# HOLOGRAPHIC PRINCIPLE AND ITS APPLICATION TO STRONGLY COUPLED SYSTEMS

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#### OUTLINE

Motivation

#### • Basics

 Applications: holographic hydrodynamics, holographic QCD, non-relativistic holography, holographic anyons

### NOTIVATION



 Many interesting physical systems are strongly interacting, and most of them are intractable even with ab initio computer simulations because of the sign problem.

• QCD without quenching, frustrated spin systems, Quantum Hall effect, High Tc superconductor, Cold atoms, graphene, etc.

• Some are beyond the category of Landau-Ginzburg theory, i.e., non-Fermi liquid.

#### NOTIVATION

- Besides some exactly solvable systems such as 2d CFTs.
- The only hope is to study some weakly coupled dual systems, and map back to the strongly coupled ones.

MOREOVER,

- The strongly coupled system is usually also strongly correlated, i.e. near critical point, so it can be approximately captured by low energy effective field theories.
- Besides, one like to study the real time dynamical properties such as transport phenomena, which is usually beyond the lattice computations.

### HOLOGRAPHIC PRINCIPLE

- General statements: A field theory can be described by a co-dim. 1 gravitational system, and vice vera.
- It is NOT a squared-root principle, it is holographic.
- It is a strong/weak duality.
- It is in perfect RG sense of QFT, as we shall see.

#### THESIMPLEST

- A 4+1 dim. AdS gravity is dual to a 3+1 dim.
   SU(N) Superconformal YM theory living on the boundary.
- The isometry of bulk geometry is the superconformal symmetry of the SYM.

#### THEORIGIN

- Therefore, the gravity and SYM are two sides of a coin---D-brane.
- Its worldvolume theory is described by low energy of open strings --- SYM.
- Its mass curves the geometry--- low energy of closed strings.
- Near the its horizon, open and closed strings decouple, and dual to each other.

#### D-BRANE



### ANTI-DE SITTER

- The AdS space is characterized by the AdS radius R, and the metric is  $ds^2 = \frac{R^2}{\tau^2}(dz^2 + \eta_{\mu\nu}dx^{\mu}dx^{\nu})$
- Its boundary (at z=0) is time-like. Instead the Minkowski space has null boundary.
- The radial scale is a RG scale of SYM, i.e. UV-IR connection.



#### THE DICTIONARY

- Kinematics:  $R^4 = g^2_{YM} N \equiv \lambda$  ---> Weak/Strong
- Large N limit to suppress loop corrections
- Gauge inv. operators of SYM (conformal dim.)
   <--> elementary bulk fields (mass)
- GKP/W prescription:

$$Z(\phi_i^{(0)}) = \exp\left(-S_{bulk}^{(on-shell)}[\phi_i]\right) = \left\langle \exp\int\phi_i^{(0)}O_i \right\rangle_{CFT}$$



#### CORRELATORS

- Solve the Klein-Gordon eqn. in AdS space
- We get  $\phi \sim z^{d-\Delta} \phi^{(0)} + z^{\Delta} \phi^{(1)} + ..., z \sim 0.$
- From on-shell bulk action, we will get powerlaw behavior.
- UV divergences  $(1/z^n)$  can be subtracted by counter terms which are covariant boundary actions, i.e.  $\int \sqrt{\gamma} F(\phi^{(0)}, R^{(0)})$

#### BLACK HOLE

- The finite temperature of boundary SYM is dual to AdS Black hole.
- Their thermodynamics match. Note the area law of black hole entropy implies holography.
- The Hawking-Page transition(thermal gas v.s. black hole) is dual to confinementdeconfinement transition of SYM.

# \* HOLOGRAPHIC QCD

 Introduce ``quarks": D7/D3 & D8/D4, these provide the large N QCD. D8/D4 can geometrically realize chiral symmetry breaking.

• The bulk KK spectrum match well (up to 10%) with the meson's of dual QCD. Same for baryons.

• Many QCD properties can be derived holographically.





#### PARADIGM SHIFT

- The AdS/CFT helps to pin down the nature of quark-gluon plasma produced in RHIC(relativistic heavy ion collider).
- At beginning, it is believed that the plasma is in weakly coupled gas phase.
- The dual gravity shows it is in the hydrodynamical (liquid) phase, and confirmed by RHIC data later on.

#### $\eta / s \ge 1 / 4\pi$

- The sign for the hydrodynamical nature of quark-gluon plasma is the non-vanishing viscosity calculated from dual gravity.
- The above universal bound is related to the universal scattering cross-section against the black hole.  $\eta = \frac{\sigma_{abs}(\omega = 0)}{16\pi G}$
- This is considered a triumph of AdS/CFT since the real time dynamics is out of reach by lattice calculation.



#### HYDRODYNAMICS

- To obtain the hydrodynamics, we should study the linearized fluctuation of the stress tensor.
- They are classified into sound, shear and scalar channels, and the transport coefficients such as sound speed, shear and bulk viscosities are encoded in the poles of the retarded Green functions.

### FROM BULK GRAVITY

- Via AdS/CFT, the fluctuation of boundary stress tensor is sourced by the fluctuation of the bulk metric by matching the tensor structure.
- One then needs to solve the linearized Einstein equation on the AdS-BH background, and extract the boundary 2-point function.
- The point is how to get the Lorentzian Green function.

#### REALTIME DYNAMICS I

 $G^{R}_{\mathcal{O}_{A}\mathcal{O}_{B}}(\omega,k) = -i \int d^{d-1}x dt e^{i\omega t - ik \cdot x} \theta(t) \langle [\mathcal{O}_{A}(t,x), \mathcal{O}_{B}(0,0)] \rangle \,,$ 

- To calculate the retarded Green functions, we need to invoke the real time dynamics at finite temperature.
- In QFT, this is done via Schwinger-Keldysh formulation by analytically continuated into complex time plane.



$$iG_{ab}(x-y) = \frac{1}{i^2} \frac{\delta^2 \ln Z[\phi_1, \phi_2]}{\delta \phi_a(x) \,\delta \phi_b(y)} = i \begin{pmatrix} G_{11} & -G_{12} \\ & \\ -G_{21} & G_{22} \end{pmatrix}$$

$$\begin{aligned} G_{11}(k) &= \operatorname{Re} G_R(k) + i \operatorname{coth} \frac{\omega}{2T} \operatorname{Im} G_R(k) \,, \qquad \omega \equiv k^0 \,, \\ G_{12}(k) &= \frac{2ie^{-(\beta - \sigma)\omega}}{1 - e^{-\beta\omega}} \operatorname{Im} G_R(k) \,, \\ G_{21}(k) &= \frac{2ie^{-\sigma\omega}}{1 - e^{-\beta\omega}} \operatorname{Im} G_R(k) \,, \\ G_{22}(k) &= -\operatorname{Re} G_R(k) + i \operatorname{coth} \frac{\omega}{2T} \operatorname{Im} G_R(k) \,. \end{aligned}$$

## REALTIME DYNAMICS 11

- The upper and lower time contours are dual to the R and L boundary, and solve the EOM in both regions.
- Choose appropriate b.c. on the horizon to extend the solutions from R to L, and then match them.
- Use GKP/W, one will reproduce the real time dynamics with retarded Green function calculated.

b.c.: in-falling modes are of positive energy



MOREOVER,

- Performing the systematical derivative expansion of the bulk metric order by order, one can show
- Perturbed *Einstein* equation yields *Navier*-*Stokes* equation.
- We may wonder if we can see the turbulence in dual gravity?

### ELECTRICITY

- One can also introduce bulk U(1) gauge field, which acts as the source to the boundary EM current.
- Then, the retarded Green function for the (bulk not boundary) gauge field fluctuation will yield the conductivity.
- The holographic conductivity bears the similar feature of the graphene.

#### CHARGED ADS4 BH & GRAPHENE?!









Hartnoll

# (NON)-FERMI LIQUID

- One can also turn on the bulk fermi field which will couple to fermionic source of dual field theory.
- The corresponding spectral function (retarded Green function) shows the Fermi surface.
- However, its scaling behavior is different from the one of Fermi liquid.

#### NON-RELATIVISTIC HOLOGRAPHY

- Many condensed matters show non-relativistic scaling behavior. What are their gravity duals?
- There are two ways in consideration.
- One is the Lifshitz type: the scaling behavior break Lorentz behavior but no Galilean symmetry.
- The other has Schrodinger symmetry.

#### GRAVITY DUALS

Lifshitz type:  

$$t \to \lambda^{z} t, \ \vec{x} \to \lambda \vec{x}, \ r \to \frac{r}{\lambda}$$
  
 $ds^{2} = R^{2}(-r^{2z}dt^{2} + r^{2}(dx^{2} + dy^{2}) + \frac{dr^{2}}{r^{2}})$ 

$$\langle O(x,t)O(0,0)\rangle \to \frac{const}{|x|^{2\Delta}}, \quad |x| \to \infty$$

RG:z=2-->z=1 $ds^{2} = R^{2}(-r^{4}f(r)^{2} + r^{2}(dx^{2} + dy^{2}) + g(r)^{2}dr^{2}/r^{2})$ 



Schrodinger type: Besides, plus the Galilean boost and special conformal transformations (symmetry of cold atoms at Feshbach resonance, or fermions at unitarity.)

$$ds^{2} = R^{2}(-r^{2z}dt^{2} + r^{2}(d\xi dt + dx^{2} + dy^{2}) + \frac{dr^{2}}{r^{2}})$$

This is a co-dim. 2 holography. One need to perform the KK reduction along the null-like Killing direction.

$$g^{\mu\nu}\partial_{\mu}\partial_{\nu} \to g\partial_i\partial_j - im\partial_t - m^2$$

$$\langle O(x,t)O(0,0)\rangle \propto \frac{\exp(-isx^2/2|t|)}{|t|}$$

#### HOLOGRAPHICANYONS

- Besides, one can also try to realize some exotic particles in condensed matters such as anyons via holography.
- The procedure is to introduce some particlelike brane by wrapping the internal space.
- When exchanging two particle, one will invoke nontrivial AB phase via EOMs of form fluxes in supergravity.
- We have realized this in ABJM, i.e.  $AdS_4 \times CP^3$ .

#### CONCLUSION

- The AdS/CFT duality is amazing since it relates two most important theories(GR & YM) in a nontrivial way.
- Moreover, it serves as a precious tool to tackle the strong coupled / correlated systems.
- Most important, it seems we are still in very primitive stage of its usage. Sure to be more for you to explore.