Precision Measurement of Lithium Hyperfine Structure

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Historical review: spectroscopy of simple atoms
Examples (H, He, Li)
What physics to study?
Experiment in NTHU
Our results and outlook
To the extent atomic structure is calculable, precision measurement test fundamental physics

Electromagnetic interaction well known

Advances in atomic theory

Simple atoms, e.g.: H, He, Li, etc...

What fundamental physics to study
Hydrogen

- non-relativistic

\[ En = -\alpha^2 mc^2 \left( \frac{1}{2n^2} \right) = -\frac{13.6 eV}{n^2} \]

- relativistic correction

\[ -\alpha^4 mc^2 \frac{1}{4n^2} \left[ \frac{2n}{(l+1/2)} - \frac{3}{2} \right] \]

- spin-orbit interaction (L-S, fine structure)

\[ -\alpha^4 mc^2 \frac{1}{4n^2} \left[ \frac{2n}{(j+1/2)} - \frac{3}{2} \right] \]

- QED effect (Lamb shift)

\[ \alpha^5 mc^2 \frac{1}{4n^3} \left\{ k(n,l) \pm \frac{1}{\pi(j+1/2)(l+1/2)} \right\} \]

- nuclear magnetic moment (hyperfine structure)

\[ \left( \frac{m}{m_p} \right) \alpha^4 mc^2 \frac{4\gamma_p}{3n^2} \left[ f(f+1) - \frac{3}{2} \right] \]

- nuclear size effect

\[ \frac{2\pi}{3} Ze^2 |\psi(0)|^2 \left\langle r^2 \right\rangle_{proton} \]

- \[ E = E_{Bohr} + E_{rel} + E_{ls} + E_{Darwin} + E_{HF} + E_{QED} + E_{nuclear} \]
The Lamb Shift

Willis Eugene Lamb
Nobel Prize in Physics 1955
"for his discoveries concerning the fine structure of the hydrogen spectrum"

1947 by Lamb ~1060 MHz
now: 1057.846(4) MHz

LS QED

1 S₁/₂

2 S₁/₂

2 P₁/₂

2 P₃/₂

1 S₁/₂

1 S₁/₂

H₂ source

Microwave region

Detector

e⁻ gun

1947 by Lamb ~1060 MHz
now: 1057.846(4) MHz

W. Eugene Lamb
Nobel Prize in Physics 1955
"for his discoveries concerning the fine structure of the hydrogen spectrum"

g-2 experiment of electron and muon:

gₑ (exp) = 2.0023193043617(15), gₑ (th) to determine fundamental constant

gₑ (exp) = 2.0023318416(12)
gₑ (th) = 2.0023318367(13)

Nuclear size effect

Transition from $s$ to $p$ state

- Decrease transition frequency

a) point nucleus
   $V \sim -1/r$

b) finite size nucleus

transition from $s$ to $p$ state

$\rightarrow$ decrease transition frequency
Measure total transition frequency: uncertainty: theory $\sim 10$ kHz, experiment $\sim 20$ kHz

$\rightarrow$ charge radius of proton

$R_p = 0.883(14)$ fm \[1, 2\]

$= 0.890(14)$ fm \[3\]

Reference:
Electron scattering

Electron beam on nucleus

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} = \frac{Z^2 \alpha^2 (\hbar c)^2}{4 E^2 \sin^4 \frac{\theta}{2}}
\]

Nuclei with finite size:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{exp}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} \cos^2 \frac{\theta}{2} \left| G_E(q^2) \right|^2
\]

\[
< r^2 > = -6 \hbar^2 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2=0}
\]

Note: not the shape to fit, but the slope at \( q^2 = 0 \)
Discrepancy

\[ R_p = 0.862(12) \text{ fm} \]


\[ R_p = 0.883(14) \text{ fm} \] \[ = 0.890(14) \text{ fm} \] \[ = 0.879(8) \text{ fm}, \text{ new experiment by GSI} \]

2\sigma \text{ deviation}

0.895(18) \text{ fm, re-analysis of world data} \\

0.879(8) \text{ fm, new experiment by GSI} \\
Muonic hydrogen

- $m_\mu/m_e \sim 200$
- Bohr radius $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$
- Energy level $E_n = -\frac{m_e}{2(4\pi\epsilon_0\hbar)^2} \frac{m_e}{n^2}$
- Wave function $\Psi(r) \sim a_0^{-3/2} e^{-r/a_0}$
- Energy shift due to nuclear size $\sim |\Psi(0)|^2 \left\langle r^2 \right\rangle$
- Sensitivity $\sim (m_\mu/m_e)^2$

at PSI, reaching 0.1 % precision

$R_{rms} = 0.84184(67) \text{ fm}$

Nature 466, 213–216, 2010
Measure isotope shift, $1s \rightarrow 2s$ at 121 nm

$\Delta \nu = 670\ 994.33464(15)\ MHz$

- Nuclear mass
- Nuclear magnetic susceptibility
- Nuclear polarizability
- Nuclear size

Summary of QED uncertainty

- H: 10 kHz, He: 1 MHz, Li: > 10 MHz
- Typical nuclear effect: several MHz
- G. Drake: QED uncertainty largely cancel in isotope shift (IS) and fine structure (FS) splitting measurement
- Uncertainty in isotope shift:
  - H: <1 kHz, He: <10 kHz, Li: 100 kHz
- Total transition frequency: test QED calculation
- Isotope shift: nuclear property
- FS and HFS: nuclear moment and many-body calculation
Two-electron system

- non-relativistic ($\sim 1/n^2$)
- relativistic correction
- spin-orbit interaction
- nuclear magnetic moment
- QED effect
- nuclear size effect (e- inside the nucleus)
- nuclear polarizibility
- many-body ($e^2/r_{12}$)
- recoil correction ($p_1p_2$)
Previous attempt for helium

Fine structure of helium-4

- Interval 29 GHz and 2.3 GHz
- Use one for constraining fine structure constant $\alpha$; another one for checking theory calculation
Previous attempt for helium

- uncertainty of theory and exp < 1 kHz
- difference = 10 kHz and 20 kHz

\[ 2^3P_{0-1} \]

\[ 2^3P_{1-2} \]

Frequency - 2291000 kHz

Frequency - 29616000 kHz
Lithium

- Lithium, more complicated
- Experimental discrepancy between York and Austria measurement
Discrepancy in lithium

- Test QED calculation, but discrepancy also exist
Energy level of Li-6,7

<table>
<thead>
<tr>
<th>Shift (MHz)</th>
<th>$F$</th>
<th>$^6\text{Li}$</th>
<th>$^7\text{Li}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2.762</td>
<td>$1/2$</td>
<td>$2P_{3/2}$</td>
<td>$2P_{3/2}$</td>
</tr>
<tr>
<td>+1.255</td>
<td>$3/2$</td>
<td>$2P_{3/2}$</td>
<td>$2P_{3/2}$</td>
</tr>
<tr>
<td>-1.757</td>
<td>$5/2$</td>
<td>$2P_{1/2}$</td>
<td>$2P_{1/2}$</td>
</tr>
<tr>
<td>+8.697</td>
<td>$3/2$</td>
<td>$2P_{1/2}$</td>
<td>$2P_{1/2}$</td>
</tr>
<tr>
<td>-17.394</td>
<td>$1/2$</td>
<td>$2S_{1/2}$</td>
<td>$2S_{1/2}$</td>
</tr>
</tbody>
</table>

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<th>$F$</th>
<th>$^6\text{Li}$</th>
<th>$^7\text{Li}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+76.068</td>
<td>$3/2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-152.136</td>
<td>$1/2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$D_1$ and $D_2$ notation for the splitting of the energy levels.
- D2 line not well resolved
- Exhibit quantum interference
- Large polarization dependence
- Isotope shift: different velocity may cause systematic effect
- FS and HFS in one isotope: almost immune to beam alignment
Our approach

Diode Laser 2 @ 671 nm

Measure beat frequency

Diode Laser 1 @ 671 nm

To atomic beam

Lock to I$_2$ transition

<table>
<thead>
<tr>
<th>No.</th>
<th>Transition energy level</th>
<th>Center(MHz)</th>
<th>Width(MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^7$Li $2S_{1/2} F=2 \rightarrow 2P_{1/2} F=2$</td>
<td>6285.173(23)</td>
<td>11.060(75)</td>
</tr>
<tr>
<td>2</td>
<td>$^6$Li $2S_{1/2} F=1/2 \rightarrow 2P_{3/2} F=1/2,3/2$</td>
<td>6345.231(34)</td>
<td>15.103(1.053)</td>
</tr>
<tr>
<td>3</td>
<td>$^7$Li $2S_{1/2} F=2 \rightarrow 2P_{1/2} F=1$</td>
<td>6377.143(18)</td>
<td>10.003(55)</td>
</tr>
<tr>
<td>4</td>
<td>$^7$Li $2S_{1/2} F=1 \rightarrow 2P_{1/2} F=2$</td>
<td>5481.770(15)</td>
<td>11.322(48)</td>
</tr>
<tr>
<td>5</td>
<td>$^7$Li $2S_{1/2} F=1 \rightarrow 2P_{1/2} F=1$</td>
<td>5573.584(45)</td>
<td>9.247(147)</td>
</tr>
</tbody>
</table>
Spectroscopy laser
Reference laser
The transitions and derivative signal of $^{127}\text{I}_2$ at 671 nm:

**Parameters**

- Pump Power = 2.8 mW
- Probe Power = < 1 mW
- Signal slope: 695 mV/MHz
- Signal-to-noise ratio: ~ 670 @ 300 ms
**Iodine measurement**

- Frequency-comb measurement of the iodine transition
- Pressure shift, pump power shift, etc.

Unit: MHz

<table>
<thead>
<tr>
<th>line</th>
<th>Result</th>
<th>IodineSpec</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>446806191.649(23)</td>
<td>446806194.57</td>
<td>2.92</td>
</tr>
<tr>
<td>a10</td>
<td>446806778.709(33)</td>
<td>446806781.53</td>
<td>2.82</td>
</tr>
<tr>
<td>a15</td>
<td>446807072.397(33)</td>
<td>446807075.27</td>
<td>2.87</td>
</tr>
</tbody>
</table>
Typical signal for lithium-7

D1 line: hyperfine structure well resolved

D2 line: only the ground state hyperfine structure resolved
Data analysis

- Difference between two peaks
- Scan up and down
- Quick scan, ≈1 point/sec
- $I_2$ instability $\rightarrow \chi^2 > 1$

- Consistent in different dates
- $I_2$ drift and cavity drift
Laser power dependence

- AC Stark effect from other levels
- Photon momentum $\rightarrow$ cooling or heating
Results

## Error Budgets

<table>
<thead>
<tr>
<th>Sources</th>
<th>Magnitude (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical error</td>
<td>5</td>
</tr>
<tr>
<td>Laser power effect</td>
<td>5</td>
</tr>
<tr>
<td>Beam misalignment</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>B field effect</td>
<td>&lt; 3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>8</strong></td>
</tr>
</tbody>
</table>

Iodine instability included in statistical error

Note: the hyperfine splitting does not simply relate to A and B coefficients as:

\[
\Delta E_{\text{HFS}} = \frac{1}{2} AK + \frac{1}{4} B^2 \frac{3K(K + 1) - 2(I + 1)J(J + 1)}{I(2I - 1)J(2J - 1)}
\]

where \( K = F(F + 1) - J(J + 1) - I(I + 1) \)

\[
\frac{HFS}{2} + 27 \text{ kHz} = A \\
HFS = 2A
\]

2nd order effect from state mixing

Summary

- For H, better theory desired
  also muonic hydrogen problem
- For He, discrepancy in fine structure

Li spectroscopy: (with optical frequency comb)

- measure Li-6,7 isotope shift and HFS in atomic beam, vapor cell and MOT
- Li\(^{+}\) spectroscopy in discharge, ion beam, and trap

People involved

- Lithium D line: 駱瑋駿, 黃耀欽, 郭彥廷
- Iodine spectroscopy: 蕭伃真
- Li\(^{+}\): 高政揚

Former member: 王宥人, 曾安廷, 陳柏安

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