbar\omega_i \hat{\sigma}_{ii}^{l} + \hbar\omega_s \sum_{l=-M}^{M} \hat{a}_{s,l}^{\dagger} \hat{a}_{s,l} + \hbar \sum_{l,l'} \omega_{ll'} \hat{a}_{s,l}^{\dagger} \hat{a}_{s,l'} \\ & + \hbar\omega_i \sum_{l=-M}^{M} \hat{a}_{i,l}^{\dagger} \hat{a}_{i,l} + \hbar \sum_{l,l'} \omega_{ll'} \hat{a}_{i,l}^{\dagger} \hat{a}_{i,l'},\end{aligned}$$

$$\begin{aligned}\hat{H}_I = -\hbar \sum_{l=-M}^{M} \Big\{ & \Omega_a(t) \hat{\sigma}_{01}^{l\dagger} e^{ik_a z_l - i\omega_a t} + \Omega_b(t) \hat{\sigma}_{32}^{l\dagger} e^{-ik_b z_l - i\omega_b t} \\ & + g_s \sqrt{2M+1} \hat{\sigma}_{12}^{l\dagger} \hat{a}_{s,l} e^{-ik_s z_l} + g_i \sqrt{2M+1} \hat{\sigma}_{03}^{l\dagger} \hat{a}_{i,l} e^{ik_i z_l} + h.c. \Big\}\end{aligned}$$

Maxwell-Bloch equations: atomic part

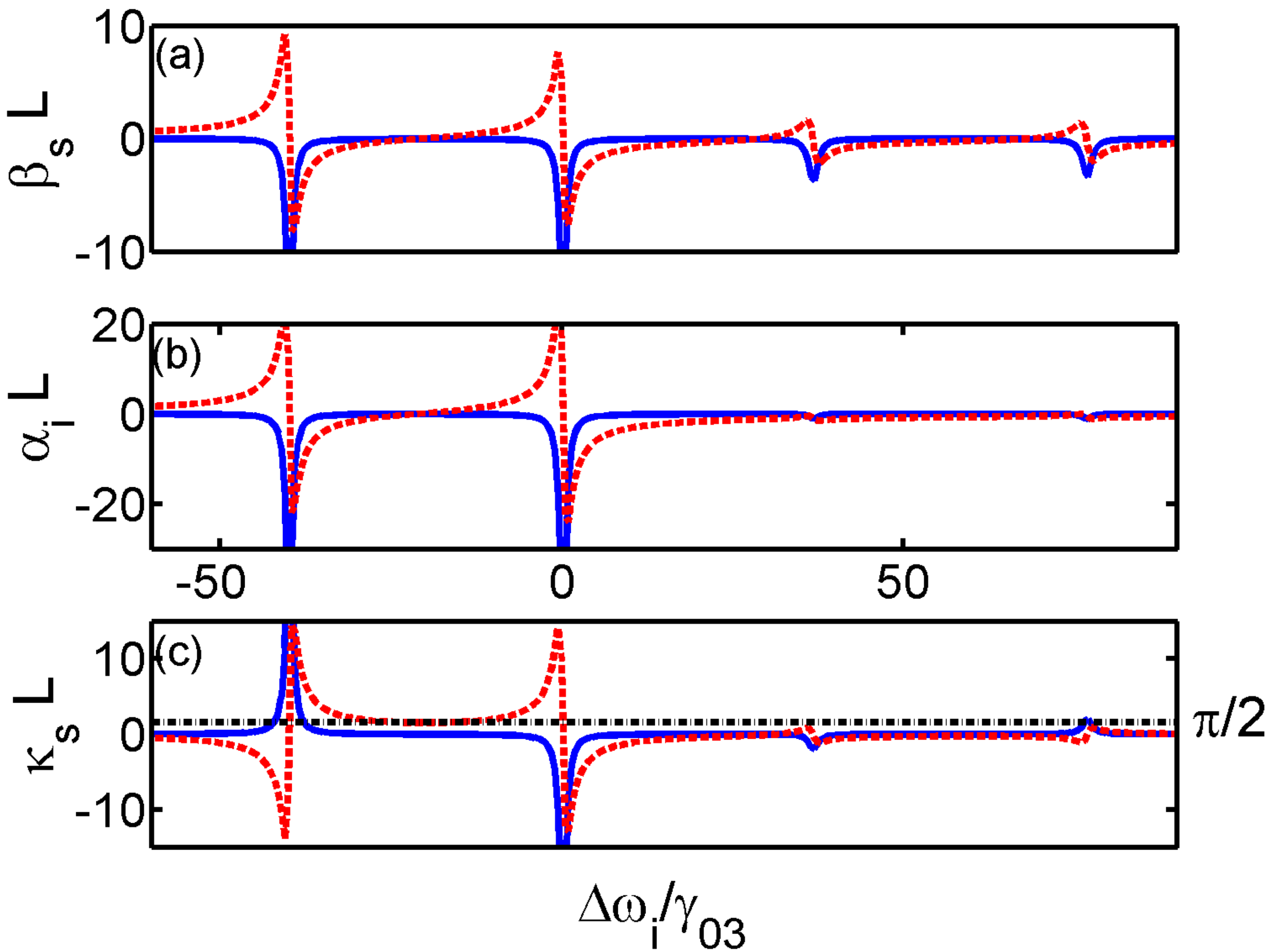
$$
\begin{aligned}
\frac{\partial}{\partial \tau}\tilde{\sigma}_{01} &= (i\Delta_1 - \frac{\gamma_{01}}{2})\tilde{\sigma}_{01} + i\Omega_a(\tilde{\sigma}_{00} - \tilde{\sigma}_{11}) + ig_s^*\tilde{\sigma}_{02}E_s^- - ig_i\tilde{\sigma}_{13}^\dagger E_i^+, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{12} &= (i\Delta\omega_s - \frac{\gamma_{01}+\gamma_2}{2})\tilde{\sigma}_{12} - i\Omega_a^*\tilde{\sigma}_{02} + ig_s(\tilde{\sigma}_{11} - \tilde{\sigma}_{22})E_s^+ + iP^*\Omega_b\tilde{\sigma}_{13}, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{02} &= (i\Delta_2 - \frac{\gamma_2}{2})\tilde{\sigma}_{02} - i\tilde{\sigma}_{12}\Omega_a + ig_s\tilde{\sigma}_{01}E_s^+ + iP^*\tilde{\sigma}_{03}\Omega_b - iP^*g_i\tilde{\sigma}_{32}E_i^+, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{11} &= -\gamma_{01}\tilde{\sigma}_{11} + \gamma_{12}\tilde{\sigma}_{22} + i\Omega_a\tilde{\sigma}_{01}^\dagger - i\Omega_a^*\tilde{\sigma}_{01} - ig_s\tilde{\sigma}_{12}^\dagger E_s^+ + ig_s^*\tilde{\sigma}_{12}E_s^-, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{22} &= -\gamma_2\tilde{\sigma}_{22} + ig_s\tilde{\sigma}_{12}^\dagger E_s^+ - ig_s^*\tilde{\sigma}_{12}E_s^- + i\Omega_b\tilde{\sigma}_{32}^\dagger - i\Omega_b^*\tilde{\sigma}_{32}, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{33} &= -\gamma_{03}\tilde{\sigma}_{33} + \gamma_{32}\tilde{\sigma}_{22} - i\Omega_b\tilde{\sigma}_{32}^\dagger + i\Omega_b^*\tilde{\sigma}_{32} + ig_i\tilde{\sigma}_{03}^\dagger E_i^+ - ig_i^*\tilde{\sigma}_{03}E_i^-, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{13} &= (i\Delta\omega_i - i\Delta_1 - \frac{\gamma_{01}+\gamma_{03}}{2})\tilde{\sigma}_{13} - i\Omega_a^*\tilde{\sigma}_{03} - iPg_s\tilde{\sigma}_{32}^\dagger E_s^+ + iP\Omega_b^*\tilde{\sigma}_{12} \\
&\quad + ig_i\tilde{\sigma}_{01}^\dagger E_i^+, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{03} &= (i\Delta\omega_i - \frac{\gamma_{03}}{2})\tilde{\sigma}_{03} - i\Omega_a\tilde{\sigma}_{13} + iP\Omega_b^*\tilde{\sigma}_{02} + ig_i(\tilde{\sigma}_{00} - \tilde{\sigma}_{33})E_i^+, \\
\frac{\partial}{\partial \tau}\tilde{\sigma}_{32}^\dagger &= (-i\Delta_b - \frac{\gamma_{03}+\gamma_2}{2})\tilde{\sigma}_{32}^\dagger - iP^*g_s^*\tilde{\sigma}_{13}E_s^- + i\Omega_b^*(\tilde{\sigma}_{22} - \tilde{\sigma}_{33}) + iP^*g_i\tilde{\sigma}_{02}^\dagger E_i^+
\end{aligned}
$$

Maxwell-Bloch equations: field part

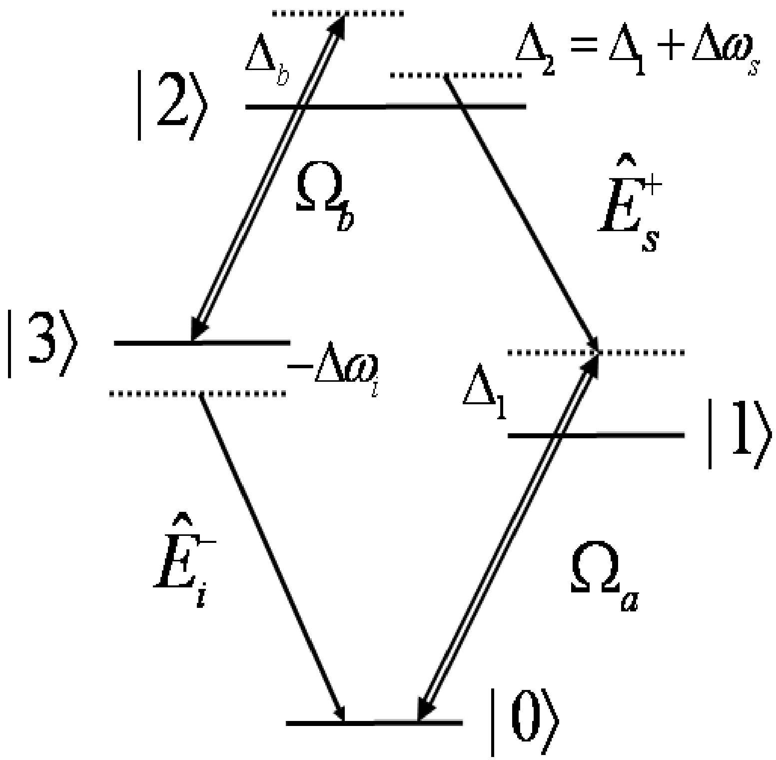
$$\frac{\partial}{\partial z}E_s^+ = \beta_s E_s^+ + \kappa_s E_i^+$$
$$\frac{\partial}{\partial z}E_i^+ = \kappa_i E_s^+ + \alpha_i E_i^+.$$

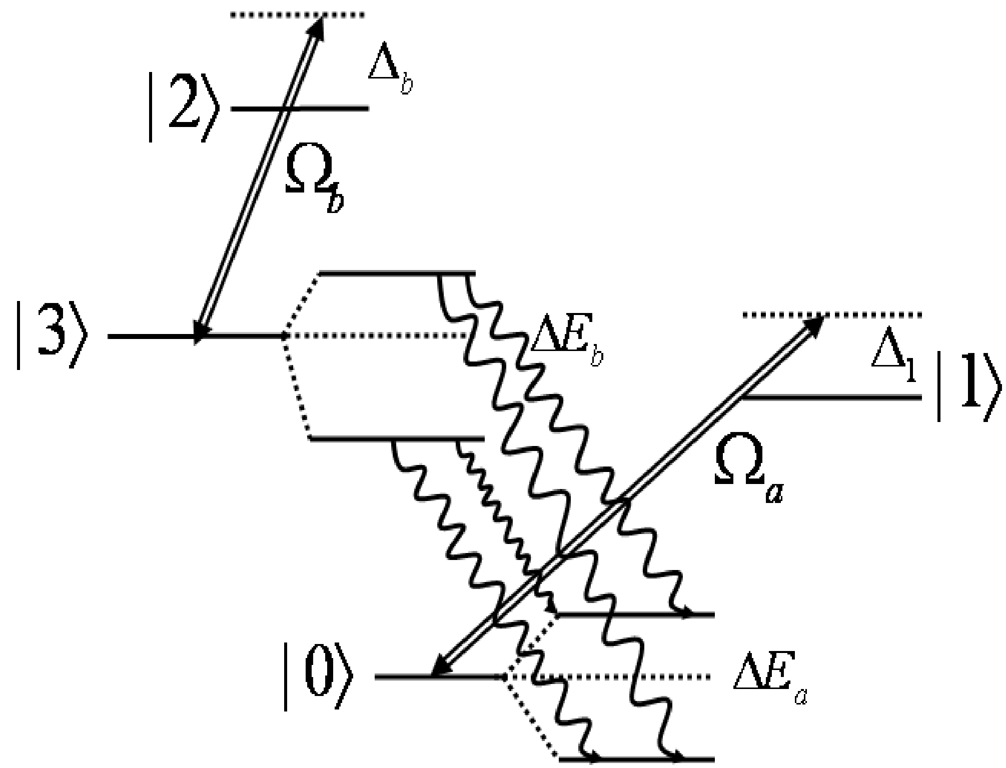
$$\eta_{\rm d} = \left|\frac{E_s^+(L)}{E_i^+(0)}\right|^2 = \left|\frac{\kappa_s}{2w}e^{(\alpha_i+\beta_s)L/2}(e^{wL} - e^{-wL})\right|^2$$
$$T_{\rm d} = \left|\frac{E_i^+(L)}{E_i^+(0)}\right|^2 = \left|\frac{e^{(\alpha_i+\beta_s)L/2}}{2w(w+q)}[\kappa_s\kappa_i e^{wL} + (q+w)^2 e^{-wL}]\right|^2.$$

Self- and cross-coupling coefficients



Diamond configuration: dressed state picture

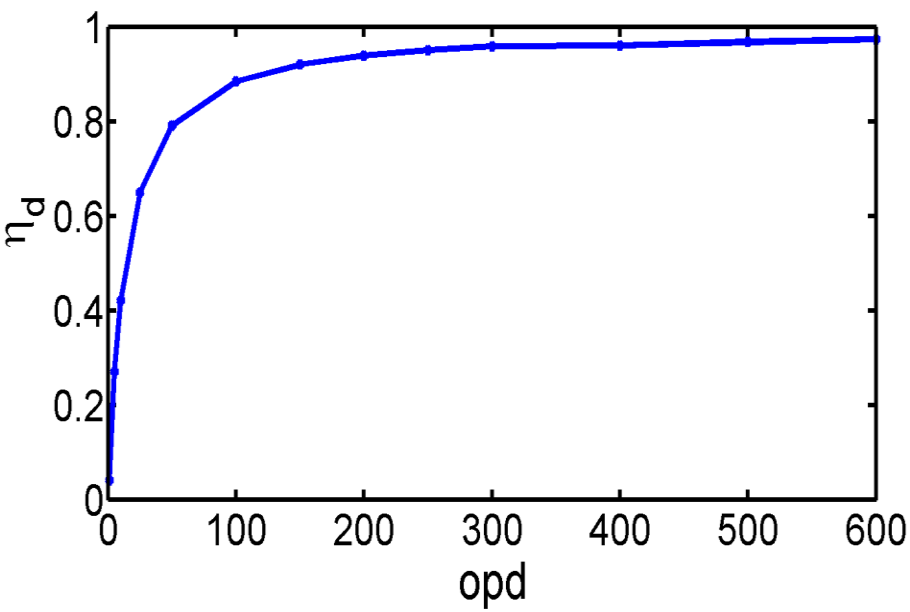




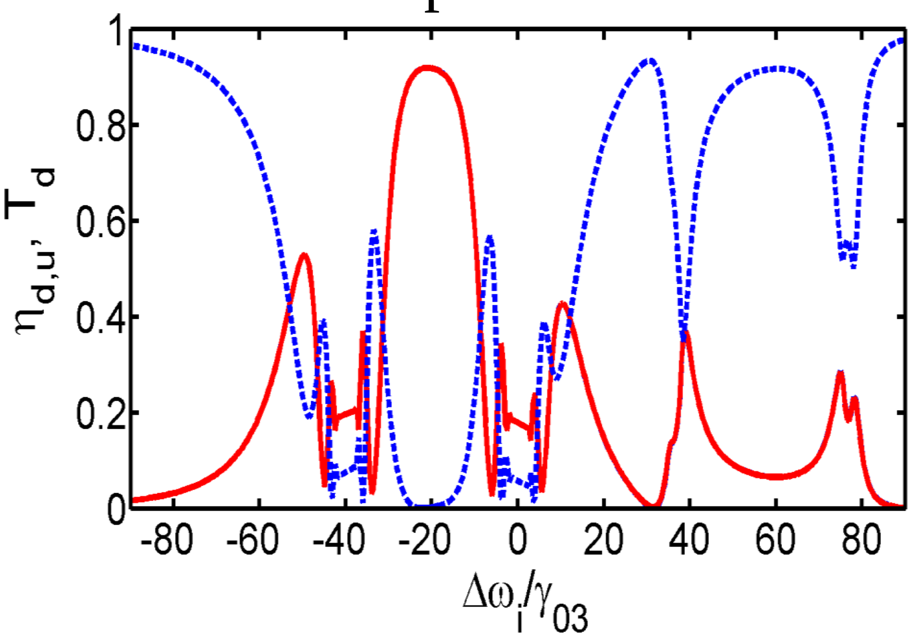
Optimal frequency conversion

Five variational parameters:

$$\Omega_a, \Omega_b, \Delta_1, \Delta_b, \Delta\omega_i.$$



opd = 150



H. H. Jen and T. A. B. Kennedy, *PRA* 82, 023815 (2010)

Pulse conversion: numerical solution

Spatial and temporal integrations:

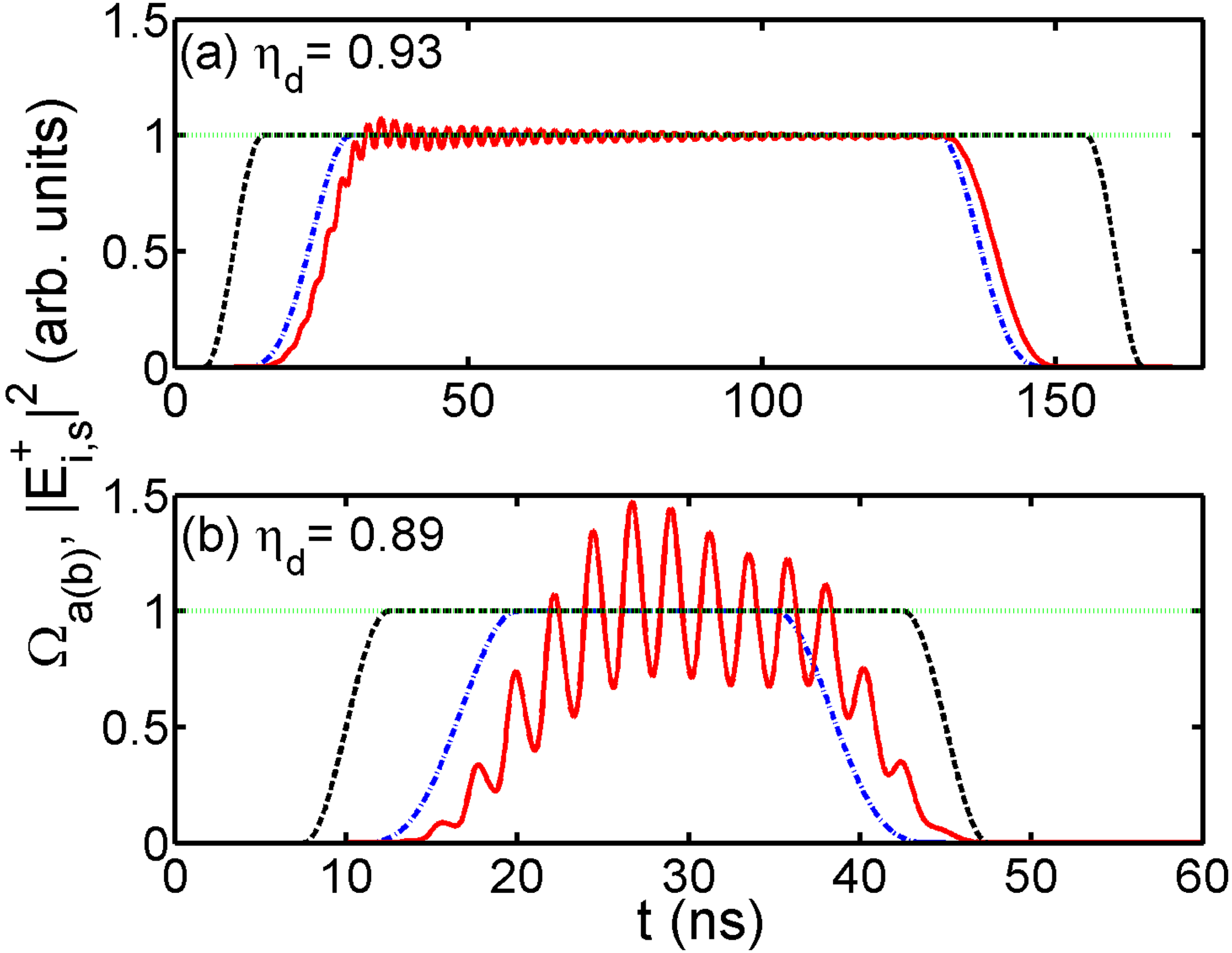
semi-implicit difference (midpoint) method.

Atomic ensemble in experiment:

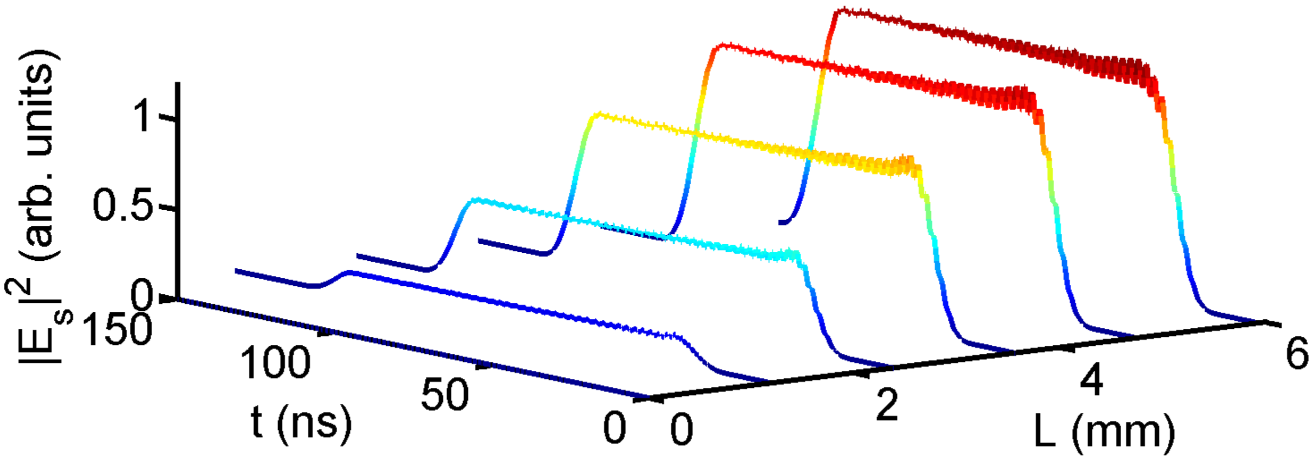
$\rho = 1.7 \times 10^{11}\,\mathrm{cm}^{-3}, L = 6\,\mathrm{mm}.$

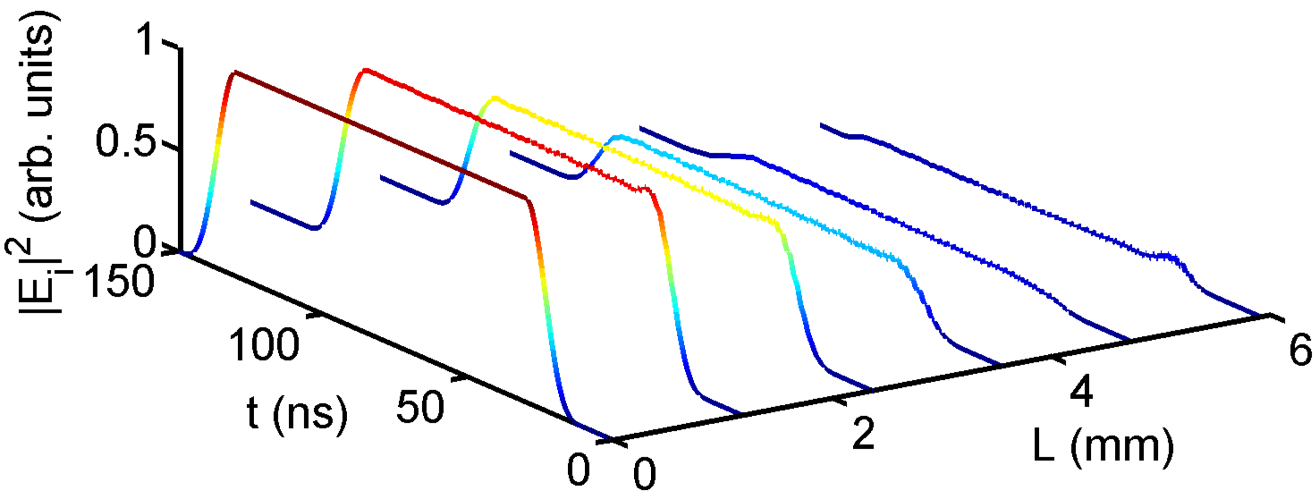
$$\eta_{\mathrm{d}} = \frac{\int |E_s^+(z = L, \tau)|^2 d\tau}{\int |E_i^+(z = 0, \tau)|^2 d\tau}.$$

Down-converted pulse conversion



Down-converted pulse conversion





$\eta_d = 0.93,$

$\text{opd} = 150.$

Heisenberg-Langevin equations: quantum fluctuations

$$\frac{\partial}{\partial z}E_s^+ = \beta_s E_s^+ + \kappa_s E_i^+ + \hat{f}_s, \quad \left\langle \tilde{\mathcal{F}}_i^\dagger(t,z) \right\rangle = \left\langle \tilde{\mathcal{F}}_i(t',z') \right\rangle = 0$$

$$\frac{\partial}{\partial z}E_i^+ = \kappa_i E_s^+ + \alpha_i E_i^+ + \hat{f}_i, \quad \left\langle \tilde{\mathcal{F}}_i^\dagger(t,z)\tilde{\mathcal{F}}_j(t',z') \right\rangle = \frac{L}{N}\delta(t-t')\delta(z-z')\hat{D}_{ij}$$

$$\begin{aligned}\hat{f}_s = \; & \frac{iNg_s^*}{cD}[(T_{03} + \frac{|\Omega_a|^2}{T_{13}} + \frac{|\Omega_b|^2}{T_{02}})\tilde{\mathcal{F}}_{12} + (\frac{\Omega_a^*\Omega_b}{T_{02}} + \frac{\Omega_a^*\Omega_b}{T_{13}})\tilde{\mathcal{F}}_{03} \\ & + \frac{i\Omega_b}{T_{13}}(T_{03} + \frac{|\Omega_b|^2 - |\Omega_a|^2}{T_{02}})\tilde{\mathcal{F}}_{13} + \frac{i\Omega_a^*}{T_{02}}(-T_{03} + \frac{|\Omega_b|^2 - |\Omega_a|^2}{T_{13}})\tilde{\mathcal{F}}_{02}] + \tilde{\mathcal{F}}_s,\end{aligned} \tag{6.21}$$

$$\begin{aligned}\hat{f}_i = \; & \frac{iNg_i^*}{cD}[(\frac{\Omega_a\Omega_b^*}{T_{02}} + \frac{\Omega_a\Omega_b^*}{T_{13}})\tilde{\mathcal{F}}_{12} + (T_{12} + \frac{|\Omega_a|^2}{T_{02}} + \frac{|\Omega_b|^2}{T_{13}})\tilde{\mathcal{F}}_{03} \\ & + \frac{i\Omega_a}{T_{13}}(-T_{12} + \frac{|\Omega_b|^2 - |\Omega_a|^2}{T_{02}})\tilde{\mathcal{F}}_{13} + \frac{i\Omega_a^*}{T_{02}}(T_{12} + \frac{|\Omega_b|^2 - |\Omega_a|^2}{T_{13}})\tilde{\mathcal{F}}_{02}] + \tilde{\mathcal{F}}_i\end{aligned} \tag{6.22}$$

Quantum fluctuations

$$\begin{aligned}
\text{(i)}\ \hat{D}_{12,12} &= \gamma_{01}\left\langle\tilde{\sigma}_{22}\right\rangle \approx \gamma_{01}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{12,03} &= \hat{D}_{12,02} = 0; \\
\hat{D}_{12,13} &= \gamma_{01}\left\langle\tilde{\sigma}_{23}\right\rangle \approx \gamma_{01}\tilde{\sigma}_{23,s} = 0; \\
\hat{D}_{12,s} &= \hat{D}_{12,i} = 0; \qquad (6.34) \\
\text{(ii)}\ \hat{D}_{03,03} &= \gamma_{32}\left\langle\tilde{\sigma}_{22}\right\rangle \approx \gamma_{32}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{03,02} &= \hat{D}_{03,13} = 0; \\
\hat{D}_{03,s} &= \hat{D}_{03,i} = 0; \qquad (6.35) \\
\text{(iii)}\ \hat{D}_{02,02} &= \hat{D}_{02,13} = 0; \\
\hat{D}_{02,s} &= \hat{D}_{02,i} = 0; \qquad (6.36) \\
\text{(iV)}\ \hat{D}_{13,13} &= \gamma_{01}\left\langle\tilde{\sigma}_{33}\right\rangle + \gamma_{32}\left\langle\tilde{\sigma}_{22}\right\rangle \approx \gamma_{01}\tilde{\sigma}_{33,s} + \gamma_{32}\tilde{\sigma}_{22,s} = 0; \\
\hat{D}_{13,s} &= \hat{D}_{13,i} = 0; \qquad (6.37) \\
\text{(V)}\ \hat{D}_{s,s} &= \hat{D}_{s,i} = 0; \qquad (6.38) \\
\text{(Vi)}\ \hat{D}_{i,i} &= 0; \qquad (6.39)
\end{aligned}$$

Conclusion

- A demonstration of quantum correlation with telecom wavelength conversion, which provides low-loss quantum network communication.
- Parametric equations for the probe fields are derived and used to compute conversion efficiencies.
- Dressed-state picture tells us the optimum conversion happens in EIT window.
- Numerical solution indicates that for shorter pulses, pump pulse induced modulation may reduce the conversion efficiency.

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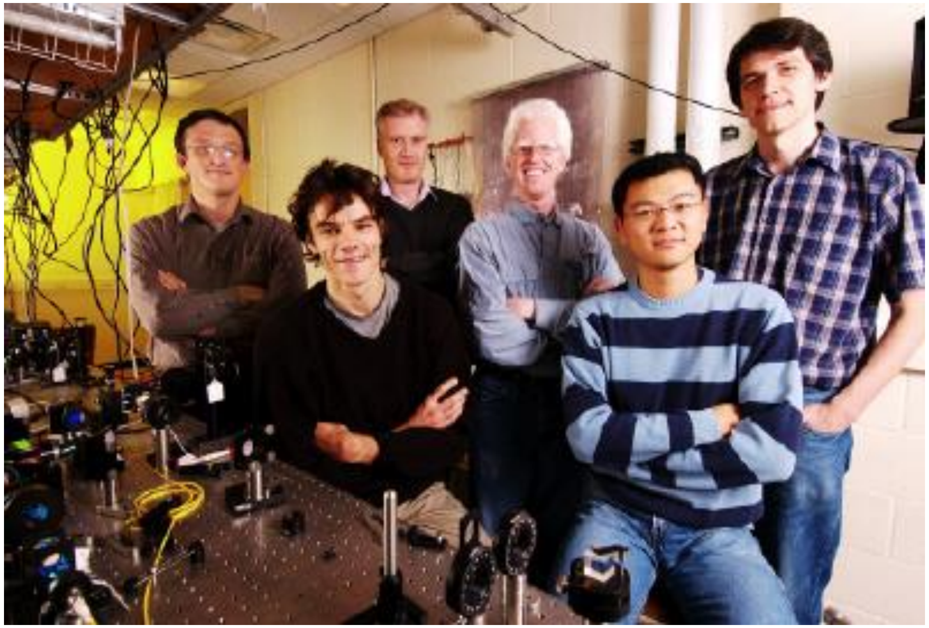
Dzmitry Matsukevich (N. U. Singapore)



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