Quantum Instabilities of deSitter Spacetime

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deSitter Spacetime

Solution of Einstein's equations with a positive cosmological constant

Global deSitter spacetime is maximally symmetric

10 Killing vectors in 4 dimensions,
Same as Minkowski spacetime

A portion of deSitter spacetime describes inflationary expansion

$$ds^2 = -dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

$$a(t) = \mathrm{e}^{Ht}$$

Why is the stability of deSitter space important?

Instabilities might alter the predictions of inflationary models.

Instabilities might lead to a natural resolution of the cosmological constant problem.

decaying cosmological constant models

e.g., Dolgov, Barr, LF, ect

Part I: Gravitons in deSitter Spacetime

quantize linear perturbations -
active fluctuations of geometry

Write $$g_{\mu\nu} = \gamma_{\mu\nu} + h_{\mu\nu}$$

background (deSitter) metric

metric perturbation -
tensor modes

Impose the transverse tracefree (TT) gauge:

$$h^{\mu\nu}{}_{;\nu} = 0 \qquad h = h^{\mu}_{\mu} = 0 \qquad h_{\mu\nu}\, u^{\mu} = 0$$

covariant derivative on the background

a timelike vector; here the comoving obverver 4-velocity

Result: tensor modes behave as massless scalars

Lifshitz 1946

$$\Box_S \, h^{\mu}_{\nu} = 0$$

scalar wave operator

Consequences:

deSitter space is classically stable

gravitons are equivalent to a pair of massless scalar fields

$$\Box_S \, \varphi = \varphi^{;\nu}{}_{;\nu} = 0$$

Infrared divergences

$\langle \varphi(x) \varphi(y) \rangle$ is not defined in the deSitter invariant state (Bunch-Davies) vacuum, but can be defined in a class of states which break deSitter symmetry

Linde, Starobinsky, Vilenkin & LF

Consequence: linear growth in comoving time

$$\langle \varphi^2 \rangle \sim \frac{H^3 t}{4\pi^2}$$

$$\langle h^{\mu\nu} h_{\mu\nu} \rangle \sim \frac{H^3 t}{2\pi^2}$$

Is this growth an instability of deSitter space?

One loop level: No, gauge invariant quantities do not grow   LF 1985

Two loop level: Controversial

Tsamis & Woodard (1996) claim to find cosmological constant damping

$$\Lambda_{eff} = \Lambda(1 - \pi^{-2}\,\ell_p^4\,H^6\,t^2) \qquad \Lambda = 3\,H^2$$

Planck length

This result disputed by others (Garriga & Tanaka)

An alternative model: gravitons coupled to photons

J.T. Hsiang, D.S. Lee, H.L. Yu & LF

Preliminaries: stress tensor renormalization in curved spacetime

Divergent parts:

Einstein tensor

regulator parameter

$$\langle T_{\mu\nu}\rangle \sim A\,\frac{g_{\mu\nu}}{\sigma^2} + B\,\frac{G_{\mu\nu}}{\sigma} + \left(C_1 H^{(1)}_{\mu\nu} + C_2 H^{(2)}_{\mu\nu}\right)\,\ln\,\sigma$$

$$H^{(1)}_{\mu\nu} \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\left[\sqrt{-g}R^2\right] = 2\nabla_\nu\nabla_\mu R - 2g_{\mu\nu}\nabla_\rho\nabla^\rho R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}$$

scalar curvature

$$H^{(2)}_{\mu\nu} \equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\left[\sqrt{-g}R_{\alpha\beta}R^{\alpha\beta}\right] = 2\nabla_\alpha\nabla_\nu R^\alpha_\mu - \nabla_\rho\nabla^\rho R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla_\rho\nabla^\rho R - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 2R^\rho_\mu R_{\rho\nu}$$

Ricci tensor

Add R^2 and $R_{\alpha\beta}R^{\alpha\beta}$ counterterms in the gravitational action and write Einstein's equations as

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} + \alpha_0 H^{(1)}_{\mu\nu} + \beta_0 H^{(2)}_{\mu\nu} = 8\pi G_0 \langle T_{\mu\nu} \rangle$$

Remove the divergent parts of $\langle T_{\mu\nu} \rangle$ by a renormalization of $\mathrm{G}_0,\ \Lambda_0,\ \alpha_0,\ \beta_0$

We want the renormalized values of $\alpha_0,\ \beta_0 = 0$ to avoid a fourth order equation.

In general, $\langle T_{\mu\nu} \rangle_{ren}$ is not expressible in terms of geometric quantities.

An exception: conformally invariant fields (e.g., photons) in a conformally flat spacetime (e.g. deSitter space). (in the vacuum state)

Here $\langle T_{\mu\nu} \rangle_{ren} = C \; B_{\mu\nu}$

Bunch, Davies, Brown & Cassidy

where

$$B_{\mu\nu} = \frac{1}{2} R_{\alpha\beta} R^{\alpha\beta} \, g_{\mu\nu} + \frac{2}{3} R R_{\mu\nu} - R^{\rho}_{\mu} R_{\rho\nu} - \frac{1}{4} R^2 \, g_{\mu\nu}$$

$$C = \frac{31}{1440\pi^2}$$ (photons)

$B^{\mu}_{\mu} \neq 0$ conformal anomaly

In deSitter space, $\langle T_{\mu\nu} \rangle_{ren} \propto g_{\mu\nu}$ so just shifts Λ

Our model: assume that this form for $\langle T_{\mu\nu} \rangle_{ren}$ holds in perturbed deSitter spacetime.

Self consistent: $B_{\mu\nu}$ is still a conserved tensor.

Key result: equation for the metric perturbations becomes

$$\Box_S \, h^{\mu}_{\nu} + 48\pi \, C \, \ell_p^2 \, H^4 \, h^{\mu}_{\nu} = 0$$

tachyonic mass

and has an exponentially growing solution:

$$h^{\mu}_{\nu} \propto \mathrm{e}^{16\pi C \, \ell_p^2 \, H^3 \, t}$$

Gauge invariant quantities, such as the Ricci tensor, also grow.

Time scale for the onset of the instability is

$$\tau = \frac{1}{16\pi C\, \ell_p^2\, H^3} \approx 10^4 \frac{E_p^2}{E_I^2}\, H^{-1}$$

$E_p =$ Planck energy $E_I =$ energy scale of inflation

Implications:

Allows adequate inflation to solve horizon & flatness problems

Does not allow for eternal inflation.

Part II: Quantum Stress Tensor Fluctuations in deSitter Spacetime

C.H. Wu, K.W. Ng & LF; also work in progress with S.P. Miao & R. Woodard

Basic idea look at the effects of fluctuations of the vacuum electromagnetic field stress tensor.

Stress tensor and expansion fluctuations

u^α = 4-velocity of a congruence of timelike geodesics

$\theta = u^\alpha_{;\alpha}$ = expansion of the congruence

Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu}$$

$$R_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$$

Ordinary matter: focussing

$T_{\mu\nu}$ Fluctuations $\longrightarrow$ θ Fluctuations

Conservation law for a perfect fluid:

$$\dot{\rho} + \theta(\rho + p) = 0$$

E.g., Robertson-Walker universe:

Stress tensor fluctuations imply density fluctuations

(Not necessarily for the same field)

Robertson-Walker Spacetime

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)\,.$$

$$dt = a(\eta)d\eta$$

$dt =$ comoving time

$d\eta =$ conformal time

Conformally invariant fields:

$$C^{RW}_{\mu\nu\alpha\beta}(x,x') = a^{-4}(\eta)\, a^{-4}(\eta')\, C^{flat}_{\mu\nu\alpha\beta}(x,x')$$

Stress tensor correlation function

$$C_{\mu\nu\alpha\beta}(x,x') = \langle T_{\mu\nu}(x)\, T_{\alpha\beta}(x')\rangle - \langle T_{\mu\nu}(x)\rangle\langle T_{\alpha\beta}(x')\rangle$$

(conformal anomaly cancels)

θ fluctuations

Assume $\sigma_{\mu\nu} = \omega_{\mu\nu} = 0,$ so that

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}u^\mu u^\nu - \frac{1}{3}\theta^2$$

Let $\theta = \theta_0 + \theta_1,$ where $\theta_0 = 3\dot{a}/a,$ and

$$\frac{d\theta_1}{dt} = -\left(R_{\mu\nu}u^\mu u^\nu\right)_q - \frac{2}{3}\theta_0\theta_1$$

$$\Longrightarrow \quad \theta_1(t) = -a^{-2}(t)\int_{t_0}^{t} dt'\, a^2(t')\,\left(R_{\mu\nu}u^\mu u^\nu\right)_q$$

Expansion correlation function:

$$\langle\theta(\eta_1)\,\theta(\eta_2)\rangle - \langle\theta(\eta_1)\rangle\langle\theta(\eta_2)\rangle = \ (8\pi)^2 \times$$
$$a^{-2}(\eta_1)\,a^{-2}(\eta_2)\int_{\eta_0}^{\eta_1} d\eta\,a^{-1}(\eta)\int_{\eta_0}^{\eta_2} d\eta'\,a^{-1}(\eta')\,\mathcal{E}(\Delta\eta,r)$$

$\mathcal{E}(\Delta\eta, r)$ = flat space energy density correlation function

$$\mathcal{E}_{em} = \frac{(r^2+3\Delta\eta^2)^2}{4\pi^4(r^2-\Delta\eta^2)^6}$$

Treat as distributions - integrate by parts

Inflationary expansion followed by reheating and a radiation dominated universe

$$a(\eta) = \frac{1}{1 - H\eta}, \qquad \eta_0 < \eta < 0, \quad \eta_0 = \text{conformal time when inflation begins}$$

$$a(\eta) = 1 + H\,\eta, \qquad \eta > 0,$$

$$t_R = \text{ reheating time in comoving time}$$

$$a(t) = \mathrm{e}^{H(t - t_R)}, \quad t \le t_R\,,$$

$$a(t) = \sqrt{1 + 2H(t - t_R)}, \qquad t \ge t_R.$$

Effect of expansion fluctuations on redshifting after reheating

Let $p = w\rho$ and integrate the conservation law to find the density fluctuations:

$$\left\langle \left(\frac{\delta\rho}{\rho} \right)^2 \right\rangle = (1+w)^2 \int_0^{\eta_s} d\eta_1 a(\eta_1) \int_0^{\eta_s} d\eta_2 a(\eta_2) \left(\langle \theta(\eta_1)\, \theta(\eta_2) \rangle - \langle \theta(\eta_1) \rangle \langle \theta(\eta_2) \rangle \right)$$

$\eta_s = \text{conformal time of last scattering}$

Power spectrum of the density fluctuations:

$$\left\langle \left(\frac{\delta\rho}{\rho} \right)^2 \right\rangle = \int d^3k \, e^{i\,\vec{k}\cdot\Delta\vec{x}} \, P_k(\eta_s)$$

(Non-Gaussian fluctuations)

Result:

$$P_k = -\ell_p^4 (1+w)^2 \, \frac{k^4 \, |\eta_0|^3}{480\pi^2} \, \ln^2[a(\eta_s)]$$

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$P_k \propto |\eta_0|^3$ Grow as the duration of inflation increases- a different instability of deSitter space.

Interpret as due to non-cancellation of anti-correlated θ fluctuations.

Interpret the sign as telling us whether the density fluctuations on a given scale are correlated or anti-correlated.

density perturbation:

a quantum gravity effect

$$\frac{\delta\rho}{\rho} \propto \ell_p^2 \left(\frac{S}{\lambda}\right)^{3/2}$$

$S = H|\eta_0| =$ expansion factor during inflation

$\lambda =$ length scale of the perturbation

Constraint on the duration of inflation:

$$\frac{\delta\rho}{\rho} < 10^{-4} \quad \Rightarrow \quad S < 10^{37} \left(\frac{10^{12}\mathrm{GeV}}{E_R}\right)^{5/3}$$

reheating energy

Allows enough inflation to solve the horizon and flatness problems

Opens the possibility of observing quantum gravity effects as a non-Gaussian, non-scale invariant component in the large scale structure.

Summary

1) deSitter space is classically stable.

2) In pure quantum gravity, it is stable at the one loop level.

3) This may not hold in higher orders?

4) In a simple model with gravitons + photons, there is a homogeneous instablity. No eternal inflation.

5) When stress tensor fluctuations are included, there is also an inhomogeneous instability.

6) The latter could produce observable features in large scale structure.