Colloquium@NDHU
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Carrier-Mediated Ferromagnetism in Spintronics

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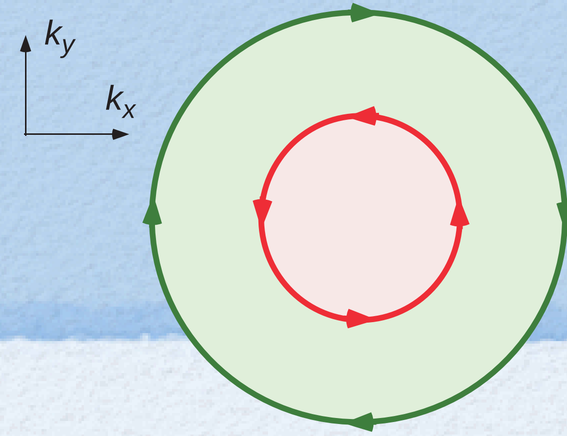
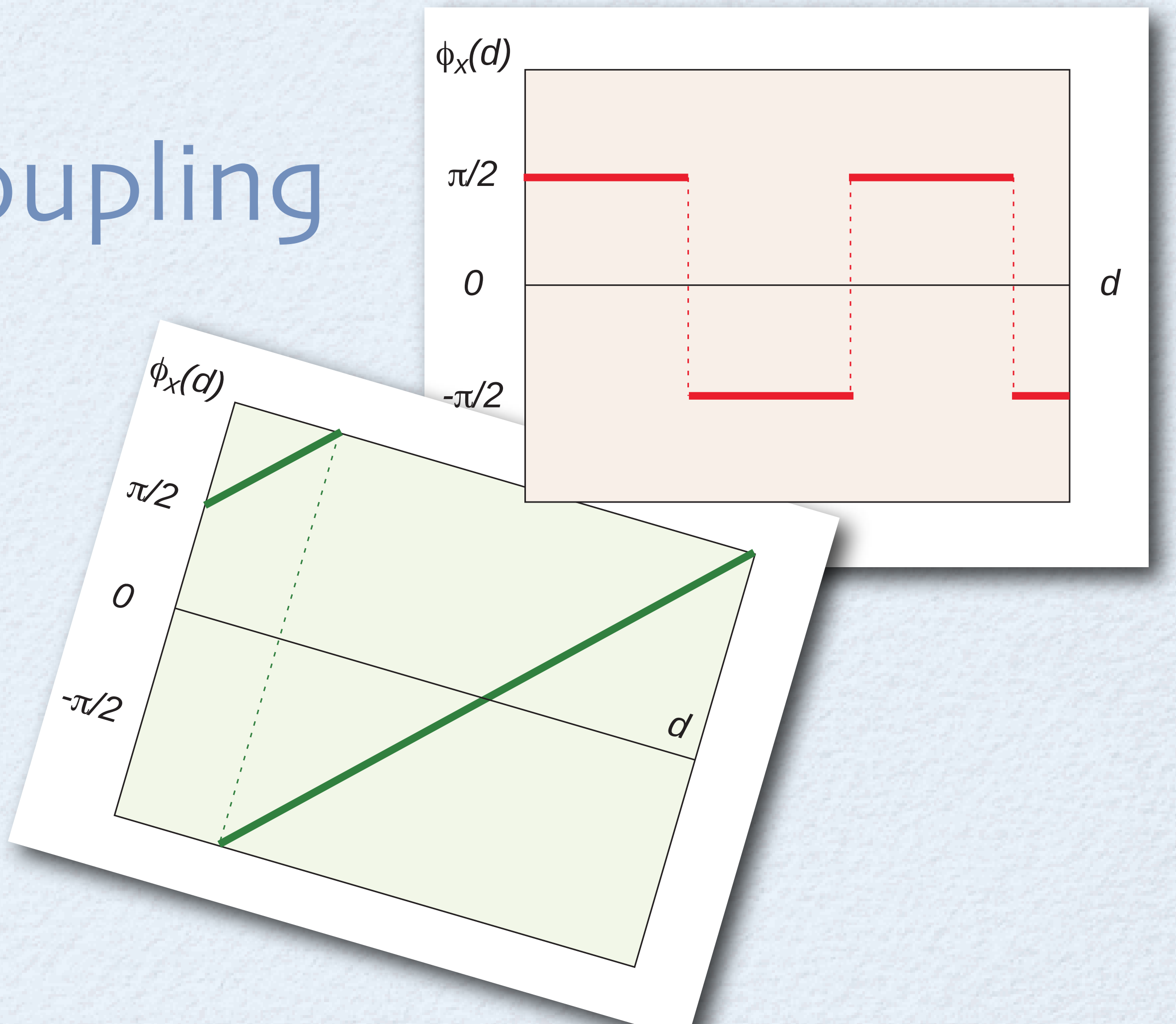


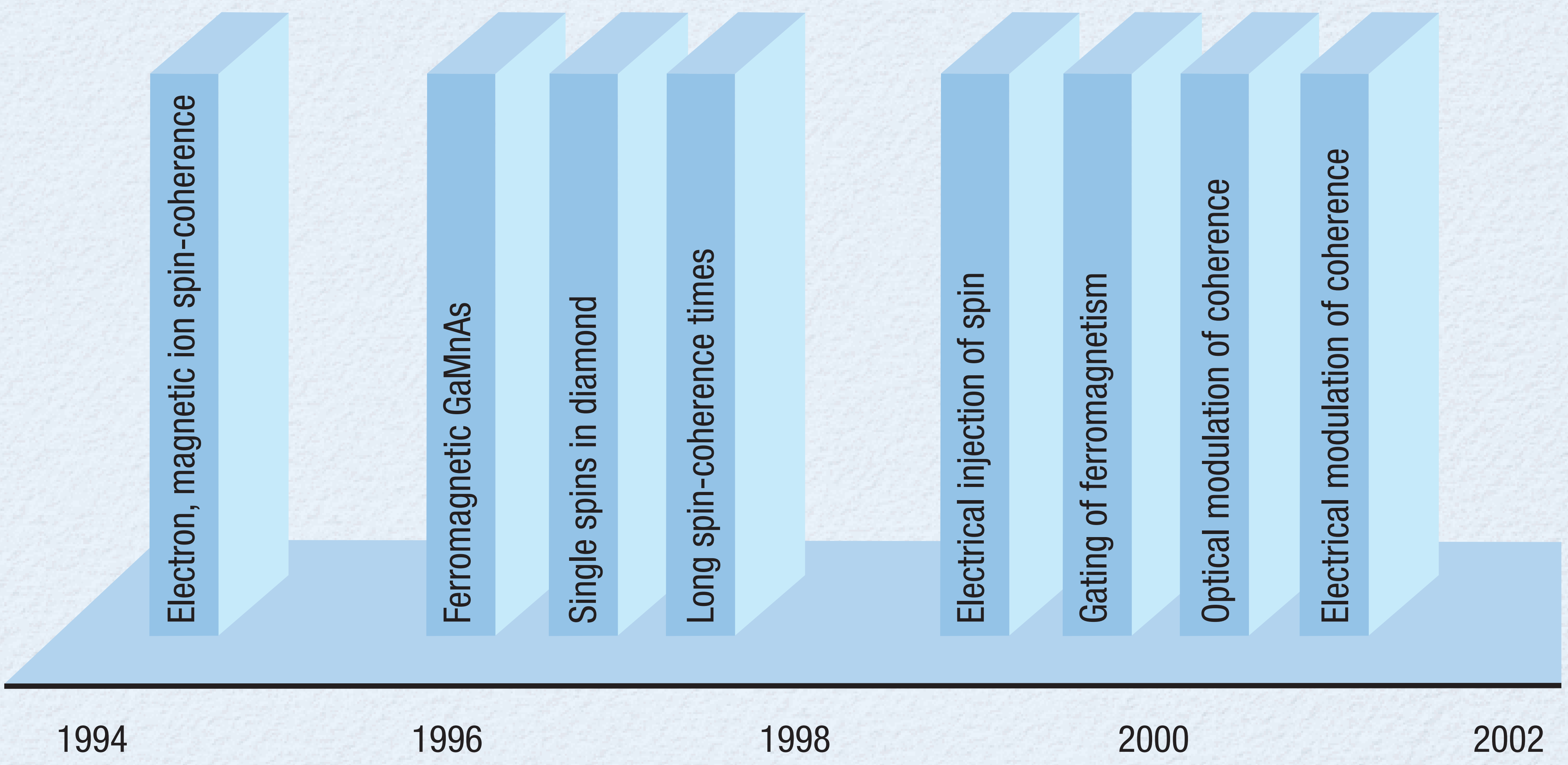
OUTLINE

- Carrier-Mediated Exchange Coupling
- How it works in diluted magnetic semiconductor
- Noncollinear exchange coupling
- Fermi surface topology
- Conclusion



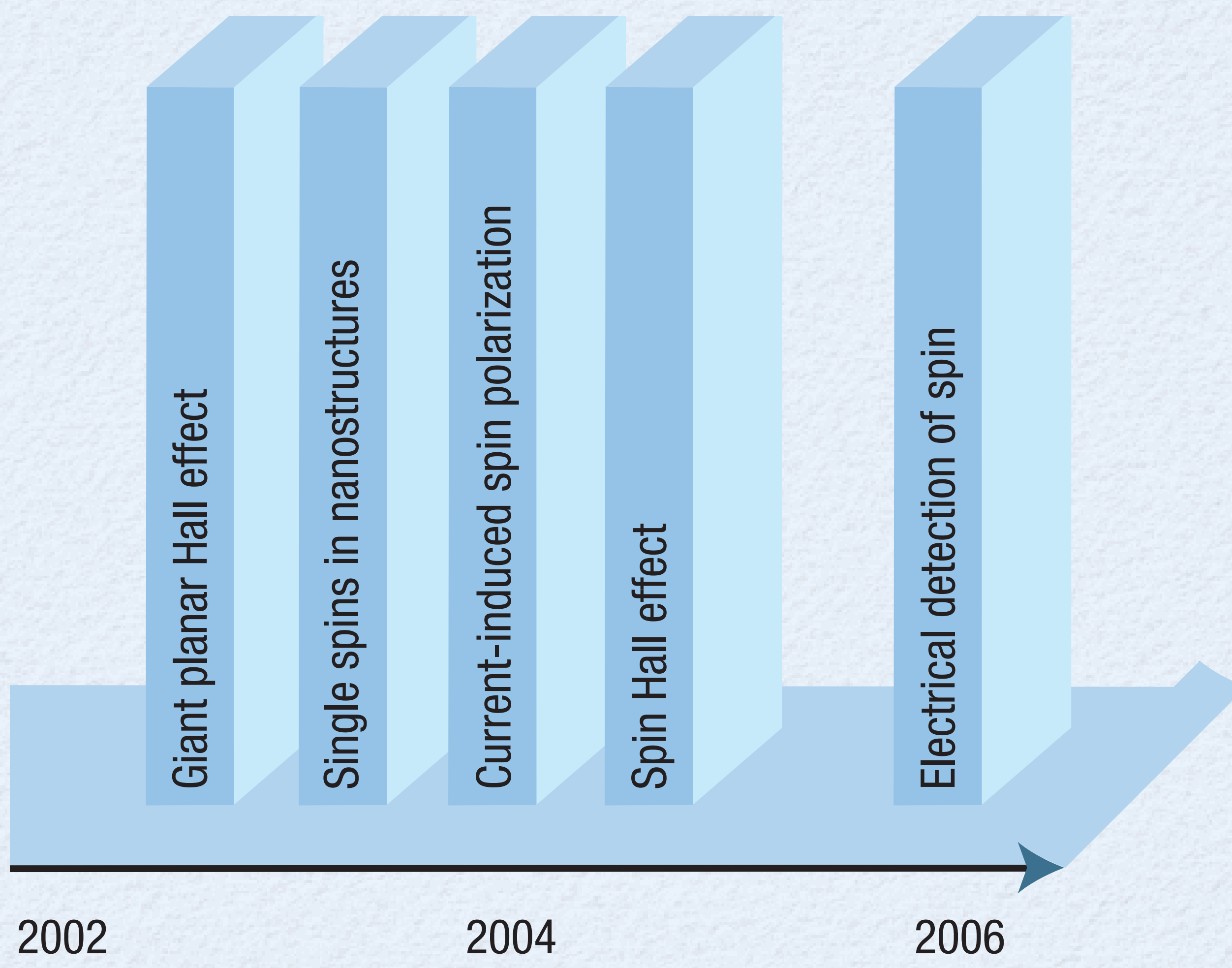
Semicond Spintronics

Timeline of key experimental discoveries in semiconductor spintronics

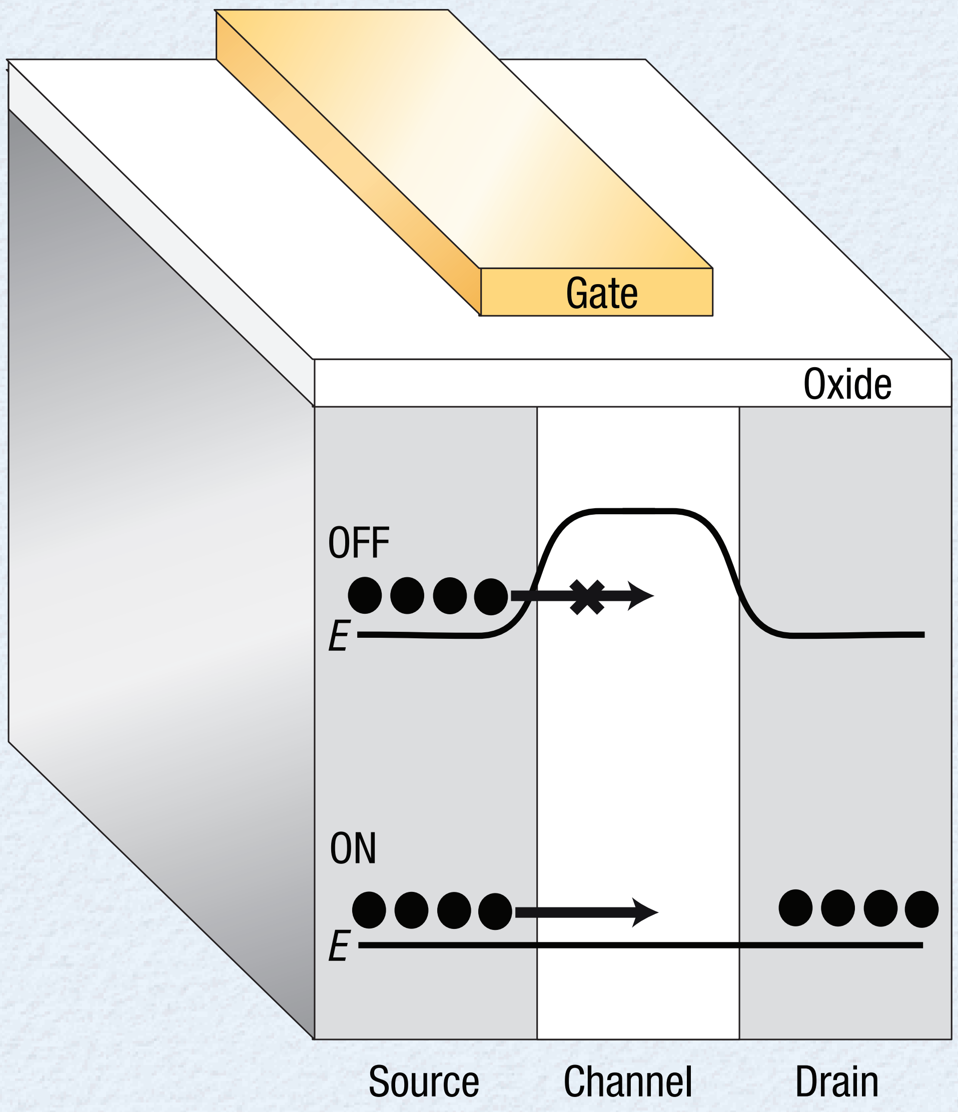


Semicond Spintronics

Timeline of key experimental discoveries in semiconductor spintronics



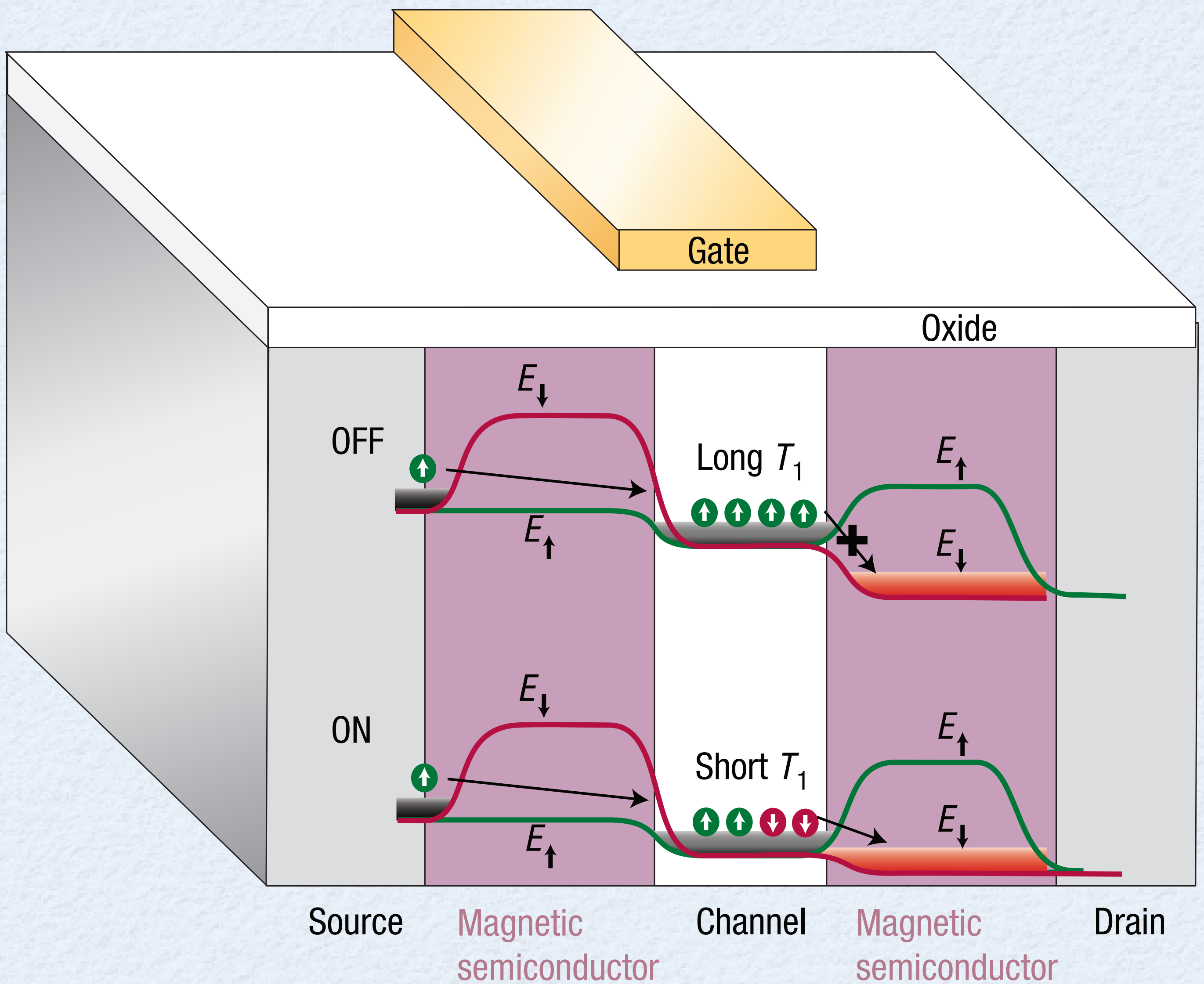
MOSFET V.S. SPIN FET

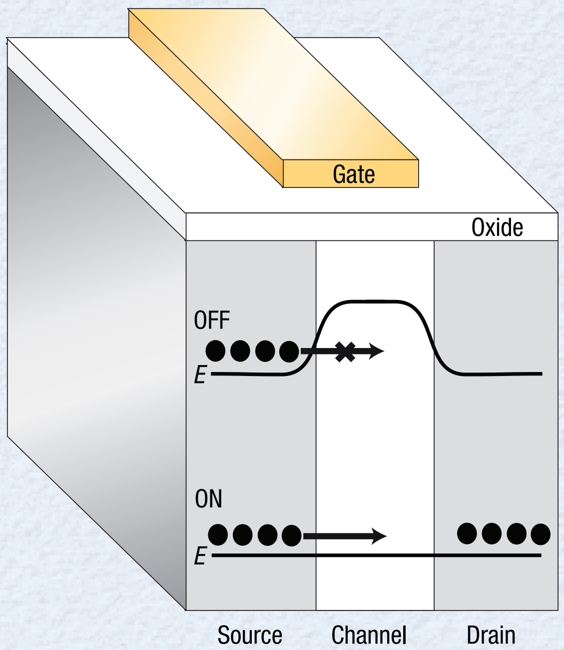


MOSFET (charge)

MOSFET V.S. SPIN FET

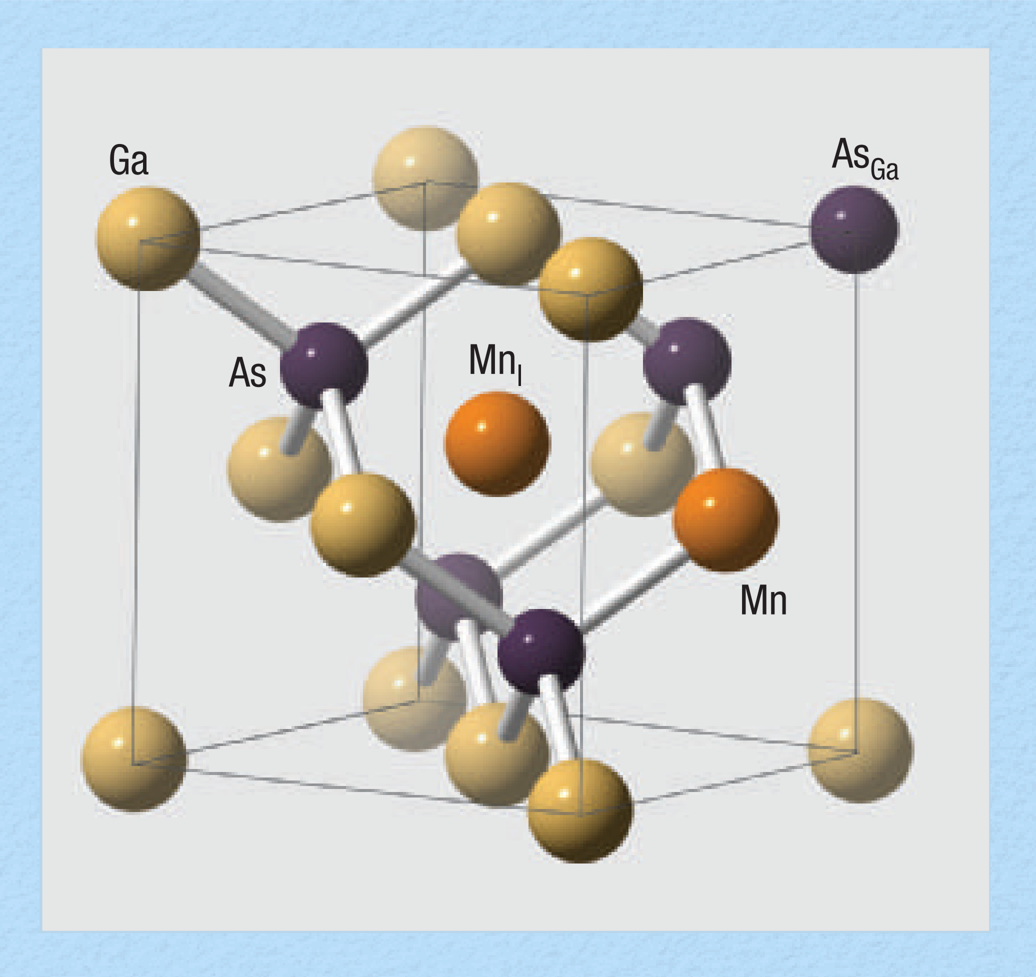
Spin FET (spin)





DOPING III-V...

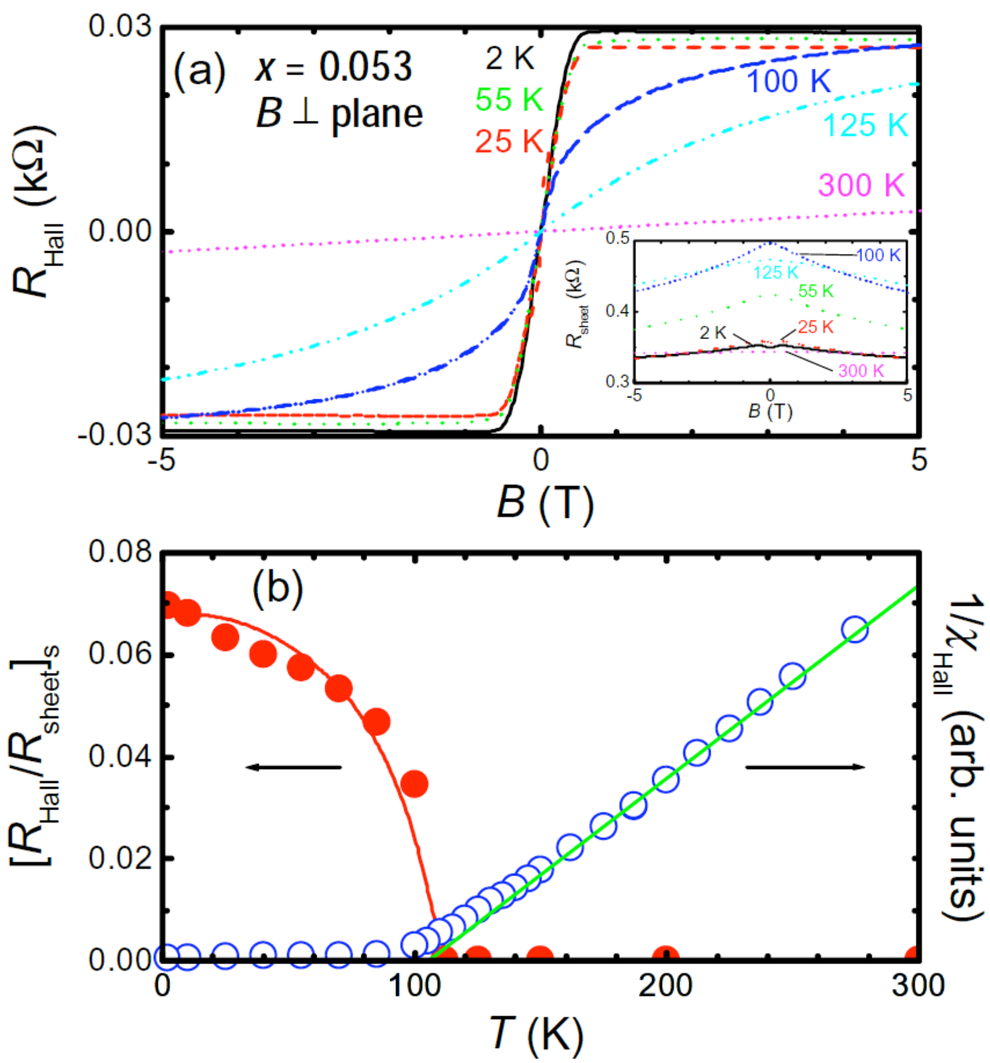
(Ga, Mn) As becomes ferromagnetic below Curie temperature Tc

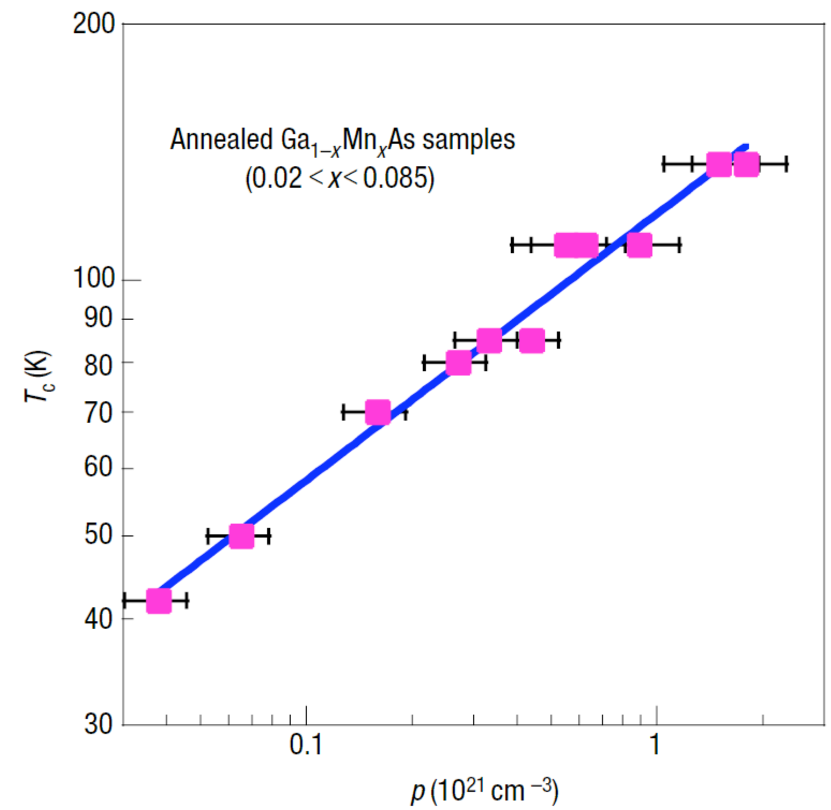


Zincblende structure of GaAs

Ferromagnetic (Ga,Mn) As

Magnetization is measured through anomalous Hall effect.





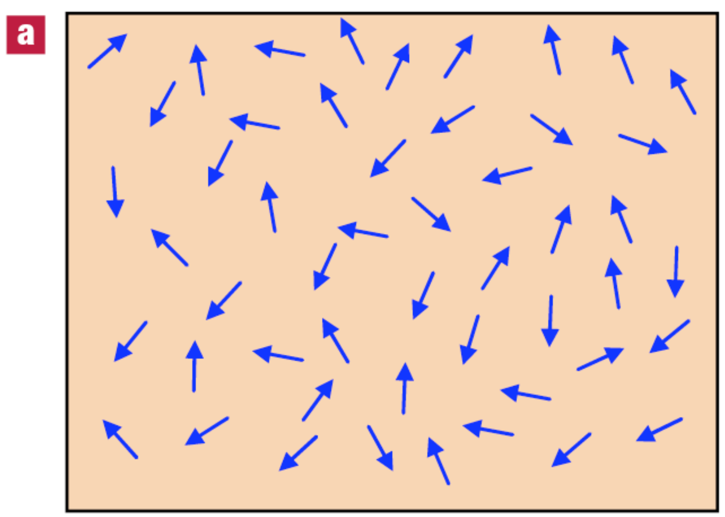
The trend of Curie temperature can be fitted rather well by the mean-field theory prediction,

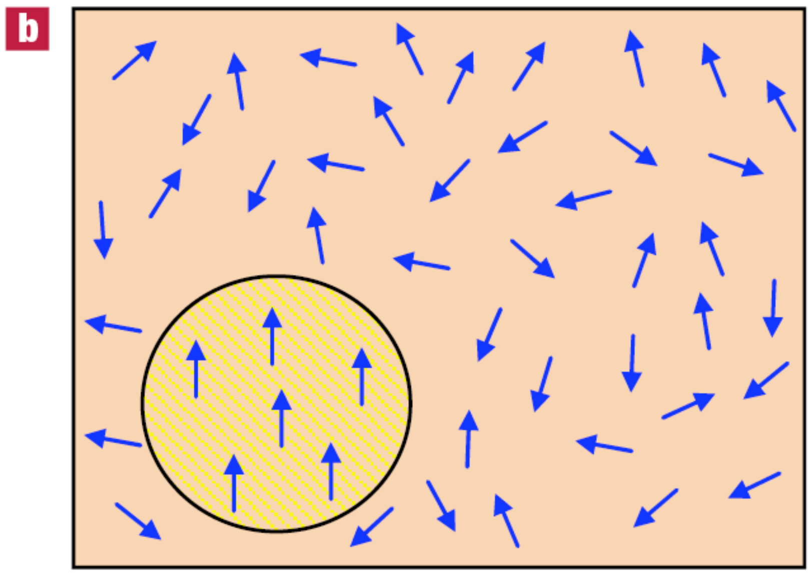
$$T_c \sim n_h^{1/3}$$

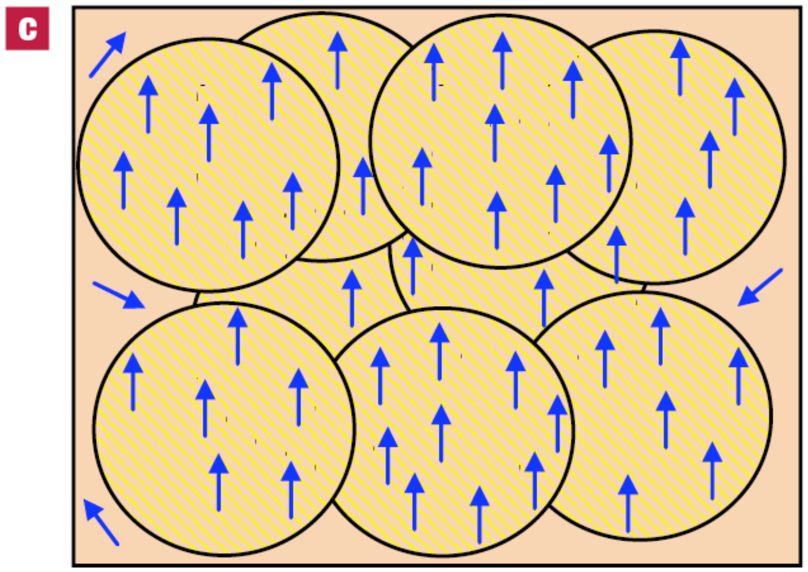
Carrier-Mediated Ferromagnetism

MacDonald et al.
Nature Materials 4, 195 (2005)

(a) At finite temperature, impurity spins prefer random orientations to maximize thermal entropy.



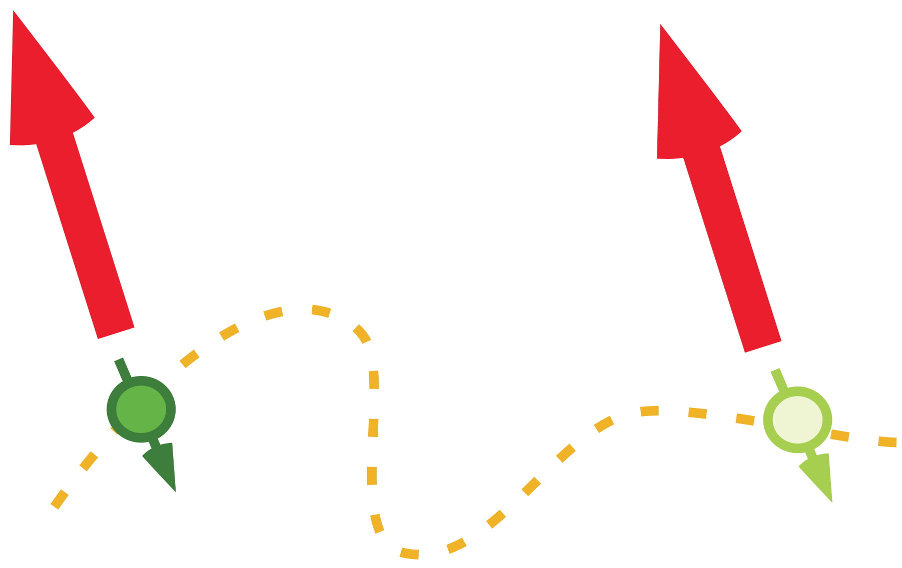




(b) The itinerant carriers like to align the impurity spins so that the kinetic energy is lowered.

(c) Delocalization of itinerant carriers leads to ferromagnetism.

Collinear Model



$$\boldsymbol{S}(r) = \sum_I \delta^3(r - R_I)\boldsymbol{S}_I$$

$$\boldsymbol{s}(r) = \frac{1}{2}\psi^*_\alpha(r)\boldsymbol{\sigma}_{\alpha\beta}\psi_\beta(r)$$

Phys. Rev. Lett. 84, 5628 (2000)
Appl. Phys. Lett. 78, 1550 (2001)

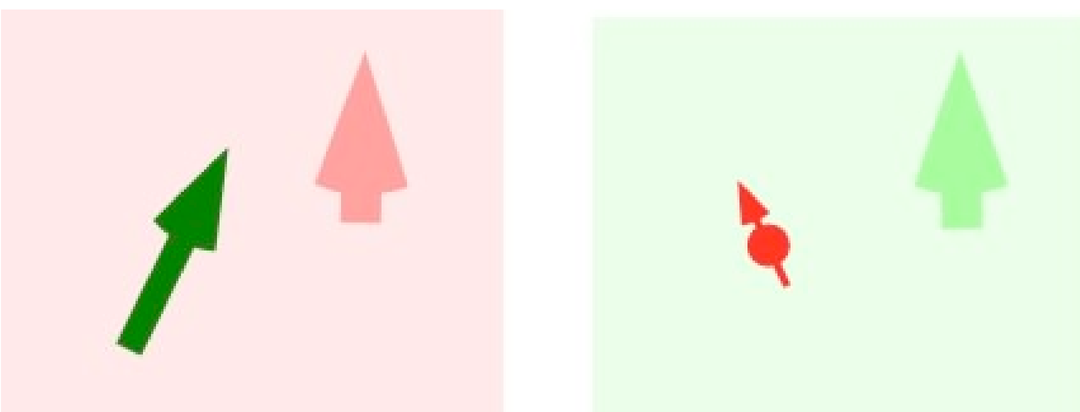
The collinear Zener model includes the kinetic energy of the itinerant carriers and the exchange energy between the localized and itinerant spin densities,

$$\begin{aligned} E &= (\text{kinetic energy}) + (\text{exchange energy}) \\ &= \int d^3r \left[\psi^*(r)\frac{p^2}{2m}\psi(r)\right] + J\int d^3r\, \boldsymbol{S}(r)\cdot\boldsymbol{s}(r), \end{aligned}$$

where J is the exchange coupling between localized impurity and itinerant spin densities.

Mean-Field Prediction

The polarization can be evaluated in mean-field limit by replacing all other spins with an effective magnetic field.



Self-consistent equations at $T=T_c$

$$\langle S_z \rangle = n_I \frac{S(S+1)}{3kT_c} J \langle s_z \rangle$$

$$\langle s_z \rangle = \left[\frac{\chi_P}{(g^* \mu_B)^2} \right] J \langle S_z \rangle$$

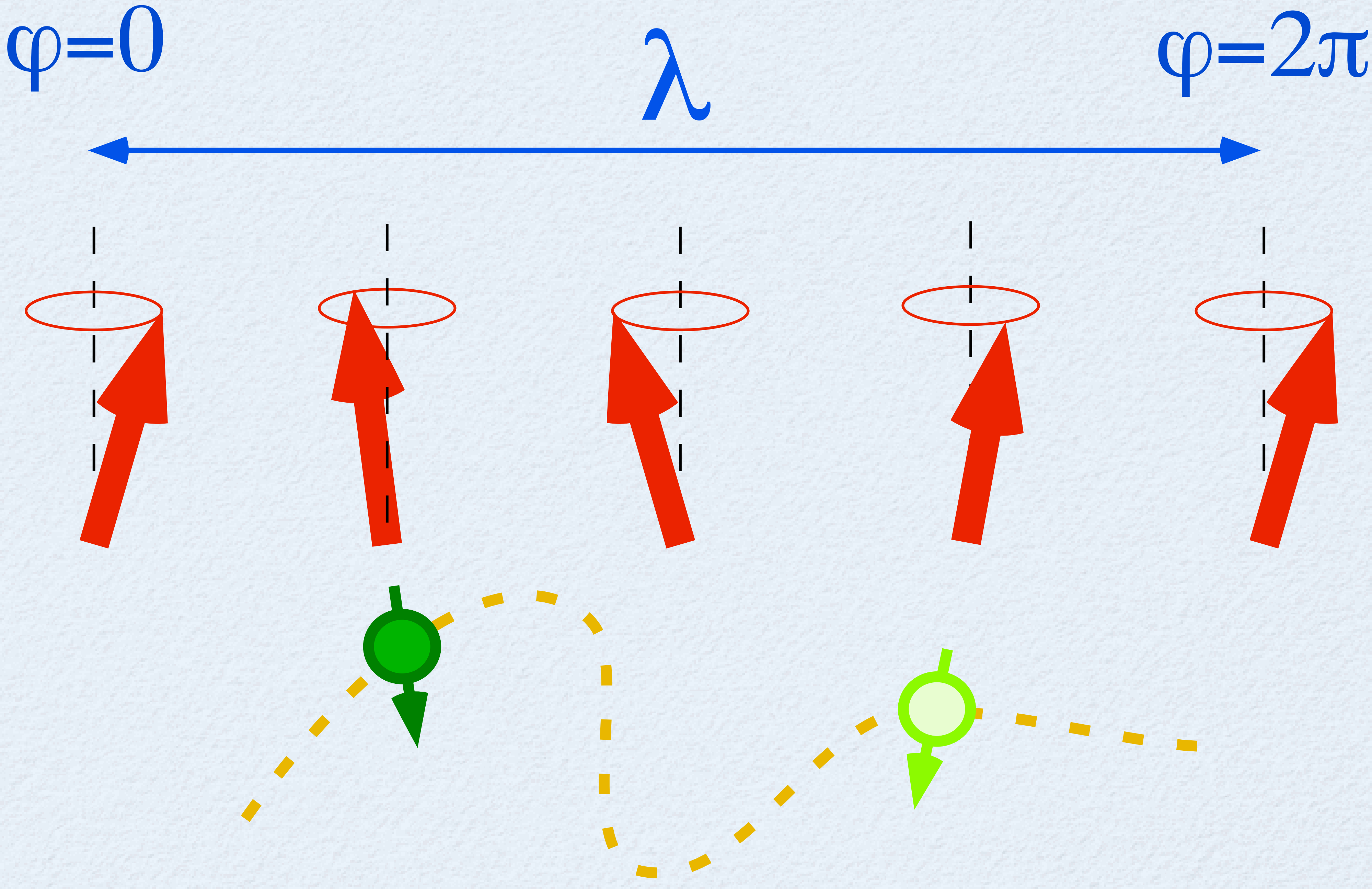
$$\langle S_z \rangle = \chi_C H = n_I \frac{S(S+1)}{3kT} g \mu_B H$$

$$\langle s_z \rangle = \chi_P h = \left[\frac{\chi_P}{(g^* \mu_B)^2} \right] g^* \mu_B h$$

The spin polarizations of Mn ions and itinerant holes under external magnetic field are described by **Curie** and **Pauli** susceptibilities.

$$kT_c = \frac{S(S+1)}{3} J^2 n_I \left[\frac{\chi_P}{(g^* \mu_B)^2} \right]$$

Spin waves



Integrating out itinerant carriers...

Write down the partition function of the Zener model and perform the partial summation over the itinerant spins,

$$Z = \sum_S \sum_s \langle Ss|e^{-\beta H}|Ss\rangle = \sum_S \left(\sum_s e^{-\beta E[S,s]}\right)$$
$$= \sum_S W_{\rm eff}[S] = \sum_S e^{-\beta(-kT \ln W_{\rm eff}[S])}.$$

We end up with the effective Hamiltonian for the localized spin,

$$E_{\rm eff}[S] = \langle S|H_{\rm eff}|S\rangle = -kT \ln W_{\rm eff}[S].$$

Path-integral formalism

Writing down the path integral for the HP bosons and the itinerant carriers,

$$\begin{aligned} Z &= \int D[\bar{z}z] \int D[\overline{\psi}\psi] e^{-S[\overline{\psi}\psi,\bar{z}z]}, \\ &= \int D[\bar{z}z] \int D[\overline{\psi}\psi] e^{-\int_0^\beta d\tau \int d^3r \; \mathcal{L}[\overline{\psi}\psi,\bar{z}z]} \end{aligned}$$

where $\mathcal{L} = \sum_\sigma [\overline{\psi}_\sigma(r,\tau)\partial_\tau\psi_\sigma(r,\tau) + \bar{z}(r,\tau)\partial_\tau z(r,\tau)] + H[\overline{\psi}\psi,\bar{z}z]$ is the Lagarangian density in the imaginary-time formalism.

Since the action is quadratic in $\overline{\psi}\psi$, we can integrate out the itinerant carriers,

$$\begin{aligned} Z &= \int D[\bar{z}z] e^{-S_z[\bar{z}z]} \int D[\overline{\psi}\psi] e^{-S_\psi[\overline{\psi}\psi\bar{z}z]} \\ &= \int D[\bar{z}z] e^{-S_z} \det G^{-1}(\bar{z}z) = \int D[\bar{z}z] e^{-S_{\text{eff}}[\bar{z}z]}, \end{aligned}$$

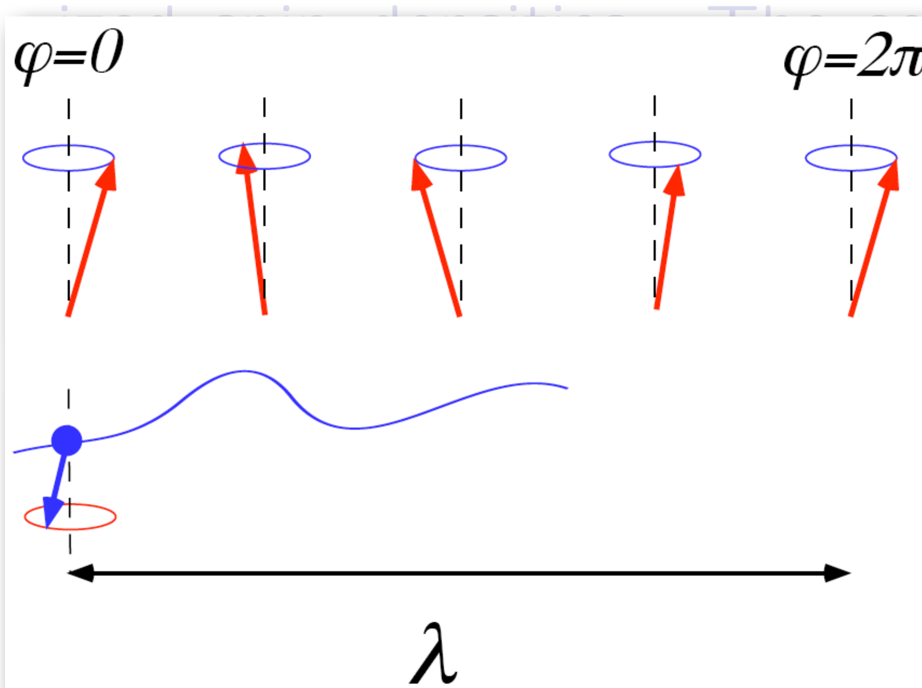
where $S_{\text{eff}}[\bar{z}z] = \left[\int d\tau \int d^3r \;\; \bar{z}(r,\tau)\partial_\tau z(r,\tau)\right] - \ln\det G^{-1}(\bar{z}z)$ is the effective action for the HP bosons.

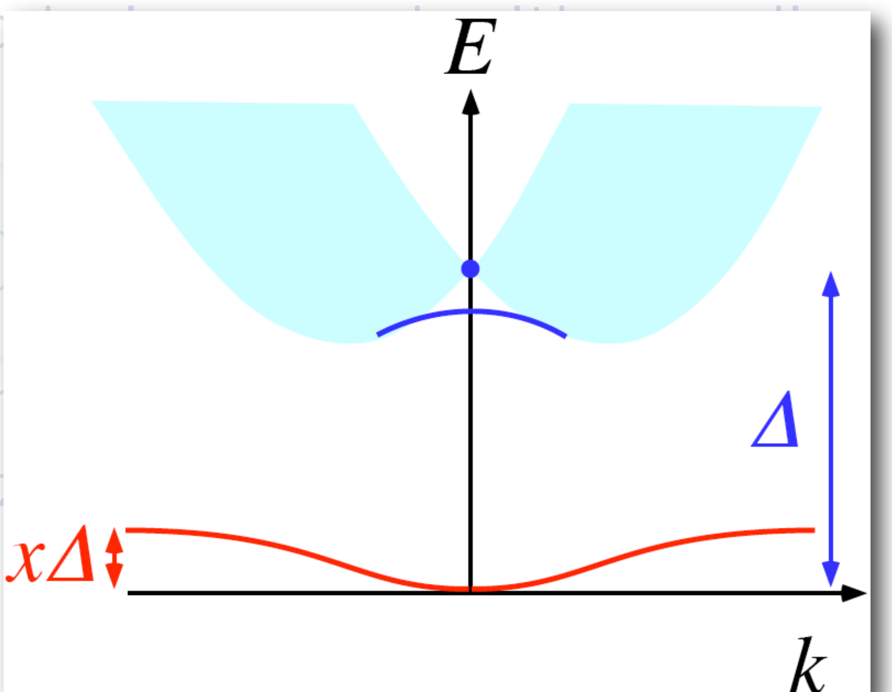
Collective Excitations

The dispersions of the collective excitations can be obtained by looking for the poles of the spin-wave propagator $D(p, \nu_n)$. After lengthy algebra, one will find two modes: the first mode is the usual spin wave of the localized spins,

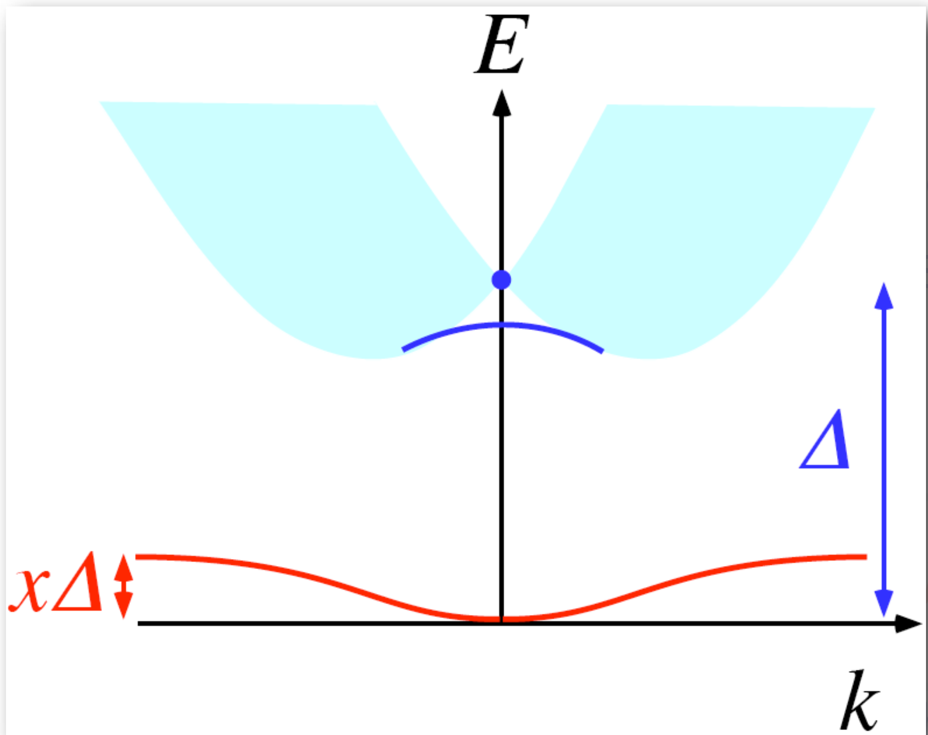
$$E_1(k) = \frac{\gamma}{1-\gamma}\epsilon(k)\left[1 - \frac{4}{5}\frac{\epsilon_F}{\Delta}\right] + \mathcal{O}(k^4),$$

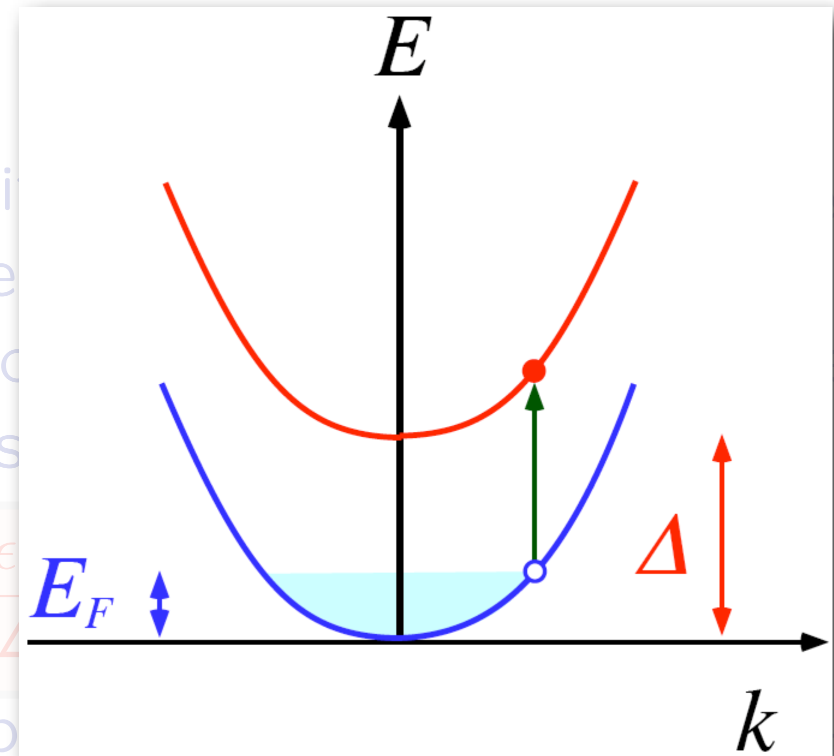
with $\gamma = n_h/2n_I S$ denotes the ratio between itinerant and local-





Collective Excitations





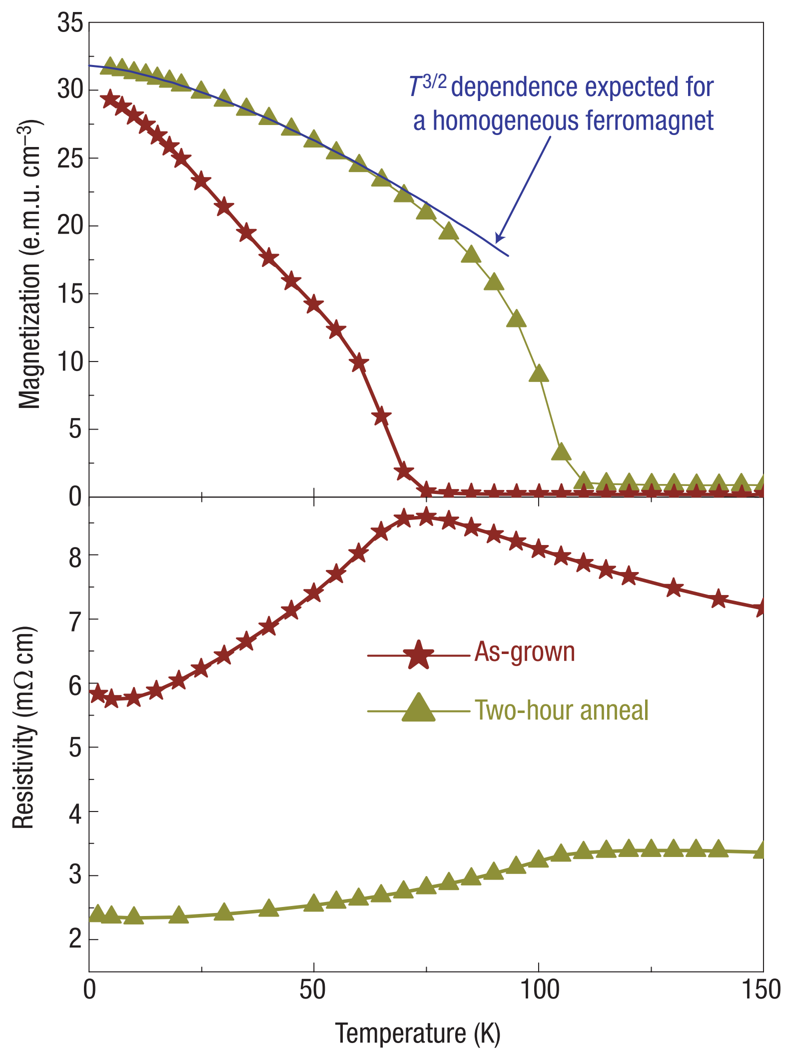
ized spin densities. The second mode is gapped with peculiar "banana"-shape dispersion,

$$E_2(k) = \Delta(1-\gamma) - \frac{1}{\gamma(1-\gamma)}\epsilon(k)\left[\frac{4}{5}\frac{\epsilon_F}{\Delta} - \gamma\right] + \mathcal{O}(k^4).$$

This mode comes from the *Stoner continuum of (electronic) magnons* that couple with the localized spins.

Transport and Field Effect

MacDonald, Schiffer & Samarth
Nature Mat. 4, 195 (2005)

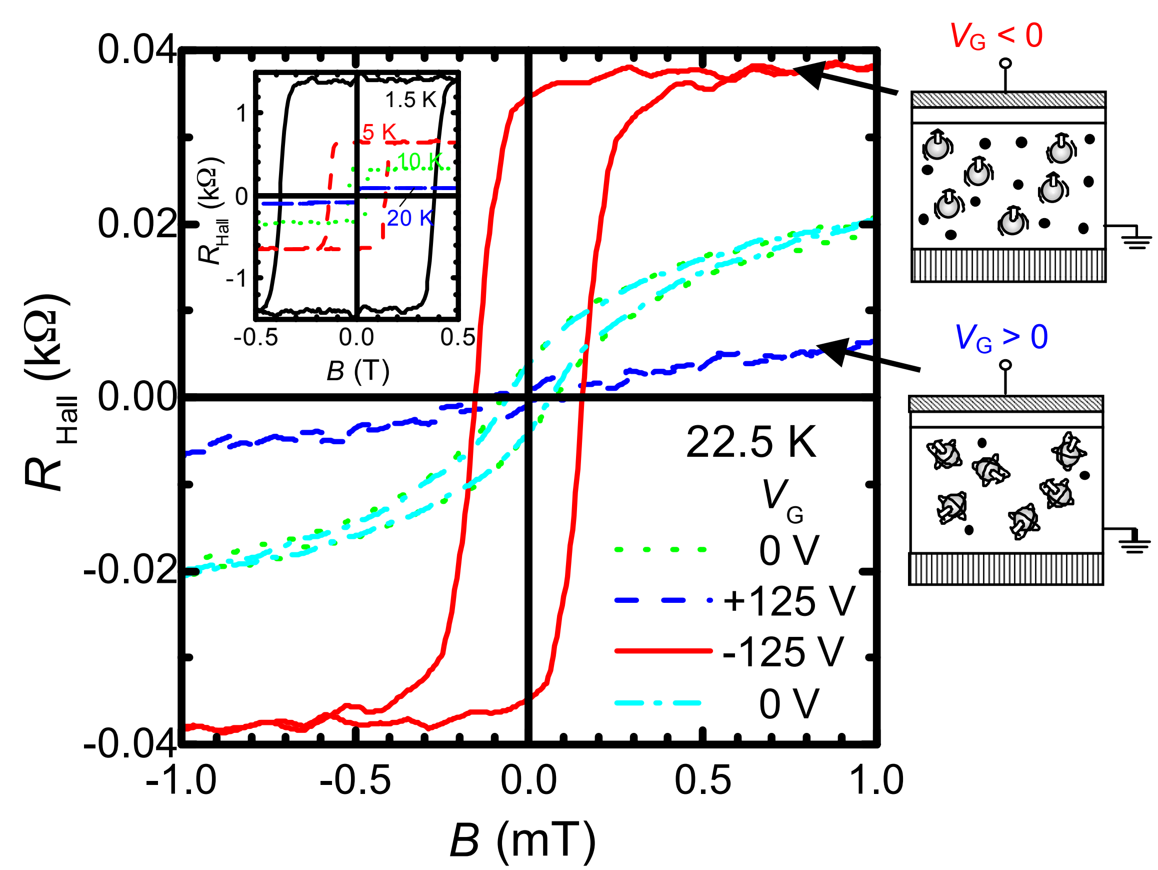


Is it the Fisher-Langer anomaly?
We have a story…

Phys. Rev. Lett. 86, 5637 (2001)
Appl. Phys. Lett. 89, 072101 (2006)

Ohno's Group
Nature 408, 944 (2000)

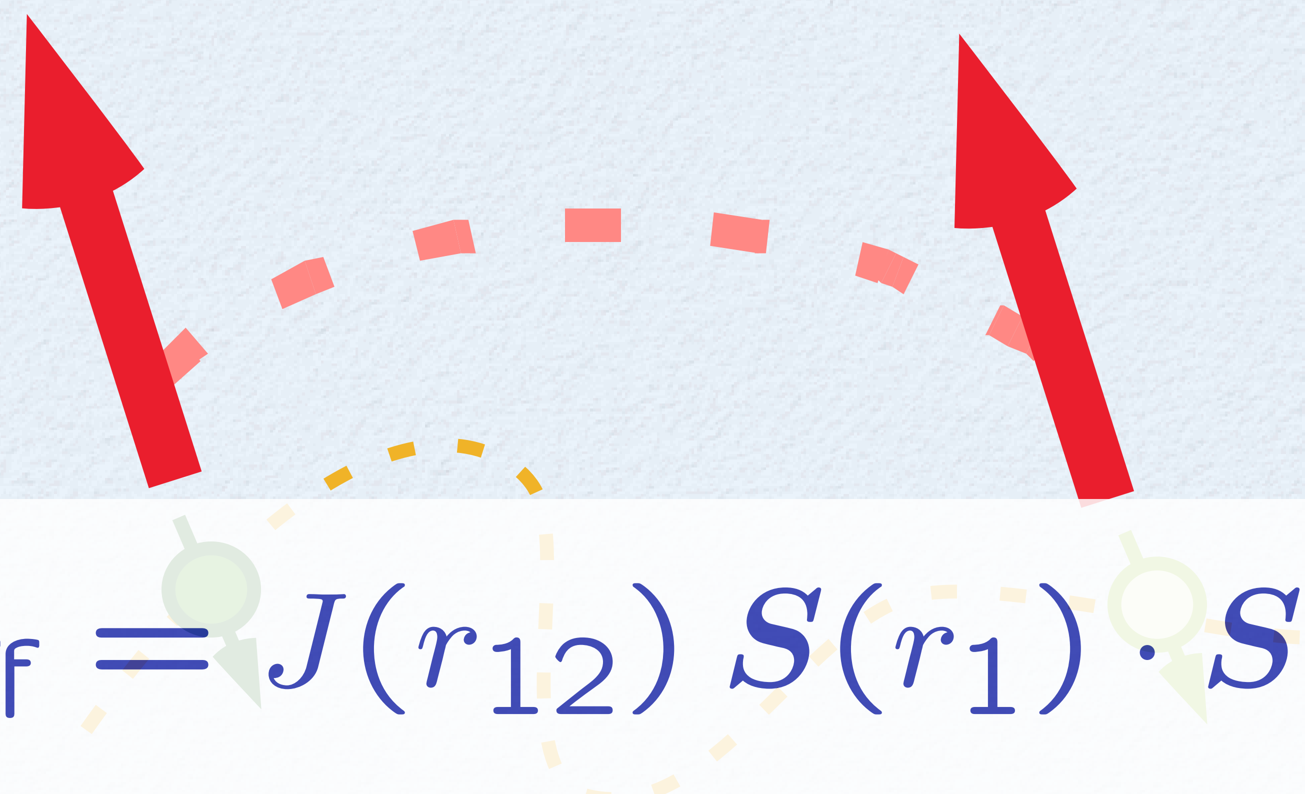
By varying the gate voltage, one can manipulate the concentration of itinerant carriers.



HOW TWO SPINS TALK...

Carrier-mediated Exchange Coupling:

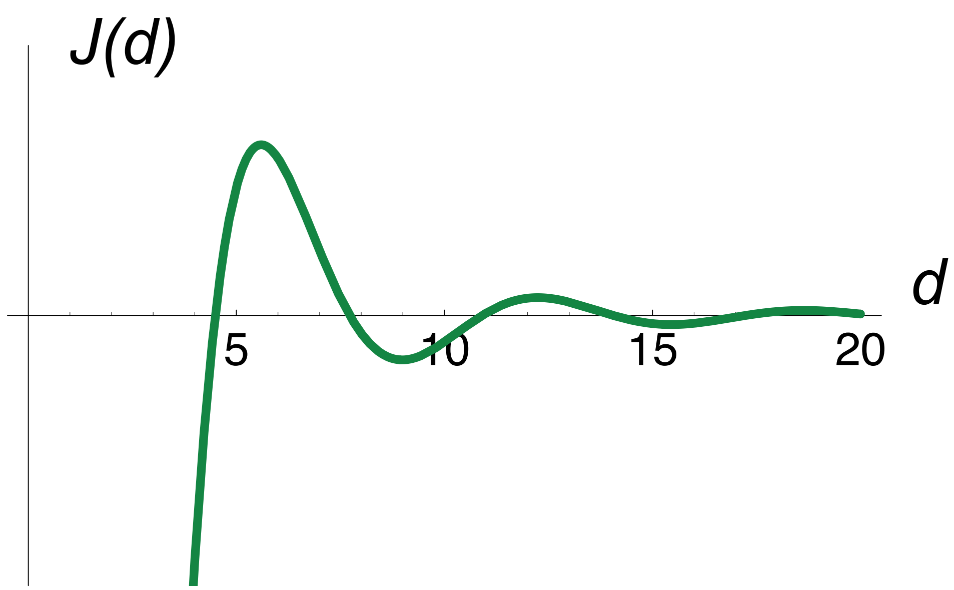
Integrating out the itinerant carriers to derive the effective Hamiltonian for the two spins

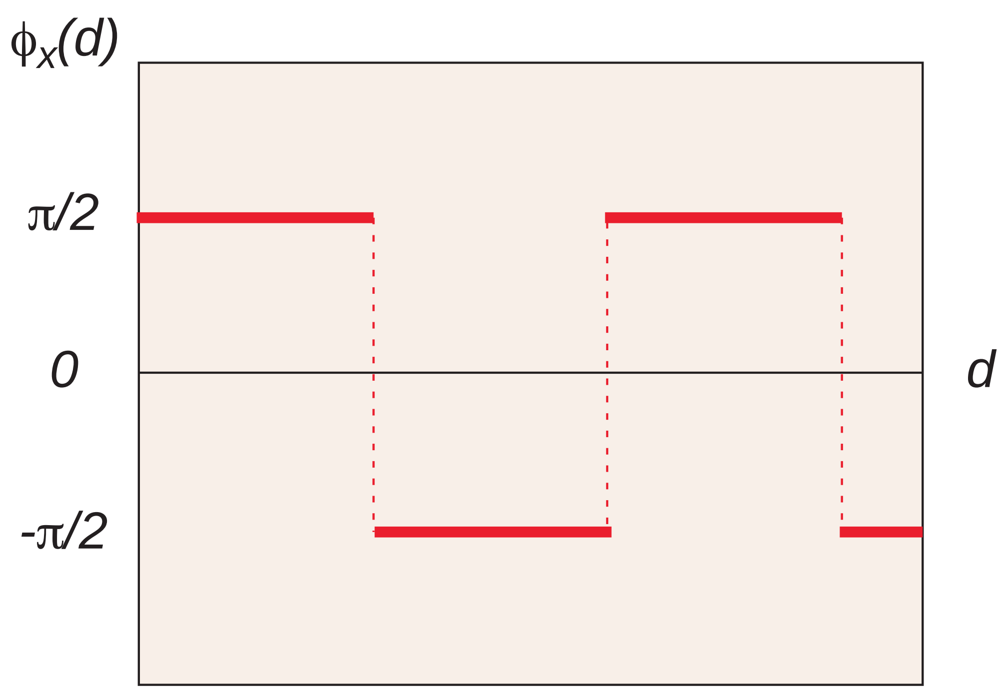


$$H_{\text{eff}} = J(r_{12})\, \boldsymbol{S}(r_1) \cdot \boldsymbol{S}(r_2)$$

Collinear RKKY Interaction

Thus, our conventional wisdom tells us that the mediated effective coupling should have RKKY oscillations…





RKKY interaction:

It can be viewed as the quantum interferences due to patches of the Fermi surface related by the time-reversal symmetry.

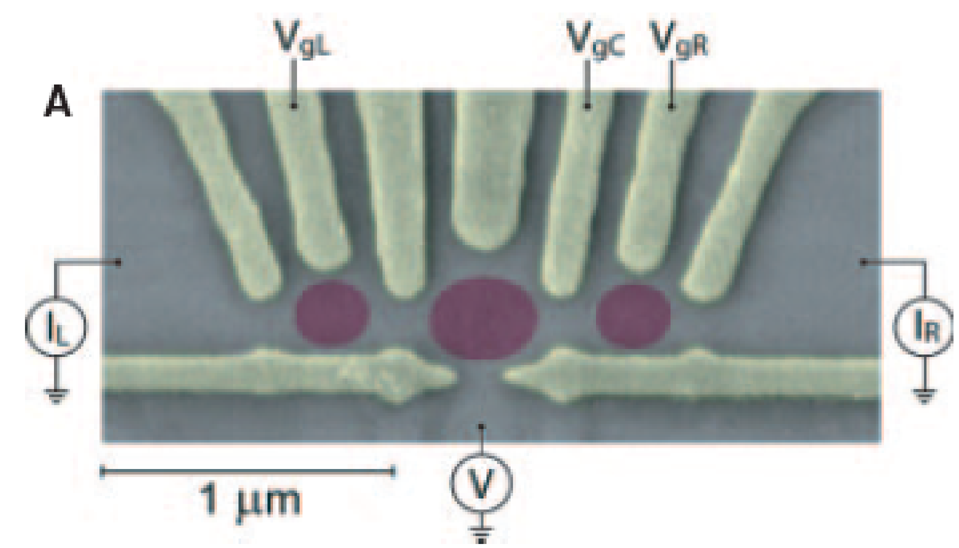
Kondo v.s. RKKY

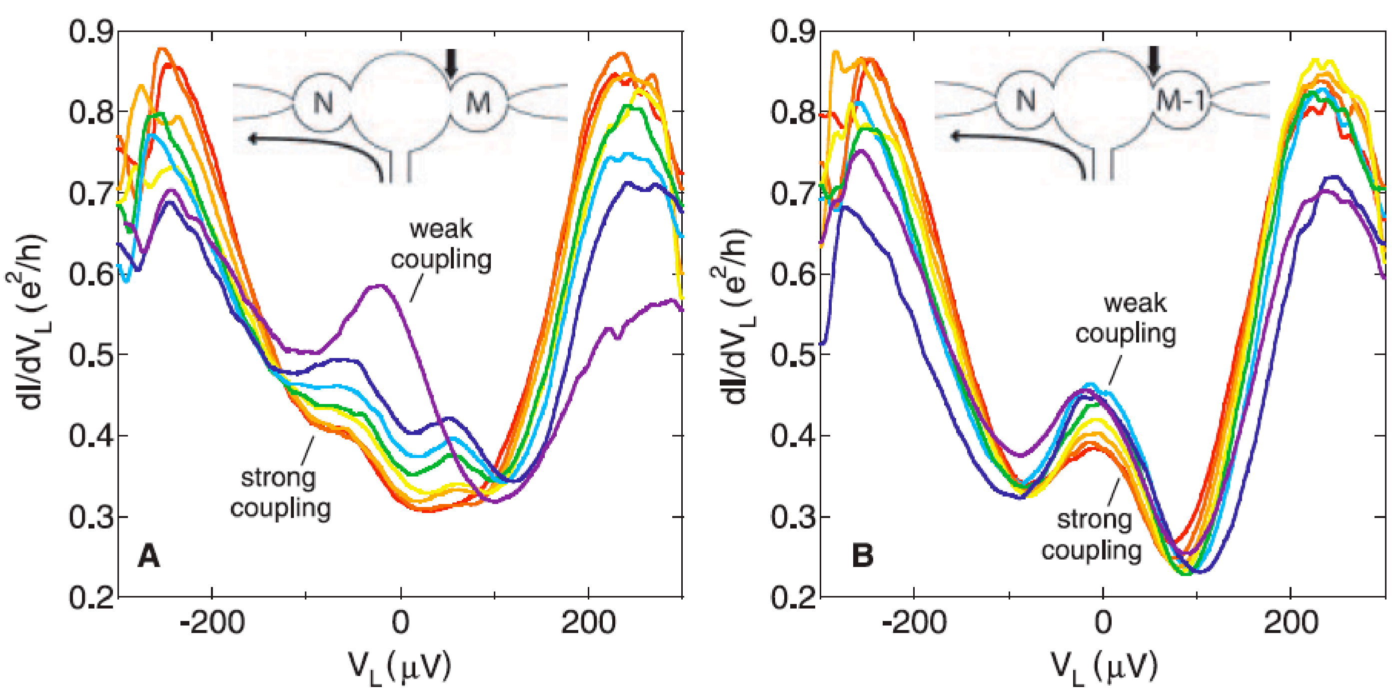
C. M. Marcus' group
Science 304, 565 (2004)

REPORTS

Tunable Nonlocal Spin Control in a Coupled–Quantum Dot System

N. J. Craig,[1] J. M. Taylor,[1] E. A. Lester,[1] C. M. Marcus,[1*]
M. P. Hanson,[2] A. C. Gossard[2]





(a) carrier-mediated RKKY exchange coupling,

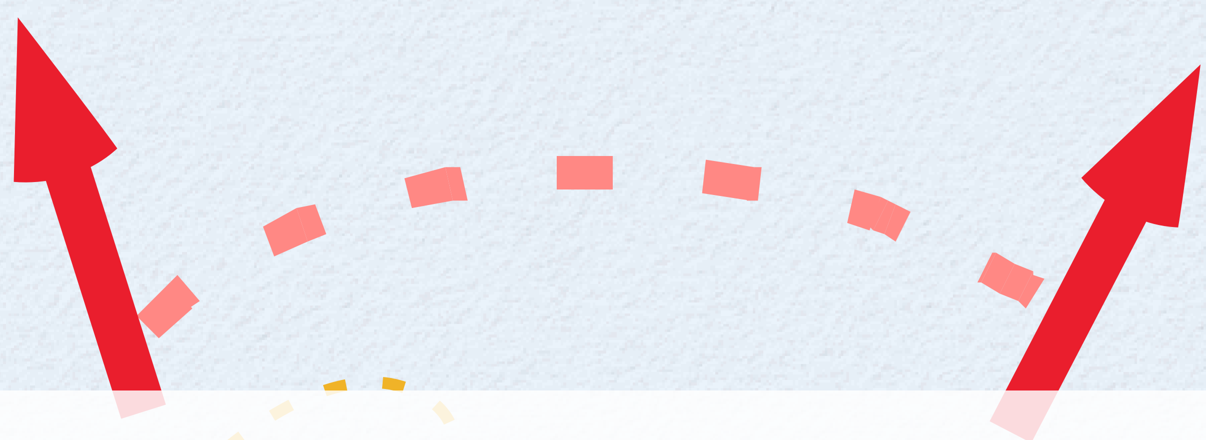
(b) competition between Kondo screening and RKKY interaction,

(c) evolution of the tunneling conductance peak.

NONCOLLINEAR ONE?

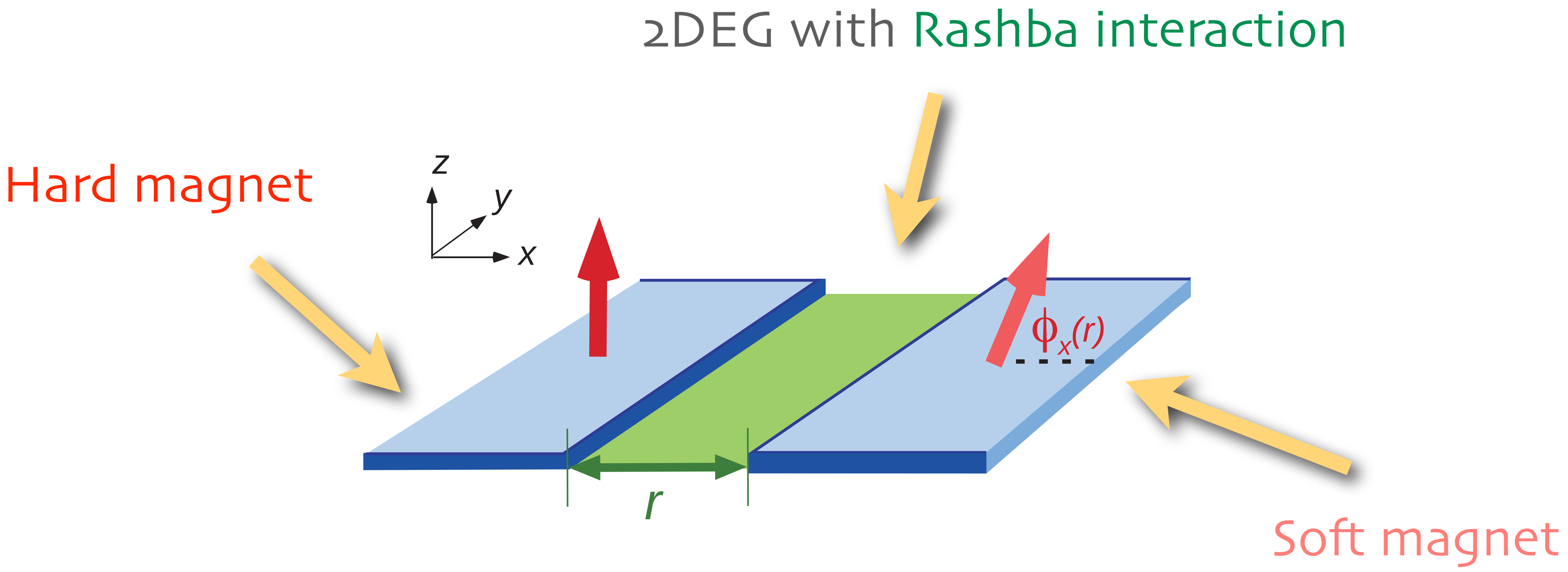
Carrier-mediated Exchange Coupling:

Integrating out the itinerant carriers to derive the effective Hamiltonian for the two spins



$$H_{\text{eff}} = \sum_{a,b} J_{ab}(r_{12})\, S_a(r_1)\, S_b(r_2)$$

Trilayer Magnetic Junction

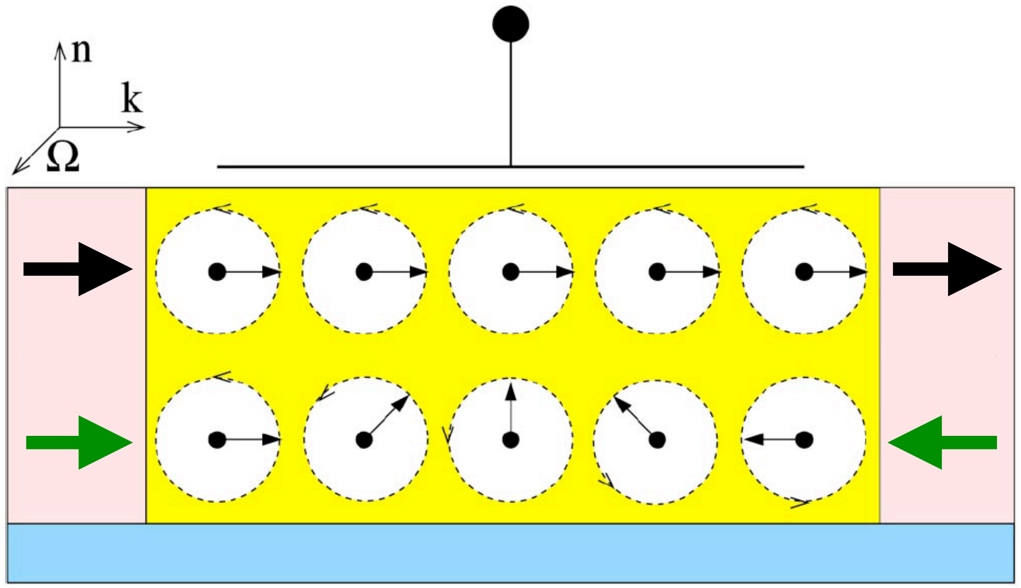


we model the intermediate layer by the Rashba Hamiltonian,

$$H = \int d^2r \, \Psi^\dagger \left[\frac{k^2}{2m^*}\mathbf{1} + \gamma_R(k_y\sigma^x - k_x\sigma^y) \right] \Psi,$$

where γ_R is the strength of the Rashba interaction.

Non-Collinear Spiral Angle?

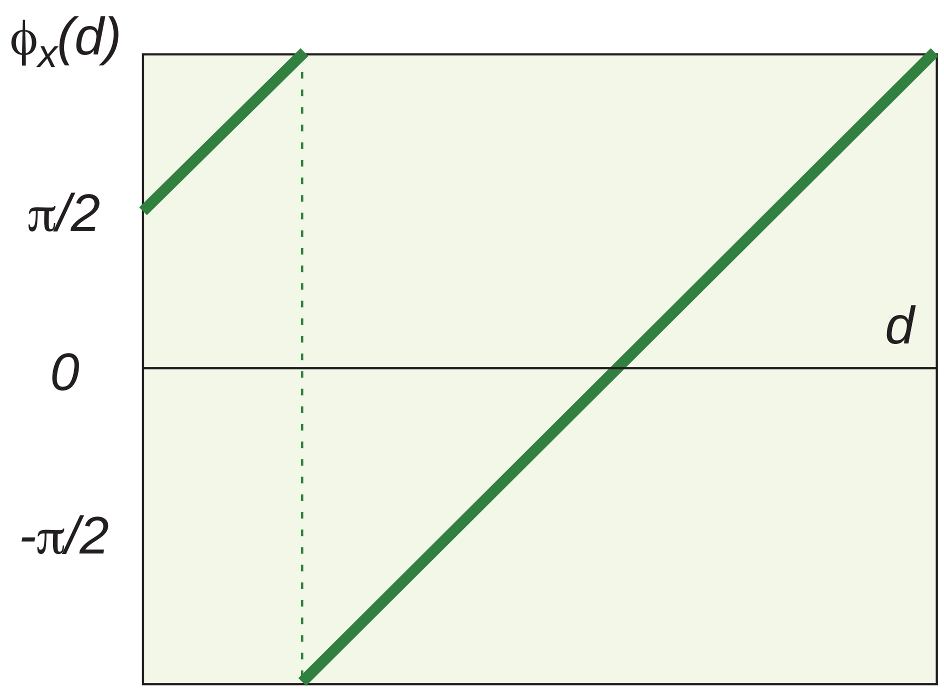


As the spins of the itinerant carriers precess, it is possible to mediate noncollinear exchange coupling.

We try out the idea for Zeeman Hamiltonian and it seems to work…

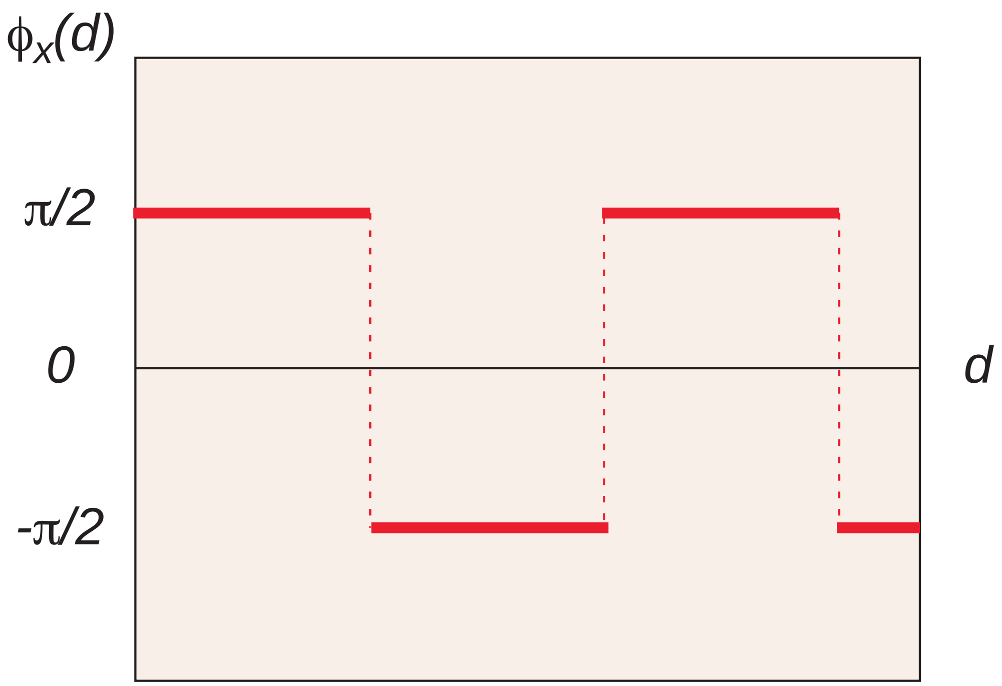
Appl. Phys. Lett. 84, 2862 (2004)
Appl. Phys. Lett. 89, 032503 (2006)

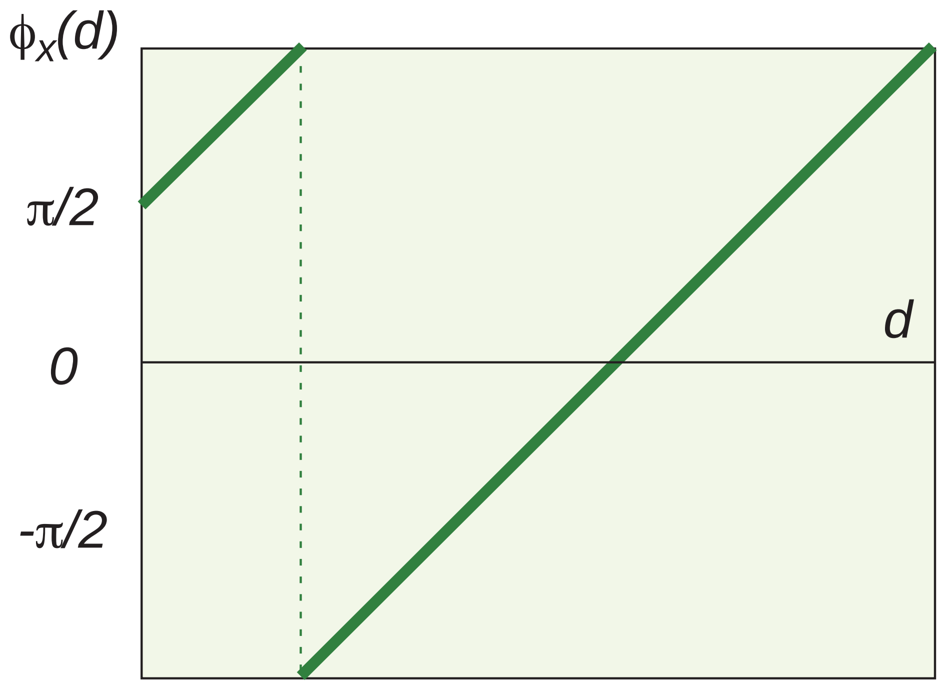
The spin of the itinerant carriers will align the soft magnet with the same spiral angle.



Therefore, we expect an effective ***non-collinear*** exchange coupling!

Which one is correct?



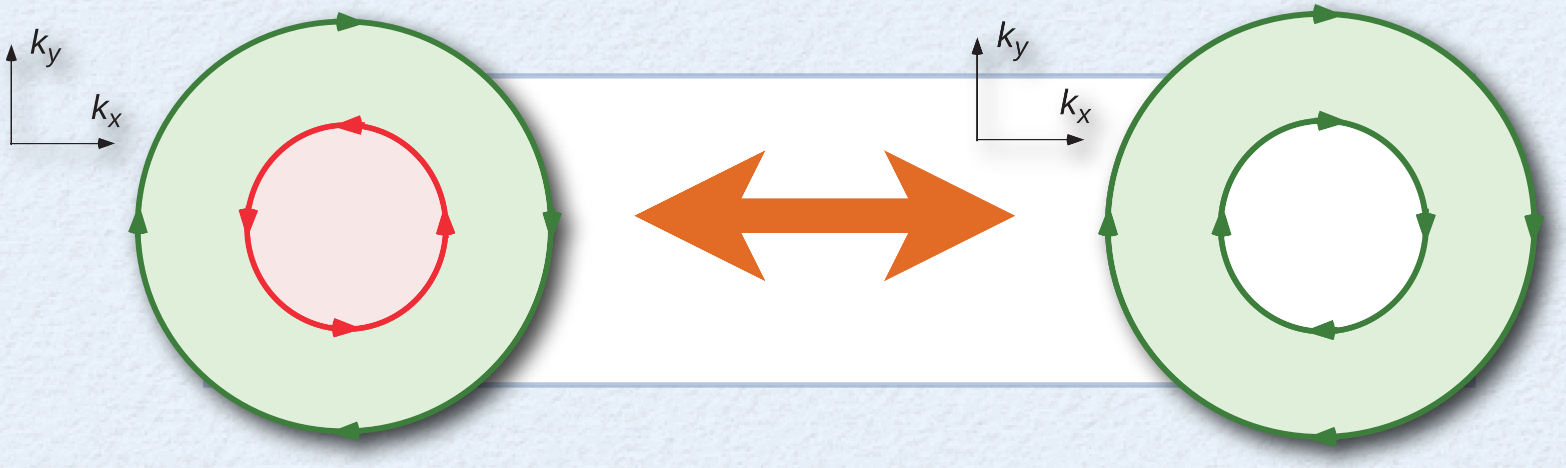


RKKY?

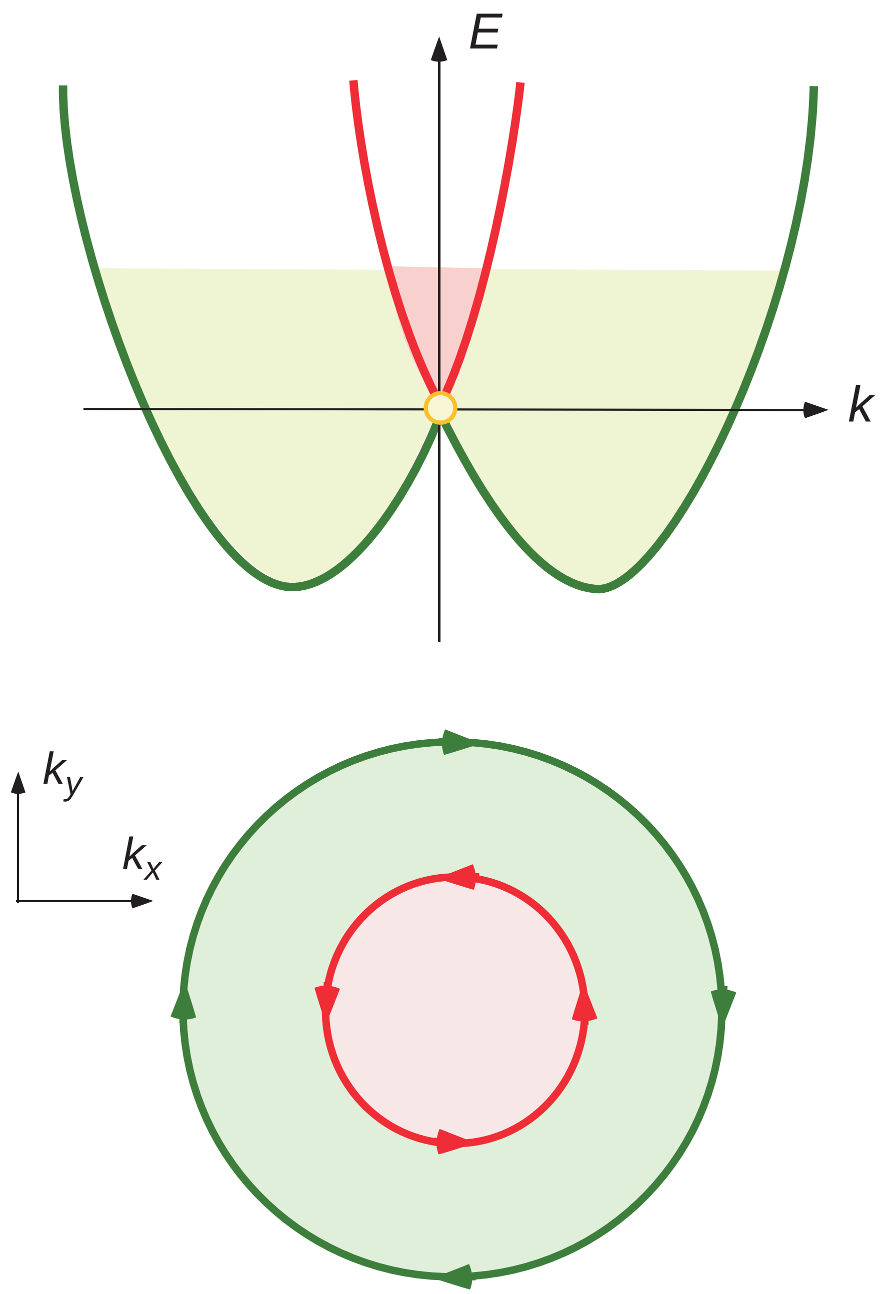
Spiral?

Fermi surface topology

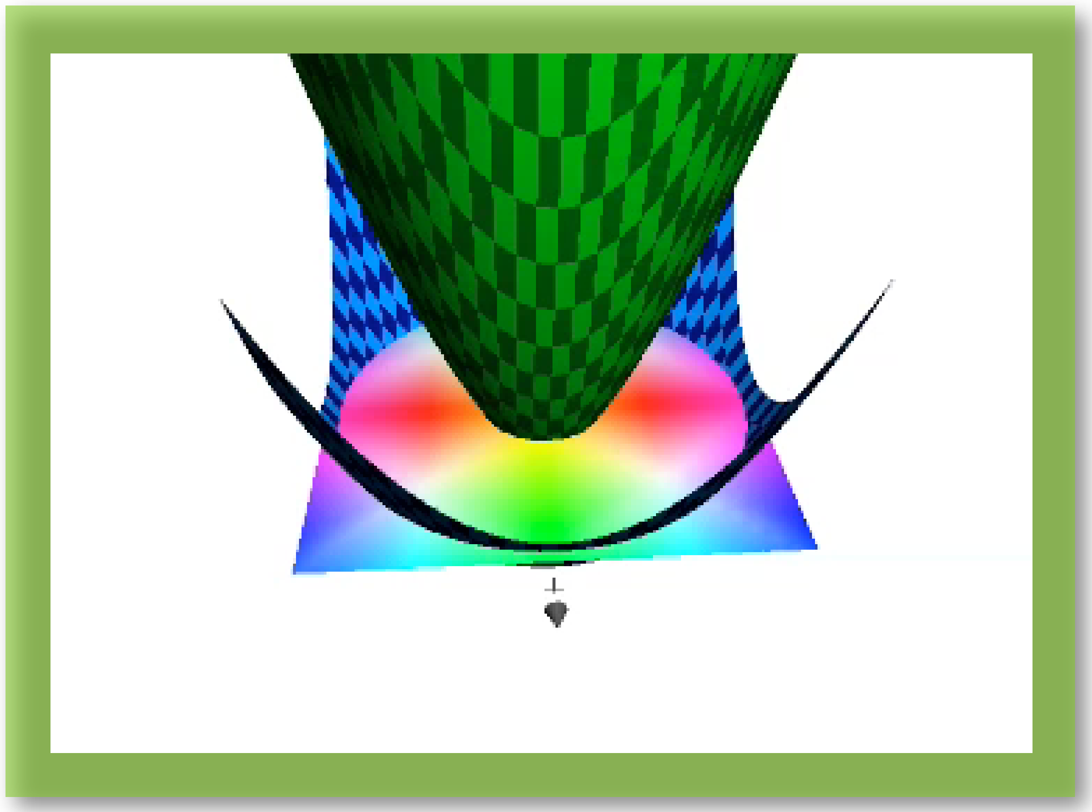
By changing the density, the Fermi surface topology changes as well.



FS Topology 1: Wedding Cake



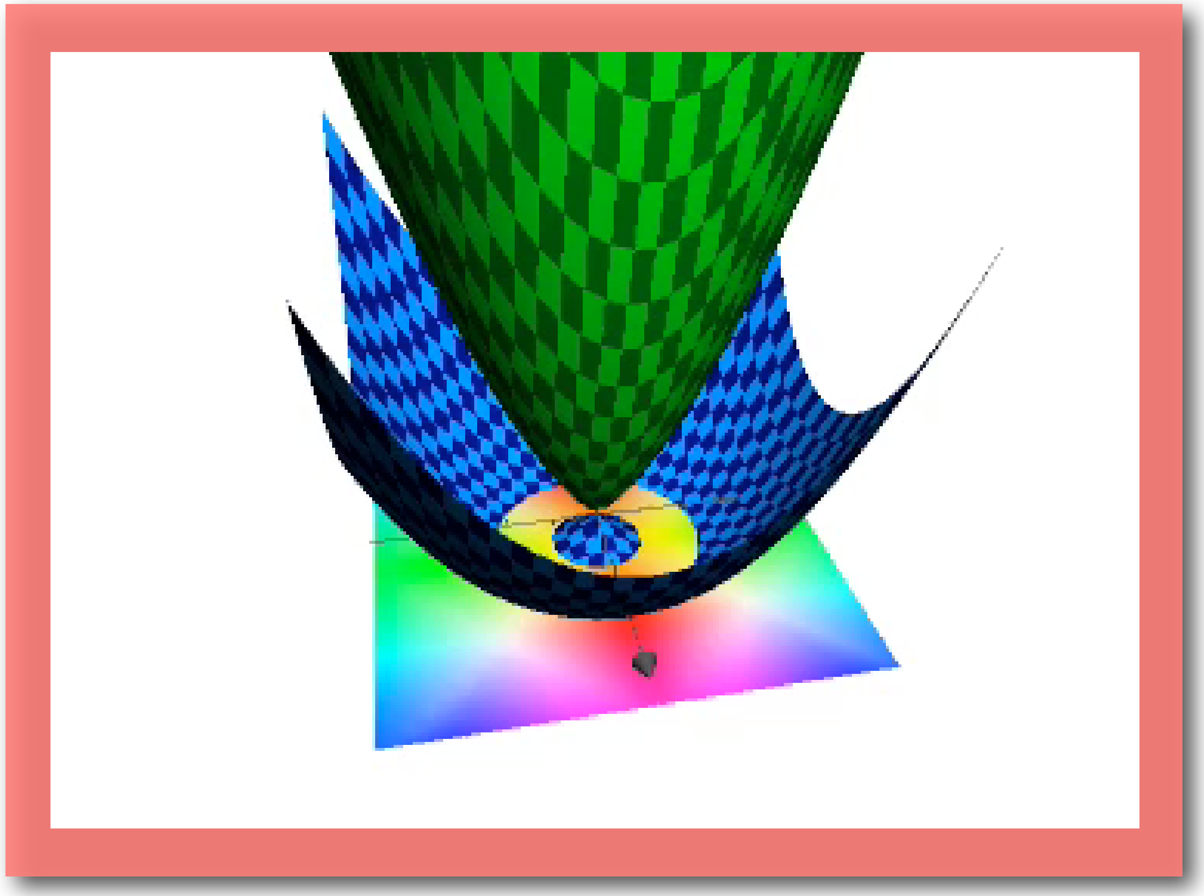
When Rashba coupling is small (compared with the Fermi energy), the Fermi surfaces consist of two particle-like circles with opposite chiralities.

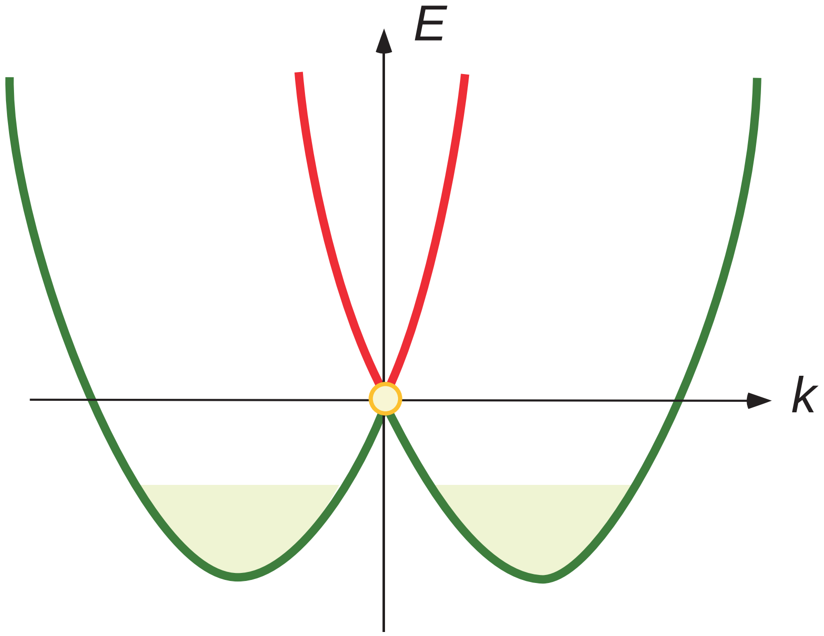


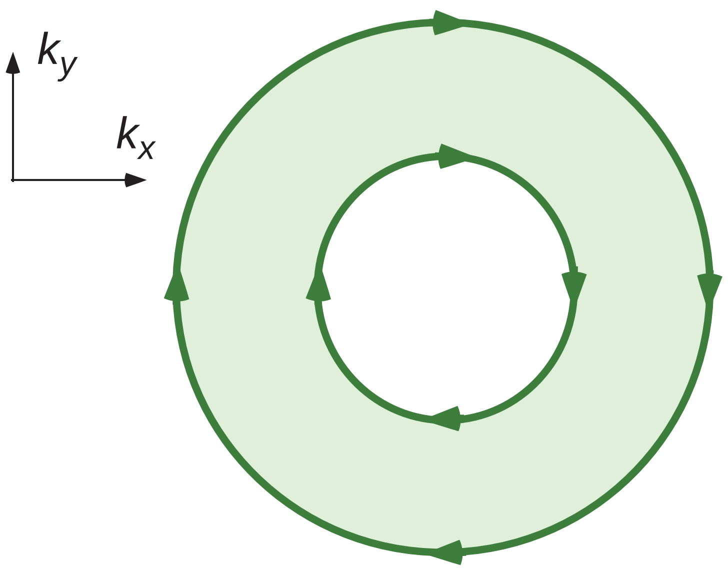
Weak Rashba regime with $\Delta_R/\epsilon_F < 1$

FS Topology 2: Bagel

When the Fermi energy is small, the Fermi surfaces consist of one particle-like and one hole-like circles with the same chiralities.







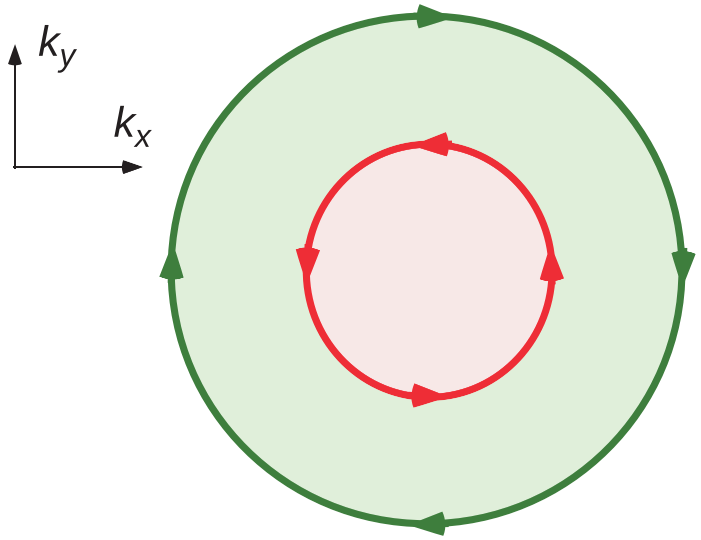
Dilute density regime with $\Delta_R/\epsilon_F > 1$

Rashba Hamiltonian for Dummies...

Due to the spin-orbital interaction, spin is no longer the good quantum number but replaced by the chirality instead,

$$\lambda = (\hat{\boldsymbol{k}} \times \hat{\boldsymbol{s}}) \cdot \hat{\boldsymbol{z}} = \pm 1.$$

It is important to remind the readers that, under the time reversal transformation, both momentum and spin reverse their directions and make the chirality invariant.



The Rashba Hamiltonian can be brought into its eigenbasis in momentum space,

$$\varphi_{k\lambda}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_\lambda(\phi) = \frac{e^{i\vec{k}\cdot\vec{r}}}{\sqrt{2}} \begin{pmatrix} -i\lambda e^{-i\theta_k} \\ 1 \end{pmatrix}$$

where $\theta_k = \tan^{-1}(k_y/k_x)$ with dispersion $\epsilon_{k\lambda} = k^2/2m^* - \lambda\gamma_R k.$

Noncollinear Exchange Coupling

Phys. Rev. B 73, 241307(R) (2006)

(1) Integrate out the itinerant carriers → the effective Heisenberg Hamiltonian between the ferromagnets, $H_{\text{eff}} = \sum_{ij} J_{ij} S_L^i S_R^j$.

(2) Within the linear response theory, J_{ij} is proportionally to the spin susceptibility tensor,

$$\chi_{ij}(\vec{r}) = \int_0^\infty dt \left\langle\left\langle i \left[\sigma^i(\vec{r},t), \sigma^j(0,0)\right]\right\rangle\right\rangle e^{-\eta t}.$$

(3) Transforming into the eigenbasis, the susceptibility tensor can be expressed as summations of the product of a weight function and the particle-hole propagator over all possible quantum numbers,

$$\chi_{ij}(\vec{r}) = \sum_{k_1\lambda_1} \sum_{k_2\lambda_2} W_{ij}(\vec{r}) \left[\frac{f(\epsilon_{k_1\lambda_1}) - f(\epsilon_{k_2\lambda_2})}{\epsilon_{k_2\lambda_2} - \epsilon_{k_1\lambda_1} - i\eta}\right].$$

The weight function is $W_{ij}(\vec{r}) = (u_{\lambda_1}^\dagger \sigma^i u_{\lambda_2})(u_{\lambda_2}^\dagger \sigma^j u_{\lambda_1}) e^{i(\vec{k_2}-\vec{k_1})\cdot\vec{r}}$, and $\epsilon_{k\lambda} = k^2/2m^* - \lambda k \gamma_R$ is the dispersion for the particle with momentum k and chirality λ.

Symmetries I

Let's take $\chi_{xy}(\vec{r}) = \chi_{xy}(r, \theta)$ as a working example.

(1) **Rotational Symmetry:** Since the operators σ_x, σ_y carry $m = \pm 1 \rightarrow \chi_{xy}(r, \theta)$ contains linear combinations of $m = 0, \pm 2$,

$$\chi_{xy}(r, \theta) = f_0(r) + f_2(r) \cos 2\theta + g_2(r) \sin 2\theta.$$

(2) **Parity Symmetry:** Furthermore, applying the parity symmetry in y direction, it requires $\chi_{xy}(r, \theta) = -\chi_{xy}(r, -\theta)$ and enforces the functions $f_0(r), f_2(r)$ to vanish,

$$\chi_{xy}(r, \theta) = g_2(r) \sin 2\theta.$$

(3) **Time-Reversal Symmetry:** Finally, the Onsager relation from the time-reversal symmetry indicates $\chi_{yx}(\vec{r}) = \chi_{xy}(-\vec{r})$,

$$\chi_{xy}(r, \theta) = \chi_{yx}(r, \theta) = g_2(r) \sin 2\theta.$$

Symmetries II

Utilizing the rotational SO(2), parity P_y (or equivalently P_x), and time reversal symmetries, one can work out the remaining components of the susceptibility tensor,

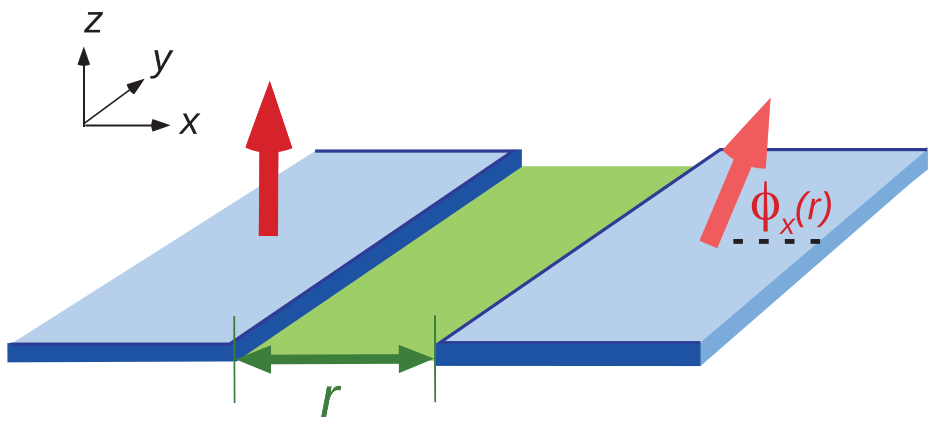
$$\chi_{ij}(r,\theta) = \begin{bmatrix} g_0 + g_2\cos 2\theta & g_2 \sin 2\theta & g_1 \cos\theta \\ g_2 \sin 2\theta & g_0 - g_2 \cos 2\theta & g_1 \sin\theta \\ -g_1 \cos\theta & -g_1 \sin\theta & h_0 \end{bmatrix}.$$

It is rather remarkable that the symmetry arguments reduce the numerical task down to evaluation of FOUR real scalar functions, $g_0(r)$, $g_1(r)$, $g_2(r)$, $h_0(r)$.

The Rashba Hamiltonian we study here further constrains $h_0(r) = g_0(r) + g_2(r)$, which reduces the number down to THREE.

Spiral Angle

$$\chi_{ij}(r,\theta) = \begin{bmatrix} g_0 + g_2\cos 2\theta & g_2 \sin 2\theta & g_1 \cos\theta \\ g_2 \sin 2\theta & g_0 - g_2 \cos 2\theta & g_1 \sin\theta \\ -g_1\cos\theta & -g_1 \sin\theta & g_0 + g_2 \end{bmatrix}$$

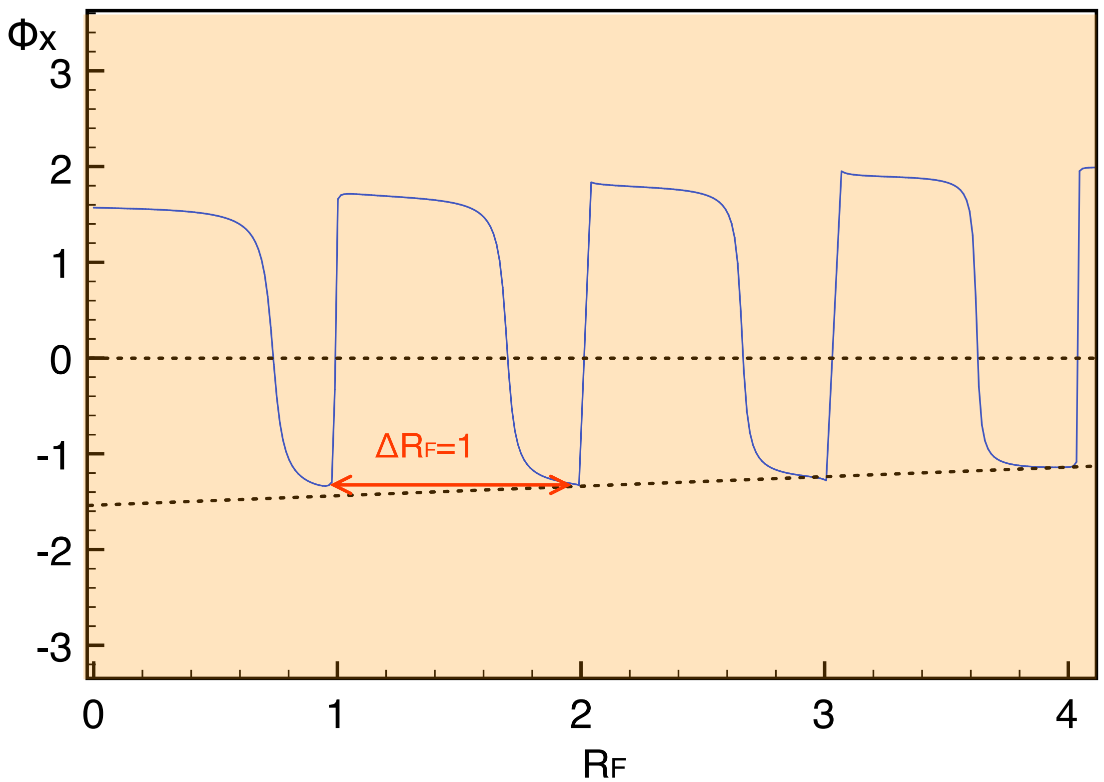


Suppose the ferromagnet on the left of the TMJ is aligned along the z-axis, we are interested in the mediated non-collinear exchange coupling proportional to $\chi_{iz}(r,0)$.

Since $\chi_{yz}(r,0) = 0$, the orientation of the induced moment is captured by the spiral angle,

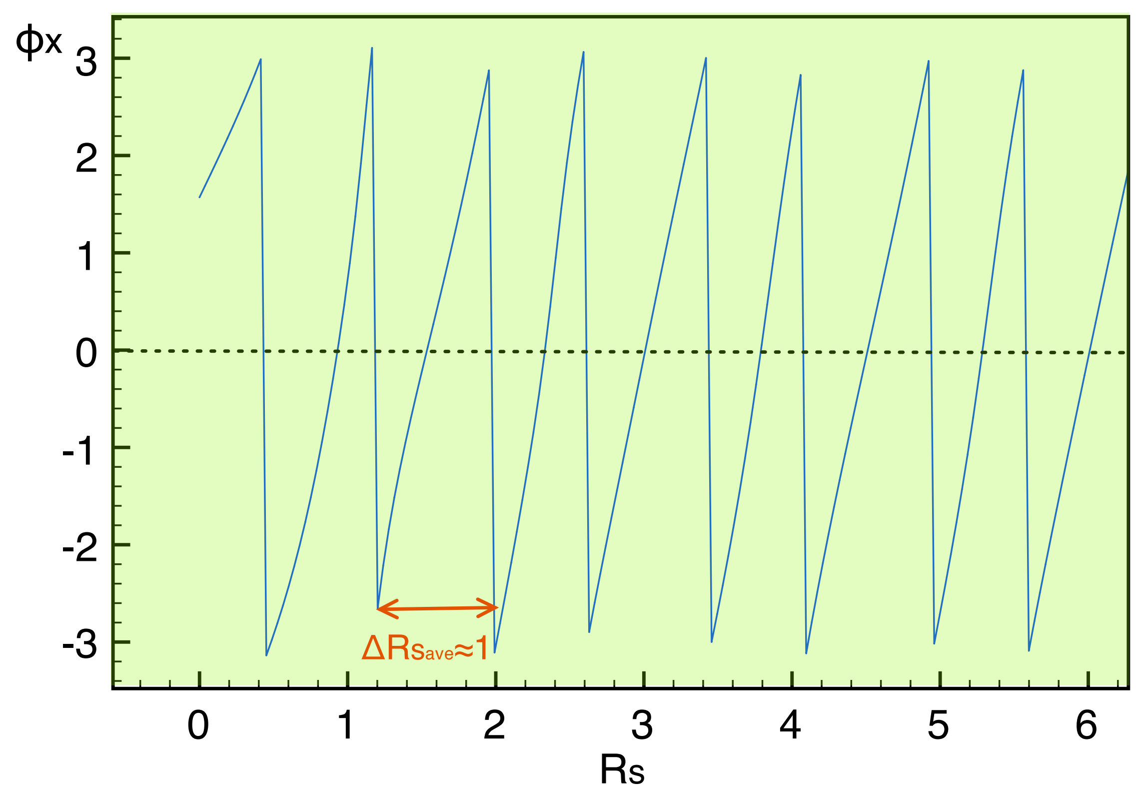
$$\phi_x(r) = \tan^{-1}\left[\frac{\chi_{zz}(r,0)}{\chi_{xz}(r,0)}\right] = \tan^{-1}\left[\frac{g_0(r)+g_2(r)}{g_1(r)}\right].$$

Numerical Results



Weak Rashba regime with $\Delta_R/\epsilon_F < 1$

Robust spiral backbone with minor oscillatory residues resembling the RKKY oscillations.



Dilute density regime with $\Delta_R/\epsilon_F > 1$

Modified RKKY oscillation with a gradual upwinding trend due to Rashba interaction.

Connection to 1D Rashba?

(1) In the asymptotic limit $k_F r \gg 1$, the reduced spin susceptibility along the radial direction $\chi_{ab}(r)$, where $a, b = x, z$, can be well approximated as 1D Rashba system.

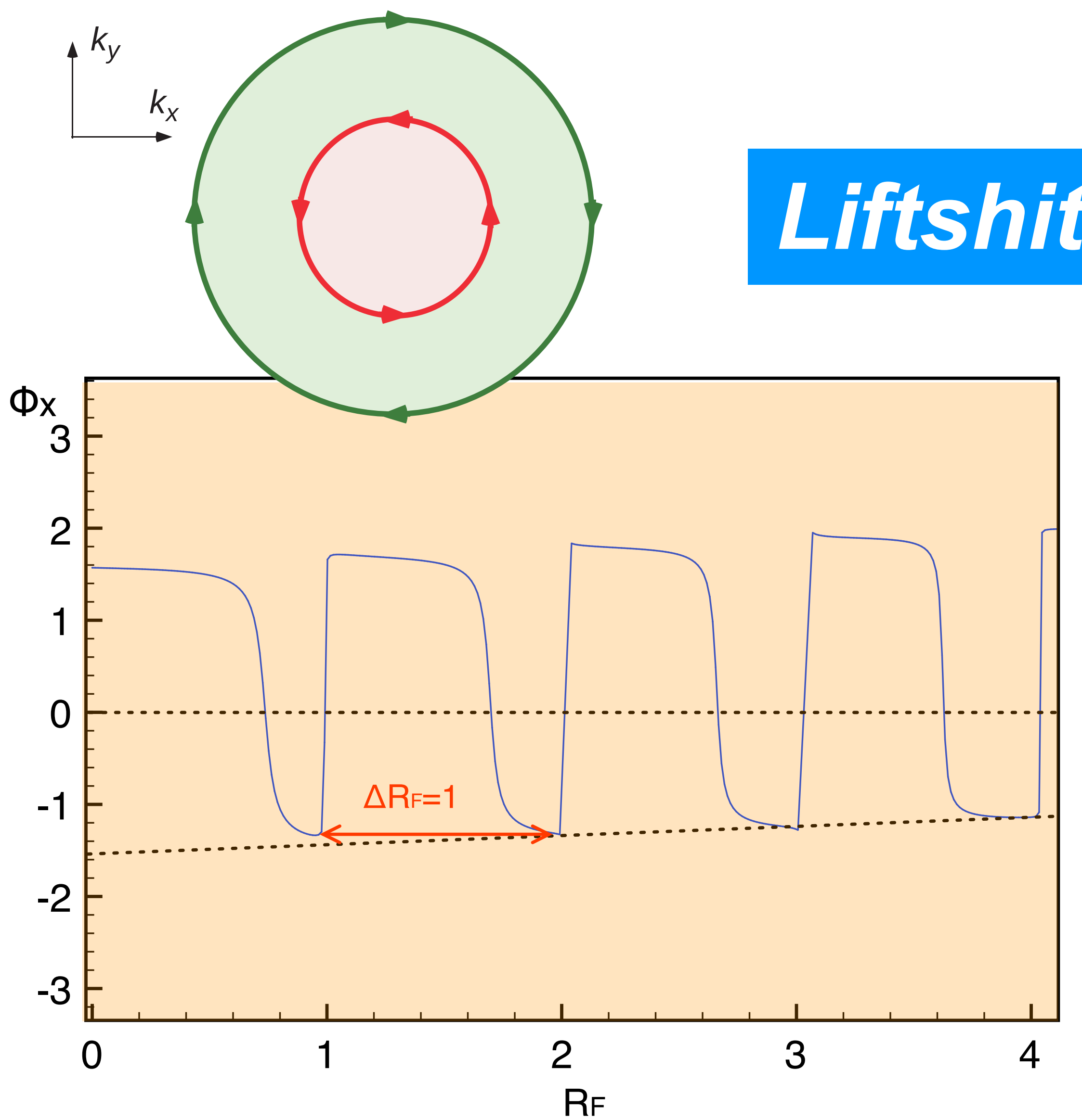
(2) Applying a local gauge transformation, $U(r) = e^{-ik_R r \sigma^y/2}$, the Rashba Hamiltonian can be mapped into the 1D free electron gas with the well-known RKKY spin susceptibility.

(3) Since the local gauge transformation is nothing but the local rotation about the y-axis with the spiral angle $\phi(r) = k_R r$, the reduced susceptibility is approximately the usual RKKY oscillation twisted by a local spiral transformation,

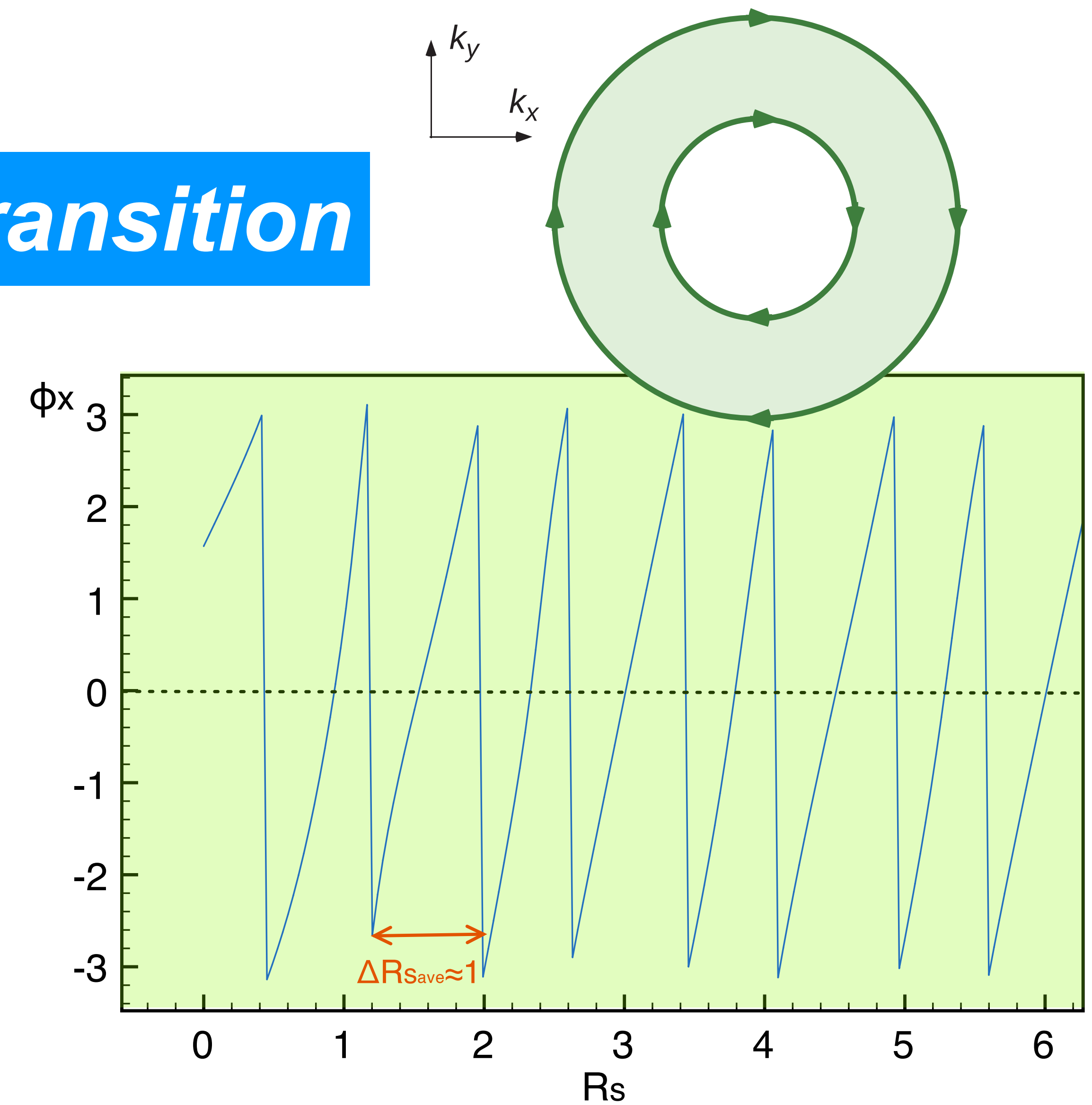
$$\chi_{ab}(r) \approx \sum_c \begin{bmatrix} \cos k_R r & -\sin k_R r \\ \sin k_R r & \cos k_R r \end{bmatrix}_{ac} \chi_{cb}^{RKKY}(r).$$

Fermi Surface Topology

By changing the carrier density, we can change the topology of the Fermi surfaces from wedding cake (with opposite chiralities) to bagel (with one chirality).

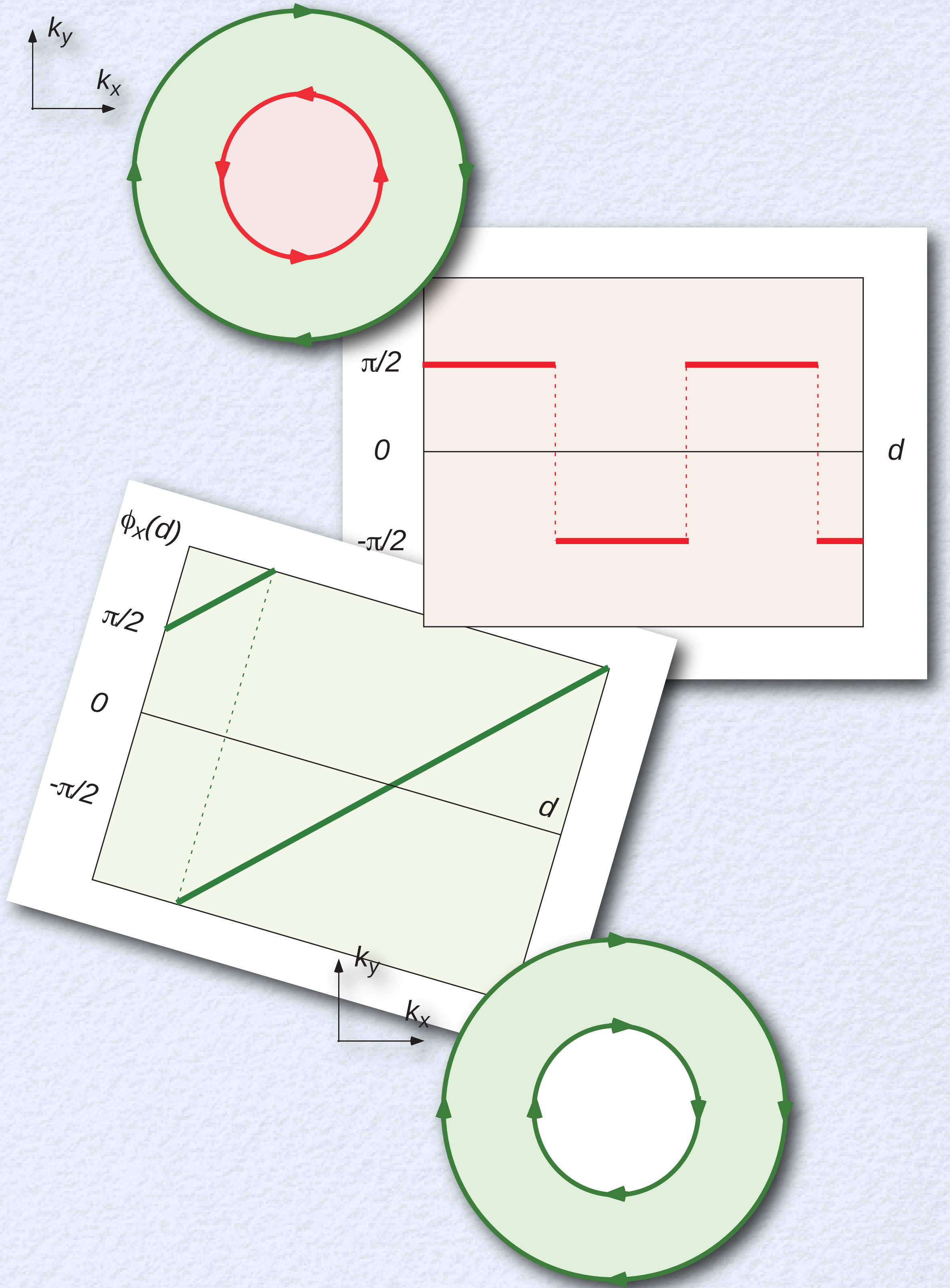


Liftshitz Transition

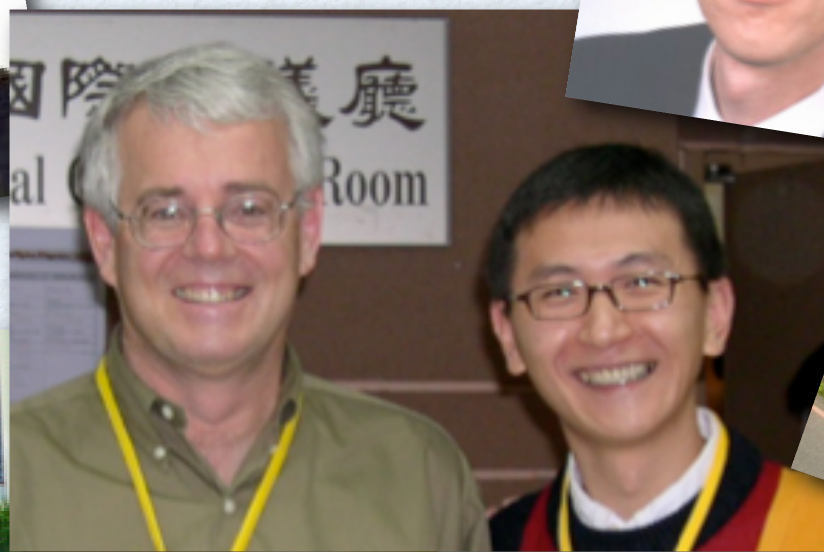


SUMMARY

- Fermi surface topology dictates the trends of the carrier-mediated exchange coupling.
- Liftshitz transition in 2DEG by measuring magnetic responses.
- Classification of magnetic behaviors solely by the winding number of the Fermi surfaces?
- With small spin relaxation, the spiral trend is favored. Not clear about the results in diffusive regime yet...







THANK YOU!!