An Introduction to Inflation and Bouncing Cosmology

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2009-11-12

Outline

(I) Why Inflation?

- The advantages and disadvantages of Standard Big Bang Theory;
- How inflation solved the major problems;
- The dynamics of inflation;
- What remained unsolved.

(II) Why Bounce?

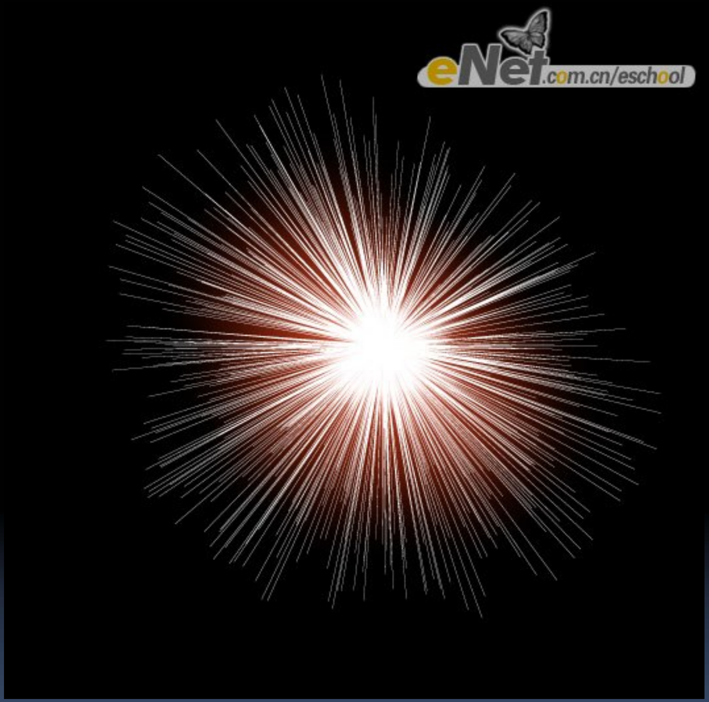
- The basic idea and general picture of bouncing scenario;
- Calculation on perturbation and signatures on observations;
- Other properties of bouncing scenario

(III) Extension of Bouncing: Cyclic Scenario

(IV) Summary

Standard Big Bang Theory

Our Universe comes from a singularity in space-time



Advantages:

(1)The age of galaxies;

(2) The redshift of the galactic spectrum;

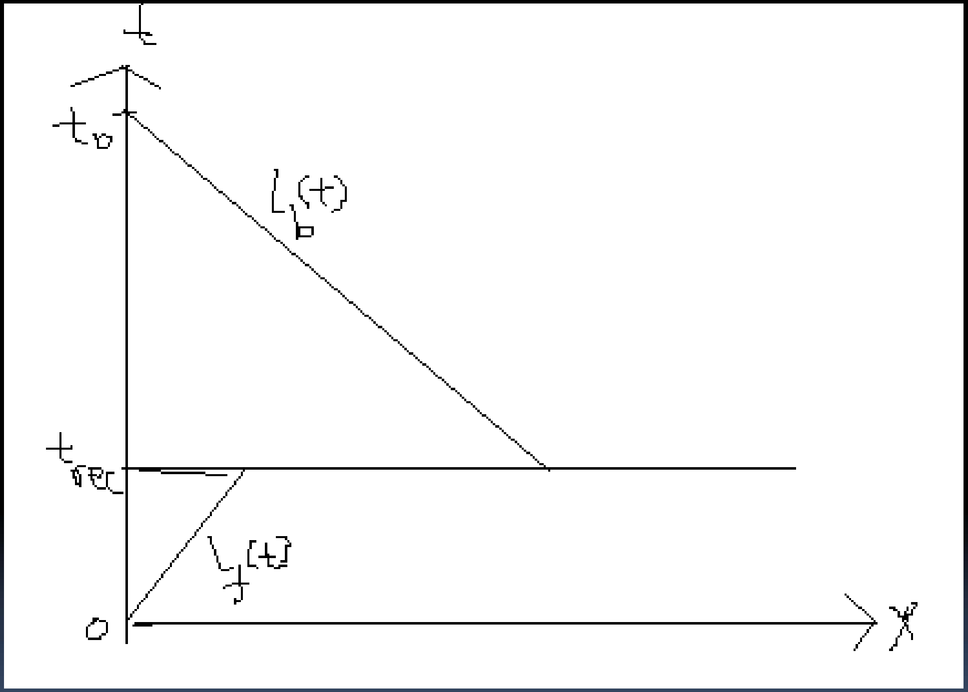
(3) The He abundance;

(4) The pridiction of CMB temperature

However, there are still many disadvantages!

Standard Big Bang Theory

Disadvantage I: Horizon Problem



The backward cone is much larger than the forward cone, indicating that the observable region today has been constituted of regions that had been causal irrelative. Then why is our universe homogeneous?

Standard Big Bang Theory

Disadvantage II: Flatness Problem

From Friedmann equation:

$$\Omega_{tot}(t) - 1 = \frac{k}{a^2 H^2}$$

$$\frac{|\Omega_{tot}(t) - 1|_{pl}}{|\Omega_{tot}(t) - 1|_0} \sim (\frac{a_{pl}^2}{a_0^2}) \sim (\frac{T_0^2}{T_{pl}^2}) \sim O(10^{-60})$$

From observation: the universe today is very flat $\Omega_{tot}(t_0) = 1.02 \pm 0.02$

So we should fine-tune the initial value of $\Omega_{tot}(t)$ to get the right value for today!

Standard Big Bang Theory

Disadvantage III: Original Structures

The existence of galaxies and obsevations from CMBR implied that there must be inhomogeneity on small scales, with tiny density perturbation $\frac{\delta\rho}{\rho} \simeq 10^{-5}$

How is it formed?

Standard Big Bang Theory

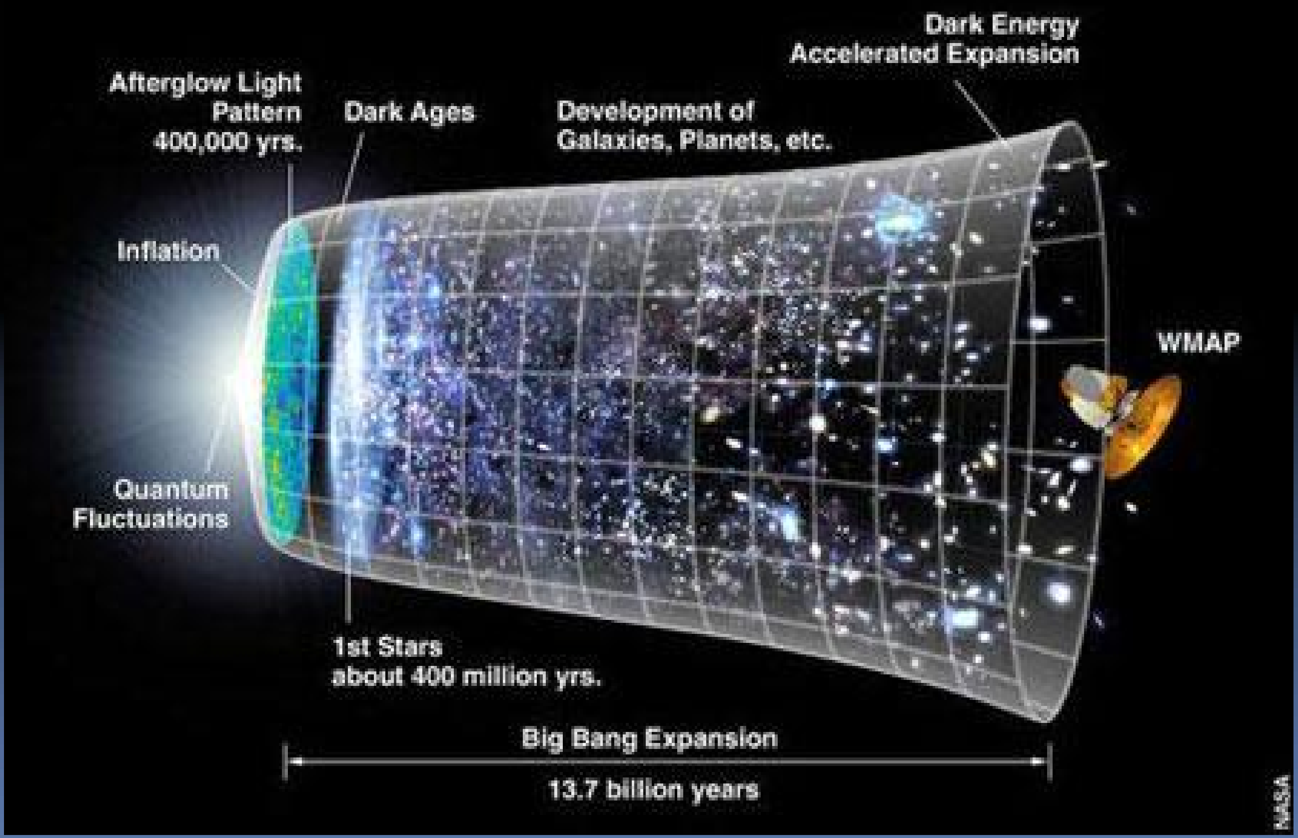
Disadvantage IV: Singularity Problem

At the very beginning of the universe, the space-time converges into a singular point, where all the physical variables blew up, and the world became unphysical.

Other disadvantages:
transplanckian problem, entropy problem, etc

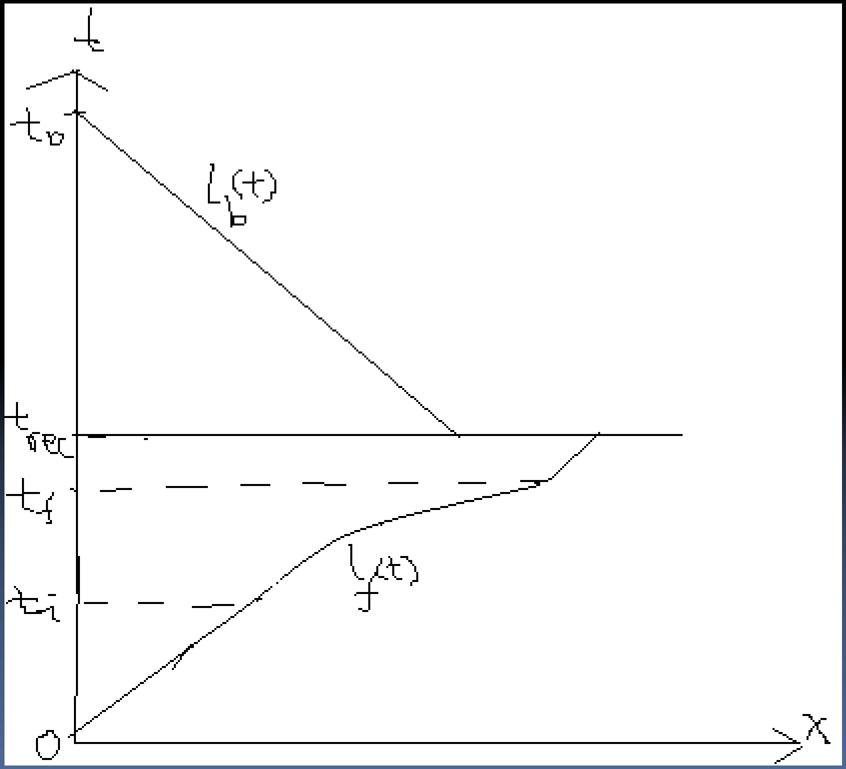
Solution to Standard Big Bang Theory: Inflation

The universe experienced a period of accelerating expansion after the big bang. During the expansion, the scale of the universe is drawn out of the horizon, all matters and radiations were diluted and the fluctuations were frozen to form today's structures.



Solution to Standard Big Bang Theory: Inflation

Solution to Horizon Problem:



Solution to Standard Big Bang Theory: Inflation

Solution to Flatness Problem:

In inflation period, $a \propto e^{Ht}$ We can define e-folding number as:

$$N \equiv \ln \frac{a_e}{a_i} = H(t_e - t_i)$$

Thus we have:

$$\frac{|\Omega - 1|_f}{|\Omega - 1|_i} \sim (\frac{a_i^2}{a_f^2}) \sim e^{-2N}$$

As long as N>70, we can get today's result assuming $|\Omega - 1|$ Before inflation being of order 1.

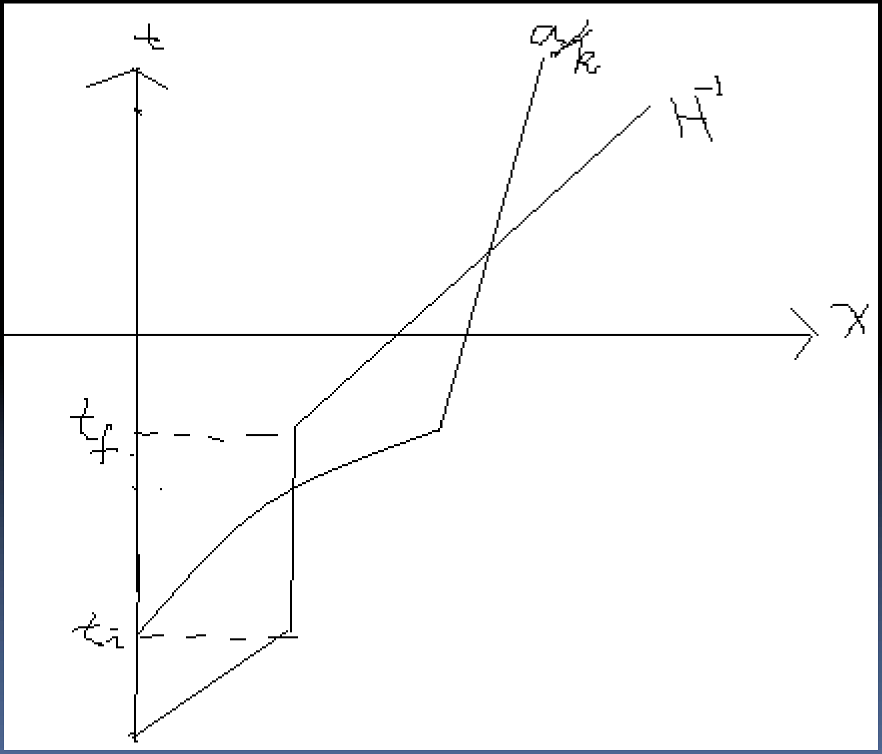
Solution to Standard Big Bang Theory: Inflation

Solution to Structure Problem:

$\lambda < l_H(t)$ Perturbation evolving in horizon

$\lambda > l_H(t)$ Perturbation driven out of horizon and frozen to form today's structure

$\lambda = a/k$ $l_H(t) = H(t)^{-1}$



The Dynamics of Inflation

The Einstein Equation: $G_{\mu\nu} = -8\pi G T_{\mu\nu}$

w. r. t. FRW metric: $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$

The 0-0 and i-i components:

$$H^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -4\pi G(\rho + 3p)$$

Conditions for acceleration:

$$\frac{\ddot{a}}{a} > 0 \Rightarrow \rho + 3p < 0$$

$$w = \frac{p}{\rho} < -\frac{1}{3}$$

Cosmological constant (with $w_\Lambda = -1$) satisfies the condition, but cannot exit to produce matter!!!

Dynamical inflation mechanisms are needed!!!

The Dynamics of Inflation

The slow-roll approximation (SRA)

Motivation: (1) provide sufficiently long period in order to solve SBB problems;
(2) produce scale-invariant power spectrum to fit today's observations

For the simplest single field inflation:

$$S = \int d^4x \sqrt{-g}[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)]$$

Define slow-roll parameter:

$$\begin{aligned}
\epsilon &\equiv -\frac{\dot{H}}{H^2} \ll 1 \,; \\
|\delta| &\equiv |\frac{\ddot{\phi}}{H\dot{\phi}}| \ll 1 \,; \\
|\eta| &\equiv |\epsilon - \delta| \ll 1 \,.
\end{aligned}$$

Or equivalently:

$$\begin{aligned}
\epsilon &\simeq \frac{1}{16\pi G}(\frac{V_\phi}{V})^2 \,; \\
\eta &\simeq \frac{1}{8\pi G}\frac{V_{\phi\phi}}{V} \,.
\end{aligned}$$

The Dynamics of Inflation

Examples of traditional inflation models:

- ### Large field inflation

Characteristic: $V''(\phi) > 0$

Example: "Chaotic" Inflation $V(\phi) = \frac{1}{2}m^2\phi^2$

Andrei D. Linde, Phys.Lett.B129:177-181,1983.

Properties: 1) destroy flatness of the potential;
2) can have large gravitational wave

- ### Small field inflation

Characteristic: $V''(\phi) < 0$

Example: CW type Inflation $V(\phi) = \frac{\lambda}{4}\phi^4(\ln\frac{|\phi|}{v} - \frac{1}{4}) + \frac{1}{16}\lambda v^4$

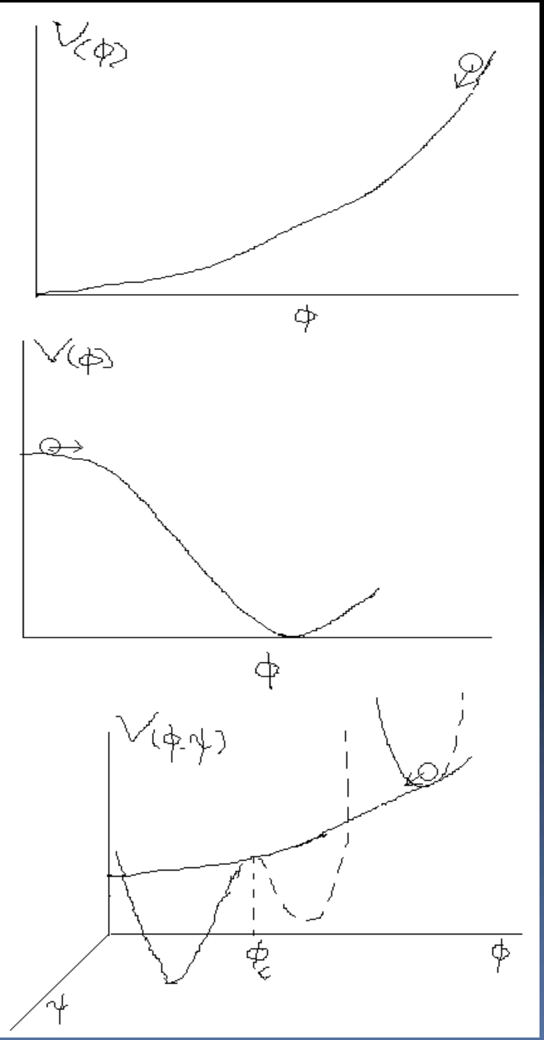
S. Coleman & E. Weinberg, Phys.Rev.D7:1888-1910,1973.

Properties: 1) initial conditions need fine-tuning;
2) cannot have large gravitational wave

- ### Hybrid inflation

$$V(\phi,\psi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}(\psi^2 - \eta^2)^2 + \frac{1}{2}g^2\phi^2\psi^2$$

Andrei D. Linde, Phys.Rev.D49:748-754,1994, astro-ph/9307002



Issues remaining unsolved by Inflation

Singularity Problem: Unsolved yet!

Possible solution to this problem:

(I)Pre-Big-Bang Scenario;
(II)Ekpyrotic Scenario;
(III) String Gas Scenario;

All go Beyond Einstein Gravity!!!

(IV) Bouncing Scenario

Within Einstein Gravity

… …

Alternative to Inflation: Bouncing Scenario

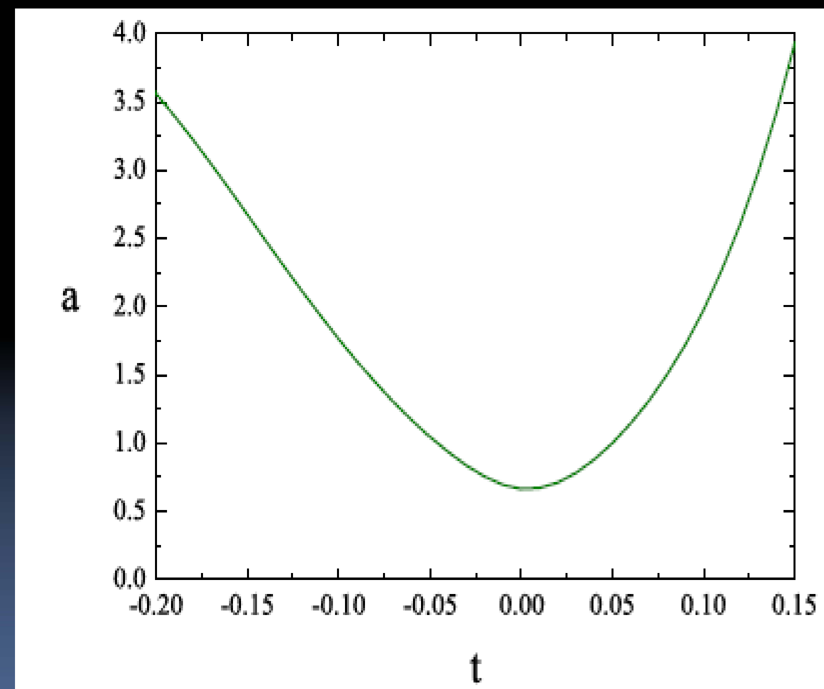
Bouncing Scenario describes a Universe transiting from a contracting period to a expanding period, or, experienced a bouncing process. At the transfer point, the scale factor a of the universe reached a non-vanishing minimum. This scenario can natrually avoid the singularity which is inevitable in Standard Big Bang Theory.

Contractio $H<0$ Expansion: $H>0$

Bouncing Po $H=0$ At the Neiborhood: $\dot{H}>0$

$$\dot{H}=-4\pi G(\rho+p)\Rightarrow w<-1$$

For a realistic scenario, one should connect this process to the observable universe (radiation dominant, matter dominant, etc), whose w>-1, so in the whole process w crosses -1!



Y. Cai, T. Qiu, Y. Piao, M. Li and X. Zhang, JHEP 0710:071, 2007

How to make EoS cross -1？No-Go Theorem

As for models, which is (1) in 4D classical Einstein Gravity, (2)described by single simple component (either perfect fluid or single scalar field with lagrangian as $\mathcal{L} = \mathcal{L}(\phi, \partial_\mu\phi\partial^\mu\phi)$), and (3) coupled minimally to Gravity, its Equation of State can never cross the cosmological constant boundary (w=-1).

The set of models with EoS crossing -1 is called Quintom!

Proof skipped, see e.g. Bo Feng et al., Phys. Lett. B 607, 35 (2005); A. Vikman, Phys. Rev. D 71, 023515 (2005); J. Xia, Y. F. Cai, Taotao Qiu, G. B. Zhao and X. M. Zhang, astro-ph/0703202; etc.

According to the No-Go Theorem, there are many models that can realize EoS crossing -1, among which the simplest one is that composed of two real scalar field (Double-field Quintom model).

The Action:

$$S = \int d^4x\sqrt{-g}(\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1 - \frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2 - V(\phi_1, \phi_2))$$

There are also other examples, e. g. vector field, field with higher derivative operators, etc.

Bouncing Scenario Given by Quintom Matter

I) Paramtrization

$$w = -0.6 - t^{-2}$$

$$H = \frac{2}{3}\frac{t}{0.4t^2+1} \quad a = \frac{(t^2+2.5)^{\frac{1}{1.2}}}{2.14594}$$

II) Double-field Quintom

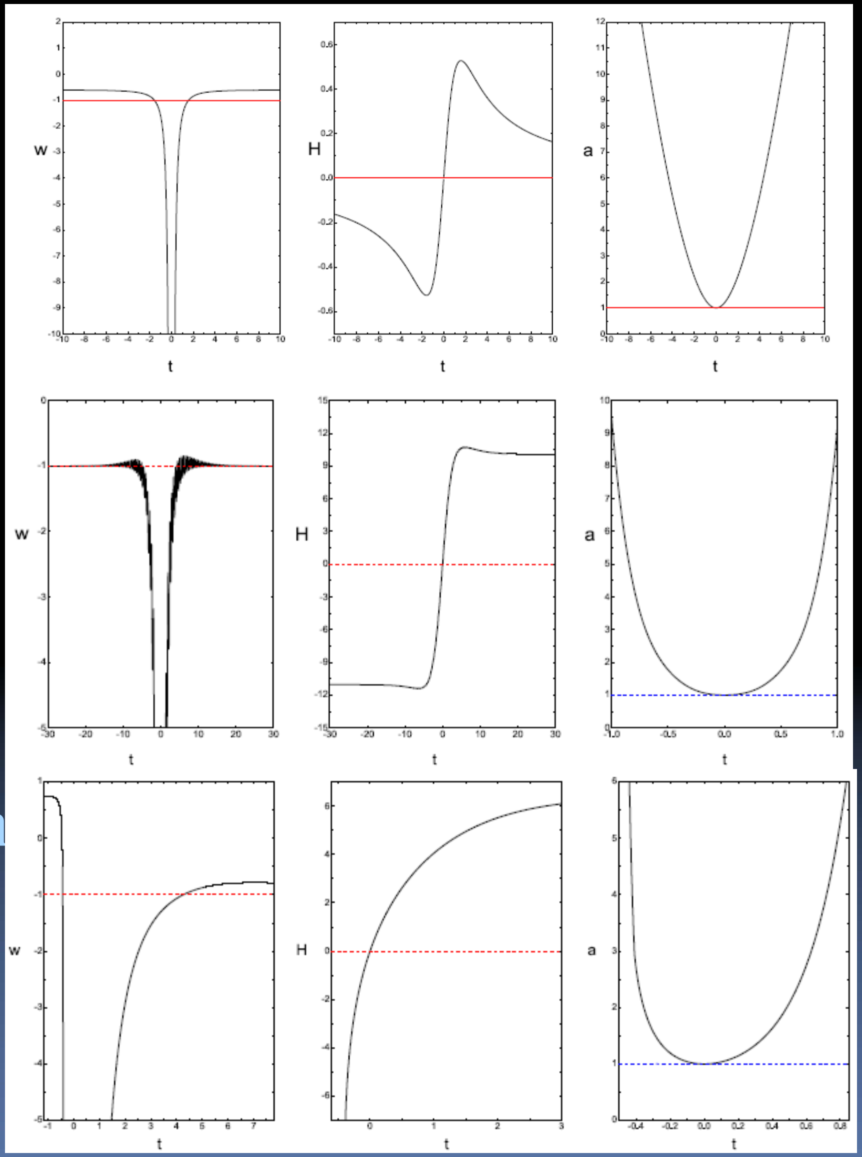
$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1 - \frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2 - V(\phi_1,\phi_2)\right)$$

$$V(\phi_1,\phi_2) = V_1\exp\left(\frac{-\lambda_1{\phi_1}^2}{{M_p}^2}\right) + V_2\exp\left(\frac{-\lambda_2{\phi_2}^2}{{M_p}^2}\right)$$

III) Single field Quintom with

$$S = \int d^4x\sqrt{-g}\left[-V(\phi)\sqrt{1-\alpha'\nabla_\mu\phi\nabla^\mu\phi + \beta'\phi\Box\phi}\right]$$

$$V(\phi) = V_0 e^{-\lambda\phi^2}$$



What’s the signature of Bouncing models?

Classification of Bouncing Models

- Bouncing Models
  - With inflation
    - Large field inflation
    - Small field inflation
    - Other inflation models
  - Without inflation

For sake of simplicity, we consider only non-interacting double-field bouncing models!

Formulae for Double-field bouncing models (I): background evolution

General Action $$S = \int d^4x\sqrt{-g}(\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1 - \frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2 - V(\phi_1,\phi_2))$$

Equations of Motion:
$$\begin{aligned}\ddot{\phi}_1 + 3H\dot{\phi}_1 + \frac{dV}{d\phi_1} &= 0\\ \ddot{\phi}_2 + 3H\phi_2 - \frac{dV}{d\phi_2} &= 0\end{aligned}$$

Stress Energy Tenso
$$\begin{aligned}\rho &= \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 + V(\phi_1,\ \phi_2)\\ p &= \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 - V(\phi_1,\ \phi_2)\end{aligned}$$

Friedmann Equatio
$$\begin{aligned}H^2 &= \frac{8\pi G}{3}(\frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 + V(\phi_1,\ \phi_2))\\ \dot{H} &= -4\pi G(\dot{\phi}_1^2 - \dot{\phi}_2^2)\end{aligned}$$

Equation of State:
$$w \equiv \frac{p}{\rho} = \frac{\dot{\phi}_1^2 - \dot{\phi}_2^2 - 2V}{\dot{\phi}_1^2 - \dot{\phi}_2^2 + 2V}$$

Formulae for Double-field bouncing models (II): Perturbative Metric

General Perturbative Metric:

$$ds^2 = a(\eta)^2(1+2A)d\eta^2 - 2(B_{,i} + S_i)d\eta dx^i - [(1-2\psi)\gamma_{ij} + 2E_{,ij} + 2F_{(i|j)+h_{ij}}]dx^i dx^j$$

where $F_{(i|j)} \equiv \frac{1}{2}(F_{i,j} + F_{j,i})$

For scalar perturbation, the gauge invariant components are constructed as:

$$\Phi = A + (1/a)[(B - E')a]'$$

$$\Psi = \psi - \mathcal{H}(B - E')$$

Taking conformal Newtonian gauge (B=E=0), we obtain metric containing those components only:

$$ds^2 = a^2(\eta)[(1+2\Phi)d\eta^2 - (1-2\Psi)dx^i dx^i]$$

Formulae for Double-field bouncing models (III): Perturbation Equations

With the above metric in hand, we can obtain perturbation equations for the gauge-invariant perturbations:

1) Perturbation Equations for metric: From perturbed Einstei

$$\delta G_{\mu\nu} = -8\pi G \delta T_{\mu\nu} \Rightarrow$$

$$\begin{aligned}
\nabla^2\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) &= 4\pi G[\phi_1'(\delta\phi_1' - \phi_1'\Phi) + a^2 V_{,\phi_1}\delta\phi_1 \\
&\quad -\phi_2'(\delta\psi_2' - \phi_2'\Phi) + a^2 V_{,\phi_2}\delta\phi_2] \\
\Psi' + \mathcal{H}\Phi &= 4\pi G(\phi_1'\delta\phi_1 - \phi_2'\delta\phi_2) \\
\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi &= 4\pi G[\phi_1'(\delta\phi_1' - \phi_1'\Phi) - a^2 V_{,\phi_1}\delta\phi_1 \\
&\quad -\phi_2'(\delta\phi_2' - \phi_2'\Phi) - a^2 V_{,\phi_2}\delta\phi_2]
\end{aligned}$$

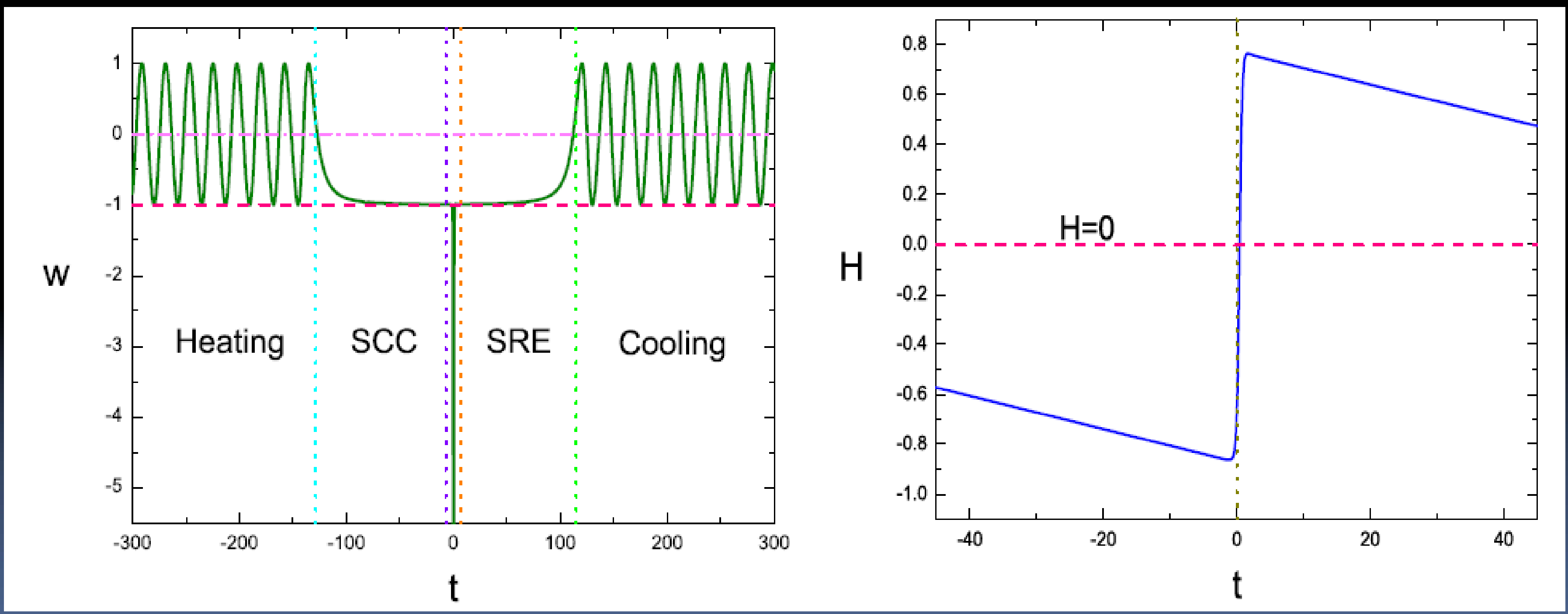
2) Perturbation Equations for field: From perturbed Equatio

$$\begin{aligned}
\delta\phi_1'' &= -2\mathcal{H}\delta\phi_1' - k^2\delta\phi_1 - a^2 V_{,\phi_1\phi_1}\delta\phi_1 - 2a^2 V_{,\phi_1}\Phi + 4\phi_1'\Phi' \ , \\
\delta\phi_2'' &= -2\mathcal{H}\delta\phi_2' - k^2\delta\phi_2 + a^2 V_{,\phi_2\phi_2}\delta\phi_2 + 2a^2 V_{,\phi_2}\Phi + 4\phi_2'\Phi' \ .
\end{aligned}$$

Calculation in Bouncing Models of Double-field Quintom

(I) Models with Large field inflation

Action: $S=\int d^4x\sqrt{-g}(\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1-\frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2-V(\phi_1))$ $V(\phi_1)=\frac{1}{2}m^2\phi_1^2$



Y. Cai, Taotao Qiu, R. Brandenberger, Y. Piao and X. Zhang, JCAP 0803:013, 2008

(Ⅰ) Models with Large field inflation

By solving perturbation equations, we obtain:

(I) Heating Phase: (in zon,

$$\Phi_k = A\eta^{-3}\frac{e^{-ik\eta}}{\sqrt{2k^3}}$$

$$\Phi_k \simeq \frac{iAk^{3/2}}{15\sqrt{2}} - \frac{3A}{\sqrt{2}}k^{-7/2}\eta^{-5}$$

(II) Slow-climbing Contracting Phase:

$$\Phi_k \simeq \sqrt{\frac{2}{\pi}}[C_k(\eta-\tilde{\eta}_c)+D_k]$$

Large k

$$\Phi_k \simeq \sqrt{\frac{2}{\pi}}[\frac{C_k}{k}\sin(k(\eta-\tilde{\eta}_c))+D_k\cos(k(\eta-\tilde{\eta}_c))]$$

(III) Bouncing Phase:

Small k

Large k

$$\Phi_k \simeq e^{-\frac{3}{4}y_1(\eta-\eta_B)^2}\left\{\tilde{F}_k\cos[k(\eta-\eta_B)]+\tilde{E}_k\sin[k(\eta-\eta_B)]\right\}$$

(IV) Slow-rolling Expanding Phase:

Large k

Small k

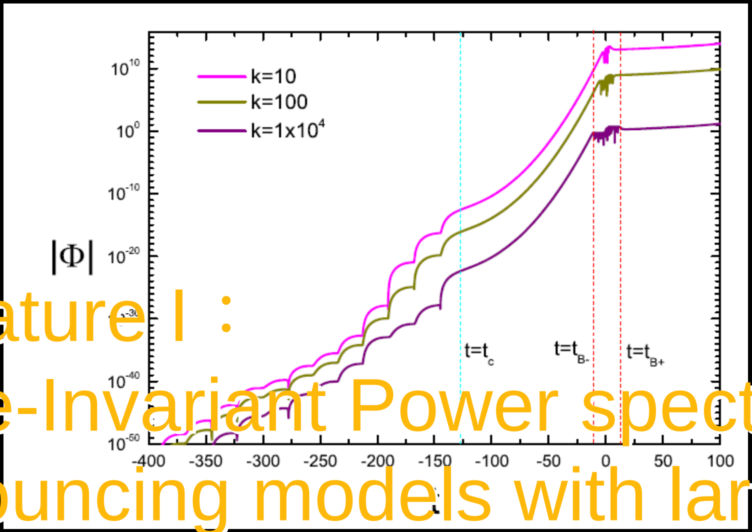
$$\Phi_k \simeq \sqrt{\frac{2}{\pi}}[\frac{G_k}{k}\sin(k(\eta-\tilde{\eta}_{B+}))+H_k\cos(k(\eta-\tilde{\eta}_{B+}))]$$

Large k

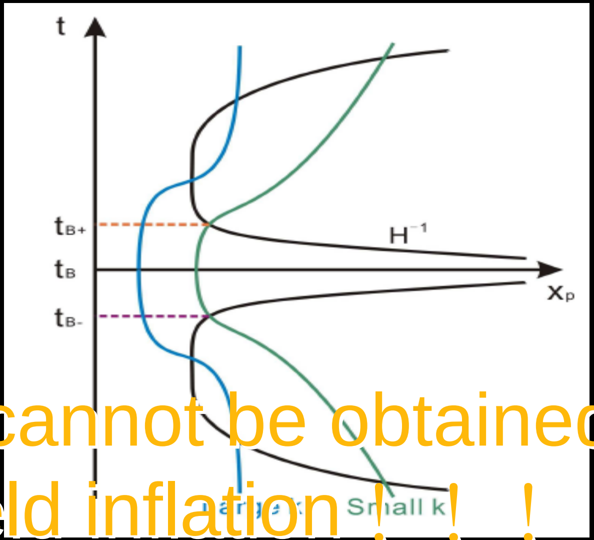
$$\Phi_k \simeq \sqrt{\frac{2}{\pi}}[G_k(\eta-\tilde{\eta}_{B+})+H_k]$$

（I）Models with Large field inflation

Numerical results:



Sketch plot of the perturba



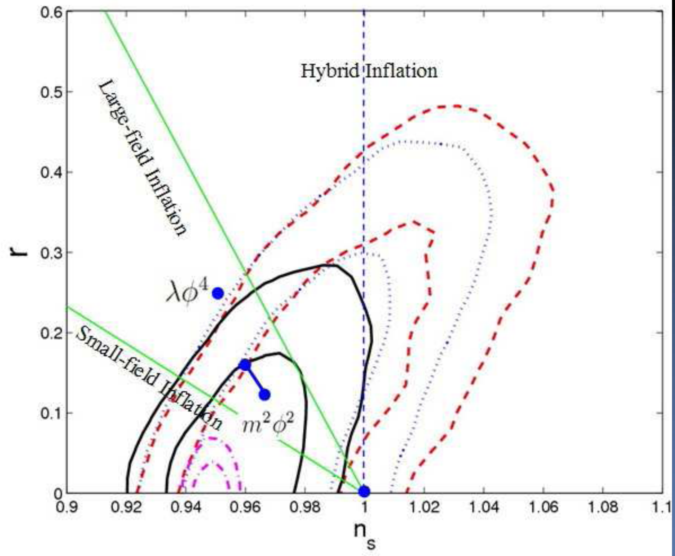
Signature I：

Scale-Invariant Power spectrum cannot be obtained by bouncing models with large field inflation！！！

Comparison: normal Large field

Inflation without Bouncing car

Scale-invariant Power Spectrur



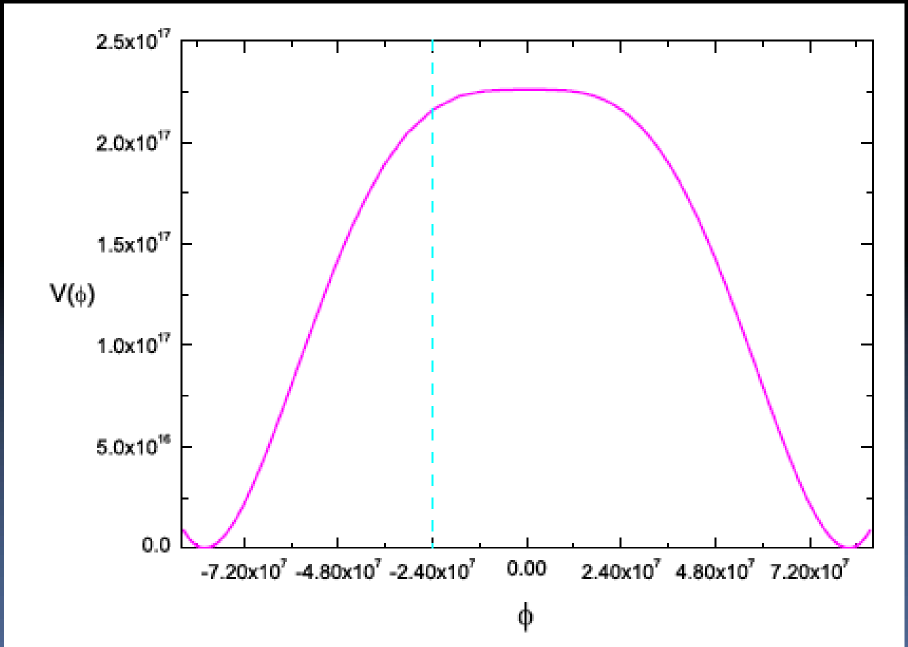
Jun-Qing Xia, Hong Li, Gong-Bo Zhao, Xinmin Zhang, Int.J.Mod.Phys.D17:2025-2048,2008,
E. Komatsu *et al.* [WMAP Collaboration], Astrophys.J.Suppl.180:330-376,2009.

(II) Models with Small field inflation

Action:

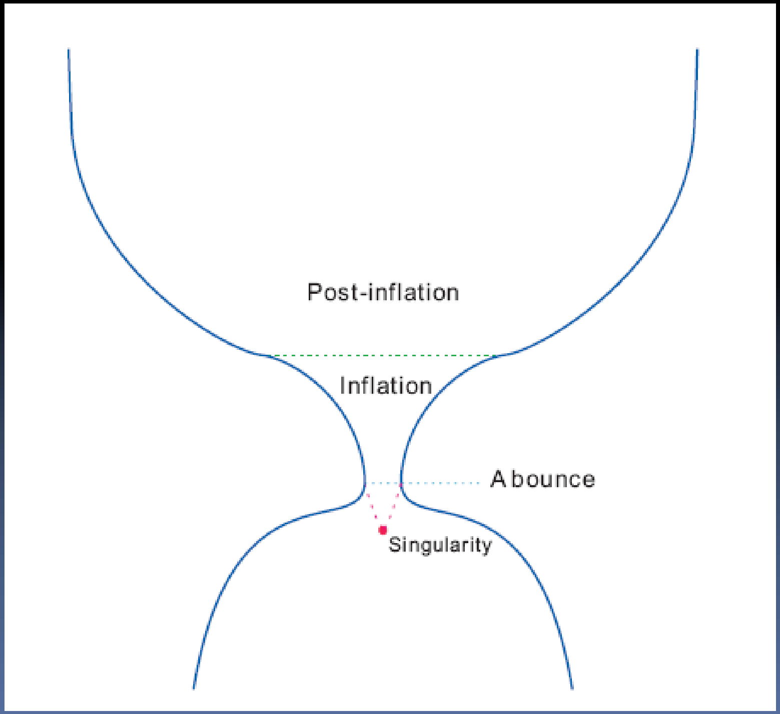
$$S = \int d^4x\sqrt{-g}(\frac{1}{2}\nabla_\mu\phi_1\nabla^\mu\phi_1 - \frac{1}{2}\nabla_\mu\phi_2\nabla^\mu\phi_2 - (\frac{1}{4}\lambda\phi_1^4(\ln\frac{|\phi_1|}{v} - \frac{1}{4}) + \frac{1}{16}\lambda v^4))$$

Sketch plot of potential:



$$V(\phi_1) = \frac{1}{4}\lambda\phi_1^4(\ln\frac{|\phi_1|}{v} - \frac{1}{4}) + \frac{1}{16}\lambda v^4$$

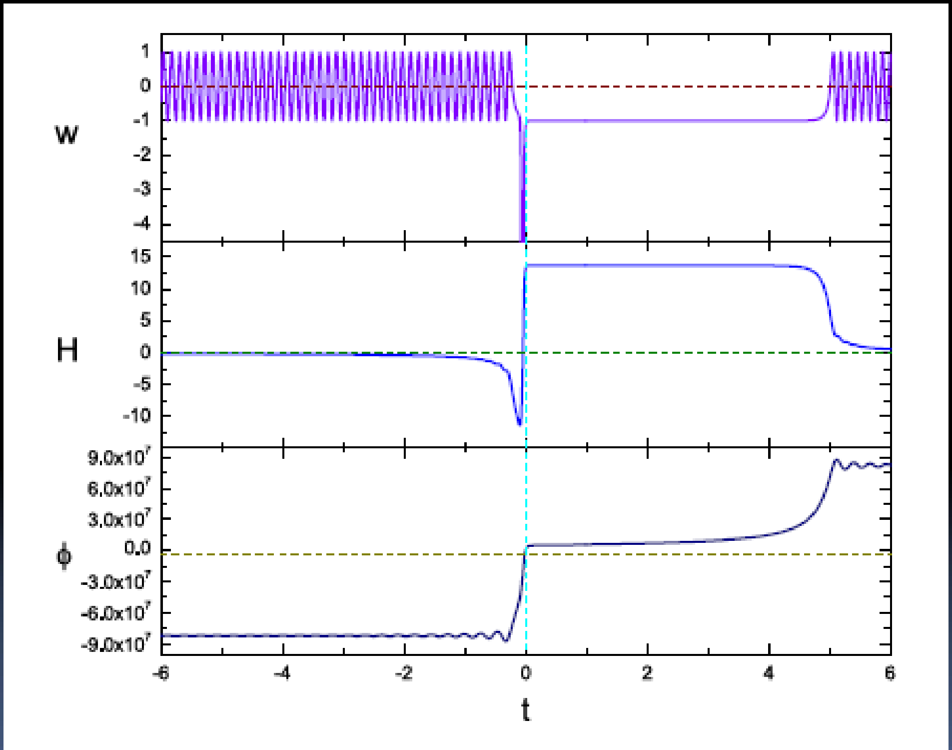
Sketch plot of scale factor

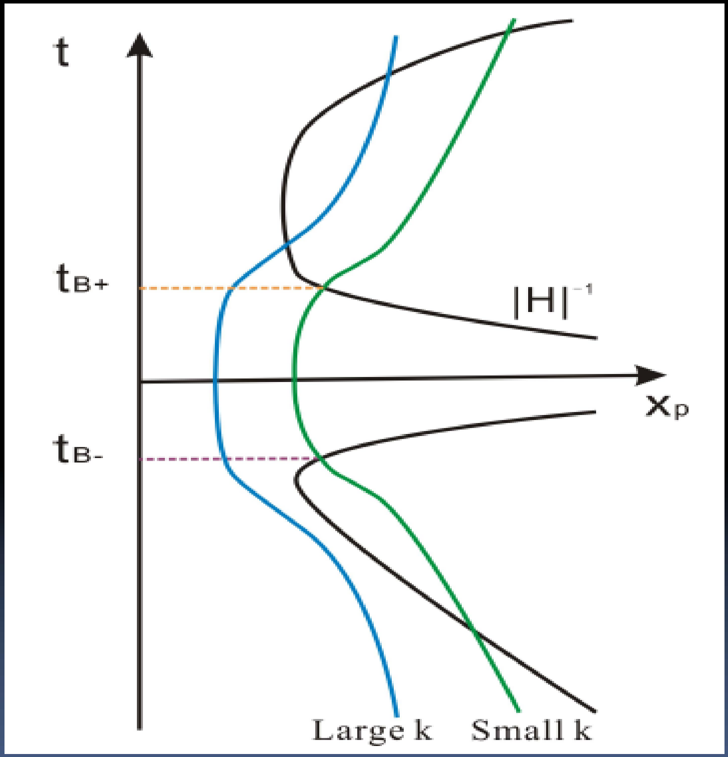


(II) Models with Small field inflation

Background evolution w. r. t. time

Sketch plot of perturbatio



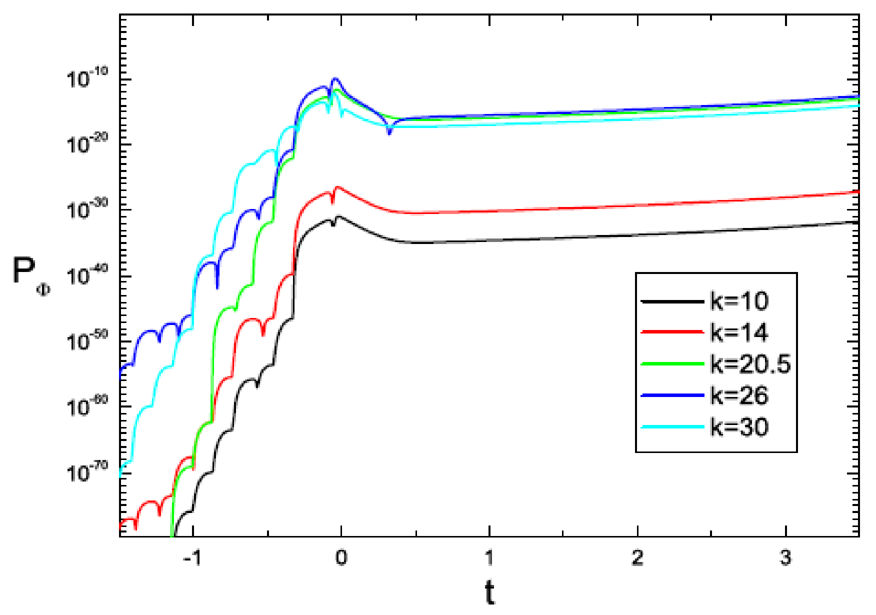


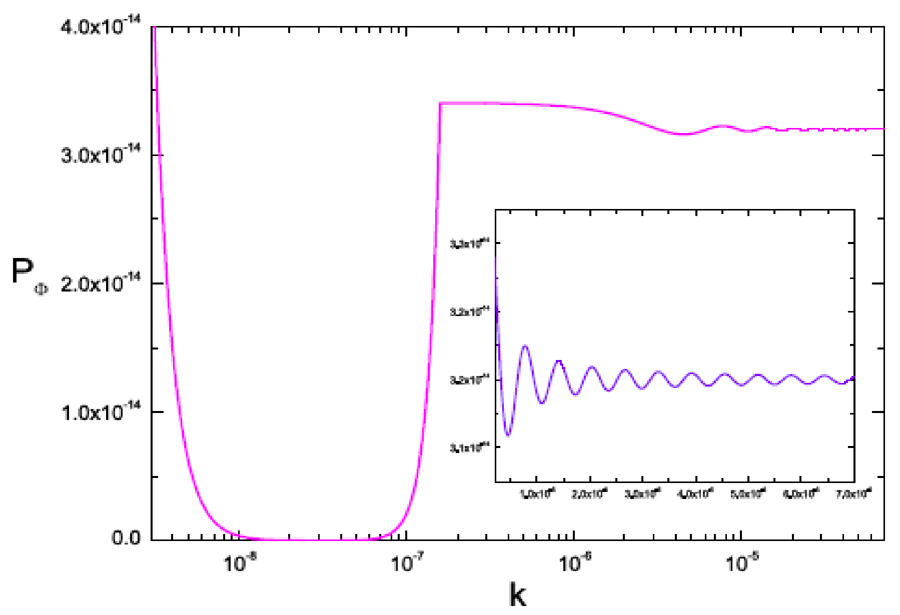
(II) Models with Small field inflation

Analytical results: nearly scale-invariant Power spectrum!

$$P_\zeta \simeq \frac{8}{3} G^2 \frac{\rho}{\epsilon}\left\{1 - \frac{3\mathcal{H}_{B+}}{2k}\sin\frac{2k}{\mathcal{H}_{B+}}\right\}$$

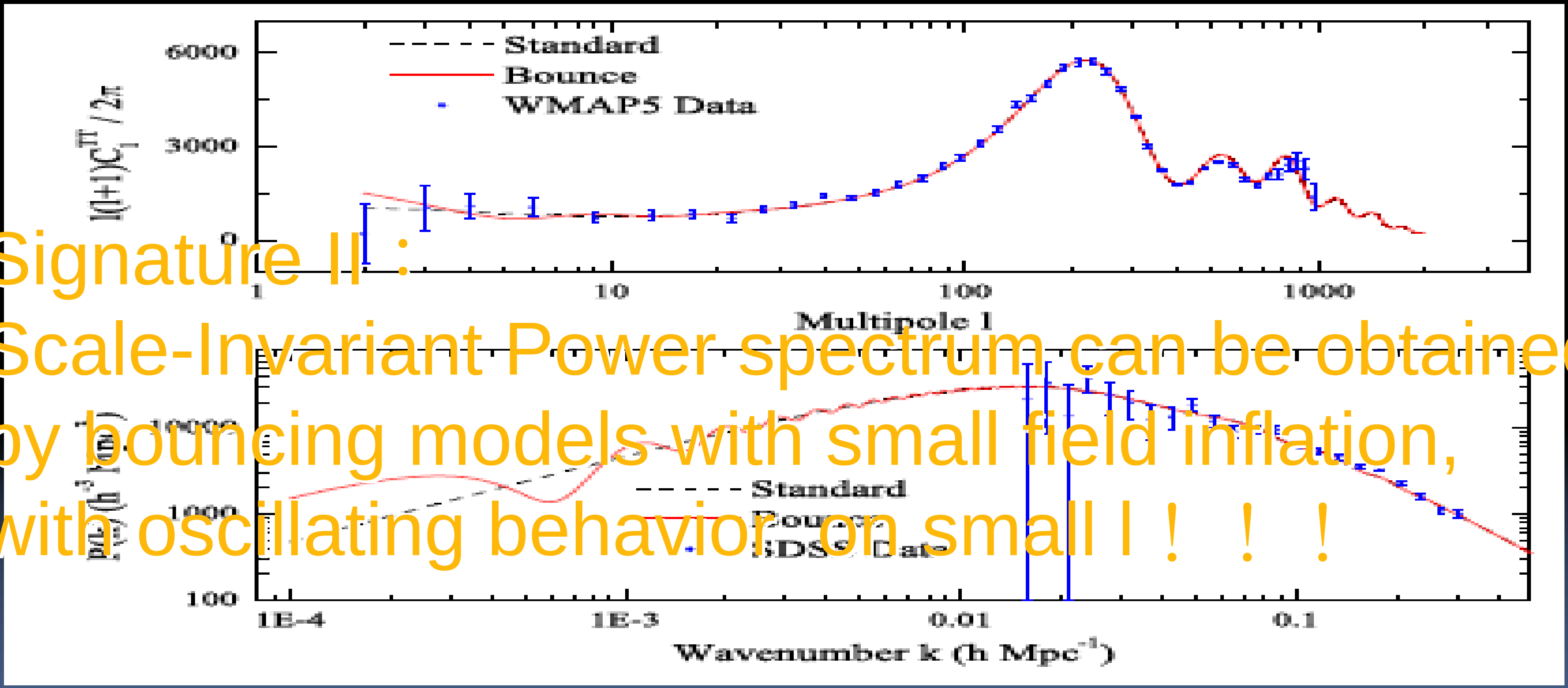
Numerical results: evolution of spectrum w. r. t. *t* and *k*





(II) Models with Small field inflation

The comparison of our result to the observational



Signature II：

Scale-Invariant Power spectrum can be obtained by bouncing models with small field inflation, with oscillating behavior on small l！！！

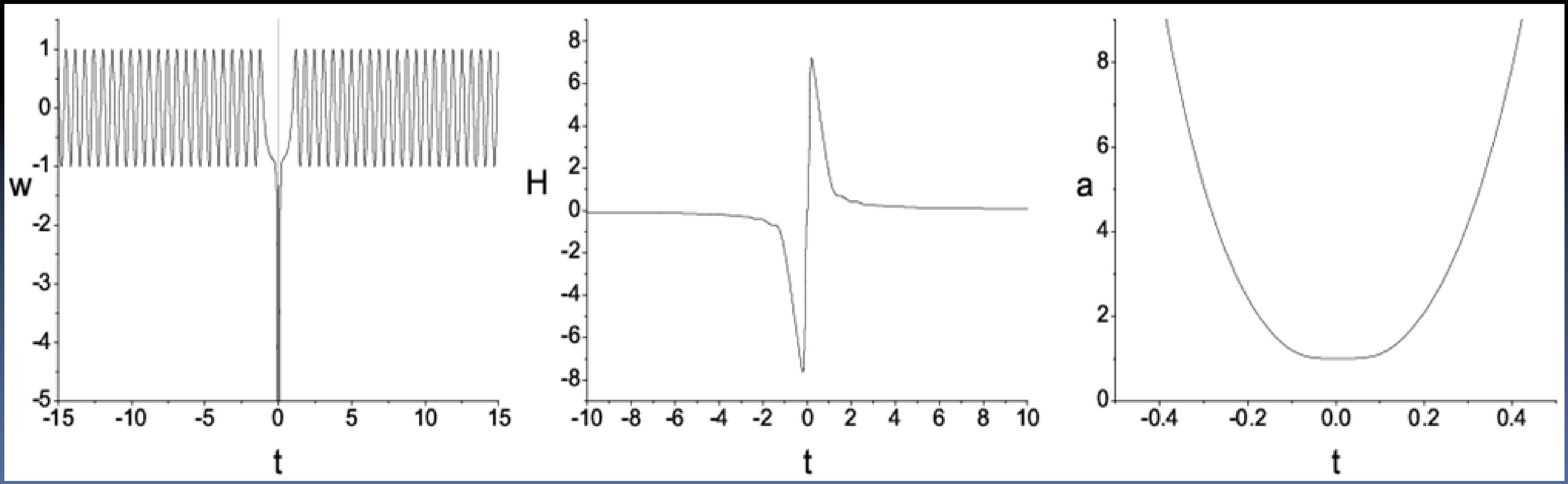
Y. Cai, Taotao Qiu, J. Xia, H. Li, X. Zhang, Phys.Rev.D79:021303,2009.

(III) Models without inflation (Lee-Wick type matter bounce)

Lagrangian: $\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2$

Or equivalentl $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2\phi^2$

Background evolution:



(III) Models without inflation

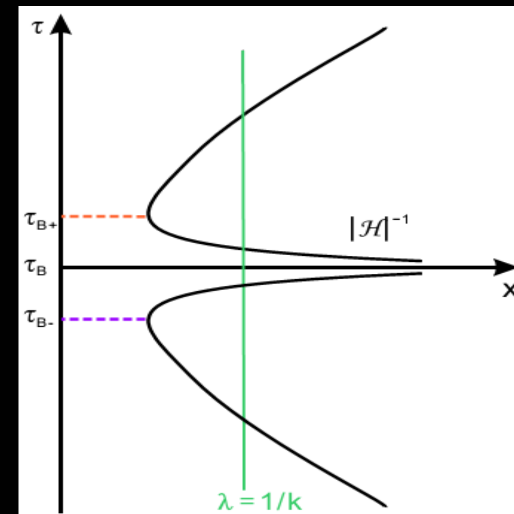
Analytical

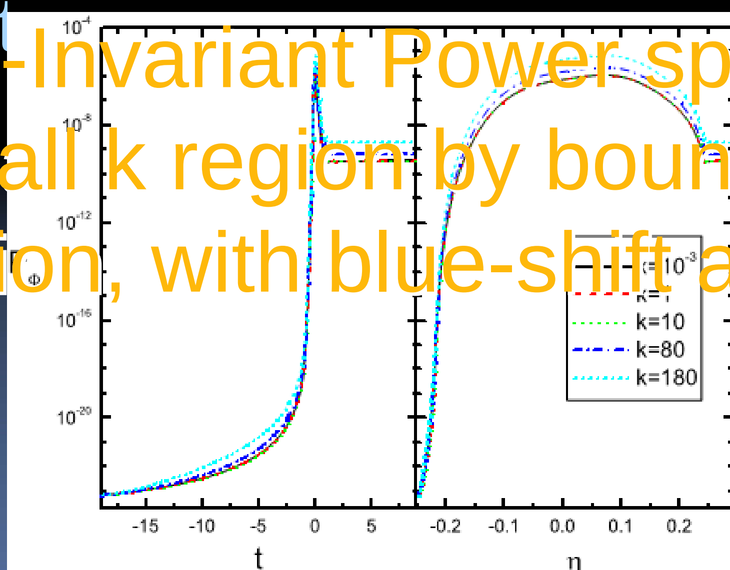
$$\Phi = \Phi_D + \Phi_S$$

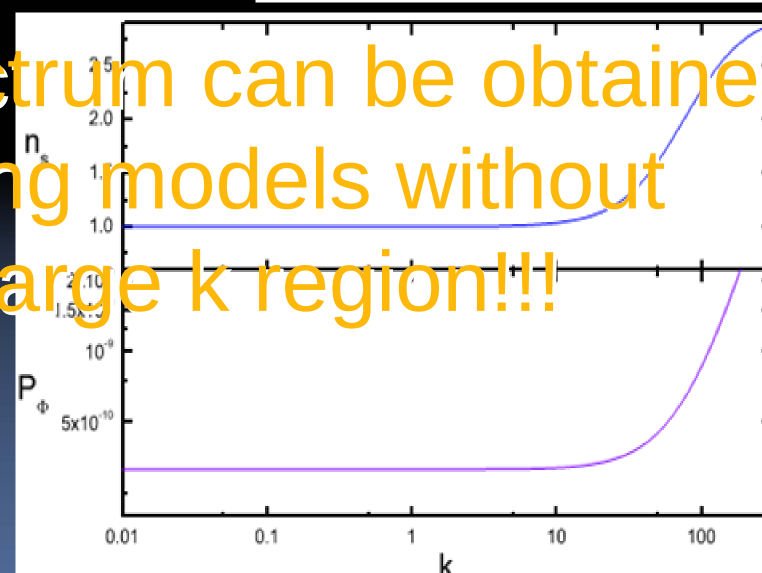
$$P_\Phi = k^3 |\Phi_D|^2 \sim k^0$$

Numerical

Sketch plot of perturbation:







Signature III :

Scale-Invariant Power spectrum can be obtained at small k region by bouncing models without inflation, with blue-shift at large k region!!!

Y. Cai, Taotao Qiu, R. Brandenberger and X. M. Zhang, arxiv: 0810.4677[hep-th].

Characteristics of (Quintom) Bouncing scenario:

(1)Can avoid Big-Bang Singularity;

(2)Multi-degree of freedom, thus may induce isotropic perturbations;

(3)May induce growth of anisotropy in contracting phase, and form black holes;

(4)May induce large non-gaussianity which cannot be ruled out by observations;

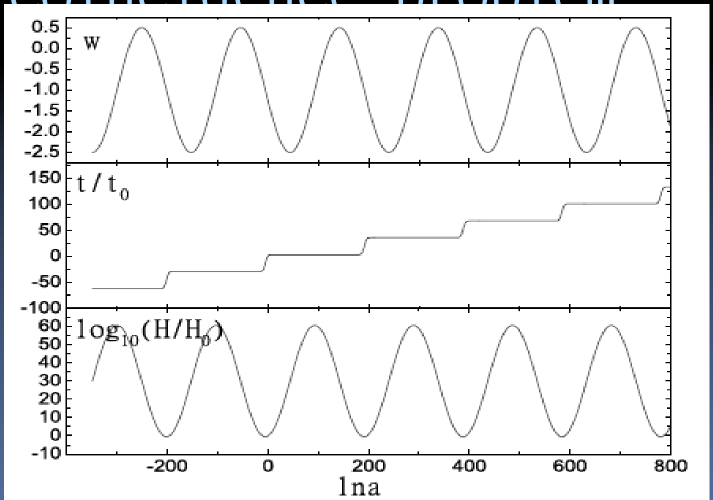
… …

The generalization of bouncing cosmology: cyclic universe

Two definitions: 1) the universe is always expanding with Hubble parameter oscillating;
2) the scale factor is oscillating, and expansion and contraction proceed alternately.

Theoretical motivation: 1) to avoid Big-Bang Singularity
2) to alleviate coincidence problem

Phenomilogical study:



$$w(\ln a) = -1 - 1.5\cos(0.032\ln a - \frac{4\pi}{9})$$

Model building:

R. C. Tolman, *Oxford U. Press, Clarendon Press, 1934.*
P. J. Steinhardt et al., Phys. Rev. D65, 126003 (2002).
L. Baum et al., Phys. Rev. Lett. 98, 071301 (2007).
T. Clifton et al., Phys. Rev. D75, 043515 (2007);
M. Bojowald et al., Phys. Rev. D70, 083517 (2004);
H. H. Xiong et al., arXiv:0711.4469 [hep-th]
……

B. Feng, M. Li, Y. Piao and X. M. Zhang, Phys.Lett.B634:101-105,2006.

Our models

instein equations:

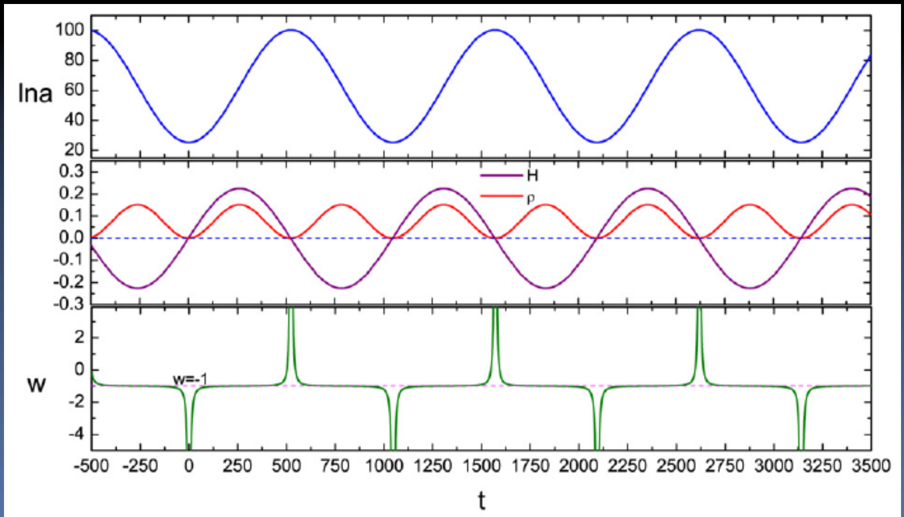
$$H^2 = \frac{8\pi G}{3}\rho$$
$$\dot{\rho} = -3H(\rho + p)$$

Lagrangian:

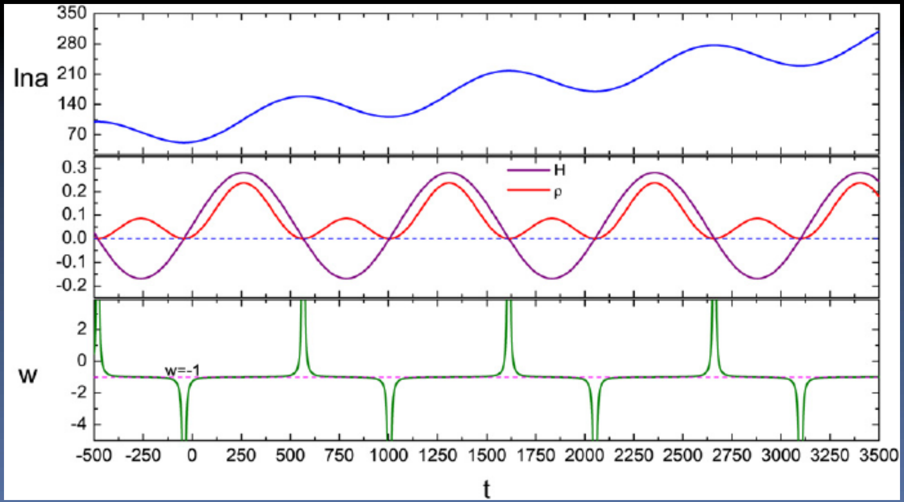
$$\mathcal{L} = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}\nabla_\mu\psi\nabla^\mu\psi - (\Lambda_0 + \lambda\phi\psi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}m^2\psi^2$$

s can be classified into 5 cases (where Λ_1 is relative to initial cond

Case I: $\Lambda_0 = 0$

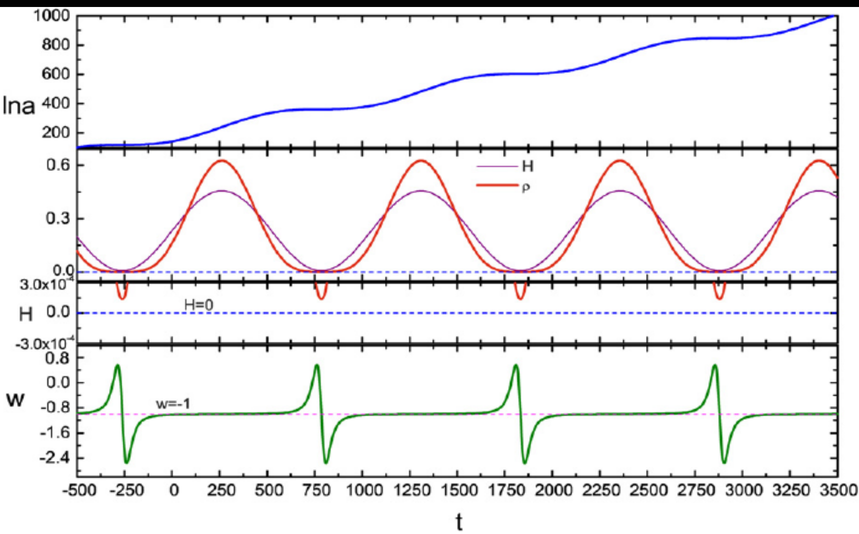


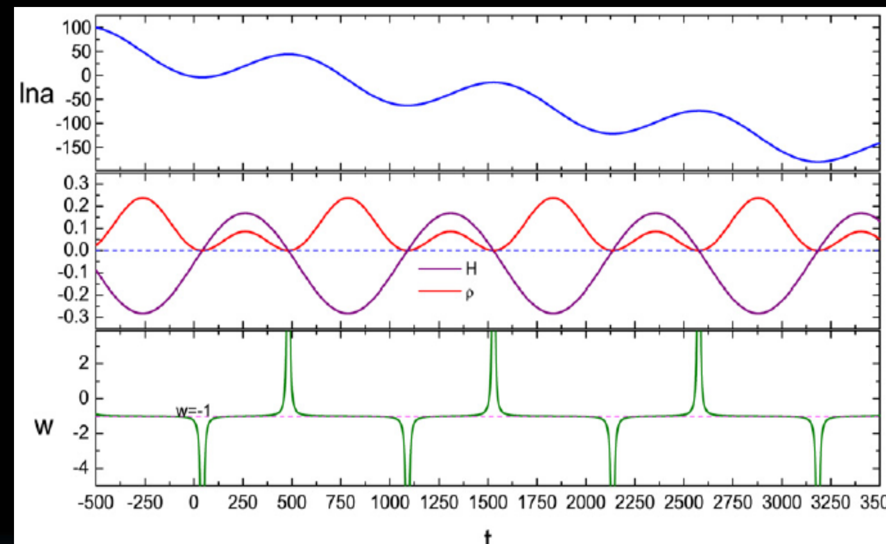
Case II: $0 < \Lambda_0 < \Lambda_1$



Case III: $\Lambda_0 \geq \Lambda_1$

Case IV: $-|\Lambda_1| < \Lambda_0 < 0$





Case V: $\Lambda_0 \leq -|\Lambda_1|$

This case corresponds to a eternally contracting universe and thus contradict with today' s reality. So we' ll not discuss about it.

H. Xiong, Y. Cai, Taotao Qiu, Y. Piao and X. M. Zhang, Phys.Lett.B666:212-217,2008.

Summary

- Standard Big Bang Theory can *almost* explain how our universe comes into being, except for some remaining issues. Inflation can solve the majority of them, but not all.

- To solve the singularity problem, a bounce process at the very beginning is generally need. Consistent perturbation theory and signatures on observations are presented.

- Extension of Bouncing Scenario: Cyclic Universe. Singularity problem can as well be solved and coincidence problem can be alleviated.

Thank you!!!