Quantum Optics with Propagating Microwaves in Superconducting Circuits

Io-Chun Hoi
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Outline

Motivation: Quantum network

Introduction to superconducting circuits

Quantum nodes

• The single-photon router
• The cross-Kerr phase shift
• The photon-number filter
• The quantum spectrum analyzer
Quantum Network

Quantum node:
Generating, processing, routing, storing, reading out quantum information.

Quantum channel:
Distributing quantum information.

Enabling large scale quantum computing and quantum communication.

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Hybrid Quantum Network

Telecom photons to distribute quantum information

Quantum node: superconducting circuits
Microwave-optical interface is needed

Y. Kubo et al. PRL 105, 140502 (2010)

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**Advantages of superconducting circuits**

Atom-light interaction on single photon level

1. Photons and “atom” interaction can be engineered
2. Standard on-chip fabrication technique
3. Tunable transition energy of the “atom”
4. Mechanical stable

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Comparison of the toolboxes

Quantum optics                      Superconducting circuits

Optical photons                     Microwave photons

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Introduction to Superconducting Circuits

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Basic Elements of Superconducting Circuits

Dissipationless!

Josephson Junction:
Non-dissipative
nonlinear inductance

\[ L_J \]

\[ L \]

\[ C \]

Capacitance

Inductance

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Basic Elements of Superconducting Circuits

Dissipationless!

Josephson Junction: Non-dissipative nonlinear inductance

Tunnel barrier between two superconductors

\[ I = I_c \sin \phi \]
\[ \frac{d\phi}{dt} = \frac{2e}{\hbar} V \]

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Artificial Atom Based on Quantized Superconducting Circuits

LC Harmonic oscillator

\[ f \sim 5\text{GHz} \sim 240\text{mK} \]
Artificial Atom Based on Quantized Superconducting Circuits

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

LC Harmonic oscillator

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Artificial Atom Based on Quantized Superconducting Circuits

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JJ is a nonlinear dissipationless inductor.
Nonlinearity makes the circuit anharmonic and addressable.

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Artificial Atom Based on Quantized Superconducting Circuits

LC Harmonic oscillator
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JJ is a nonlinear dissipationless inductor
Nonlinearity makes the circuit anharmonic and addressable.

\[ h f_{10} \gg k_B T \]
\[ T @ mK \]

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Artificial Atom Based on Quantized Superconducting Circuits

LC Harmonic oscillator
\[ f \sim 5GHz \sim 240mK \]

Tunable transition frequency by flux \( \Phi_{ext} \) through the SQUID loop.

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Resonant scattering

Resonant scattering in 3D space

Incoming light

Atom/dipole emits light

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Resonant scattering in 3D space

The extinction signal is due to interference

Spatial mode mismatch


Fig. from
U. Håkanson

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Resonant scattering in 1D waveguide

Fully coherent: no transmission, perfect reflection.

Resonant scattering in 1D waveguide

Fully coherent: no transmission, perfect reflection.

Point like atom/dipole!

$\lambda \sim cm$ Wavelength of EM field
$\delta \sim \mu m$ Size of “atom”

Relaxation dominated by transmission line.


IoChun, Hoi et al. PRL 107, 073601 (2011)
Resonant scattering in 1D waveguide

Fully coherent: no transmission, perfect reflection.

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\[ \lambda \sim cm \quad \text{Wavelength of EM field} \]
\[ d \sim \mu m \quad \text{Size of "atom"} \]

Relaxation dominated by transmission line.

IoChun, Hoi et al. PRL 107, 073601 (2011)
Transmission and reflection

\[ r = \frac{\langle V_R \rangle}{\langle V_{in} \rangle} \]

\[ t = \frac{\langle V_T \rangle}{\langle V_{in} \rangle} \]

Reflection coefficient

\[ r = -\frac{\Gamma_{10}}{2\gamma_{10}} \left[ \frac{1 - i\delta \omega_{p} / \gamma_{10}}{1 + \left( \delta \omega_{p} / \gamma_{10} \right)^2 + \Omega_{p}^2 / \Gamma_{10} \gamma_{10}} \right] \]

Transmission coefficient

In resonance, low power

\[ \left| r \left( \delta \omega_{p} = 0, \Omega_{p} \ll \gamma_{10} \right) \right| = \frac{\Gamma_{10}}{2\gamma_{10}} = \frac{1}{1 + 2\Gamma_{\phi} / \Gamma_{10}} \]

Strong interaction limit:

\[ \Gamma_{10} \gg \Gamma_{\phi} \quad \left| r \left( \delta \omega_{p} = 0, \Omega_{p} \ll \gamma_{10} \right) \right| \approx 1 \quad \text{Fully coherent.} \]

\[ \delta \omega_{p} : \text{Detuning} \]
\[ \Gamma_{10} : \text{Relaxation} \]
\[ \Gamma_{\phi} : \text{Pure dephasing} \]
\[ \gamma_{10} = \Gamma_{10} / 2 + \Gamma_{\phi} \]

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Saturation of transmission

\[ T = |t|^2 \]

Almost full transmission at high power

\[ \langle N_P \rangle = \frac{P_p}{\hbar \omega_p (\Gamma_{10} / 2\pi)} \]

Nonlinear nature of the atom!

<table>
<thead>
<tr>
<th>Sample</th>
<th>( E_J/\hbar )</th>
<th>( E_C/\hbar )</th>
<th>( E_J/E_C )</th>
<th>( \omega_{10}/2\pi )</th>
<th>( \omega_{21}/2\pi )</th>
<th>( \Gamma_{10}/2\pi )</th>
<th>( \Gamma_\phi/2\pi )</th>
<th>Ext.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.7</td>
<td>0.59</td>
<td>21.6</td>
<td>7.1</td>
<td>6.38</td>
<td>0.073</td>
<td>0.018</td>
<td>90%</td>
</tr>
<tr>
<td>2</td>
<td>10.7</td>
<td>0.35</td>
<td>31</td>
<td>5.13</td>
<td>4.74</td>
<td>0.041</td>
<td>0.001</td>
<td>99%</td>
</tr>
</tbody>
</table>
Coherent vs Incoherent scattering

\[ \Omega_p \ll \gamma_{10} \]

\[ \langle V_{in} \rangle^2 = \langle V_R \rangle^2 \approx \langle V_R^2 \rangle \quad |r_{p,1}| \sim 1 \]


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Autler-Townes Splitting

A. A. Abdumalikov, Jr et al. PRL 104, 193601 (2010)

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The Single-Photon Router

By turning on or off the control tone, we can decide which port the input photons go to.

I.-C. Hoi et al. PRL 107, 073601 (2011)

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Measuring both T and R simultaneously

Sample 1

Sample 2

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Photon-Photon interaction via a three-level atom

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Photon-Photon interaction via a three-level atom

\[ |0\rangle \xrightarrow{\omega_p} |1\rangle \xrightarrow{\omega_c} |2\rangle \]

\[ \Delta \varphi_p \]

Parameters

\[ P_P, P_C, \omega_p, \omega_C \]

\[ \omega_C = \omega_{21} \]

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Photon-Photon interaction via a three-level atom

\[ |0\rangle \rightarrow |1\rangle \rightarrow |2\rangle \]

Parameters

\( P_P, P_C, \omega_P, \omega_C \)

\( \omega_C = \omega_{21} \)

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Nonlinear interaction between two microwaves

\[ \Delta \varphi_p \approx 0.3 \]

\( \langle N_C \rangle \approx 0.3 \)

At other flux bias.

Device 1

\[ P_p = \begin{cases} 
-140 \text{ dBm} \\
-127 \text{ dBm} \\
-117 \text{ dBm} 
\end{cases} \]

\[ \langle N_C \rangle = \frac{P_C}{\hbar \omega_c (\Gamma_{21} / 2\pi)} \]

\[ \langle N_P \rangle = \frac{P_p}{\hbar \omega_p (\Gamma_{10} / 2\pi)} \]
The Giant Cross-Kerr Phase Shift

\[ r_{p,2} = 1 + i \frac{2\Gamma_{10}}{\Omega_p} \langle \sigma_- \rangle \]

\[ t_{p,1} = 1 + i \frac{\Gamma_{10}}{\Omega_p} \langle \sigma_- \rangle \]

\[ \Delta \varphi_p \propto \langle N_C \rangle \]

I.-C. Hoi et al. PRL 111, 053601 (2013)

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The Giant Cross-Kerr Phase Shift

\[ r_{p,2} = 1 + i \frac{2\Gamma_{10}}{\Omega_p} \langle \sigma_- \rangle \]

\[ t_{p,1} = 1 + i \frac{\Gamma_{10}}{\Omega_p} \langle \sigma_- \rangle \]

Kerr medium
\[ \omega_c \rightarrow \frac{\delta\omega_p}{\omega_p} \rightarrow |0\rangle \]
\[ \Delta \varphi_p \propto \left\langle N_c \right\rangle \]

I.-C. Hoi et al. PRL 111, 053601 (2013)

\[ \langle N_p \rangle \ll 1 \]

\[ \langle N_c \rangle \]


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What is the photon statistics of the scattered field?
Intensity-Intensity Correlation

Single photon source | Beam splitter | Photon counter

Source
\(I_1(t)\)
\(I_2(t)\)

Hanbury Brown-Twiss
Nature 177, 27 (1956)

Second-order correlation function

\[ g^{(2)}(\tau) = \frac{\langle I_1(t)I_2(t+\tau) \rangle}{\langle I_1(t) \rangle \langle I_2(t+\tau) \rangle} \]

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Photon statistics from second order correlation function

A comparison between different light sources:

**Thermal light**
- Intensity vs. time graph
- $g^{(2)}(\tau)$ graph

**Coherent state**
- Intensity vs. time graph
- $g^{(2)}(\tau)$ graph

**Laser light**
- Intensity vs. time graph
- $g^{(2)}(\tau)$ graph

**Single photon source**
- Intensity vs. time graph
- $g^{(2)}(\tau)$ graph

(distance photon antibunching)

**Nonclassical field!**
Photon number filter

Poisson probability distribution

$$|V_{in}\rangle = a_0 |0\rangle + a_1 |1\rangle + a_2 |2\rangle + \ldots$$


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Photon number filter

Poisson probability distribution

\[ |V_{in}\rangle = a_0 |0\rangle + a_1 |1\rangle + a_2 |2\rangle + \ldots \]

\[ |V_R\rangle = r_0 |0\rangle + r_1 |1\rangle \]

Antibunching!

\[ |V_T\rangle = t_0 |0\rangle + t_1 |1\rangle + t_2 |2\rangle + \ldots \]

Bunching!


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Second-order coherence of microwaves

Hanbury Brown-Twiss measurement of output state
Commercial "beam splitter"
Noise temperature of detection chain is about 7K
Noise of two amplifier is uncorrelated.

\[ g^{(2)}(\tau) = 1 + \frac{\langle \Delta P_1(t) \Delta P_2(t + \tau) \rangle}{\left[ \langle P_1(t) \rangle - \langle P_{1N}(t) \rangle \right] \left[ \langle P_2(t) \rangle - \langle P_{2N}(t) \rangle \right]} \]

Covariance
\[ \Delta P_1 \Delta P_2 \equiv \left[ P_1 - \langle P_1 \rangle \right] \left[ P_2 - \langle P_2 \rangle \right] \]

Gabelli et al. PRL 93, 056801(2004)

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Transmitted field: Superbunching Statistics

\[ g^{(2)}(\tau = 0) = 2.31 \pm 0.09 > 2 \]

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The antibunching behavior reveal quantum nature of light!


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Reflected field: Theory

$g^2(\tau)$

$P = -132 \text{ dBm}$

$T=50 \text{ mK}, BW=55 \text{ MHz}$

- Complete theory
- No leakage
- Partial theory

Partial theory $(T,BW)$
- $0 \text{ mK}, 55 \text{ MHz}$
- $0 \text{ mK}, 1 \text{ GHz}$

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An artificial atom in front of a mirror
An artificial atom in front of a mirror

Reflection coefficient:

$$r_p = \frac{\langle V_R \rangle}{\langle V_{in} \rangle}$$

Single ion:
J. Eschner Nature, 413, 495 (2001)

Mirror shapes the modes of the vacuum that couple to atom.

Changing the normalized distance: $L/\lambda$
Changing the spontaneous emission rate

\[ \Omega_p \ll \gamma \]

Weak drive:

\[ |r_p| \]

Experimental data

\[ \lambda(\Phi) \]

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Changing the spontaneous emission rate

$\Omega_p \ll \gamma$

Experimental data

Weak drive:

$|r_p|$

$\omega_p/2\pi$ [GHz]

$\Phi/\Phi_0$

$\lambda(\Phi)$

$L = \lambda/2$

Atom decoupled from vacuum fluctuations at node.

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Changing the spontaneous emission rate

\[ \Omega_p \ll \gamma \]

Weak drive:

\[ |r_p| \]

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\[ \omega_p/2\pi \text{ [GHz]} \]

\[ \Phi/\Phi_0 \]

\[ \lambda(\Phi) \]

\[ L = \lambda/2 \]

Atom decoupled from vacuum fluctuations at node.

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Spontaneous emission rate as a function of normalized distance

\[ \Gamma_1(\Phi) = 2\Gamma_{1,b} \cos^2[\theta(\Phi)/2] \]

\[ \theta(\Phi) = 2 \times [2\pi L/\lambda(\Phi)] + \pi \]

\( \Gamma_{1,b} \): relaxation rate of bare atom

\( \theta \): phase difference between scattered field from the same atom.
Calibrating atom-field coupling $K$
Probing quantum vacuum fluctuations from spontaneous emission rate

\[ \Gamma_1 = k^2 S \]

\[ S = 2\hbar \omega_a \cos^2 \left[ \frac{\theta(\Phi)}{2} \right] \]

\( k \): coupling constant

Conclusion

Quantum node:
Generating, processing, routing quantum information.

The photon-number filter (Generating)
The cross-Kerr phase shift (Processing: phase gate)
The single-photon router (Routing)
The quantum spectrum analyzer (Probing fluctuation)


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New lab in NTHU:
Postdoc, PhD student, Master student wanted!