RS model as a Framework for Flavor Physics

John N Ng

Triumf

NTHU Taiwan , 2008

Outline

- Brief introduction of Extra Dimensional (XD) Models and the RS1 model
  - (A) XD with flat internal space
    - (a) Scalar fields in higher dimenionaltheories
    - (b) Kaluza-Klein (KK) decomposition
    - (c) Boundary conditions
  - (B) Warped internal space
    - (a) Scalar fields
    - (b) Fermions in RS location ,location, location
    - (c) Gauge Bosons
    - (d) The need for custodial $SU(2)_R$. Bulk symmetry is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
    - (e) Quark Mass Matrices : symmetrical or asymmetrical
    - (f) KK Fermions mixing
- Some Phenomenology
- Conclusions

Intro to XD

- Compactification of internal dimensions always involve a scale R or ’volume’,V
- Two equivalent descriptions

  (A) At distances large compare to R
  4D langage is more more appropriate $\longrightarrow$ KK modes

  (B) At distances smaller than R
  Higher dim language is better $\longrightarrow$ Takes into account of all KK modes
- Use the KK langauage since it is more relevant to phenomenology.

The KK decomposition

Quantum fields in $4+n$ dimenions

$\Phi(x^\mu, y^i)(\mu = 0,1,2,3; \quad y^i$ parametrize the compact space $\quad i = 1,\cdot,n)$

We can always expand any function in any complete set of functions $f_n(y^i)$ via

$$\Phi(x^\mu, y^i) = \frac{1}{\sqrt{V}} \sum_n \phi^n(x^\mu) f_n(y^i)$$

$\phi^n \longrightarrow n^{th}$ KK mode

Choose f_n to be orthonormal (basis)functions

$$\langle f_n | f_m \rangle = \delta_{nm}$$

We allows us to think of ϕ^n as independent d.o.f

KK decomposition II

How to choose the basis is model dependent.
In general assume perturbation philosophy

- Understand the ’free’ part of the XD Lagrangian
- Add interactions later

Scalar Fields

The action of a free scalar field is

$$S = \int d^4x d^n y \ \frac{1}{2}(\partial_N \Phi \partial^N \Phi - M^2 \Phi^2) = -\int d^4x d^n y \frac{1}{2} \Phi(\Box + M^2)\Phi$$

and

$$\Box + M^2 = \partial_\mu \partial^\mu + \partial_i \partial^i + M^2$$

Use the eigenfunctions of the XD part

$$(\partial_i \partial^i + M^2) f_n = m_n^2 f_n$$

- This is linear partial differentail eqn we can solve
- Choose appropriate boundary conditions which is part of the definition of the theory

Scalar fields II

Use the KK expansion of Φ insert into S

$$-\frac{1}{V}\int d^4x d^n y \sum_{m,m'} \frac{1}{2}\phi^m(x^\mu) f_m(y^i) \left[f_{m'}(y^i)\partial_\mu\partial^\mu \phi^{m'}(x^\mu) + \phi^{m'}(x^\mu)(\partial_i\partial^i + M^2) f_{m'}(y^i)\right.$$

We get

$$-\frac{1}{V}\sum_{m,m'}\left(\int d^n y f_m(y^i) f_{m'}(y^i)\right)\int d^4x \frac{1}{2}\phi^m(x^\mu)(\partial_\mu\partial^\mu + m^2_{m'})\phi^{m'}(x^\mu)$$

Implies

$$\frac{1}{V}\int d^n y f_m f_{m'} = \delta_{m,m'}$$

Physics of KK decomposition

The theory can be written as

$$-\sum_n \int d^4x \frac{1}{2}\phi^n(\partial_\mu\partial^\mu + m_n^2)\phi^n$$

A free XD scalar is equivalent to an infinite tower of 4D scalars with masses m_n

- If $m_0 = 0$ we have a massless 4D field
- For 1 XD the eigenfunctions f_n are circular functions
- Need to impose boundary conditions to specify these functions.

Boundary Conditions

Illustrate by considering 5D or 1XD examples.

- Compactification on a circle S^1
  (A) Impose periodic b.c $\Phi(y) = \Phi(y + 2\pi R)$
  (B) If $M^2 = 0$ the KK masses are
  $m_n = \frac{n}{R}$
- Compactify on an interval i.e y extends from $y = 0$ to $y = R$. They are called fixed points where branes are situated. At the fixed points
  (A) Dirichlet b.c
  $$f_n = 0$$
  (B) Neumann b.c.
  $$\partial f = 0$$
  (C) or a mixture

Bulk Gauge Fields

Use 5D QED compactify on a circle as an example to bring out the physics

- The action is

$$S = \int d^4x \int_0^{2\pi R} -\frac{1}{4} F_{MN} F^{MN} + gaugefixing; \quad F_{MN} = \partial_M A_N - \partial_N A_M$$

- KK expand the gauge field as in the scalar case. Look at the A_μ term

$$A_\mu \sim \sum_n A_\mu^n(x) f_n(y)$$

- Becuase of bulk gauge invariance the zero mode is by

$$\partial_y f_0(y) = 0$$

- The zero mode has a constant profile $\longrightarrow$ 4D gauge invaraince.
- Other KK modes similar to scalars

Bulk Fermions

Again consider a minimal 5D Lagrangian for a fermion Ψ. The action is

$$S = \frac{i}{2} \int d^5x \left(\bar{\Psi}\Gamma^M \partial_M \Psi - \partial_M \bar{\Psi}\Gamma^M \Psi - m\bar{\Psi}\Psi \right)$$

Notice a bulk mass term is included and sign of m is not determine.

- The bulk field Ψ is a 4-component Dirac spinor.
- Decompose under 4D Lorents subgroup into a pair of Weyl spinors

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$$

- Bulk equation of motion from δS gives

$$-i\bar{\sigma}^\mu \partial_\mu \chi - \partial_y \bar{\psi} + m\bar{\psi} = 0$$

$$-i\sigma^\mu \partial_\mu \bar{\psi} + \partial_y \chi + m\chi = 0$$

Bulk Fermions II

- KK expand the 5D wavefunctions

$$\chi = \sum_n g_n(y)\chi_x(x)$$
$$\bar{\psi} = \sum_n f_n(y)\bar{\psi}_n(x)$$

- These fermions obey the 4D Dirac equations

$$-i\bar{\sigma}^\mu \partial_\mu \chi_n + m_n \bar{\psi}_n = 0$$
$$-i\sigma^\mu \partial_\mu \bar{\psi}_n + m_n \chi_n = 0$$

- Substituting back into 5D EOM we get a set of coupled eigenvalue equations

$$\partial_y g_n + m g_n - m_n f_n = 0$$
$$\partial_y f_n - m f_n + m_n g_n = 0$$

Bulk Fermions Boundary Conitions

- Consider the fix point $y = 0$ and the field takes the b.c. Dirichlet

$$\chi = 0|_0$$

- The other component must satify

$$(\partial_y + m)\chi = 0$$

  i.e. the Neumann condition

- For the zero mode the eigen equations decouple:

$$\partial_y g_0 + m g_0 = 0$$
$$\partial_y f_0 - m f_0 = 0$$

- If we choose $\psi|_0 = 0 \to f_0 = 0$ then

$$g_0 = \sqrt{\frac{1 - e^{-2mL}}{2mL}}\, e^{-my} \quad (L = 2\pi R)$$

Introduction to the Randall-Sundrum Model

- There are more than 4 dim. Indeed RS assumes $1+4$ dim with a warp or conformal metric. AdS
- 5D interval is given by

$$ds^2 = G_{AB}dx^A dx^B = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2$$

- Two branes are located at $\phi = 0$ (UV) and $\phi = \pi$ (IR).
- Metric is

$$-\pi \le \phi \le \pi\,,\ \sigma \equiv kr_c|\phi|\,,$$

$$G_{AB} = \begin{pmatrix} e^{-2\sigma}\eta_{\mu\nu} & 0 \\ 0 & -r_c^2 \end{pmatrix}\,,\ G^{AB} = \begin{pmatrix} e^{+2\sigma}\eta^{\mu\nu} & 0 \\ 0 & -\frac{1}{r_c^2} \end{pmatrix}$$

RS model as 5D field theory

- The action is generalized to 5D e.g. the bulk scalar field we have

$$
\begin{aligned}
S_5 &= \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G}(G^{MN}\partial_M\Phi\partial_B\Phi - m^2\Phi^2) \\
&= \frac{1}{2}\int d^4x \int_{-\pi}^{\pi} r_c d\phi \left[e^{-2\sigma}\eta^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{r_c^2}\Phi\partial_\phi\left(e^{-4\sigma}\partial_\phi\Phi\right) - m^2 e^{-4\sigma}\Phi^2\right]
\end{aligned} \quad (1)
$$

- Integrate over ϕ to give a 4D effective theory
- Do KK decomposition.

$$\Phi(x,\phi) = \frac{e^{\sigma}}{\sqrt{r_c}}\sum_n \Phi_n(x) y_n(\phi)$$

One recovers the canonical 4D scalar field $[\Phi_n] = 1$ and the KK eigen-mode $[y_n] = 0$. y_n is normalized by

$$\int_{-\pi}^{\pi} d\phi y_n(\phi) y_m(\phi) = \delta_{mn}$$

RS:Scalars

- The $y_n(\phi)$ satisfies the eigenvalue eqn

$$-\frac{e^{\sigma}}{r_c^2}\partial_\phi\left(e^{-4\sigma}\partial_\phi(e^{\sigma}y_n)\right)+m^2e^{-2\sigma}y_n=m_n^2y_n$$

- The 4D effective action becomes

$$S=\frac{1}{2}\sum_n\int d^4x\left[\eta^{\mu\nu}\partial_\mu\Phi_n\partial_\nu\Phi_n-m_n^2\Phi_n^2\right]$$

- $m_n=0$ are the zero modes. Identify them as SM fields.
- The solutions are exponentials

$$y_0=\frac{e^{kr_c\phi}}{N_0}\left[e^{-\nu kr_c\phi}+b_0e^{+\nu kr_c\phi}\right]\,,\ (\nu\equiv\sqrt{4+m^2/k^2})$$

More RS

- After integrating out the extra the dimension the 4D effective action is

$$S = \frac{1}{2} \sum_n \int d^4x \, [\eta^{\mu\nu} \partial_\mu \Phi_n \partial_\nu \Phi_n - m_n^2 \Phi_n^2]$$

- For $m_n \neq 0$ the solutions are given by Bessel functions of order $\nu = \sqrt{4} + \frac{m^2}{k^2}$

$$y_n(\phi) = \frac{e^\sigma}{N_n} [J_\nu(z_n) + b_n Y_\nu(z_n)]$$

- m_n and b_n are determined by boundary conditions at $\phi = 0, \pi$. The derivatives are continuous.

Gauge Fields in RS

- Take QED as the toy model. The action is

$$S_5 = -\frac{1}{4}\int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G} F^{MN} F_M{}^N$$

- Choose the unitarity gauge $A_4 = 0$ and KK decompose the gauge field

$$A_\mu(x,\phi) = \frac{1}{\sqrt{r_c}} \sum_n A_\mu^n(x)\chi_n(\phi)$$

with a normalization

$$\int_{-\pi}^{\pi} d\phi \; \chi_n \chi_m = \delta_{mn}$$

- The 4D Lagrangian is

$$\mathcal{L}_4 \supset +\frac{1}{2}\eta^{\mu\nu} \sum_{m,n} A_\mu^n A_\nu^m \int_{-\pi}^{\pi} d\phi \chi_m \partial_\phi \left(\frac{e^{-2\sigma}}{r_c^2} \partial_\phi \chi_n \right)$$

RS Gage Fields II

- The zero mode has a flat profile

$$\chi_0 = \frac{1}{\sqrt{2\pi}}$$

This preserves charge universality

- The solutions for KK excitaions are Bessel functions of order unity

$$\chi_n = \frac{m_n e^{\sigma}}{k N_n}[J_1(z_n) + a_n Y_1(z_n)]$$

- a_n and m_n are determined by b.c.c at the fixed points $\phi = 0, \pi$

Fermions in 5D Bulk

- 5D fermions are 4-component spinors i.e. vector-like fermions

$$\Psi(x^\mu, y) = \begin{pmatrix} \psi_R(x^\mu, y) \\ \psi_L(x^\mu, y) \end{pmatrix}$$

- The Dirac matrices in 5D are $\gamma^M = (\gamma^\mu,\, i\gamma^5)$
- Project out the L,R chiral states by boundary conditions or orbifold parities ,i.e. how the field transforms under $Z_2 : y \to -y$
-

$$\begin{pmatrix} \psi_R(x,y) \\ \psi_L(x,y) \end{pmatrix} \longrightarrow \pm \begin{pmatrix} \psi_R(x,-y) \\ -\psi_L(x,-y) \end{pmatrix}$$

Fermions in Warp Space

- 5D action for fermions is

$$\int d^4x d\phi \sqrt{G} E_a^A \bar{\Psi} \gamma^a D_A \Psi$$

where $E_a^A = diag\left(e^\sigma, e^\sigma, e^\sigma, e^\sigma, \frac{1}{r_c}\right)$

- Do the usual KK decomposition:

$$\Psi_{L,R}(x,\phi) = \frac{e^{\frac{3}{2}\sigma}}{\sqrt{r_c}} \sum_n \Psi_n^{L,R}(x) \hat{\phi}_n^{L,R}(\phi) \tag{2}$$

$$\int_{-\pi}^{\pi} d\phi \, \hat{\phi}_n^{L*}(\phi) \hat{\phi}_m^L(\phi) = \int_{-\pi}^{\pi} d\phi \, \hat{\phi}_n^{R*}(\phi) \hat{\phi}_m^R(\phi) = \delta_{mn}$$

- The profile of the wavefunction is controlled by $m = ck$. Enters into the order of Bessel fn.

Bulk Fermions II

- The equations are

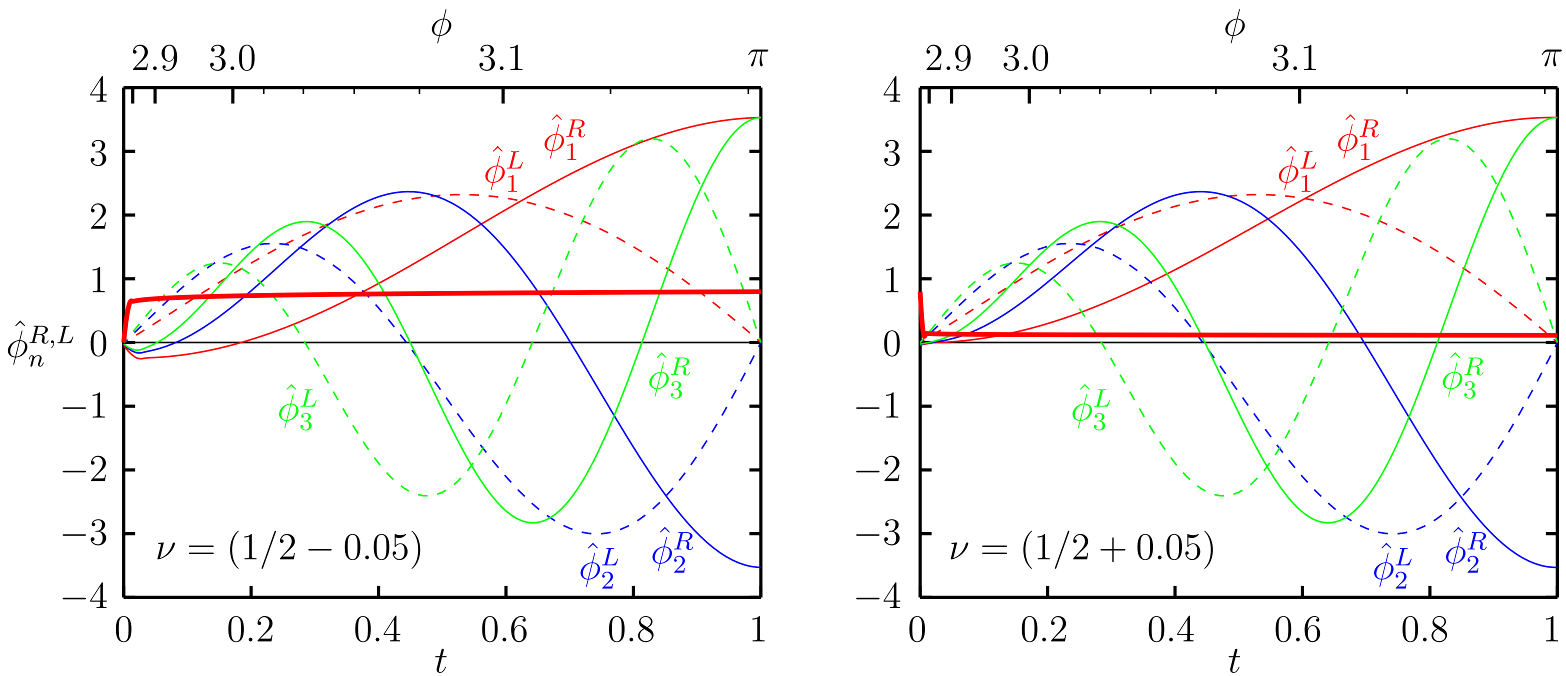
$$\left(m - \frac{k}{2} + \frac{1}{r_c}\partial_\phi\right)\hat{\phi_n^R} = m_n e^\sigma \hat{\phi_n^L}$$
$$\left(m + \frac{k}{2} - \frac{1}{r_c}\partial_\phi\right)\hat{\phi_n^L} = m_n e^\sigma \hat{\phi_n^R} \tag{3}$$

- The zero modes which we identify as SM fermions $c = \nu \equiv \frac{m}{k}$

$$\hat{\phi}_0^L = N_L^0 e^{kr_c\phi(1/2+\nu)}\,,\; N_L^0 = \sqrt{\frac{kr_c(\nu+1/2)}{e^{2kr_c\pi(\nu+1/2)}-1}}$$
$$\hat{\phi}_0^R = N_R^0 e^{kr_c\phi(1/2-\nu)}\,,\; N_R^0 = \sqrt{\frac{kr_c(1/2-\nu)}{e^{2kr_c\pi(1/2-\nu)}-1}} \tag{4}$$

- Since both solutions are Z_2 even at $\phi = 0$, only one of the two is allowed by the Z_2.
- The RH chiral zero mode lives near the UV (IR) brane if $\nu > 1/2 (\nu < 1/2)$.
- LH zero mode resides close to UV (IR) brane for $\nu < -1/2 (\nu > -1/2)$

Profiles of bulk fermions



The thick red lines are the zero mode wave function of RH chiral fermion.

$$t \equiv e^{-kr_c(\pi-\phi)}$$

Fermion Masses in RS

- The coefficients $c_{l,R}$ control the zero modes i.e peaks at UV or IR
- Localize the Higgs at the IR brane
- Have the zero modes i.e. the SM chiral fermions localize near UV brane
- The overlap after SSB will be very samll
- No need to fine tune Yukawa's.
- Quark masses are naturally small.
- If all the fermions both LH doublet and RH singlets are localized near UV then t-quark comes out too light
- t-quark or (t_L, b_L) must not be too far from IR brane

Quark Masses in RS

- The quark masses are given by

$$(M_f^{RS})_{ij} = v_W \frac{\lambda_{5,ij}^f}{kr_c\pi} f_L^0(\pi, c_{f_i}^L) f_R^0(\pi, c_{f_j}^R) \equiv v_W \frac{\lambda_{5,ij}^f}{kr_c\pi} F_L(c_{f_i}^L) F_R(c_{f_j}^R)\,, \qquad f = u,\, d\,,$$

where the label f denotes up-type or down-type quark species. $v_w = 174$ GeV.

$$f_{L,R}^0(\phi, c_{L,R}) = \sqrt{\frac{kr_c\pi(1 \mp 2c_{L,R})}{e^{kr_c\pi(1\mp 2c_{L,R})} - 1}}\, e^{(1/2 \mp c_{L,R})kr_c\phi}$$

where the upper (lower) sign applies to the LH (RH) zero mode

- The Yukawa couplings λ_{ij} are not necessarily symmetric in i, j
- $f_{L,R}$ shows that the masses are control by values of $c_{L,R}$
- The task is to configurations that fits the CKM matrix.
- Added bonus : both LH and RH quark configurations are given for each solution
- Both LH and RH rotations are given for each solution
- In the SM only LH rotations are detectable. $V_{CKM} = V_L^{u\dagger} V_L^d$.

General Configurations

- In general quark mass matrices are not symmetrical in RS. Several configurations found. One example:

$$
\begin{aligned}
c_Q &= \{0.634, 0.556, 0.256\}\,, \\
c_U &= \{-0.664, -0.536, 0.185\}\,, \\
c_D &= \{-0.641, -0.572, -0.616\}\,.
\end{aligned} \tag{5}
$$

- The u and d quark mass matrices (at TeV scale)

$$
\langle |M_u| \rangle = \begin{pmatrix} 8.97\times 10^{-4} & 0.040 & 0.767 \\ 0.01 & 0.554 & 8.69 \\ 0.16 & 9.06 & 142.19 \end{pmatrix}, \quad \langle |M_d| \rangle = \begin{pmatrix} 0.0019 & 0.017 & 0.0044 \\ 0.022 & 0.196 & 0.050 \\ 0.352 & 3.20 & 9.813 \end{pmatrix},
$$

where we have used $ke^{-kr_c\pi} = 1.5$ TeV.

RS Quark Masses contd

- The CKM matrix elements for the above

$$|V_{us}^L| = 0.16(14)\,, \qquad |V_{ub}^L| = 0.009(11)\,, \qquad |V_{cb}^L| = 0.079(74)\,,$$
$$|V_{us}^R| = 0.42(24)\,, \qquad |V_{ub}^R| = 0.12(10)\,, \qquad |V_{cb}^R| = 0.89(13)\,, \tag{6}$$

- Note the RH rotations are larger than the LH ones.
- Appears to be true from the numerical searches we found
- How to test it?

Symmetrical Mass Matrices in RS

- Most of the ’constructions’ start from conjecture assuming that they are symmetrical
- Put zeros (1 to 3) in appropriate places to fit CKM and the observed mass heirarchies.
- Can RS accomodate these without fine tuning the Yukawa couplings
- Only ONE texture zero structures are allowed.
- By construction $U_L = U_R$

All is not well

- The main problem is that the new KK modes will modify EWPT
- The S, T parameters will receive tree level corrections
- It is known that $\rho = 1$ is protected by a custodial $SU(2)$symmetry
- Promote that to a bulk gauge symmetry.
- Tree level KK gauge effects are suppressed
- The gauge symmetry is now $SU(2)_L \times SU(2)_R \times U(1)_X$
- Take $X = B - L$

Custodial RS model

- Break $SU(2)_R \to U(1)_R$ by orbifold b.c.

$$\begin{array}{lcc} & UV & IR \\ \tilde{W_\mu}^{1,2} & - & + \\ other gauge fields & + & + \end{array}$$

- $U(1)_R \times U(1)_X \to U(1)_Y$ by vev on UV brane. We have a Z' and B_μ

$$Z'_\mu = \frac{g_5 \tilde{W_\mu}^3 - g'_5 \tilde{B_\mu}}{\sqrt{g_5^2 + g_5'^2}}$$

and

$$B_\mu = \frac{g'_5 \tilde{W_\mu}^3 + g_5 \tilde{B_\mu}}{\sqrt{g_5^2 + g_5'^2}}$$

- B_μ is the SM hypercharge gauge boson and broken with $SU(2)_L$ on IR brane by Higgs.

Quark Representations

- Zero modes have parity $(++)$
- Usual assignment

$$\begin{matrix} SU(2)_L & SU(2)_R \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \begin{pmatrix} \cancel{t_R} \\ b_R \end{pmatrix} \end{matrix}$$

because t_R is a zero mode and $SU(2)_R$ is broken on UV

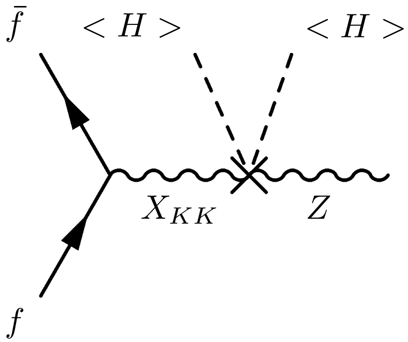
- d_R and t_R must have their own (- +) partners

$$\begin{matrix} SU(2)_L & \overbrace{\qquad\qquad SU(2)_R \qquad\qquad} \\ \begin{pmatrix} t_L \\ b_L \end{pmatrix} & \begin{pmatrix} T_R \\ b_R \end{pmatrix} \begin{pmatrix} t_R \\ B_R \end{pmatrix} \end{matrix}$$

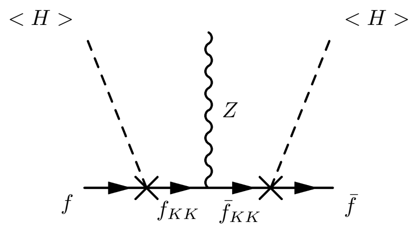
- They don't affect the quark mass matrix.

FCNC in the Minimal Constrained RS Model

- Besides the direct production of the KK Z $\gtrsim 2.5$ TeV is tree level FCNC
- FCNC $Z - Z_{KK}$ and $Z - Z'_K K$ mixing



- KK-fermion mixings



- Going to the mass basis the unitarity is broken $\longrightarrow$ FCNC

$t \to Z + jets$

- The BR is

$$Br(t \to c(u)Z) = \frac{2}{\cos^2\theta_W}\left(|Q_Z(t_L)\,\hat{\kappa}^L_{tc(u)}|^2 + |Q_Z(t_R)\,\hat{\kappa}^R_{tc(u)}|^2\right)\left(\frac{1-x_t}{1-y_t}\right)^2\left(\frac{1+2x}{1+2y}\right.$$
$$= 1.8677 \times \left(|Q_Z(t_L)\,\hat{\kappa}^L_{tc(u)}|^2 + |Q_Z(t_R)\,\hat{\kappa}^R_{tc(u)}|^2\right)$$

where $x_t = \frac{m_Z^2}{m_t^2}$ and $y_t = \frac{m_W^2}{m_t^2}$ and $Q_Z(f) = T_L^3(f) - Q\sin^2\theta_w Q_f$

- LH and RH decays are different becuase $\kappa^R > \kappa^L$ in the config we found

$$\hat{\kappa}^u_{ab} = (U^u)^\dagger_{a3}(\kappa^g_{Q^u_3} + \kappa^f_{Q^u_3})\,U^u_{3b}\,, \qquad Q^u = \{u,\, c,\, t\}$$

- $BR(t_R \to Z + c(u)_R) > BR(t_L \to Z + c(u)_L)$ by ~ 20.
- The BR is $\sim 10^{-5}$ c.f SM $\sim 10^{-13}$.
- Compare the decays in $t\bar{t}$ vs single tW channels.

Conclusions

- We have found that the RS model can have good quark mass matrices without fine tuning Yukawas
- It can accomodate symmtricall mass matrices if there is only one texture zero and not more
- For asymmetrical conf $U_R > U_L$
- Tree level FCNC best probe in $t \to Z + jets$
- BR is $\sim 10^{-5}$ makes it very exciting at the LHC
- Predicts RH decays are dominant.