Dipolar lattice and its quantum fluctuation effects

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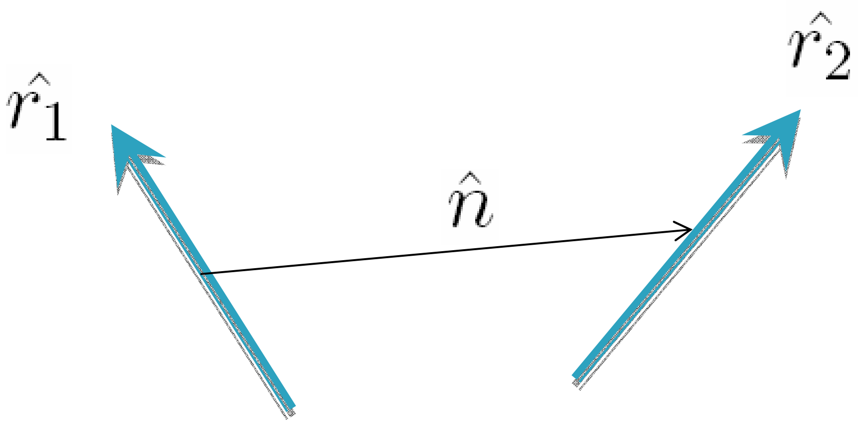
Outline

1. Introduction
2. 1D system
3. 2D system
4. Future work

Introduction

Dipole-dipole interaction: Anisotropic and long range order

$$V_{dd} = \frac{D^2}{r^3}\left(\hat{r_1}\cdot\hat{r_2} - 3(\hat{r_1}\cdot\hat{n})(\hat{r_2}\cdot\hat{n})\right)$$

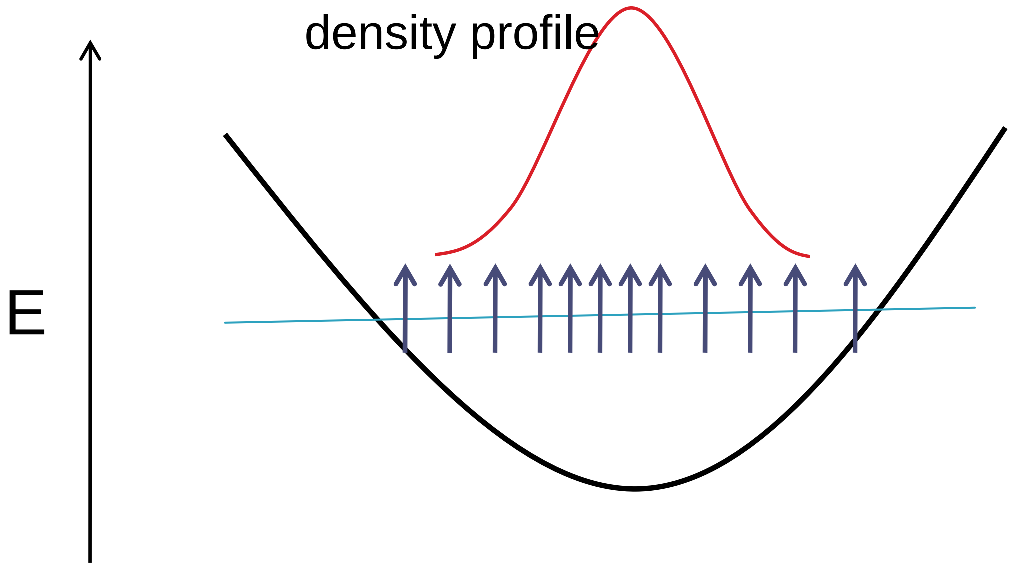


Exotic many-body states arising from this interaction

1D wire

$$H = \sum_{i}^{N} \frac{p_i^2}{2m} + \frac{1}{2}\sum_{i\neq j}^{N} \frac{D^2}{|x_i - x_j|^3} + \frac{m\omega^2}{2}\sum_{i}^{N} x_i^2.$$

$$\beta = \frac{D^2}{m\omega^2 l^5} = \frac{D^2/l^3}{m\omega^2 l^2}, \qquad l = \sqrt{\frac{\hbar}{m\omega}}.$$



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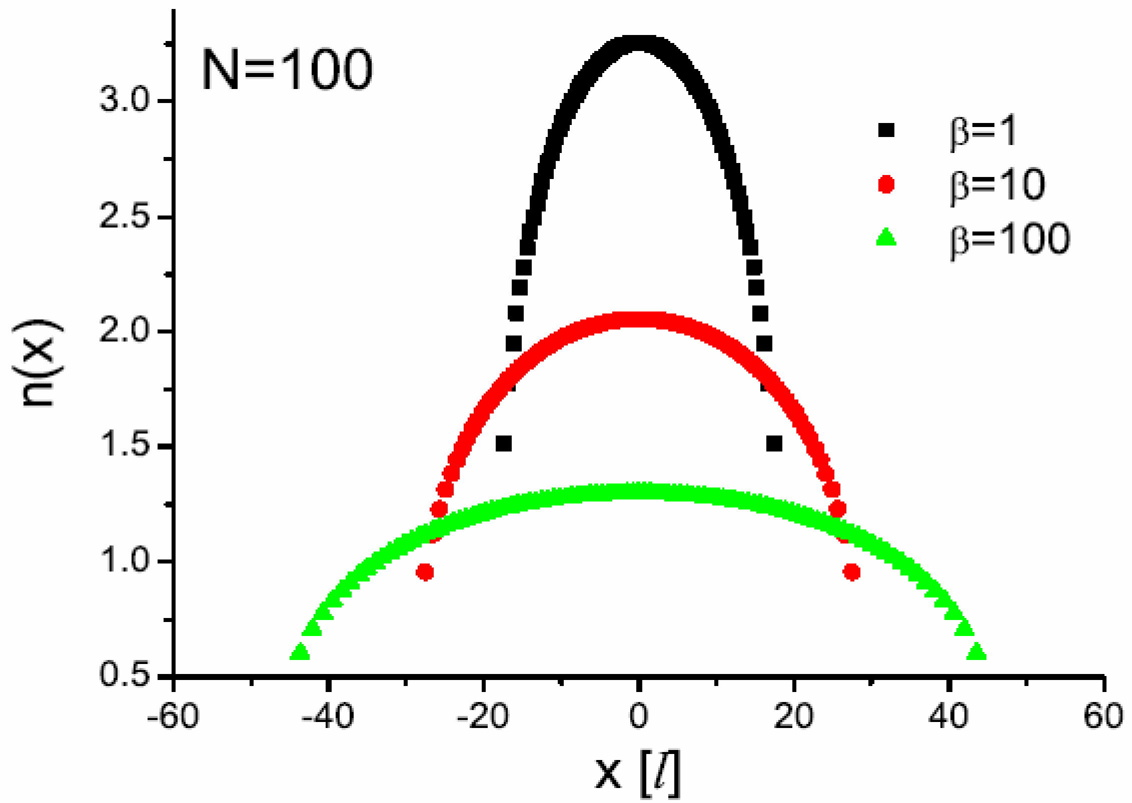


Phonon

$$H = \sum_{i}^{N} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i \neq j}^{N} \frac{D^2}{|x_i - x_j|^3} + \frac{m\omega^2}{2} \sum_{i}^{N} x_i^2.$$

$$= \sum_{i}^{N} \frac{p_i^2}{2m} + \frac{1}{2} \sum_{i,j} \frac{D^2}{l^5} M_{i,j} \delta x_i \delta x_j$$

$$= \sum_{n} \frac{\tilde{p_n}^2}{2m} + \frac{m}{2} \sum_{n} \omega_n^2 \xi_n^2,$$

$$= \sum_{n} \hbar\omega_n (b_n^\dagger b_n + 1/2).$$

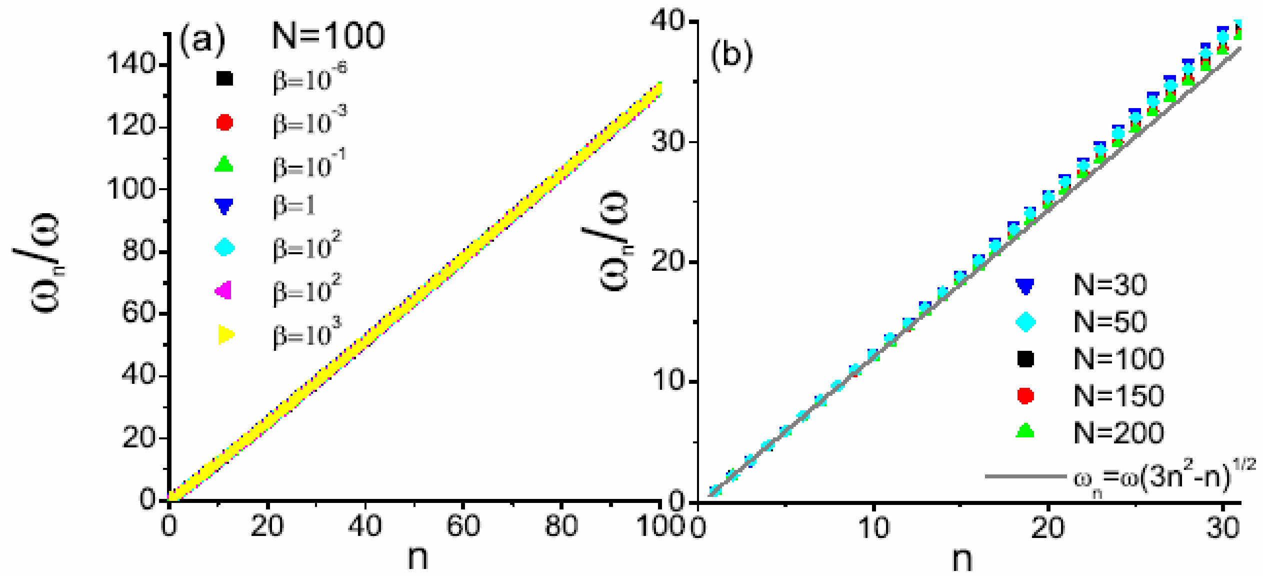
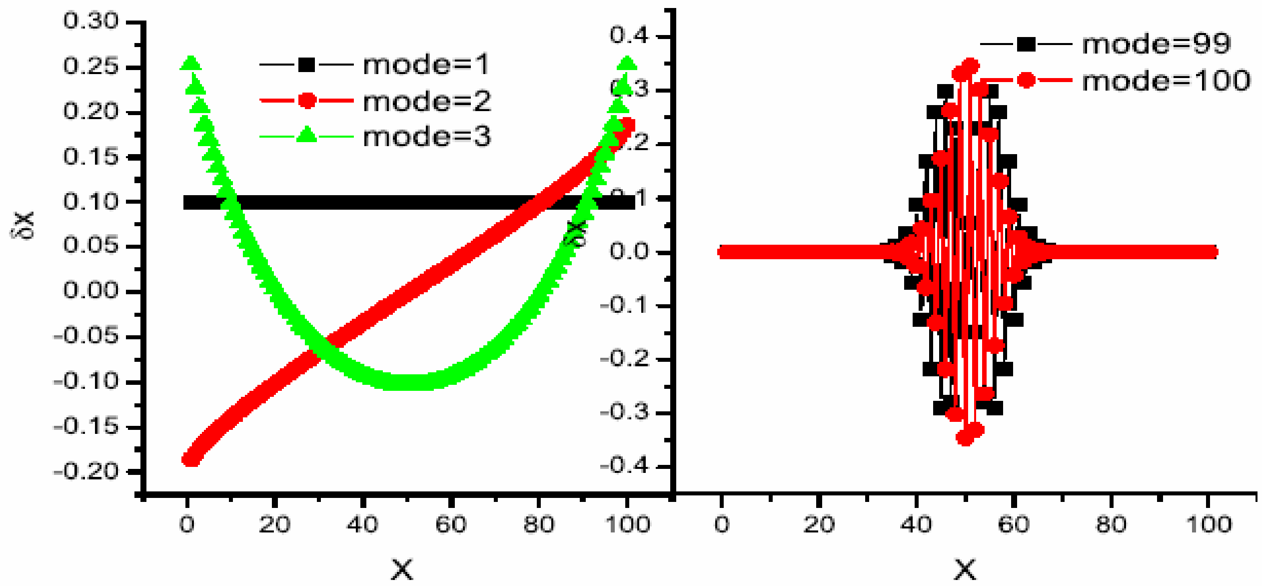
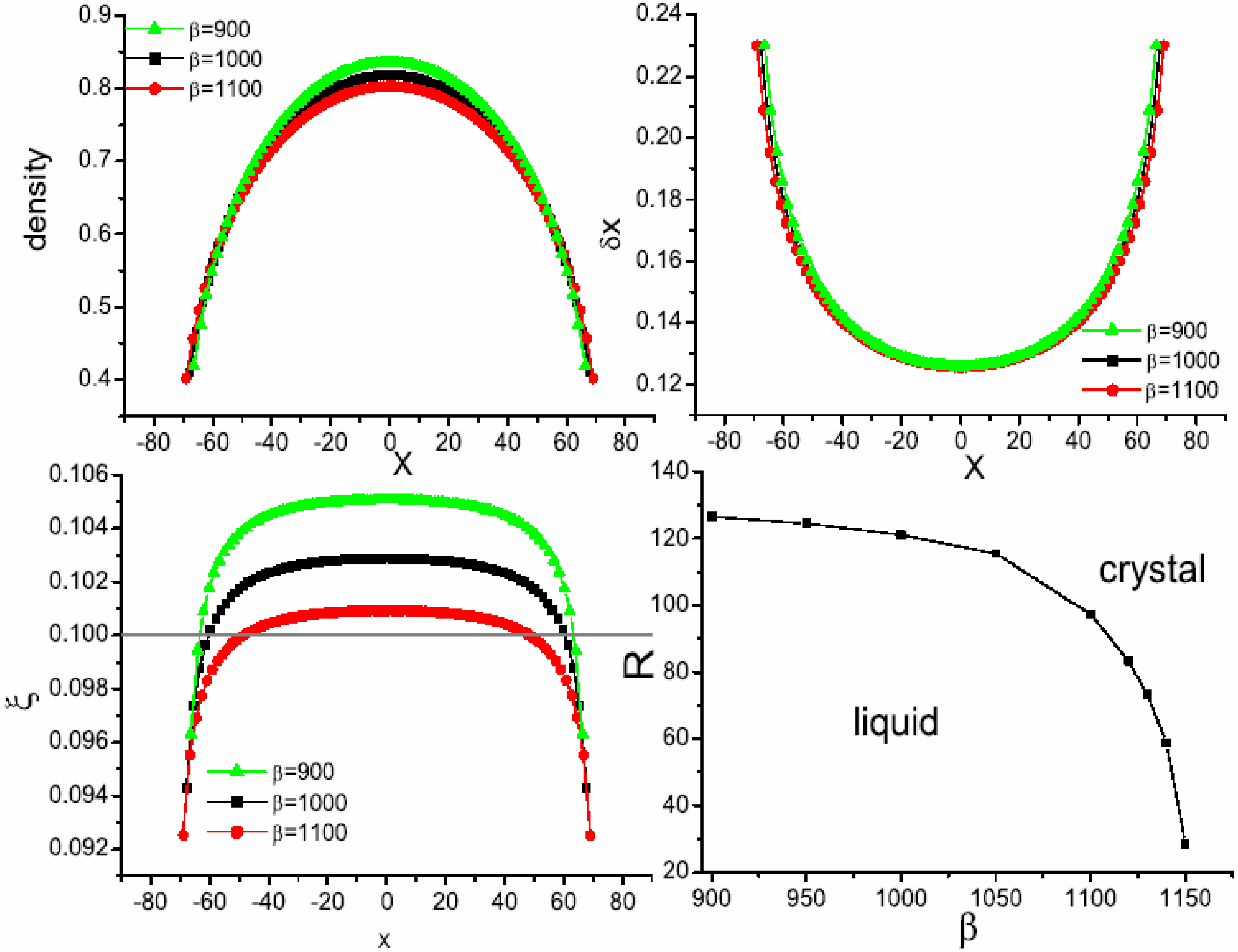


FIG. 2: (a) Phonon frequency of N=100 with different β. (b) $\beta = 1$ with different N.



According to Lindemann condition, the system is in crystal phase if

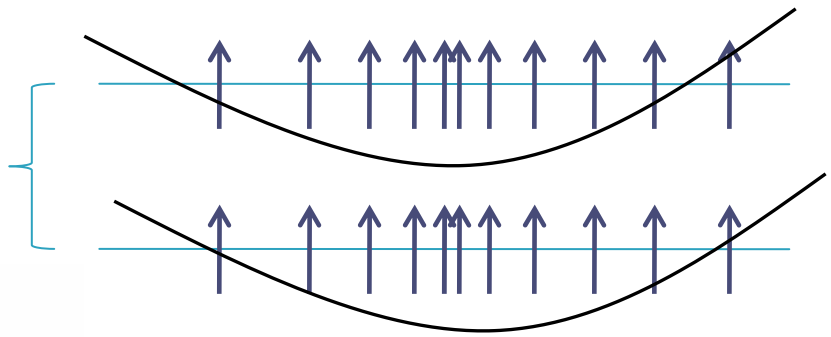
$$\xi_i \equiv \frac{\langle \delta x_i \rangle}{d_i}$$



Two wire

$$H = T + V_{intra} + V_{inter} + V_{trap}$$

d



$$T = \sum_{\alpha=1,2} \sum_{i}^{N} \frac{p_{\alpha,i}^2}{2m},$$

$$V_{intra} = \frac{1}{2} \sum_{\alpha=1,2} \sum_{i \neq j}^{N} \frac{D^2}{|x_{\alpha,i} - x_{\alpha,j}|^3},$$

$$V_{inter} = \sum_{i,j}^{N} \frac{D^2}{[(x_{1,i} - x_{2,j})^2 + d^2]^{3/2}},$$

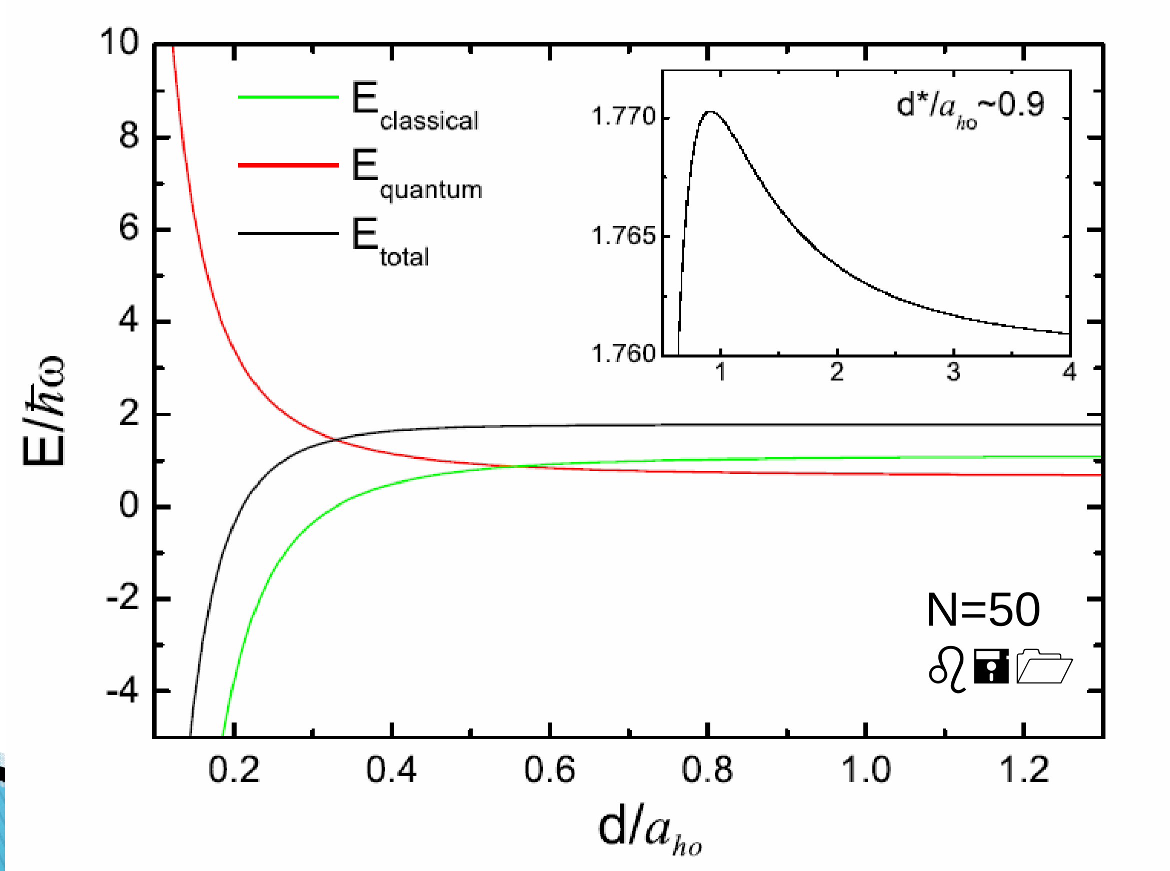
$$V_{trap} = \frac{1}{2} m\omega^2 \sum_{\alpha=1,2} \sum_{i}^{N} x_{\alpha,i}^2.$$

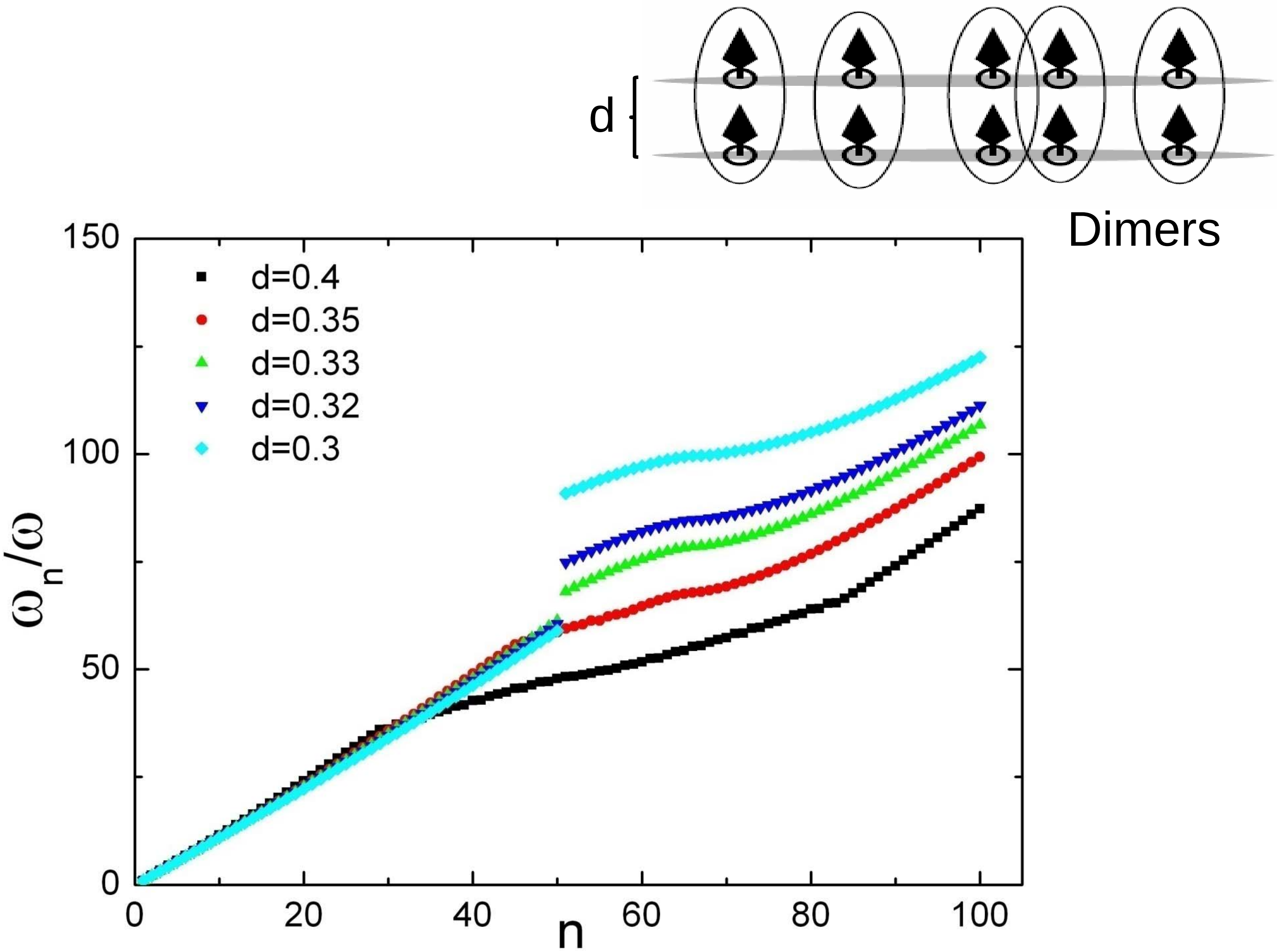
Quantum repulsion

$$H = E_{classical} + E_{quantum}$$

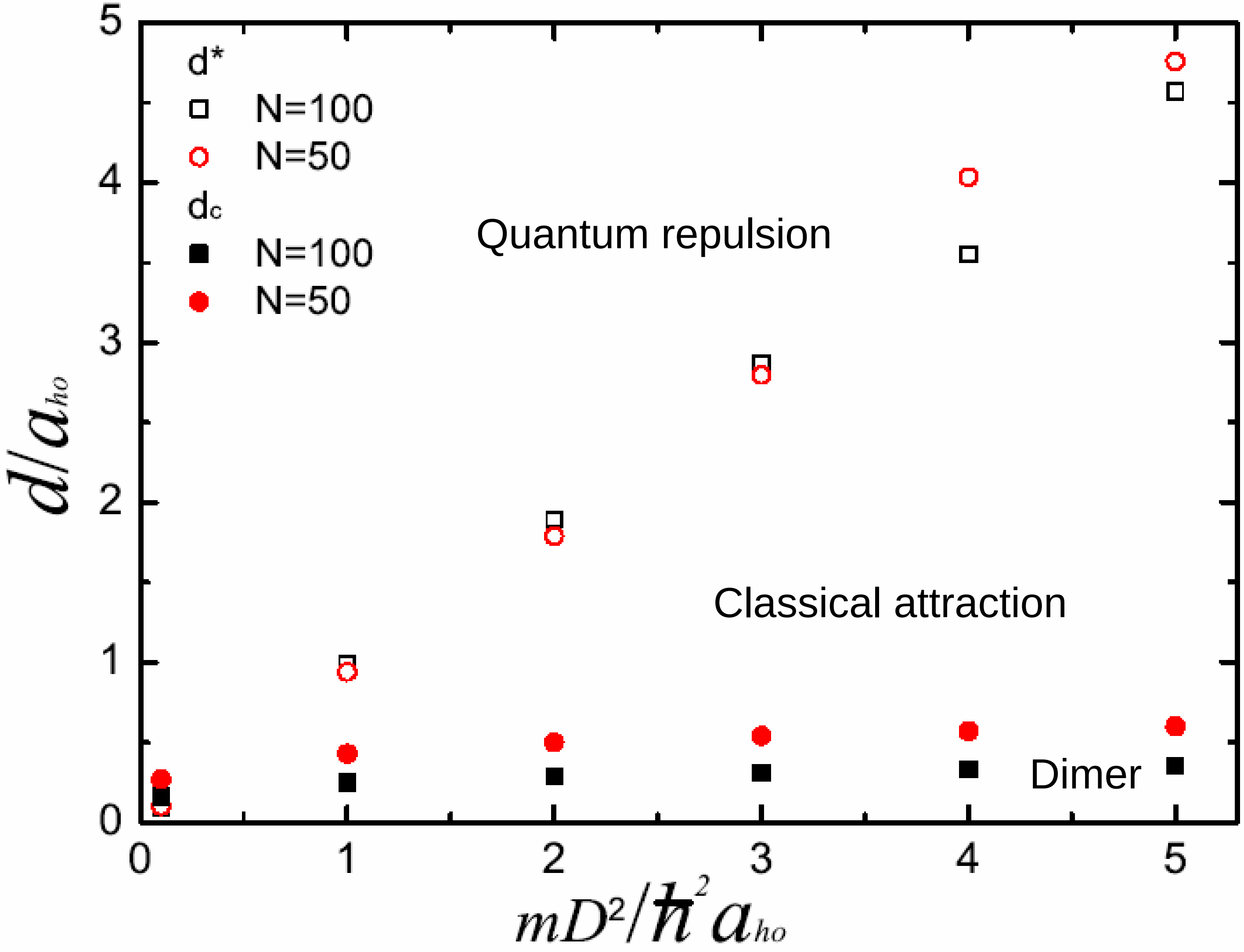
$$E_{classical} = V_{d-d}(x_0) + V_{trap}(x_0)$$

$$E_{quantum} = \sum_k \hbar\omega_k (n + 1/2)$$

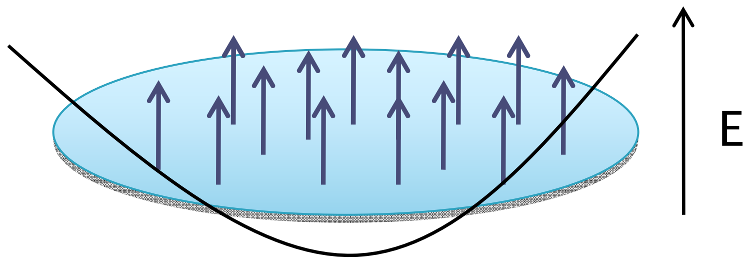




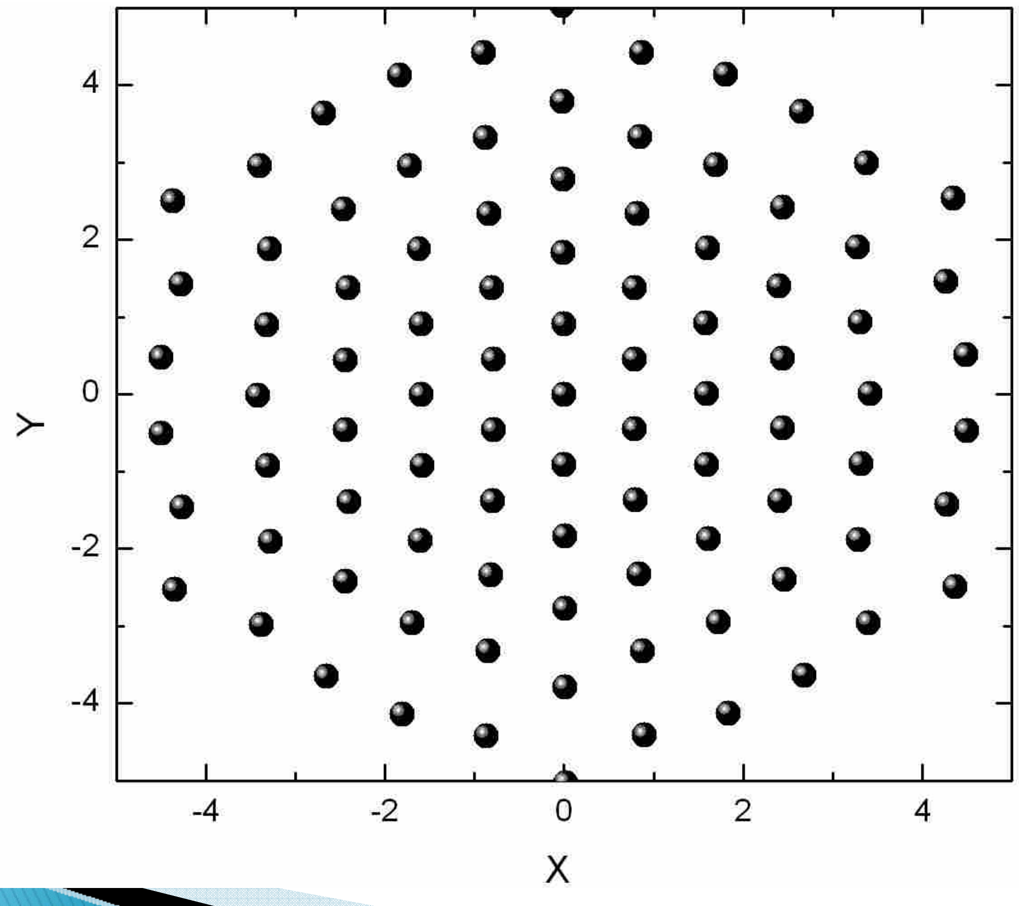
d
Dimers
d=0.4
d=0.35
d=0.33
d=0.32
d=0.3
ω_n/ω
n
0
50
100
150
0
20
40
60
80
100


d*
N=100
N=50
dc
N=100
N=50
Quantum repulsion
Classical attraction
Dimer
d/a_ho
mD²/ħ² a_ho
0
1
2
3
4
5

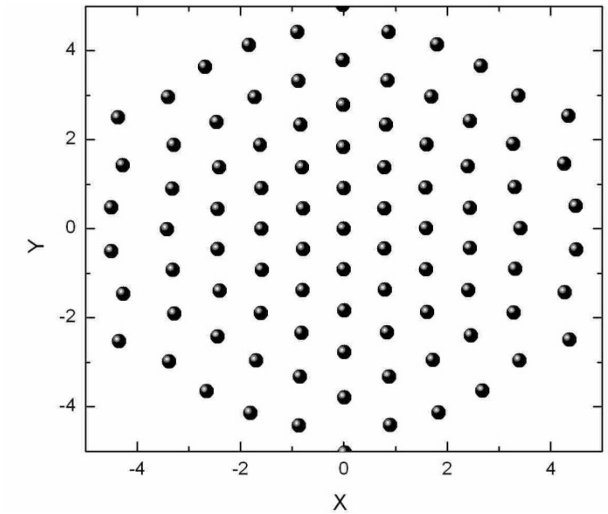
2D dipolar system



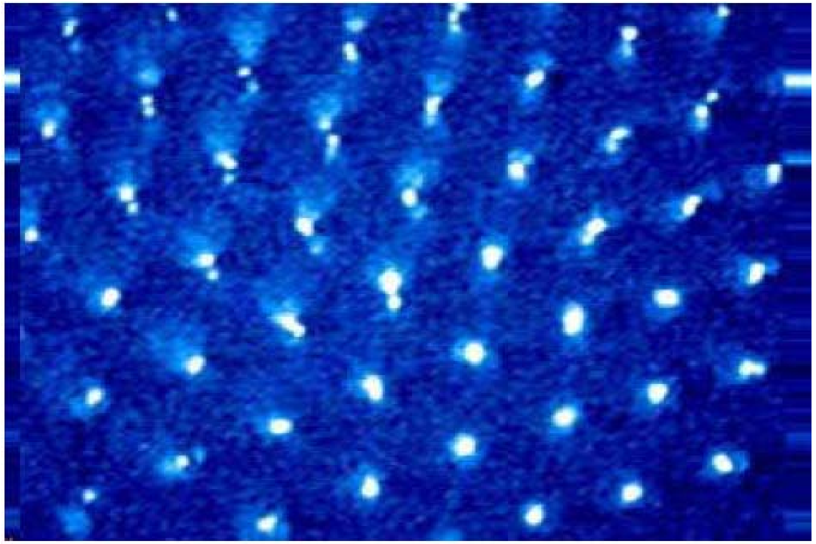
Equilibrium position



Wigner crystal

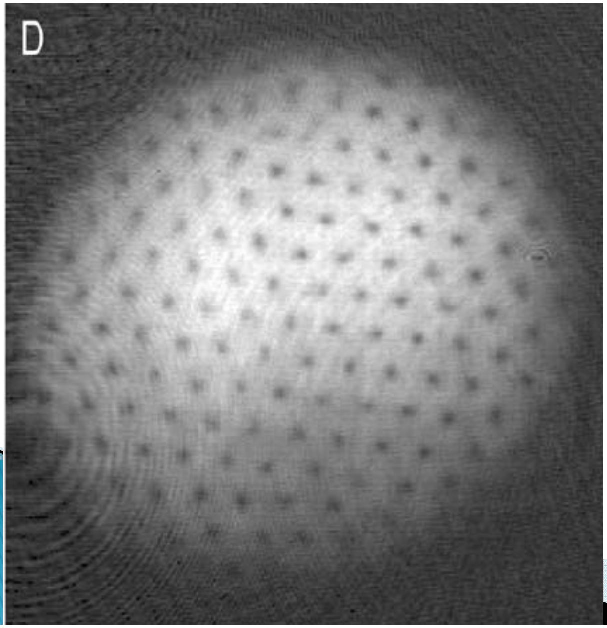


Dusty plasma

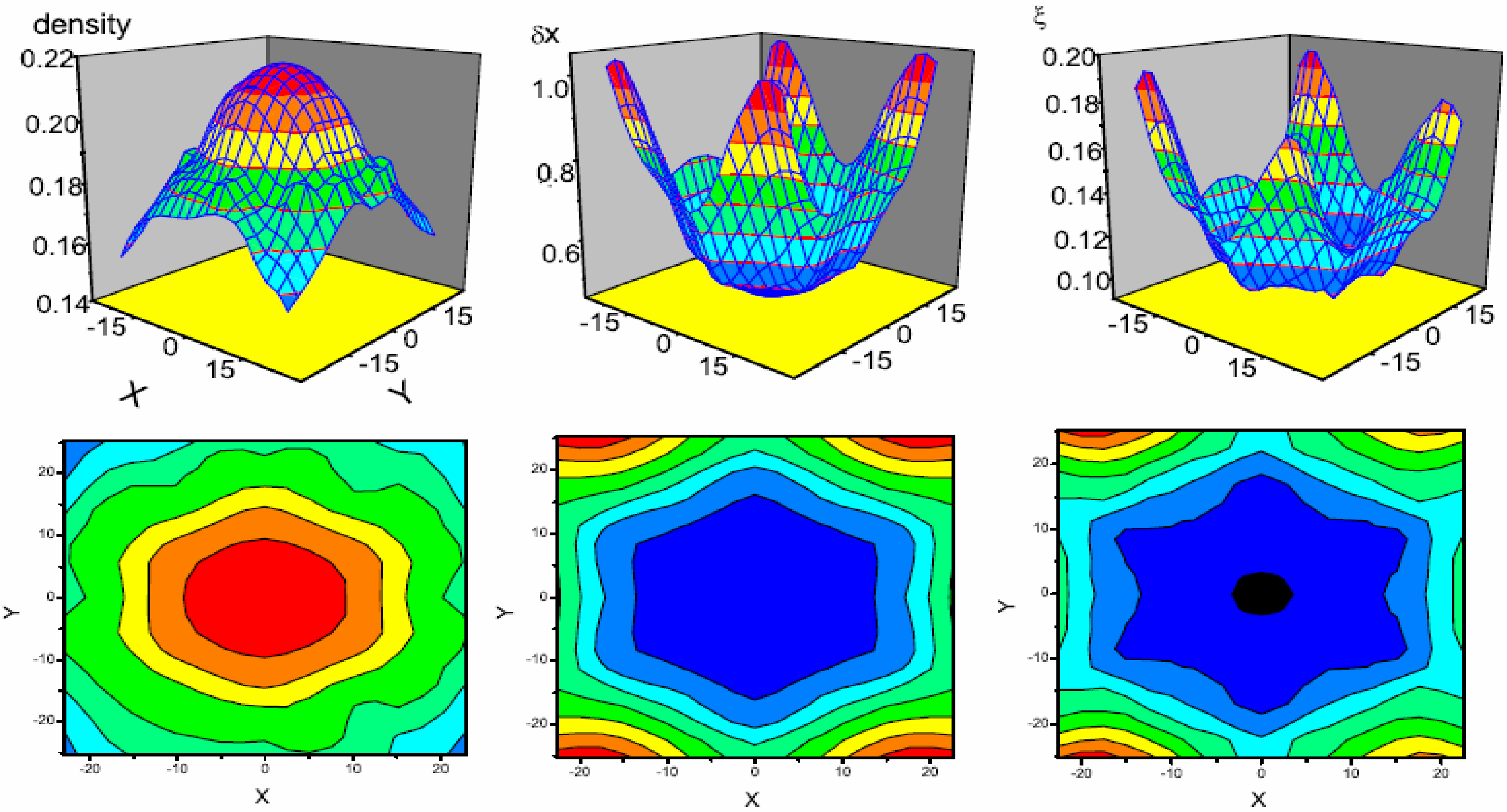


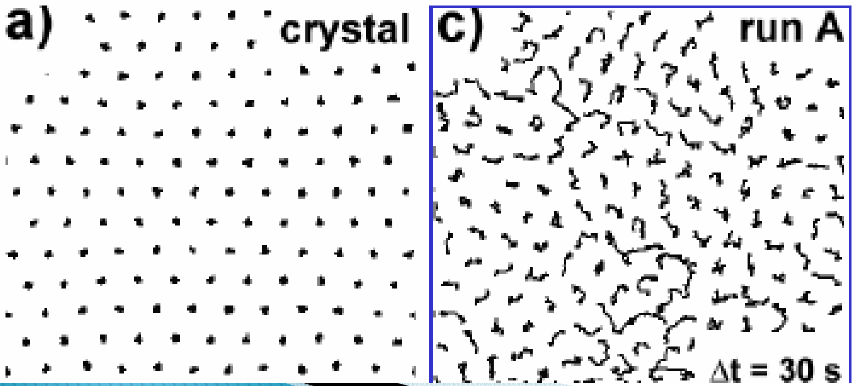
J.H.Chu and Lin I, PRL(1994)
Lin I et al, Science (1996)

BEC vortex



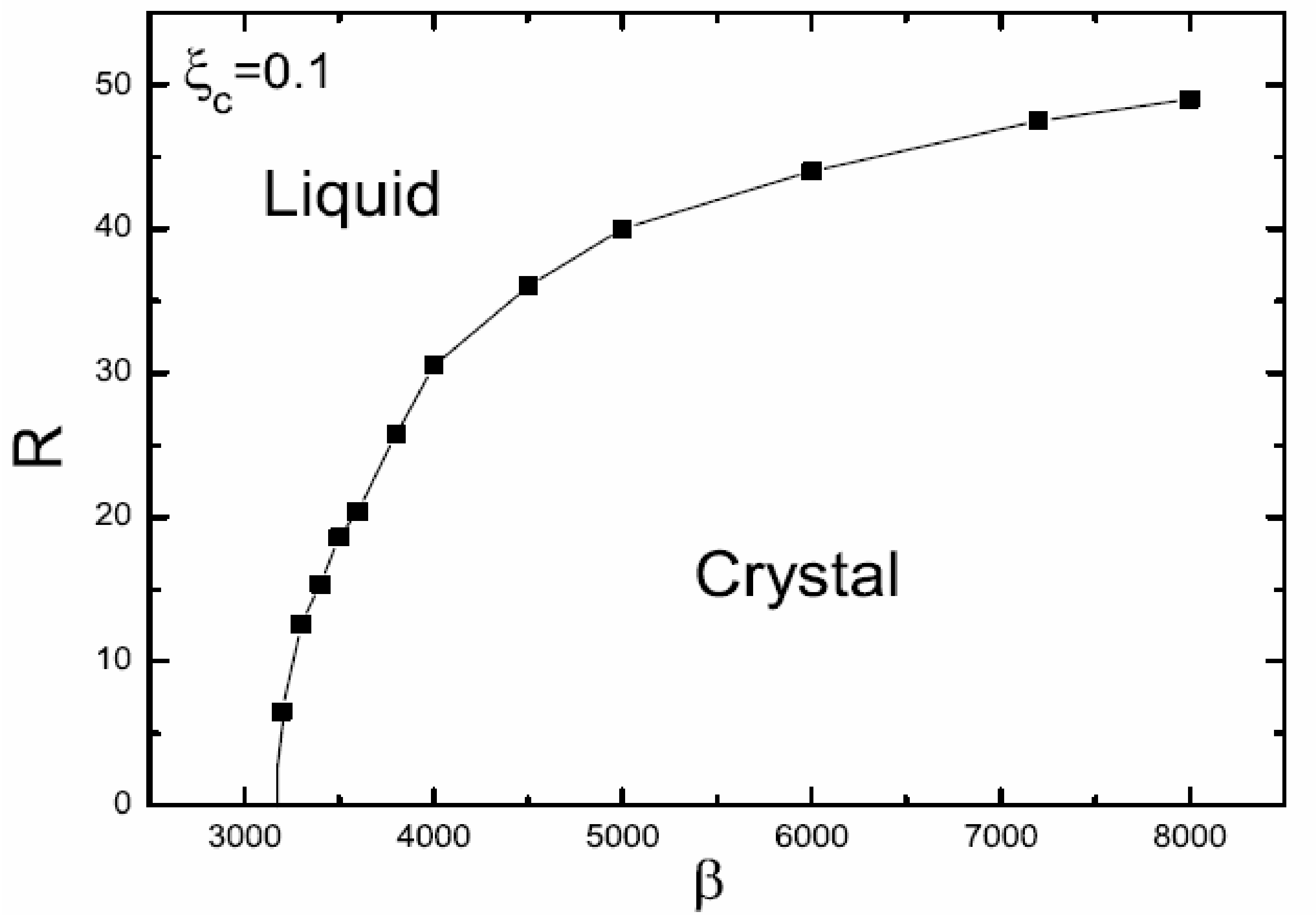
J. R. Abo-Shaeer, C. Raman,
J. M. Vogels, W. Ketterle , Science (2001)

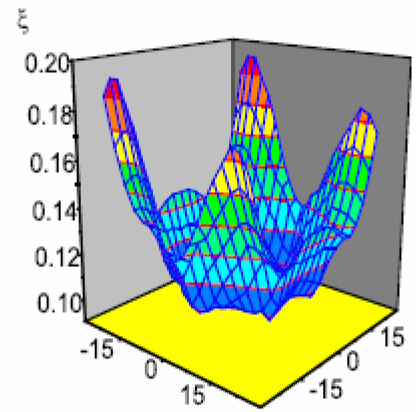


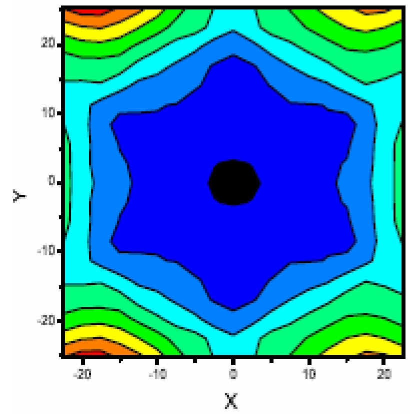


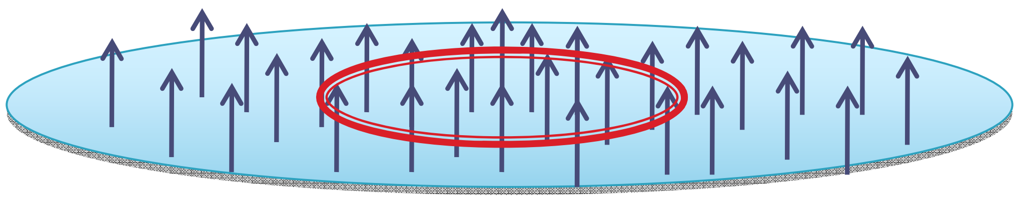
Lindemann's condition

Lin I et al, Science(1996)

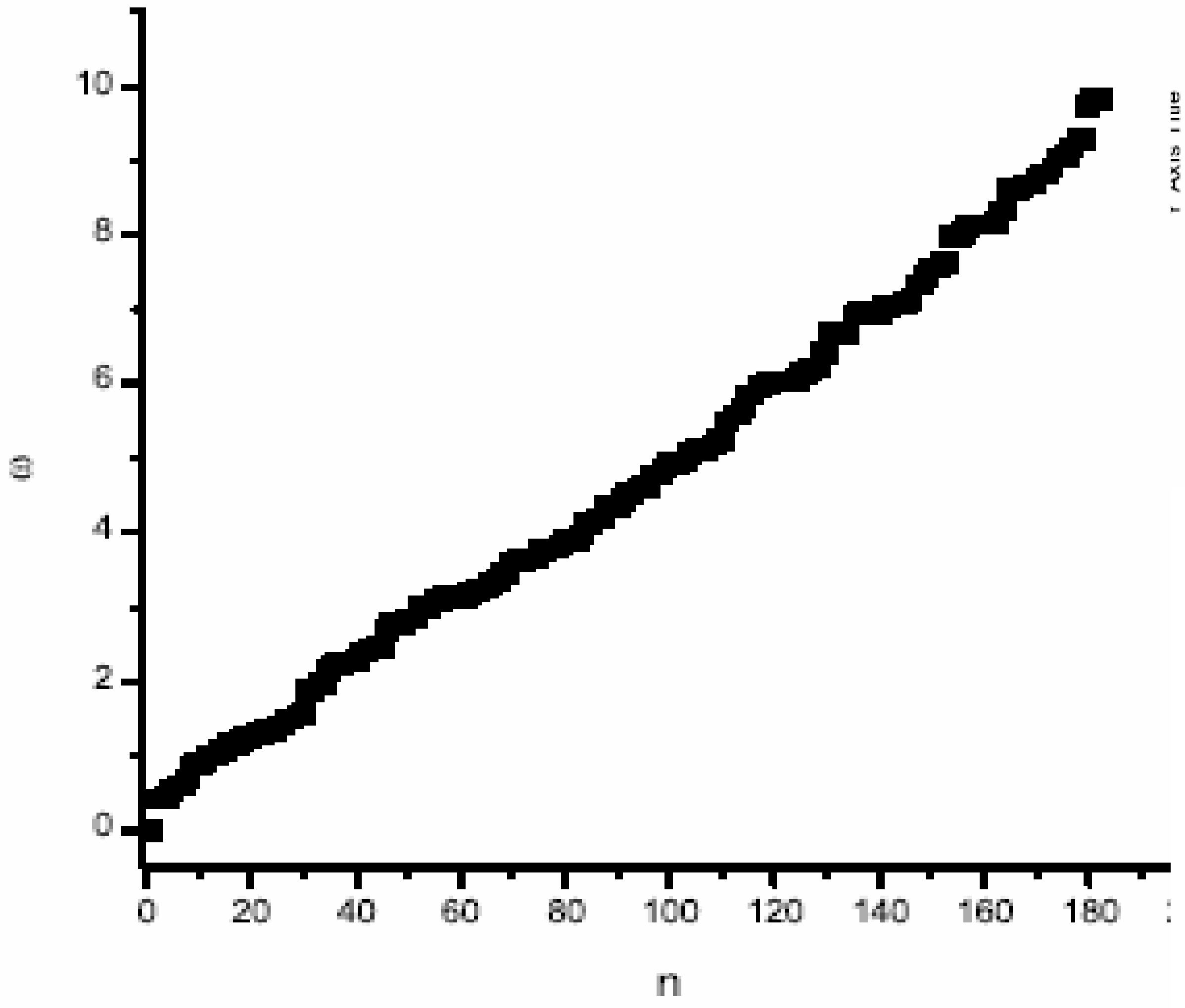

ξc=0.1
Liquid
Crystal
R
β
0
10
20
30
40
50
3000
4000
5000
6000
7000
8000

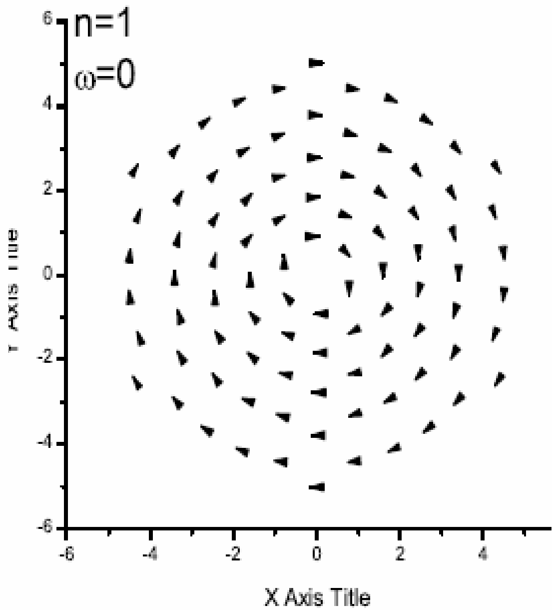

ξ
0.20
0.18
0.16
0.14
0.12
0.10
-15
0
15
15
0
-15

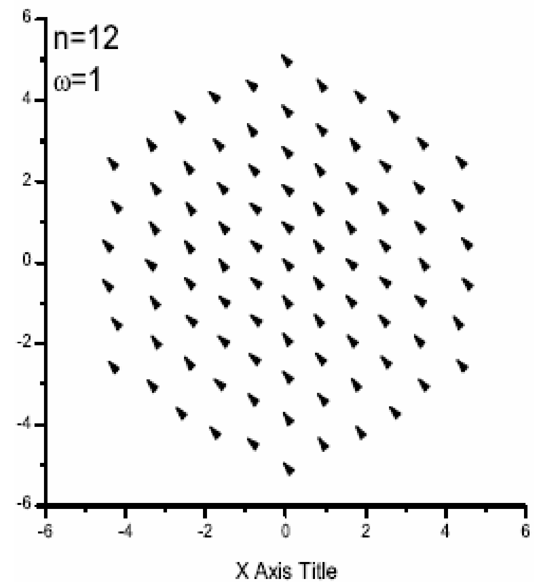

Y
X
20
10
0
-10
-20
-20
-10
0
10
20



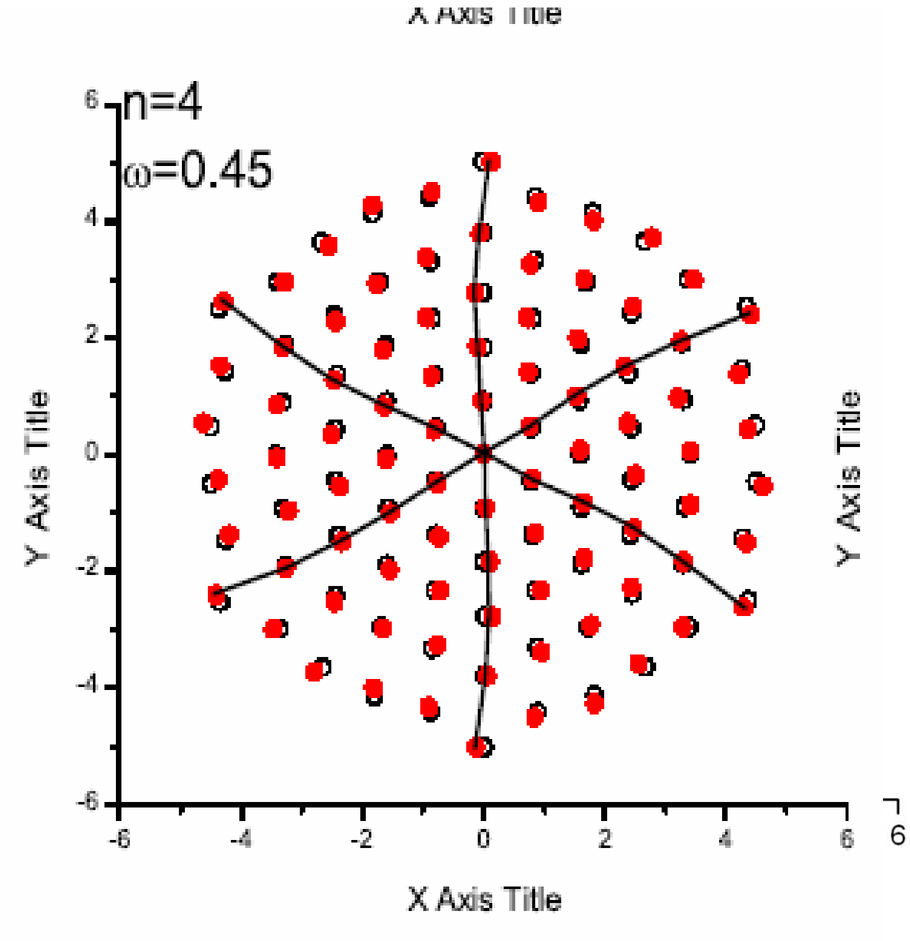
Phonon modes

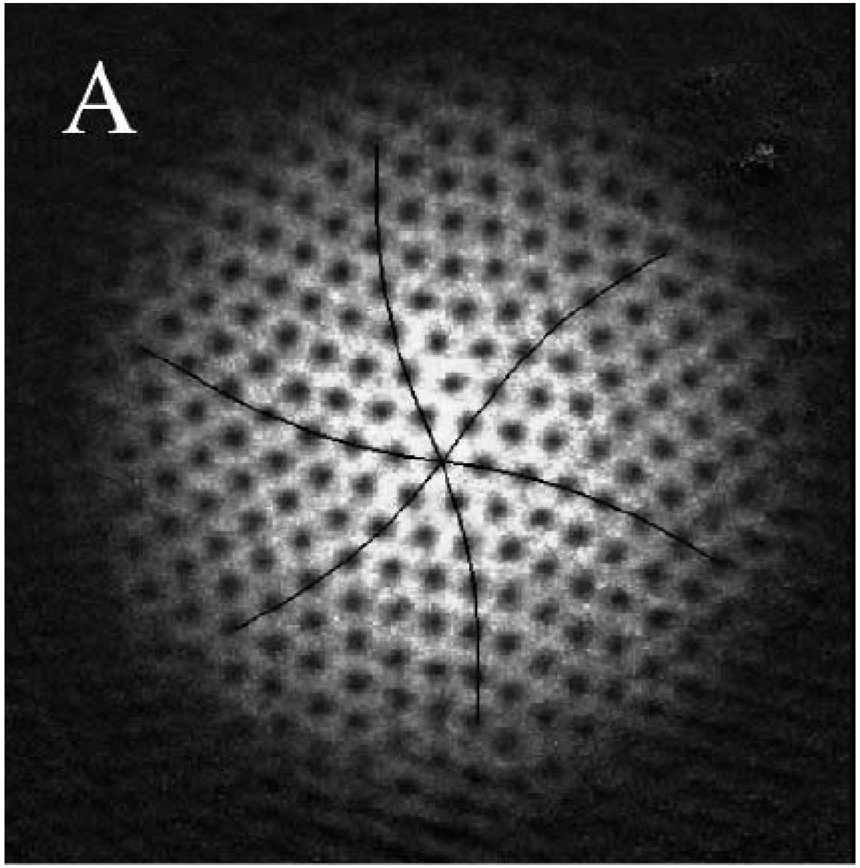






Low energy excitation: Tkachenko mode

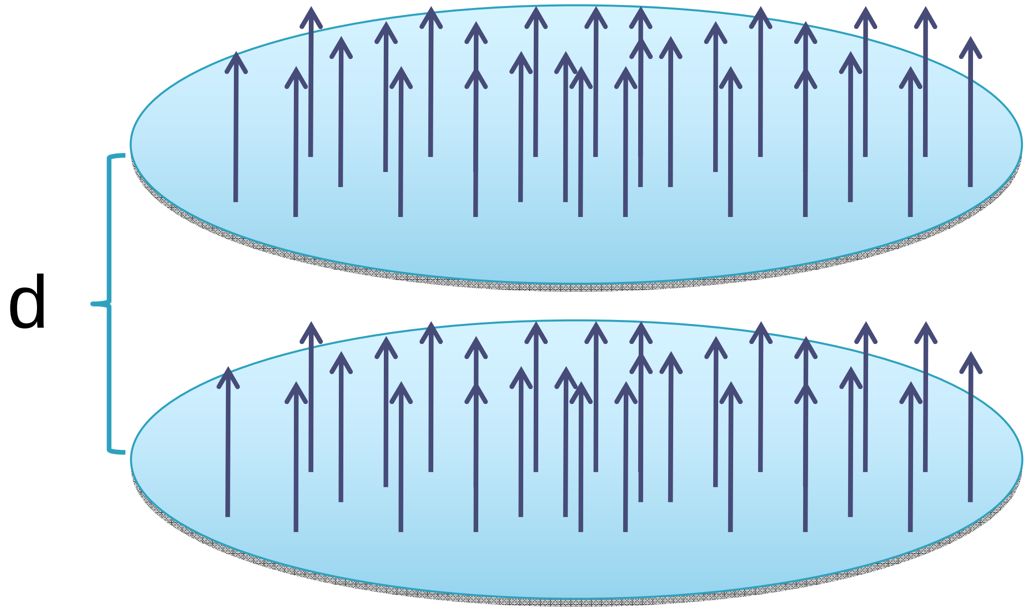




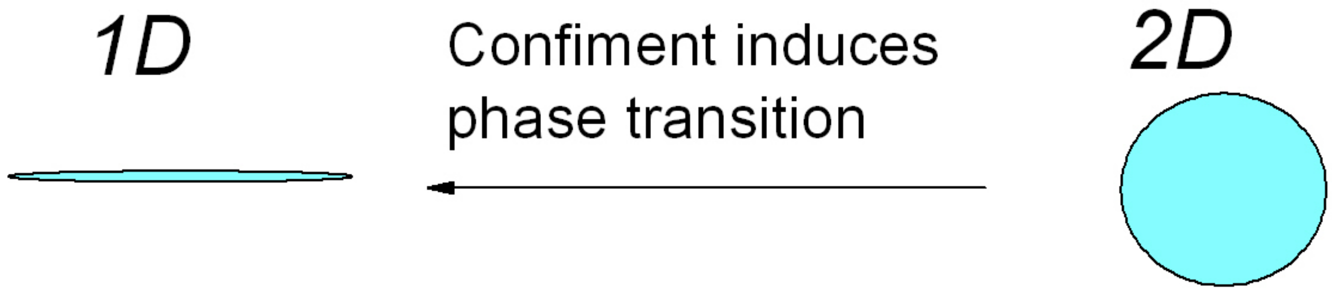
Coddington, Engels, Schweikhard and Cornell, PRL(2003)

Future study

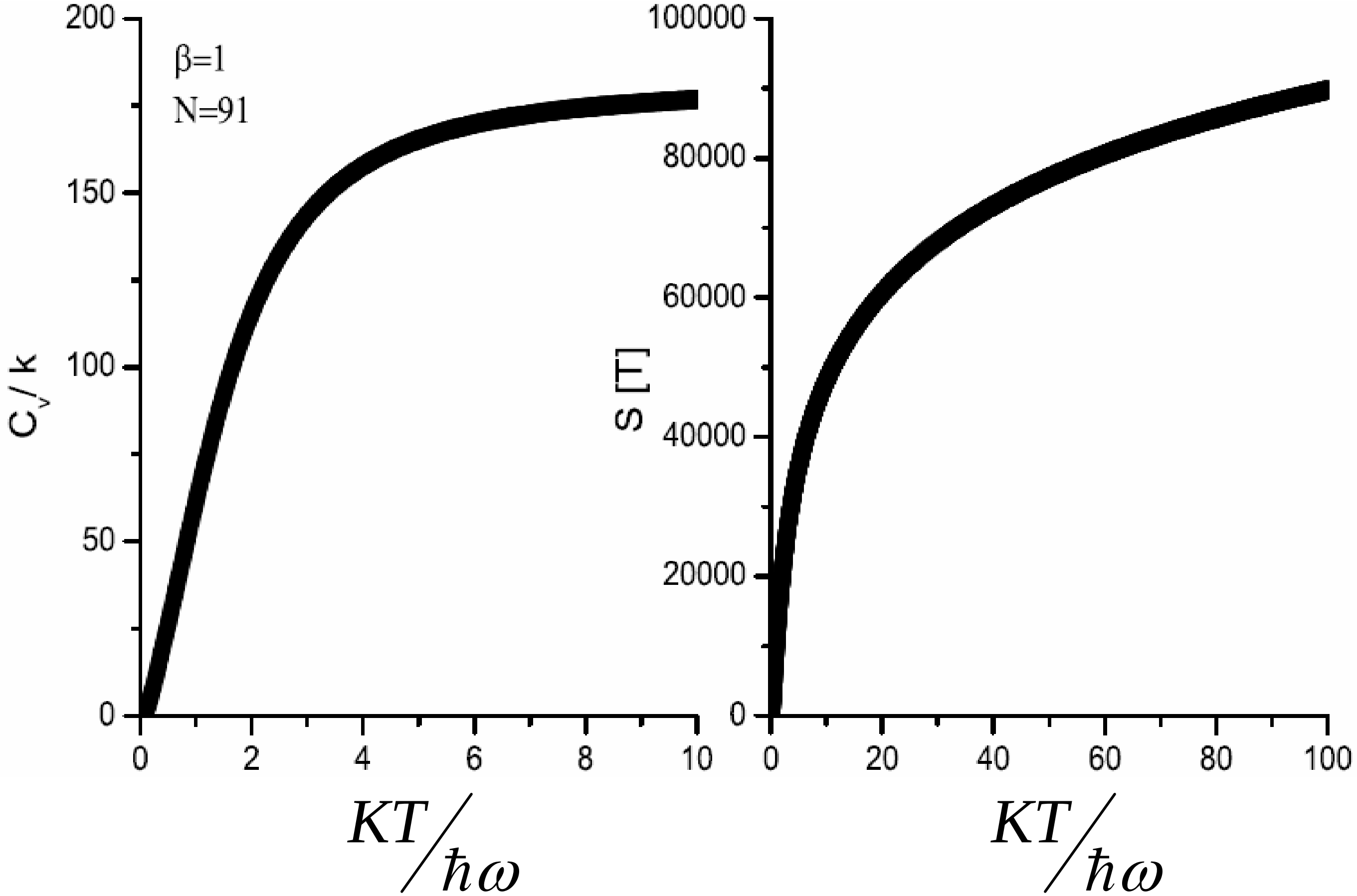
1. Two layers



2. From 2D to 1D ?



Thank you



β=1
N=91
C_v / k
200
150
100
50
0
0
2
4
6
8
10
KT/ħω
S [T]
100000
80000
60000
40000
20000
0
0
20
40
60
80
100
KT/ħω