

Ultrashort optical pulse

- Electric field
 - Carrier
 - Envelope function $a(t)$

$$E(t) = \text{Re}\{a(t) \exp^{i\omega_0 t}\}$$

$I(t) = |a(t)|^2$

Δt

$E(t) = \text{Re}\{a(t) \exp^{i\omega_0 t}\}$

- Ultrafast optics
 - Full-width half-maximum pulse duration $\Delta t < 1\text{ps}$

MPA

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Optical pulse shaping

- We can shape the optical waveform however you like

Intensity (a.u.)

Measurement

Calculation

Time (ps)

31 GHz

Frequency comb

(Amplitude + Phase) Control

Line-by-Line Pulse Shaper

25 km SMF

1 ps, 496 GHz pulse train

Delay (ps)

Jiang*, Huang*, Leaird, Weiner, Nat. Photon. 1, 463 (2007)

Chuang and Huang*, Opt. Express 18, 24003 (2010)

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Outline

- Ultrafast 3rd-order nonlinear optics
 - Self-phase modulation
 - Optical solitons
 - Raman scattering
 - Self-steepening
- Supercontinuum generation
- Spectral compression
- Summary

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Nonlinear wave propagation equation

- The inclusion of nonlinear polarization

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \bullet \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial^2 \vec{D}}{\partial t^2}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}_{(1)} + \vec{P}_{NL} = \vec{D}_{(1)} + \vec{P}_{NL}$$

$$\nabla^2 \vec{E} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu_0 \epsilon_{(1)} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}_{NL}}{\partial t^2}$$

- Scalar treatment

$$D \approx \epsilon_0 (n_0^2 + 2n_0 \delta n_{(1)} + 2n_0 \delta n_{NL}) E \approx (\epsilon_{(1)} + 2\epsilon_0 n_0 \delta n_{NL}) E$$

$$\vec{P}_{NL} \approx 2\epsilon_0 n_0 \delta n_{NL} E$$

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Nonlinear propagation equation in waveguide

- Basic formulation

$$\frac{\partial a}{\partial z} + \beta_1 \frac{\partial a}{\partial t} - \frac{j\beta_2}{2} \frac{\partial^2 a}{\partial t^2} + j\gamma |a|^2 a + \frac{\alpha}{2} a = 0$$

$$\gamma = \frac{\omega_0^2 n_0 \mathbf{n}_2}{\beta_0 c^2 A_{\text{eff}}}$$
optical Kerr term
- NonLinear Schrödinger Equation
 - Lossless
$$\frac{\partial a}{\partial z} - \frac{j\beta_2}{2T_0} \frac{\partial^2 a}{\partial \tau^2} + j\gamma |a|^2 a = 0$$
- Generalized NLSE

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} A - \frac{j}{2} \beta_2 \frac{\partial^2 a}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 a}{\partial T^3} = -j\gamma \left[|a|^2 a - T_R a \frac{\partial(|a|^2)}{\partial T} - \frac{j}{\omega_0} \frac{\partial(|a|^2 a)}{\partial T} \right]$$

dispersion SPM Raman

Self-steepening

Self-phase modulation (SPM)

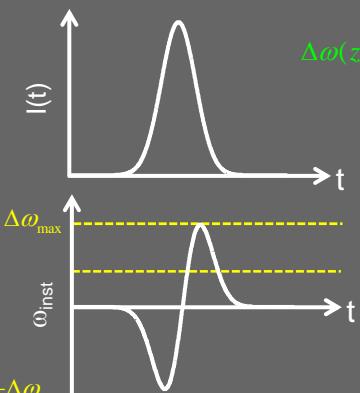
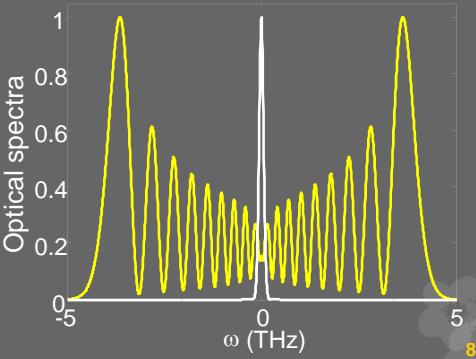
- No dispersion

Gives broadened spectrum

$$\frac{\partial a}{\partial z} = -j\gamma |a|^2 a$$

$$a(z, t) = a(0, t) \exp[-j\gamma |a(0, t)|^2 z]$$

instantaneous frequency:

$$\Delta\omega(z, t) = \frac{\partial(\Delta\varphi)}{\partial t} = -\frac{\partial}{\partial t} (\gamma |a|^2 z)$$



Optical solitons

- For anomalous dispersive materials: $\beta_2 < 0$
- Dimensionless NLSE
- Solution: solitons

$$j \frac{\partial a}{\partial \zeta} = \frac{1}{2} \frac{\partial^2 a}{\partial \tau^2} + |a|^2 a$$

$$a(z, t) = \sqrt{P_c} \operatorname{sech}\left(\frac{t}{T_0}\right) \exp\left[-\frac{j|\beta_2|z}{2T_0^2}\right]$$

- Complete balance between dispersion and SPM
- Pulse energy
- Soliton order

$$U = \frac{2|\beta_2|}{\gamma T_0}$$

$$N^2 = \frac{\gamma T_0 P_{peak}}{|\beta_2|}$$

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Adiabatic soliton temporal compression

- Dispersion control
- Energy conservation

$$U = \frac{2|\beta_2|}{\gamma T_0} \Rightarrow \frac{T_0(0)}{T_0(z)} = \frac{\beta_2(0)}{\beta_2(z)}$$

Gives broadened spectrum

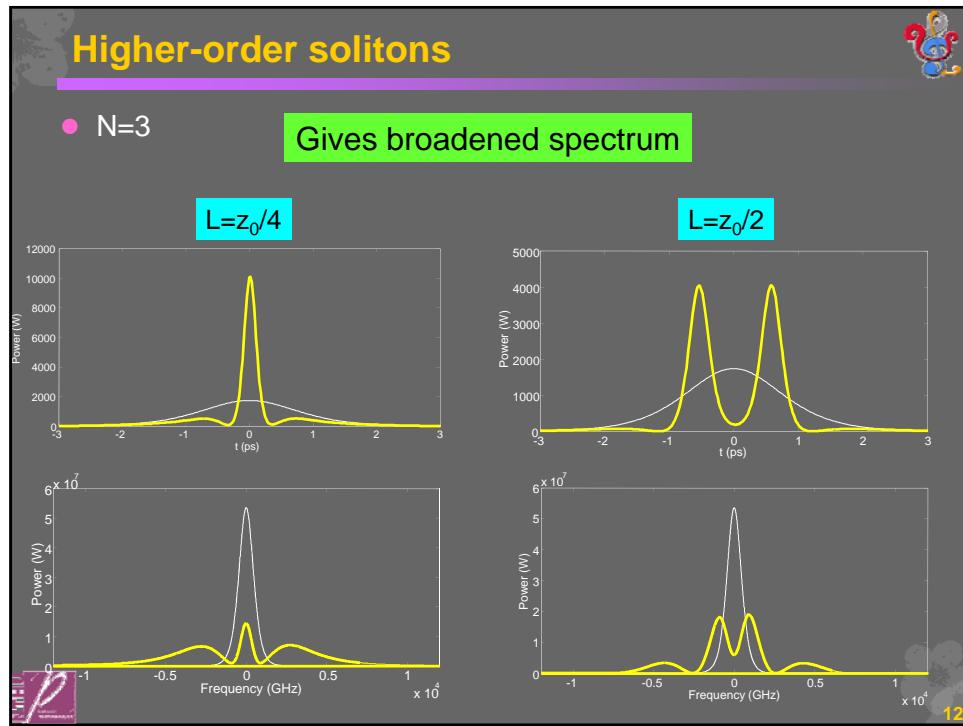
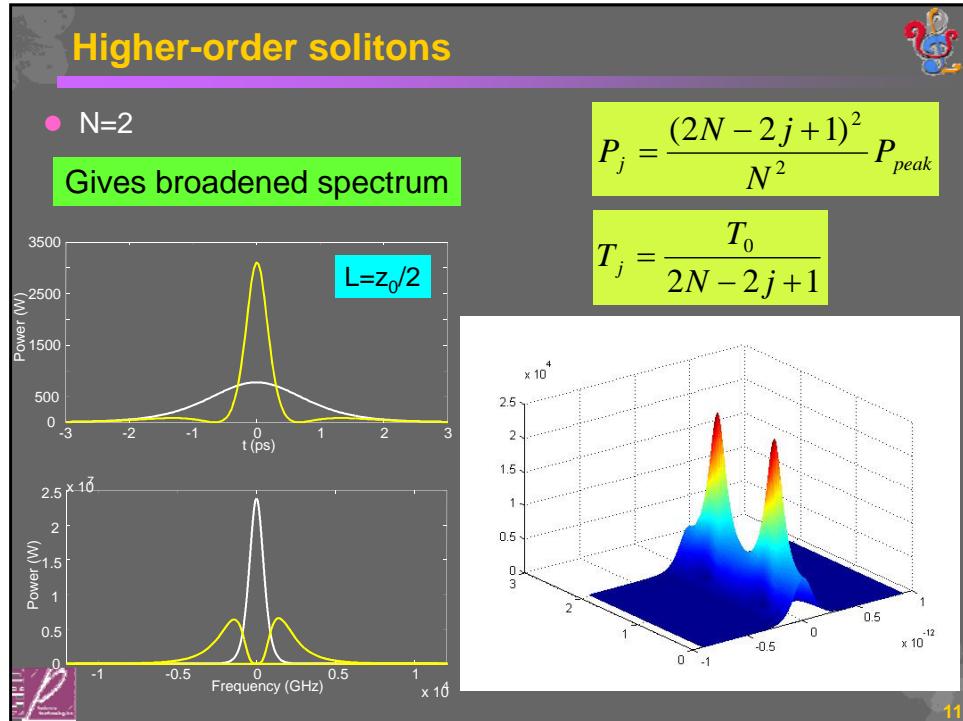
Dispersion-decreasing fiber

Wavelength (nm)

SHG (dB)

298 fs

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Raman response

- Delayed nonlinear phase → delayed instantaneous frequency
- Red-shift of the pulse power spectrum

Gives broadened spectrum

Time-domain view

A. M. Weiner, *Ultrafast Optics* (Wiley, 2009)

Fourier-transform properties

- Raman response real $\Leftrightarrow F(-\omega)=F^*(\omega)$
- A single-sided function
 - $\text{Im}\{F(\omega)\} < 0$ for $\omega > 0$
 - $\text{Im}\{F(\omega)\} > 0$ for $\omega < 0$

Frequency-domain view

$\text{Im}\{F(\tilde{\omega}_s - \tilde{\omega}_p)\}$

Self-steepening

- Gives rise to intensity-dependent group velocity
 - Dispersionless, only electronic response

$$\frac{\partial a}{\partial z} + j\gamma \left(1 - \frac{j}{\omega_0} \frac{\partial}{\partial t'} \right) |a|^2 a = 0$$

Gives broadened spectrum

$$\frac{\partial |a|}{\partial z} + \frac{3\gamma}{\omega_0} |a|^2 \frac{\partial |a|}{\partial t'} = 0$$

$$|a| \propto f(t' - \frac{z}{v}) \quad v = \frac{\omega_0}{3\gamma |a|^2}$$

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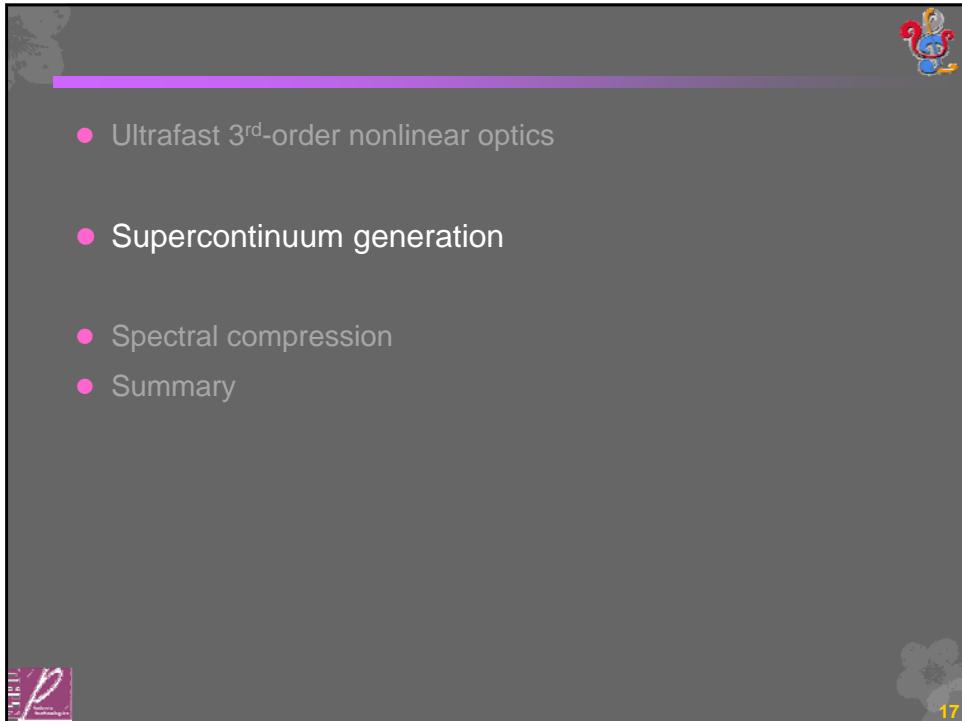
Soliton fission

- N^{th} -order solitons will break into N fundamental solitons
 - Raman: soliton self-frequency shift (SSFS)
 - Self-steepening

Gives broadened spectrum

Dudley et.al, Nat. Phys. 3, 597 (2007)

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• Ultrafast 3rd-order nonlinear optics

• Supercontinuum generation

• Spectral compression

• Summary

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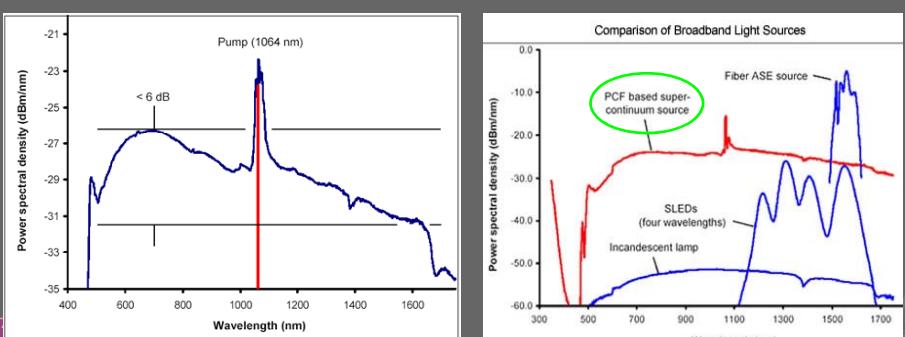
SC: Definition

• What is SC?

- Coherent light source having large bandwidth
- Broadened input spectrum by nonlinear optical processes

• Applications

- Metrology, spectroscopy, sensing, ultrashort pulse.....

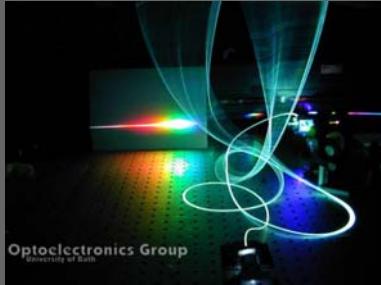


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SC: History

- Historical evolution

Year	Nonlinear medium	Laser type	Pulse width	Pulse intensity/ Peak power	SC -20 dB bandwidth
1970	Borosilicate	Nd:Glass	5 ps	1 GW/cm ²	300 nm
1977	Water	YAlG:Nd	30 ps	45 MW	600 nm
1983	Ethylene glycol	Rh6G	80 fs	3 GW	130 nm
1987	MMF	Nd:YAG	25 ps	1.5 GW/cm ²	60 nm
1995	DSF	Er ³⁺ fiber laser	1 ps	1.2 kW	300 nm
1999	MF	Ti:Sapphire	100 fs	8 kW	1200 nm



Optoelectronics Group
University of Bath

www.bath.ac.uk/physics/groups/ppmg/research_pcf_scg.html

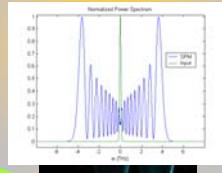
birth of optical frequency comb tightly linked

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SC: Major Nonlinear Processes

- Elastic processes
- Inelastic Process Along with SSFS + SS

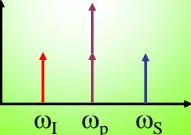
SPM

$$\Delta\omega_{inst} = -\gamma L \frac{\partial |A|^2}{\partial t}$$


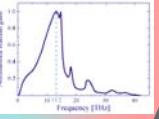
FWM

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

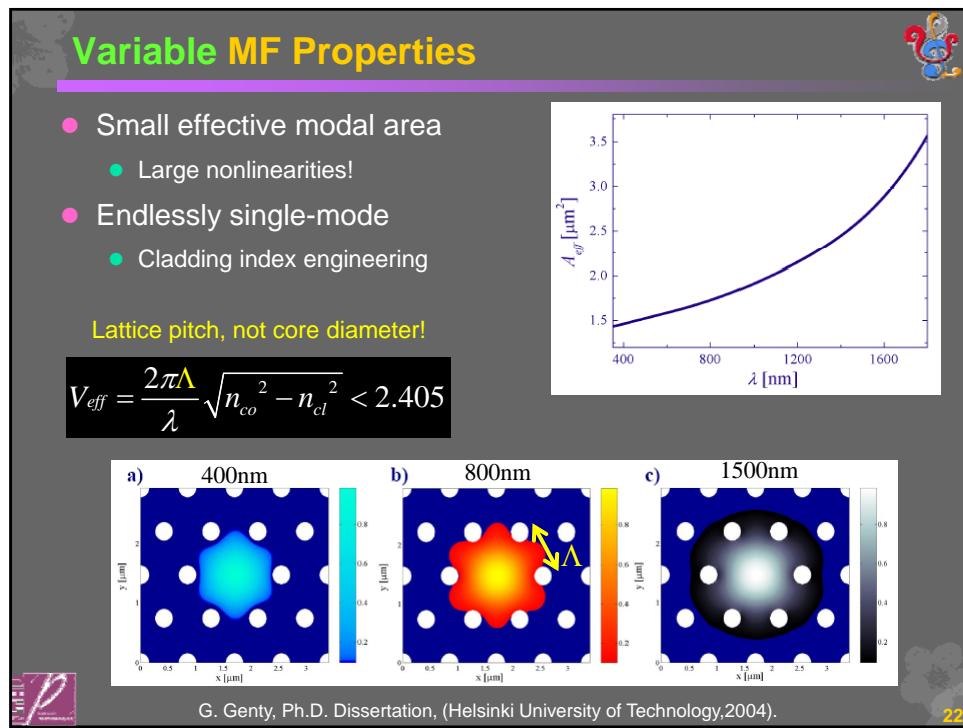
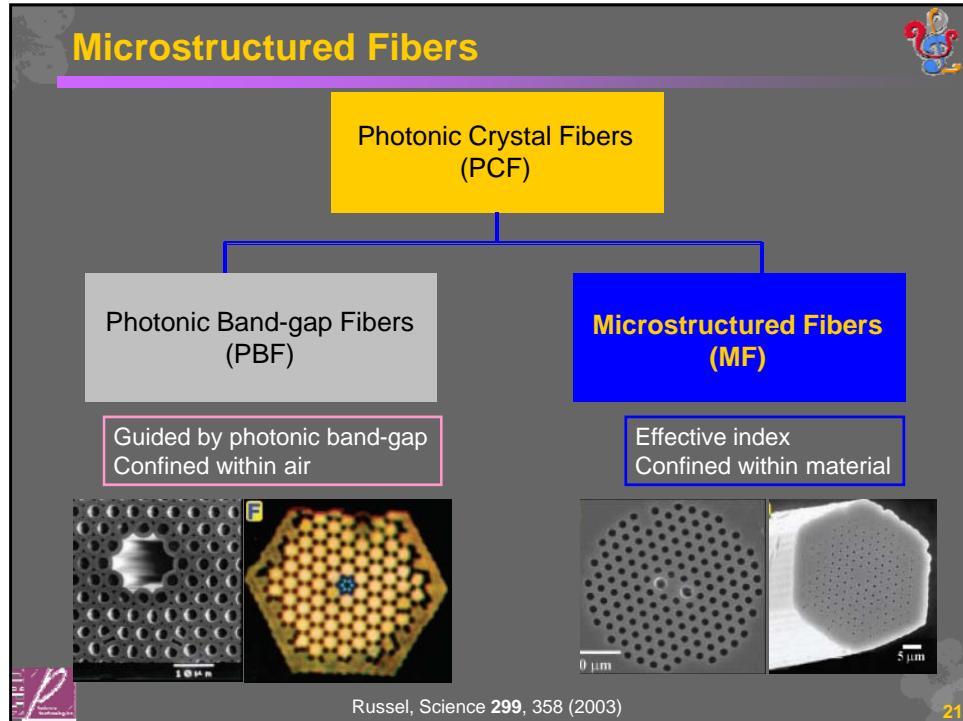
$$2\omega_p = \omega_s + \omega_l$$

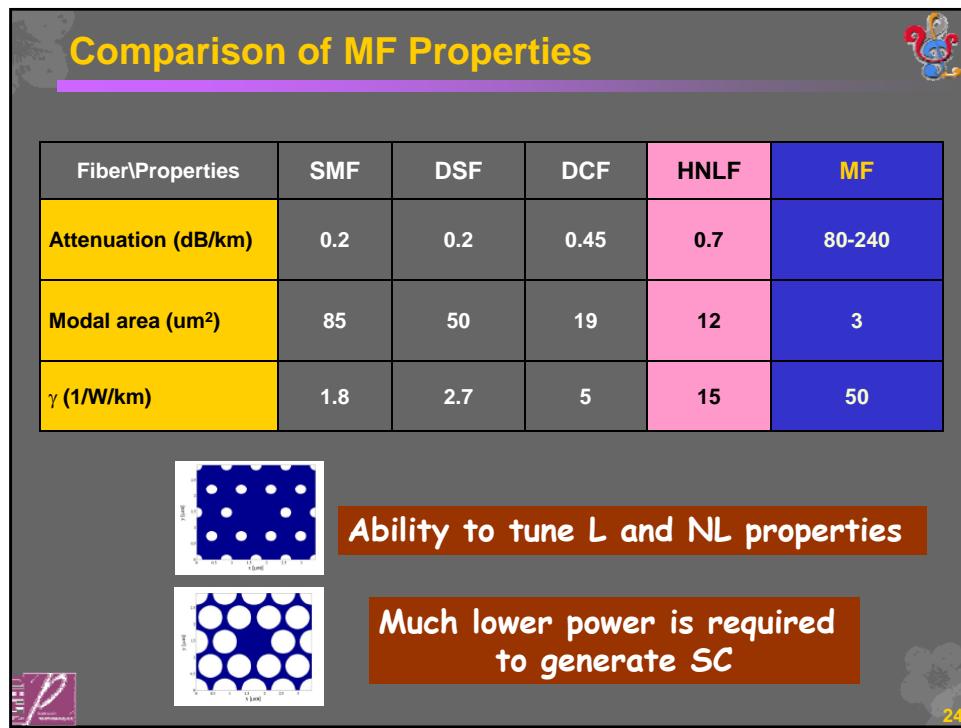
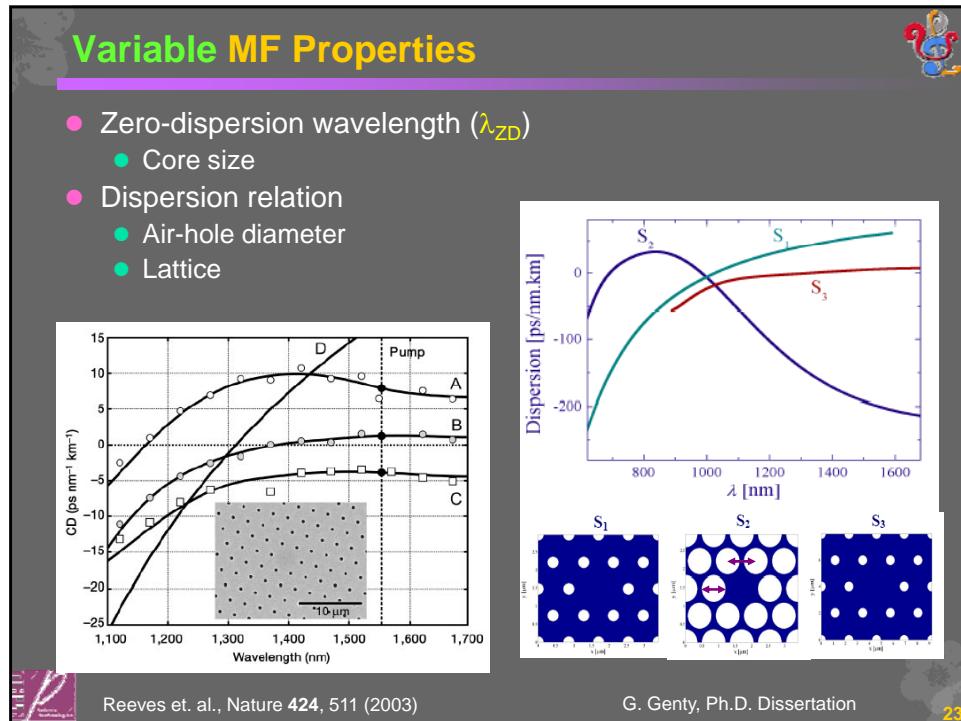
$$\Delta k_M + \Delta k_{WG} + \Delta k_{NL} = 0$$


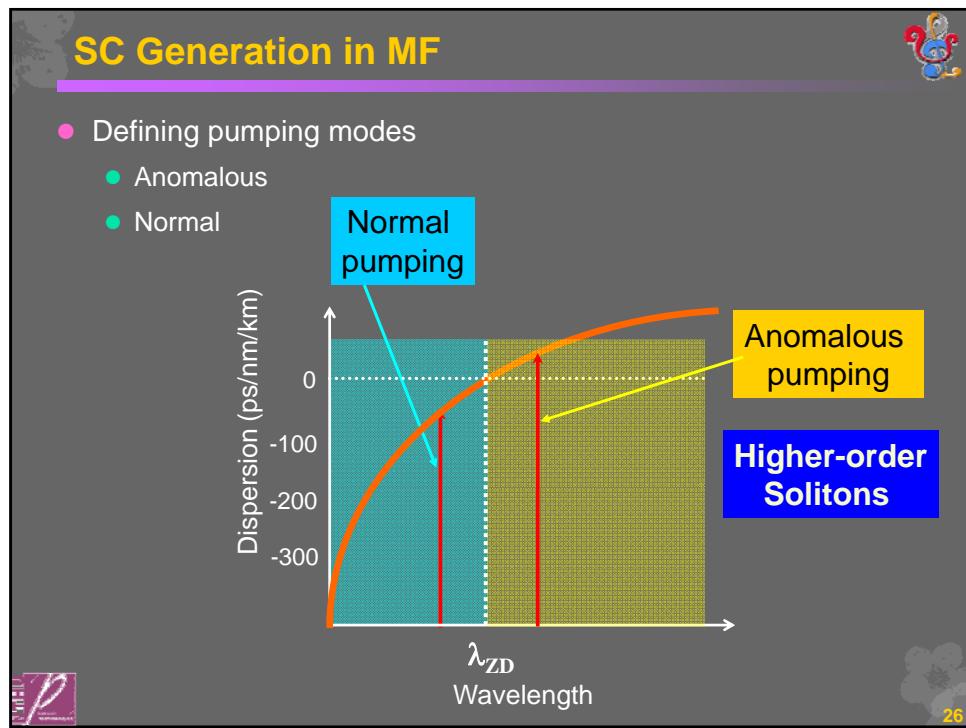
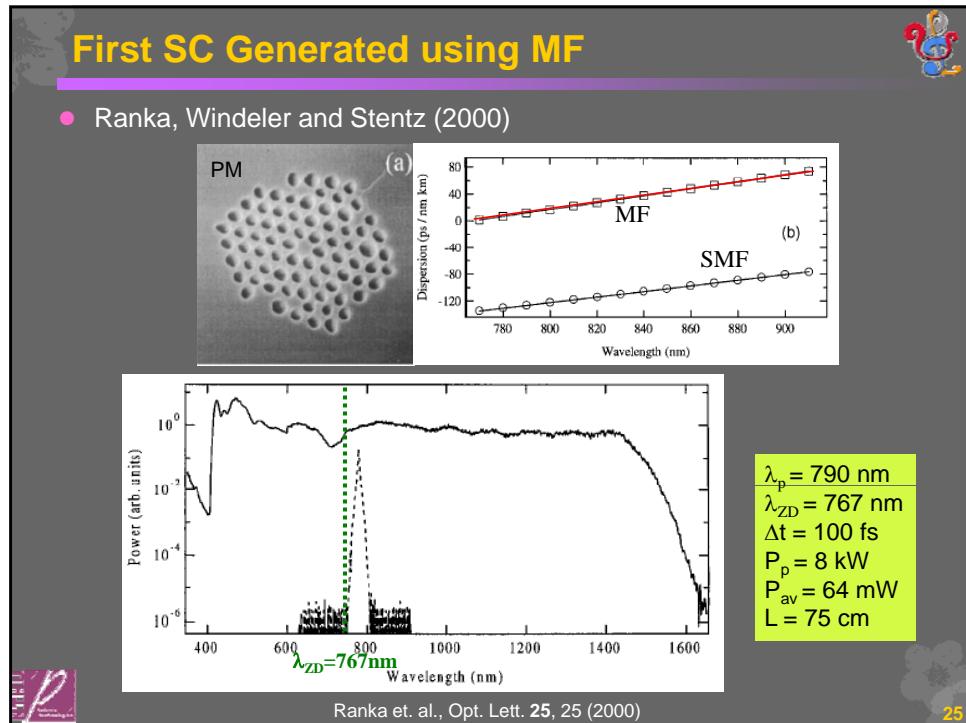
SRS

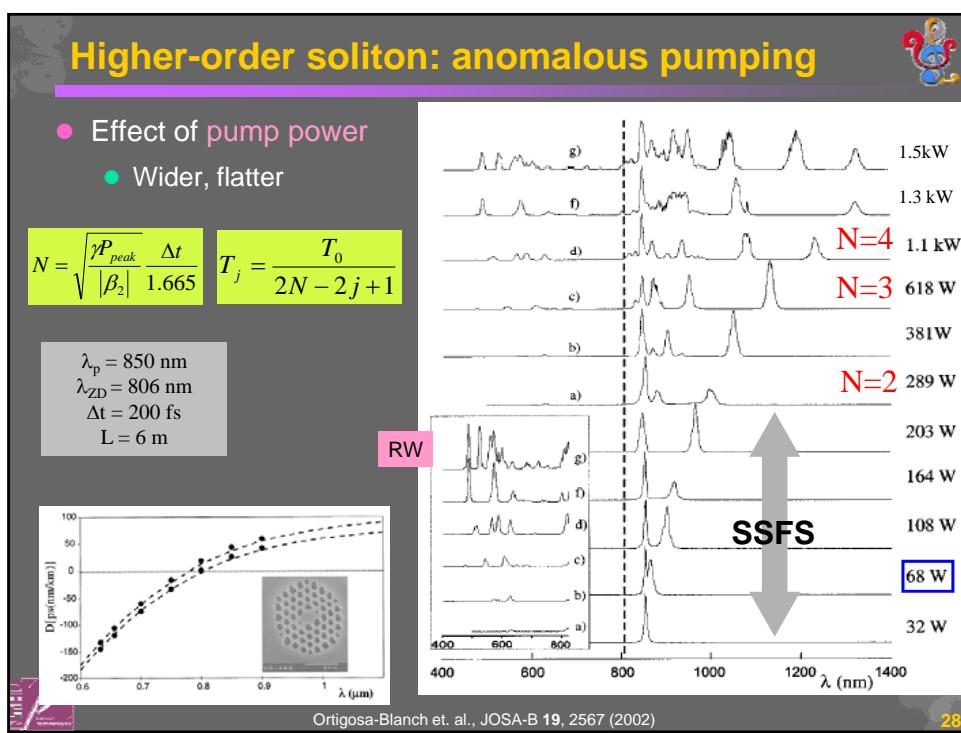
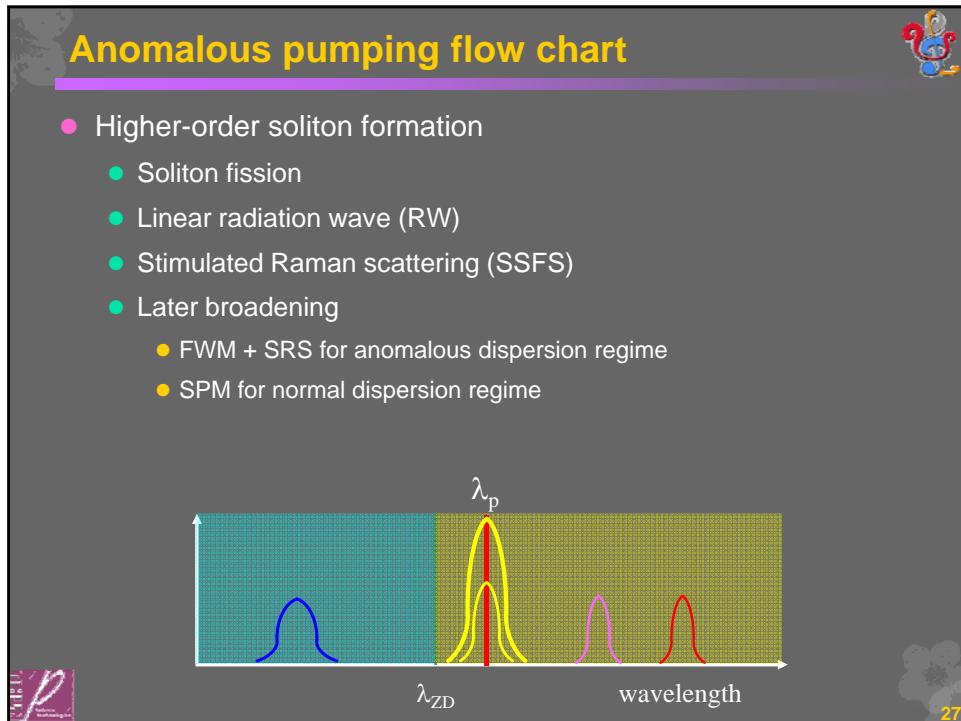
$$2\omega_p = \omega_s + \omega_{AS}$$


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- Ultrafast 3rd-order nonlinear optics
- Supercontinuum generation

- **Spectral compression**
 - Brief history
 - Our approach
 - Numerical and experimental results

- Summary

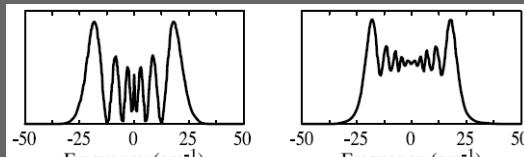


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Nonlinear processes → spectral broadening

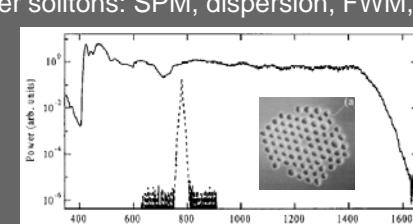


- Self-phase modulation

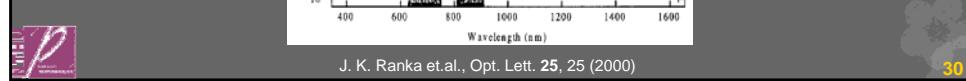


A. M. Weiner, *Ultrafast Optics* (Wiley, 2009)

- Supercontinuum generation
- Higher-order solitons: SPM, dispersion, FWM, SSFS, soliton fission



J. K. Ranka et.al., Opt. Lett. **25**, 25 (2000)



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Applications

- Highly spectrally bright coherent sources
 - Optical metrology, linear/nonlinear spectroscopy
- ps pulse trains
 - Narrow-band, CARS
- Different compared to spectral filtering
 - Energy re-distribution
 - Ideally loss-less

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Spectral narrowing/compression

- Earliest experimental observation
 - R. H. Stolen and C. Lin, Phys. Rev. A **17**, 1448 (1978)
- Physical insight explained
 - S. A. Planas et.al, Opt. Lett. **18**, 699 (1993).
 - M. Oberthaler and R. A. Höpfel, Appl. Phys. Lett. **63**, 1017 (1993).
- Instantaneous frequency
 - Pulse chirp, SPM and dispersion

$$\omega_{inst} = \frac{\partial \phi(t)}{\partial t}$$

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Prior works

- Various fiber types and input chirp conditions
- Compression ratio

Publications	Fiber type (sign of β_2)	Input chirp	Compression ratio
Shen et.al, JLT 17 , 452 (1999)	SMF (-)	+	2
Washburn et.al, OL 25 , 445 (2000)	SMF (+)	-	3.5
Limpert et.al, APB 74 , 191 (2002)	Yb-doped (+)	-	16
Andresen et.al, OL 30 , 2025 (2005)	PCF (~ 0)	-	21
Sidorov-Biryukov et.al., OE 16 , 2502 (2008)	HNPCF (-)	X	7
Fedotov et.al., OL 34 , 662 (2009)	HNPCF (-)	X	6.5
Nishizawa et.al., OE 18 , 11700 (2010)	CPF (-)	X	25.9
Andresen et.al., OL 36 , 707 (2011)	HNPCF (+)	-	5

chirp-free, below soliton P_{peak} solitons

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Soliton effect for spectral compression

- Comb-profile fiber
 - 19 segments of SMF and DSF
 - Large compression ratio of **25.9**
 - Excellent side-lobe suppression

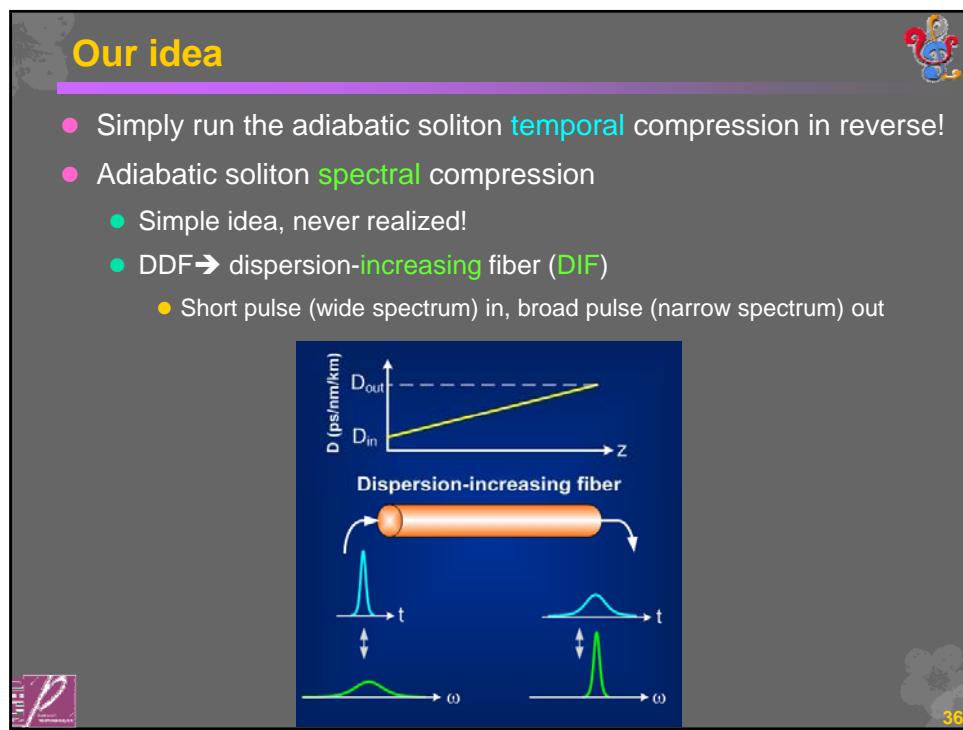
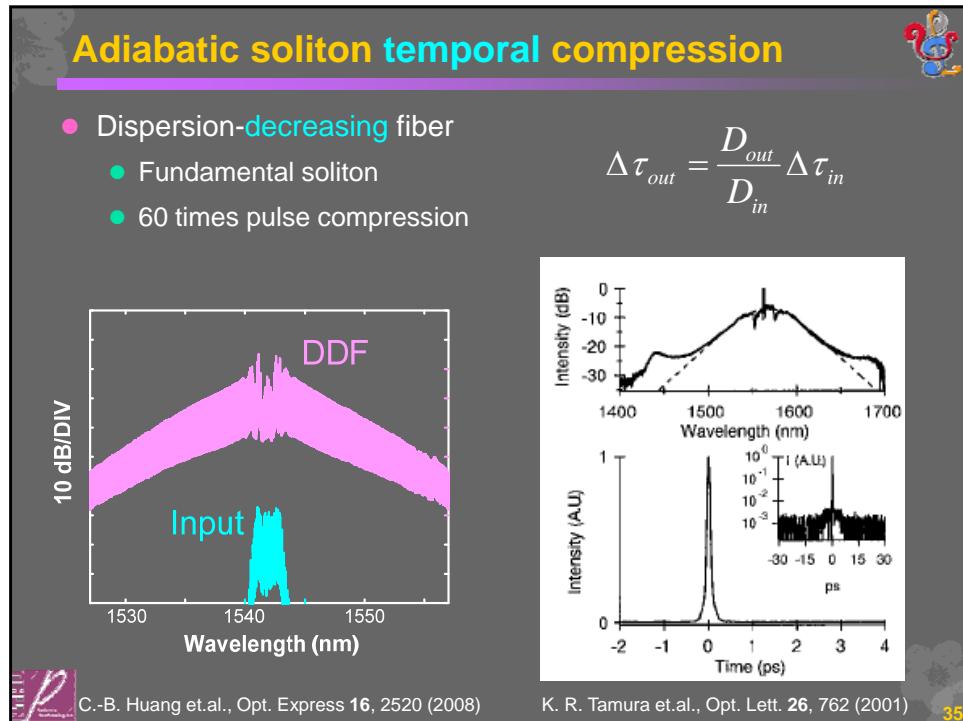
(a) Spectral intensity (a.u.) vs Wavelength (nm). The plot shows a red spectrum with a central peak at ~1620 nm and side-lobes suppressed by -19.2 dB. A dashed blue line indicates the envelope of the spectrum.

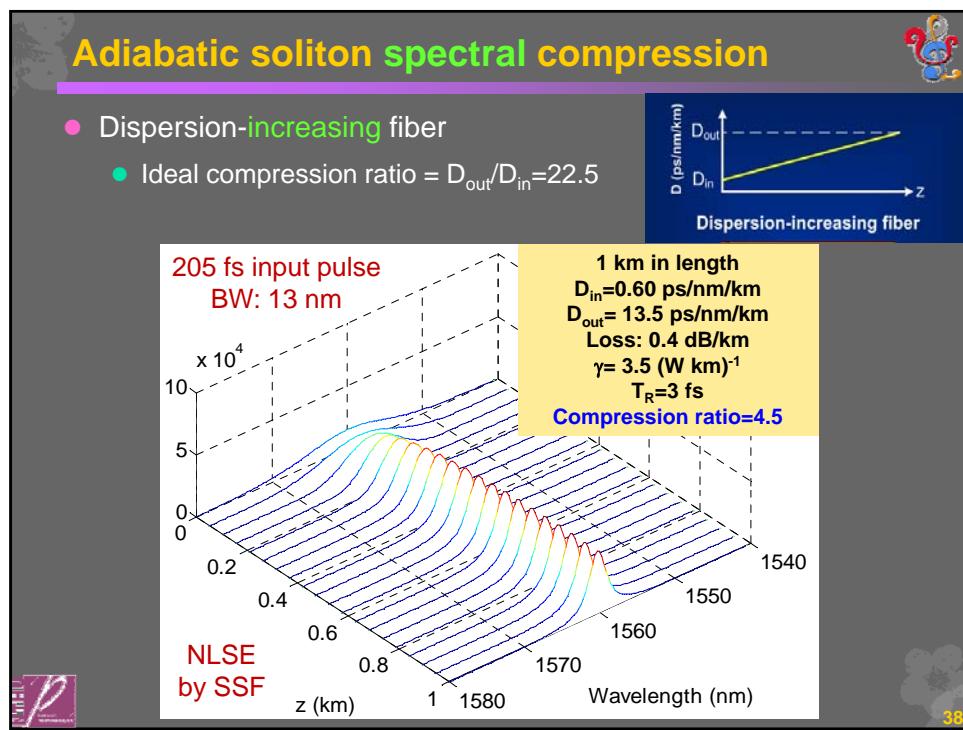
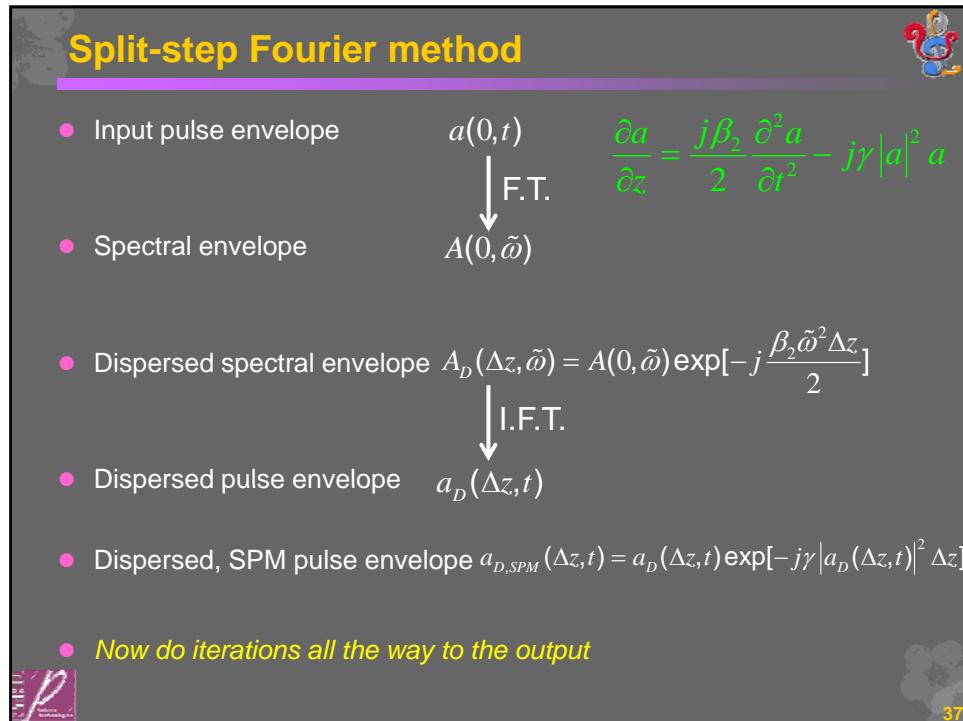
(b) Beta_2 (ps^2/km) vs Propagation length (m). The plot shows vertical bars representing the actual β_2 values and red dots representing the average β_2 values over 19 segments.

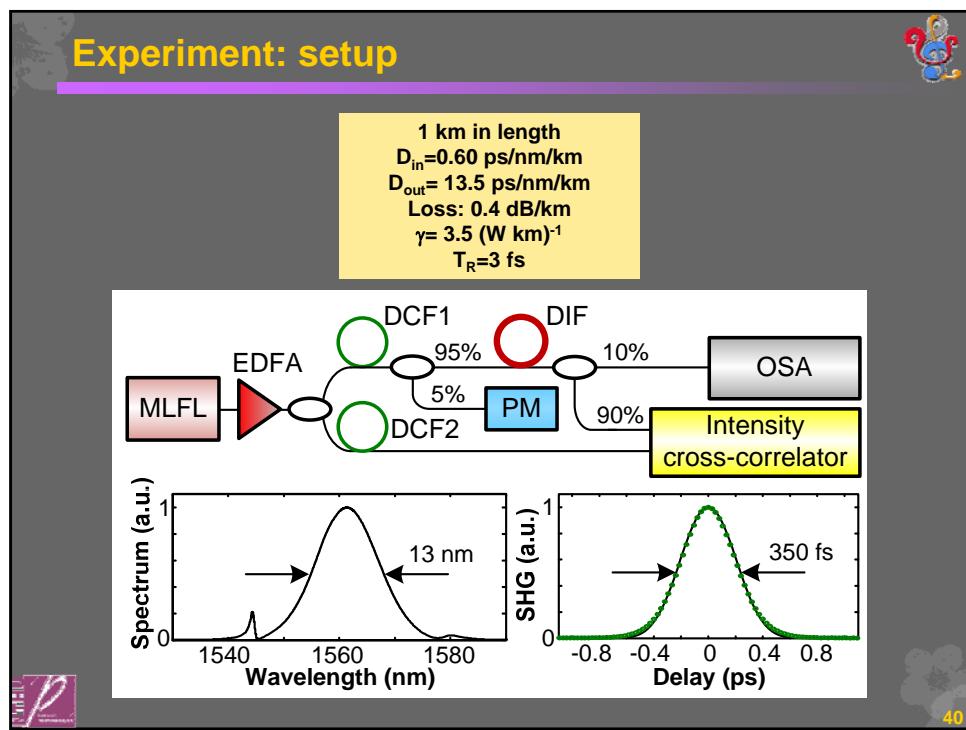
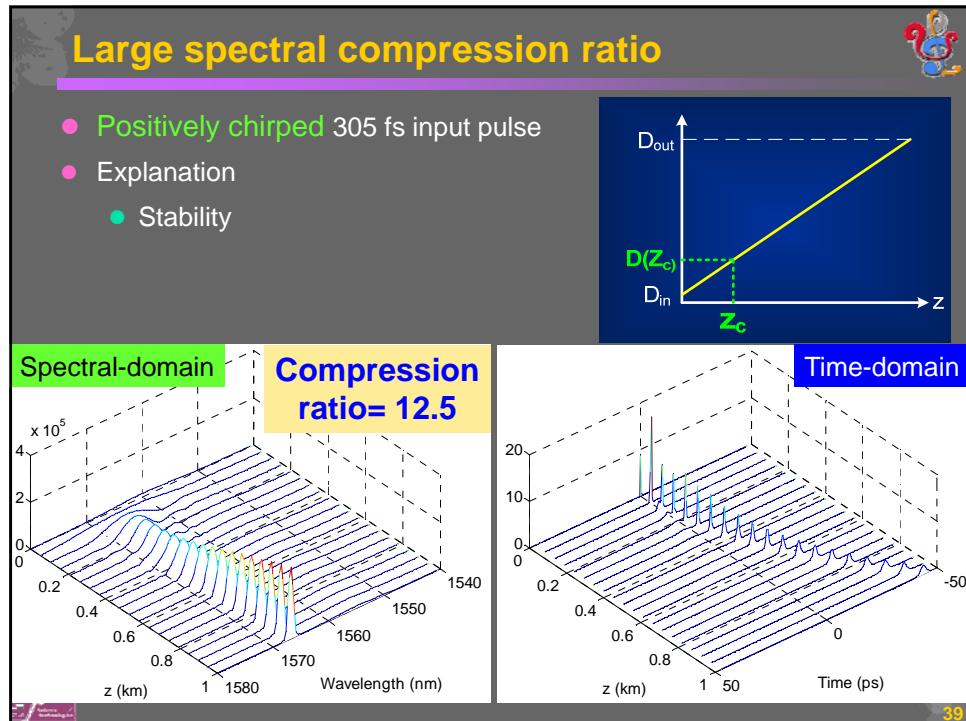
(c) Spectral width (nm) vs Propagation length (m). The plot shows the spectral width decreasing from ~10 nm at 0 m to ~1 nm at 600 m for two wavelengths: 1620 nm (red solid line) and 1770 nm (blue dashed line).

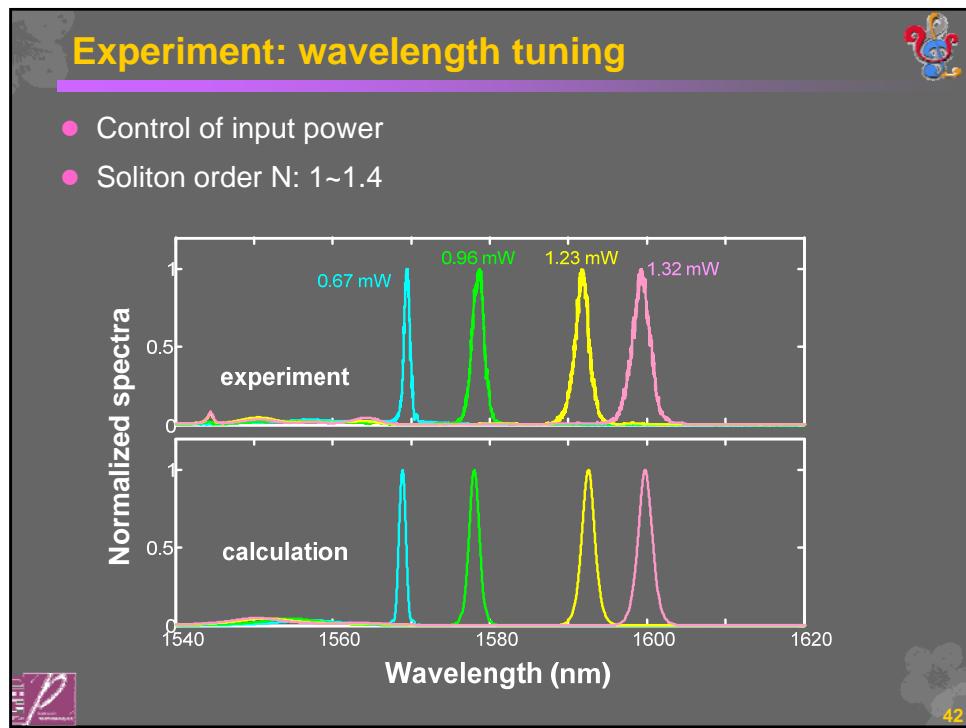
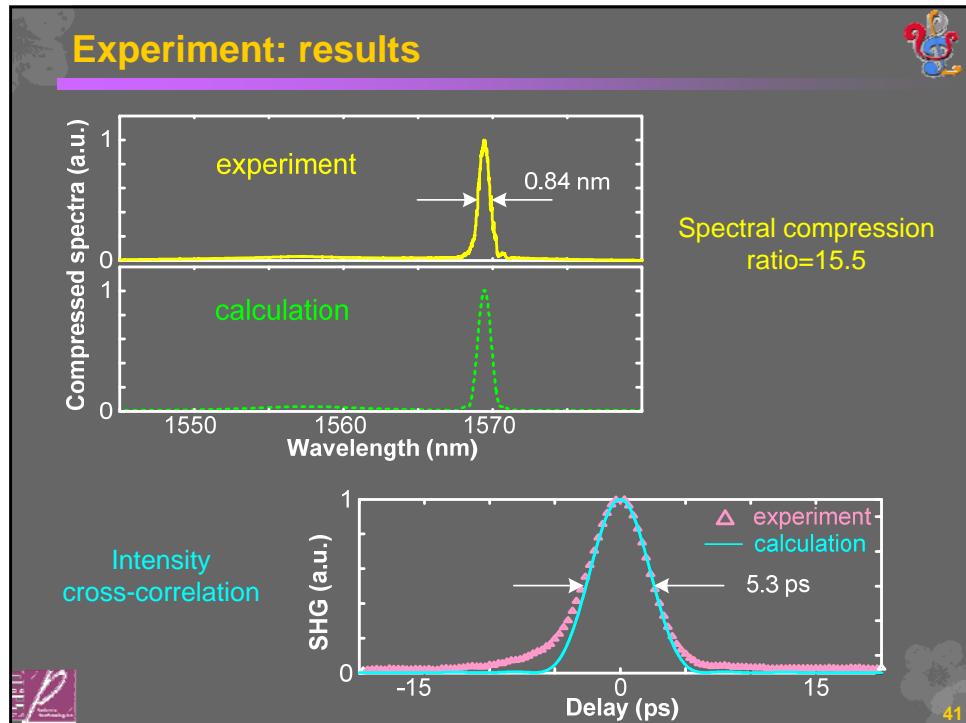
N. Nishizawa et.al., Opt. Express **18**, 11700 (2010)

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Summary

- Adiabatic soliton propagation in a dispersion-increasing fiber is a simple means in achieving spectral compression
- Positively chirped** pulse provides better spectral compression ratio if dispersion ramp is too large
- Experiments for **the first time**: 350 fs chirped pulse in a 1-km DIF
 - Spectral compression ratio of 15.5
 - Temporal broadening
 - 30 nm of wavelength tuning ability

H.-P. Chuang and C.-B. Huang, Opt. Lett. **36**, 2848 (2011)

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