Walking dynamics from lattice gauge theory

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Based on: PRD80, 034507 (2009) and arXiv:0910.4196.

Outline

- Phenomenological motivation
- The step-scaling method
- The finite-volume Wilson loop scheme

  $\longrightarrow$ Definition and pure-YM
- The twisted Polyakov loop scheme

  $\longrightarrow$ The $N_f = 12$ case
- Conclusion and outlook

Fundamental scalar Higgs in the SM

- Hierarchy problem

  $\longrightarrow m_H^2 = 2\lambda v^2 \sim \Lambda_{\rm UV}^2$.

- The theory is trivial

  $\longrightarrow$ Higgs self-coupling: $\lambda(\mu) = \frac{\lambda(\Lambda_{\rm UV})}{1+(24/16\pi^2)\lambda(\Lambda_{\rm UV})\log(\Lambda_{\rm UV}/\mu)}$

  $\longrightarrow$ The coupling vanishes for all μ when the cut-off $\Lambda \to \infty$.

- A solution: strong interactions involving fermions

  $\longrightarrow \Lambda_{\rm EW} \sim \Lambda_{\rm UV} e^{-g_c^2/g_{\rm UV}^2}$.

Motivation
Extended technicolour model

- Standard-model fermion masses

  $\longrightarrow m_f = \frac{C(\mu)}{\Lambda^2_{ETC}} \langle \bar{\psi}\psi \rangle$ via dim-6 operators $\frac{C(\mu)}{\Lambda^2_{ETC}} \bar{\psi}\psi \bar{f} f$.

  $\longrightarrow \Lambda_{ETC} \sim$ TeV via estimating $\langle \bar{\psi}\psi \rangle$ using M_W and slow running of $\bar{\psi}\psi$.

- FCNC processes

  $\longrightarrow \frac{1}{\Lambda^2_{ETC}} \bar{f} f \bar{f} f$.

  $\longrightarrow \Lambda_{ETC} \sim 10^2 \sim 10^3$ TeV from constraints imposed by $K^0 - \bar{K}^0$ mixing.

- A solution: fast running of $\bar{\psi}\psi$ (large anomalous dimension).

Motivation
Slow and fast running of the condensate

- Slow running ("TeV-QCD")

  $\longrightarrow \langle\bar{\psi}\psi\rangle_{\mathrm{ETC}} \approx \langle\bar{\psi}\psi\rangle_{\mathrm{EW}}\ [1 + \alpha\ \log(\Lambda_{\mathrm{ETC}}/\Lambda_{\mathrm{EW}})] \sim \Lambda^3_{\mathrm{EW}}.$

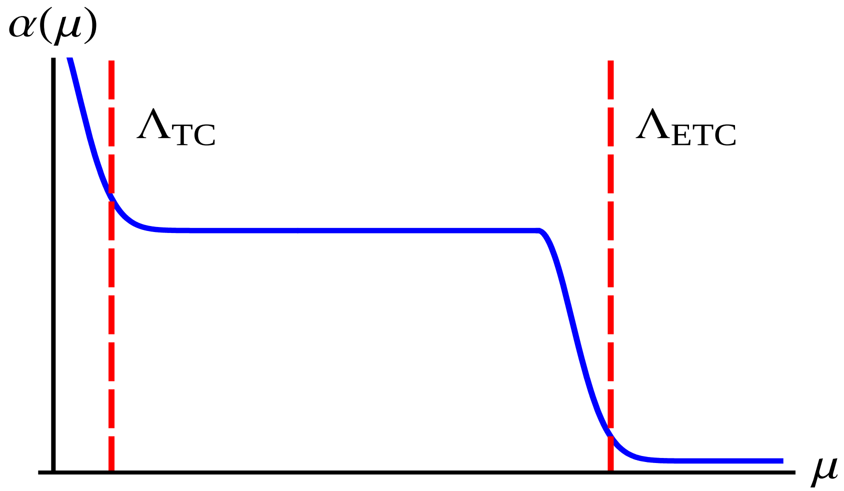
  $\longrightarrow \Lambda_{\mathrm{ETC}} \sim \Lambda_{\mathrm{EW}}(\Lambda_{\mathrm{EW}}/m_f)^{1/2}$

- Fast running (IR fixed point with $\gamma^* = 1$)

  $\longrightarrow \langle\bar{\psi}\psi\rangle_{\mathrm{ETC}} \approx \langle\bar{\psi}\psi\rangle_{\mathrm{EW}}\ (\Lambda_{\mathrm{ETC}}/\Lambda_{\mathrm{EW}}) \sim \Lambda^2_{\mathrm{EW}}\Lambda_{\mathrm{ETC}}.$

  $\longrightarrow \Lambda_{\mathrm{ETC}} \sim \Lambda_{\mathrm{EW}}(\Lambda_{\mathrm{EW}}/m_f)$

Motivation
Walking technicolour



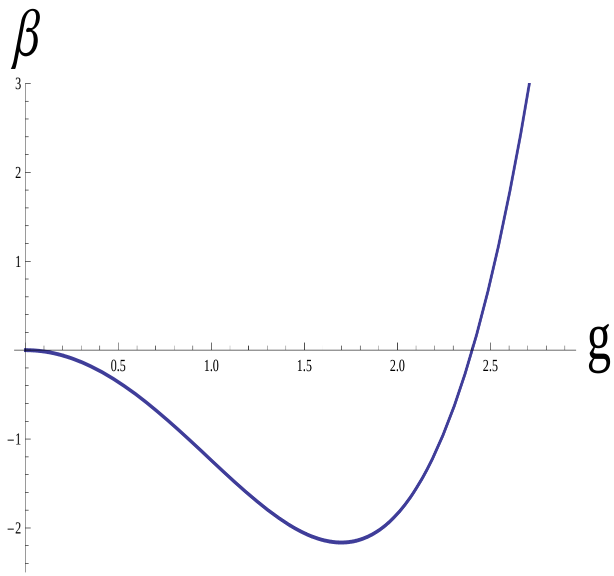
- Generates large anomalous dimension for $\bar{\psi}\psi$ to solve the FCNC problem.

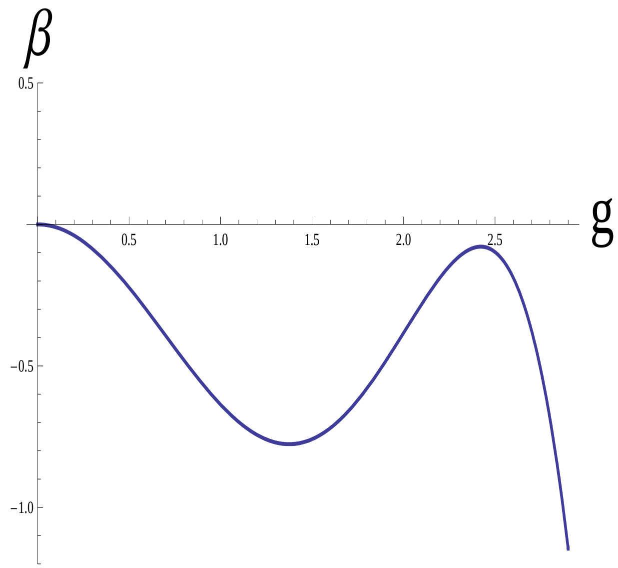
- Modifies the relevant spectral function and the OPE to elude the S-paramter criticism *a'la* Peskin and Takeuchi.

- $\Lambda_{\rm ETC}/\Lambda_{\rm TC} \sim 10^2 \sim 10^3$.

  $\longrightarrow$ Compared to the typical lattice size $L/a \sim 30$ in each direction.

Motivation
IR-conformal/Walking β function





- Need a scale to have a gap.
- How do we look for a walking theory?

Motivation
Technicolour in the twenty-first century

- Except for "TeV-QCD", technicolour has not been ruled out.
- Serious lattice calculations to support/kill technicolour
  - Hadron spectrum.

    L. Del Debbio, A. Patella and C. Pica
    S. Catterall and F. Sannino
    A. Hietanen *et al.*

  - Pheses of candidate walking theories.

    S. Catterall *at al.*
    T. DeGrand, B. Svetitsky and Y. Shamir

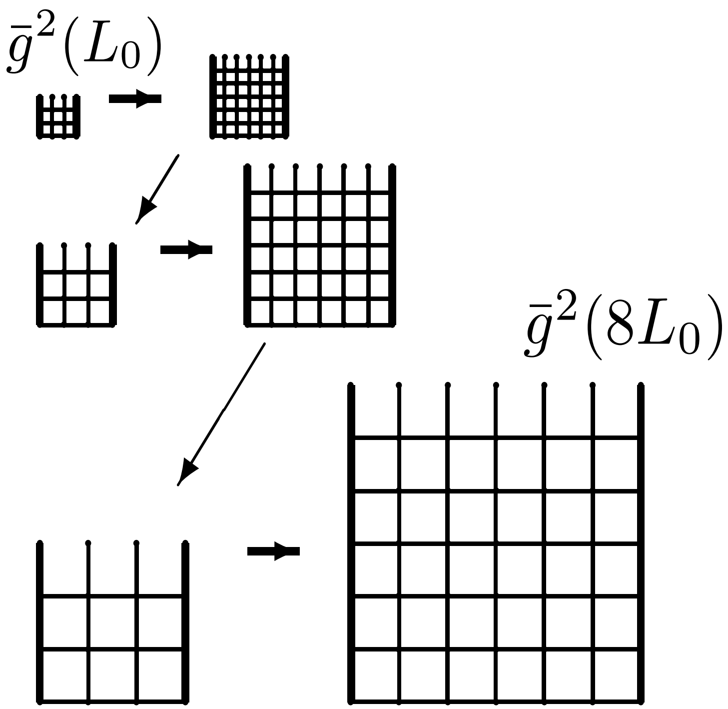
  - Spectrum of the Dirac operator.

    Z. Fodor *et al.*

  - The step-scaling method for the coupling constant.

    T. Appelquist, G. Fleming and E. Neil
    This work

The step-scaling method
The idea



- Extrapolate to the continuum limit at every step.
- Vary the scale by changing the dimensionful lattice size L_0.

The step-scaling method
Implementation: An example

1. Prepare lattices of size $\widehat{L_0}^4$:

$$\widehat{L_0} = L_0/a = 6, 8, 10, 12,$$

   and tune the lattice spacing (via varying the bare coupling).

   $\longrightarrow$ The renormalised coupling $g(L_0)$ is the same on these lattices.

2. Double the lattice size to be

$$\widehat{L_0} = L_0/a = 12, 16, 20, 24,$$

   and calculate $g(2L_0, a/L_0)$.

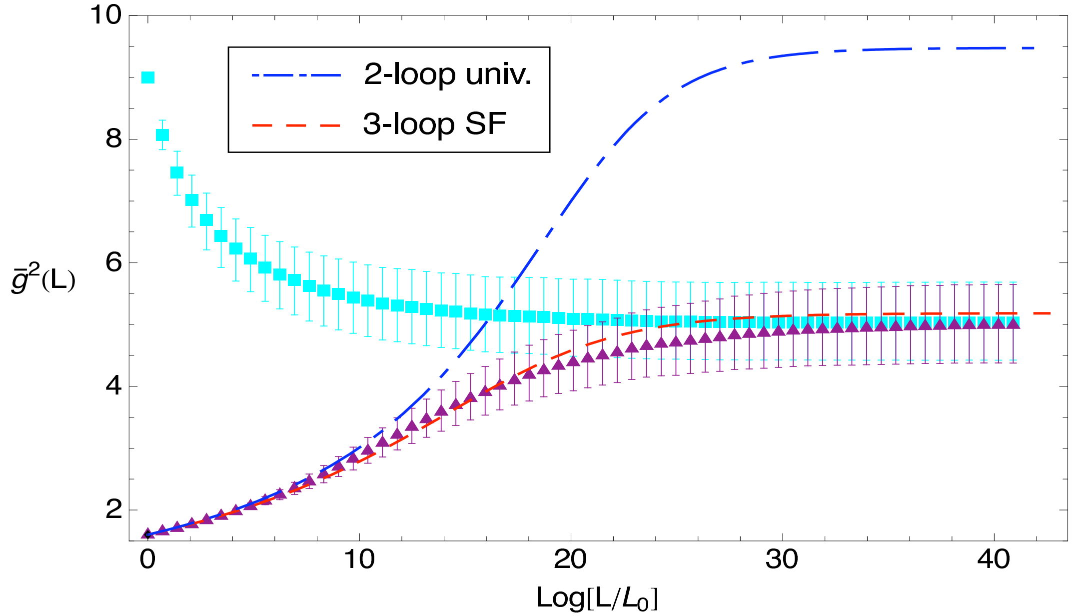
   $\longrightarrow$ Extrapolate to the continuum limit to get $g(2L_0)$.

3. Go back to the lattices in 1., and tune the lattice spacing.

   $\longrightarrow$ The renormalised coupling equals $g(2L_0)$ on all these lattices.

   $\longrightarrow$ Repeat 2. to obtain $g(4L_0)$.

The step-scaling method
Results for the Schödinger Functional scheme



T.Appelquist, G.Fleming and E.Neil, PRD79, 076010 (2009).

- SU(3) fundamental fermions, $N_f = 12$.
- Scheme dependence?
- Controversy from the study of the Dirac operator eigenvalue spectrum?

A new non-perturbative scheme
General considreations

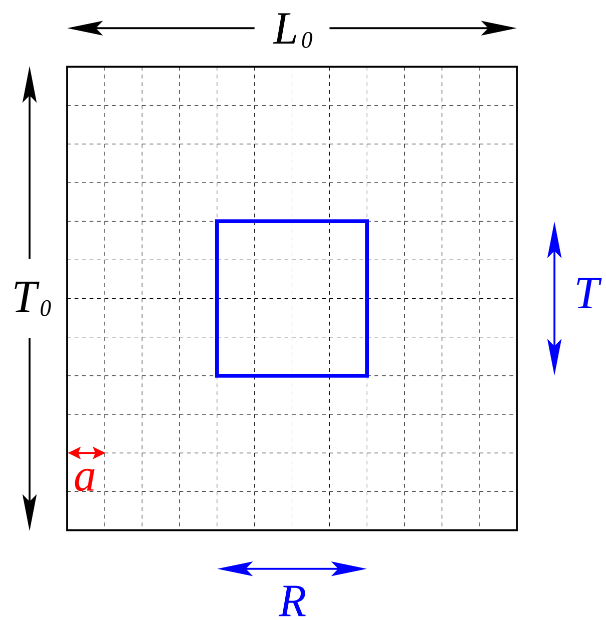
$$A_{\mathrm{LO}} = k\ g_0^2$$

$$A_{\mathrm{NP}}(\mu) = k\ g^2(\mu)$$

$$\Big\downarrow \quad \text{Lattice}$$

$$A_{\mathrm{NP}}(L,a) = k(a)\ g^2(L,a)$$

The finite-volume Wilson-loop scheme
The idea



- Set up $T_0 = L_0$ and $T = R$.
- When $a \to 0$, with fixed $\hat{r} \equiv (R + a/2)/L_0$, there is only one scale.

$\longrightarrow$ Varying $\hat{r}$ corresponds to changing scheme.

The finite-volume Wilson-loop scheme
More details

- The Creutz Ratio (CR):

$$-R^2 \frac{\partial^2}{\partial R \partial T} \ln \langle W(R,T;L_0,T_0) \rangle |_{T=R,T_0=L_0} \xrightarrow{\text{LO}} k g_0^2 .$$

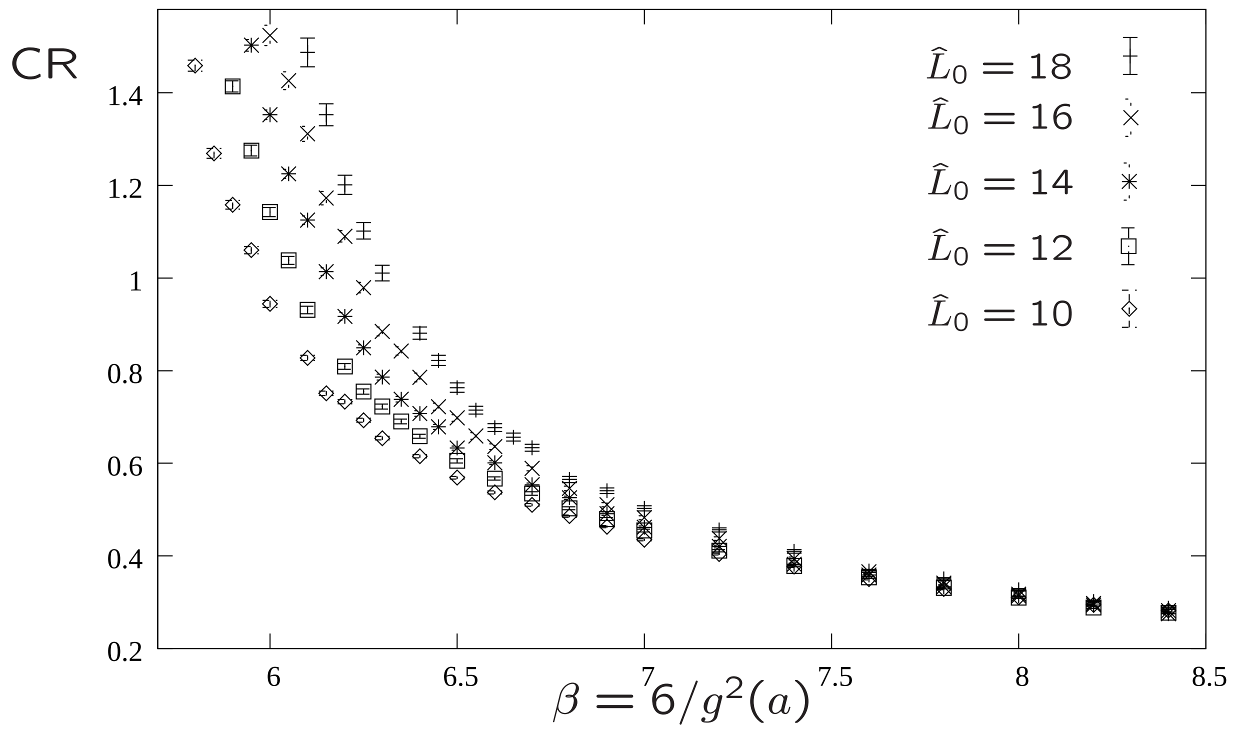
- On the lattice ($\hat{R} = R/a$ and $\hat{L}_0 = L/a$):

$$-(\hat{R}+1/2)\ \chi(\hat{R}+1/2, \hat{T}+1/2; \hat{L}_0) = -(\hat{R}+1/2)\ \ln \left[\frac{W(\hat{R}+1,\hat{T}+1;\hat{L}_0) W(\hat{R},\hat{T};\hat{L}_0)}{W(\hat{R}+1,\hat{T};\hat{L}_0) W(\hat{R},\hat{T}+1;\hat{L}_0)} \right] .$$

- The factor k can be calculated analytically (periodic BC)

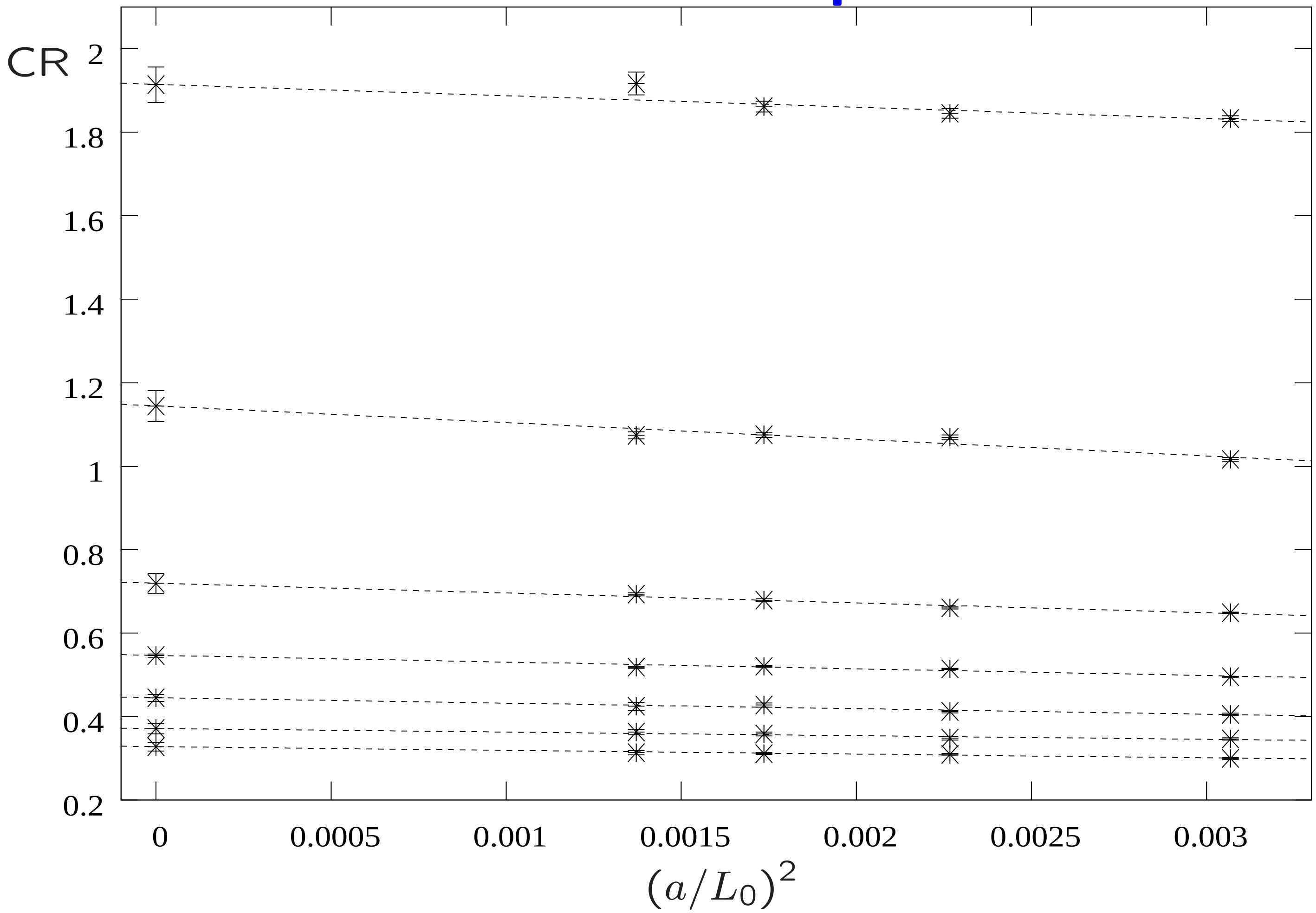
$$k = -R^2 \frac{\partial^2}{\partial R \partial T} \left[\frac{4}{(2\pi)^4} \sum_{n_\mu \neq 0} \left(\frac{\sin\left(\frac{\pi n_0 T}{L_0}\right)}{n_0} \right)^2 \frac{\mathrm{e}^{i \frac{2\pi n_1 R}{L_0}}}{n_0^2 + \vec{n}^2} \right]_{T=R} + \text{zero} - \text{mode contrib.}$$

Numerical test of the FV Wilson-loop scheme
Simulation parameters in pure-YM

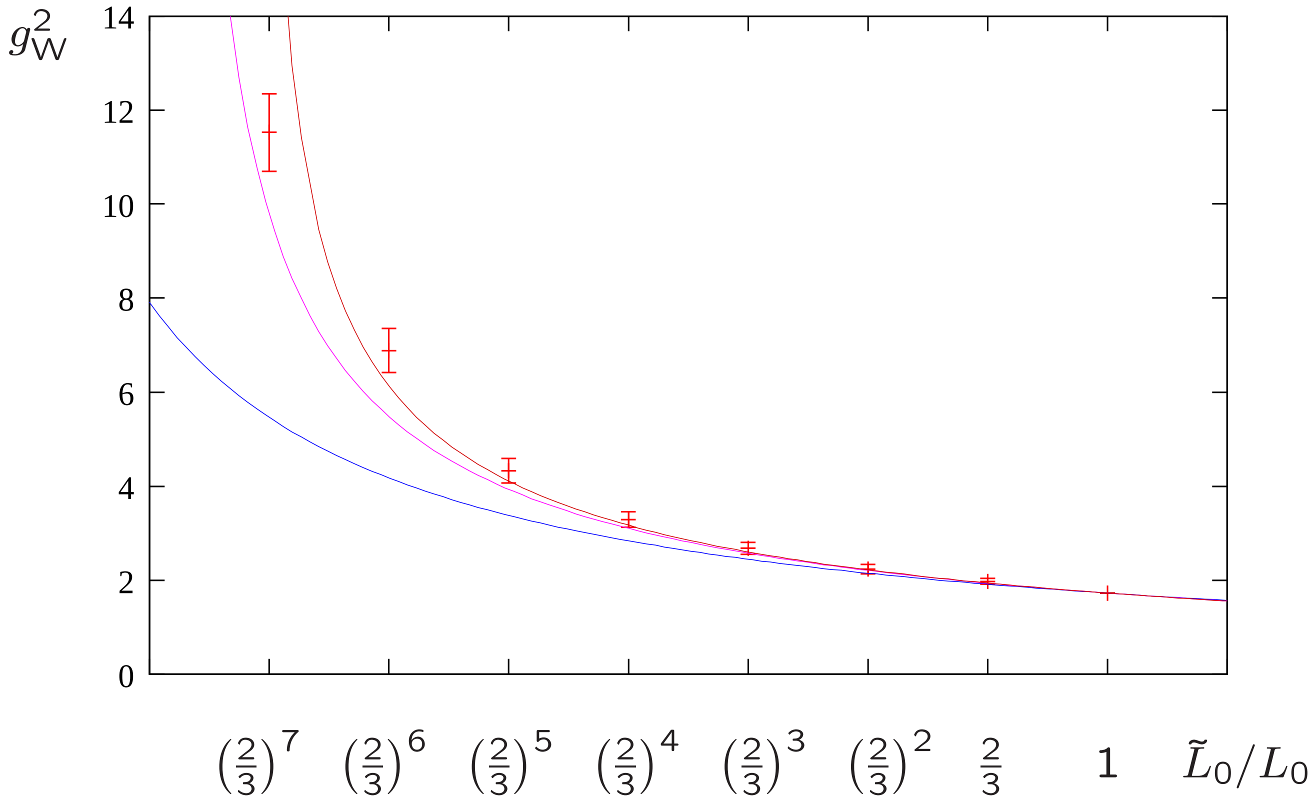


- Take CR $= 0.2871$ as the starting point (step 0, scale $\tilde{L}_0$).
- Take the step size $s = 1.5$.

Numerical test of the FV Wilson-loop scheme
Continuum extrapolation



Numerical test of the FV Wilson-loop scheme
Result for pure-YM



Twisted Polyakov Loop scheme
Twisted Boundary Condition

- $U_\mu(x+\widehat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger,\ \nu = 1, 2,$

  where the twist matrices Ω_ν satisfy

  $\Omega_1\Omega_2 = \mathrm{e}^{2i\pi/3}\Omega_2\Omega_1,\ \Omega_\mu\Omega_\mu^\dagger = 1,\ (\Omega_\mu)^3 = 1,\ \mathrm{Tr}(\Omega_\mu) = 0.$

't Hooft, 1979.

- If $\psi(x+\widehat{\nu}L) = \Omega_\nu\psi(x) \Rightarrow \psi(x+\widehat{\nu}L+\widehat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$

- The "smell" d.o.f. for fermions, $N_s = N_c$:

  $\psi_\alpha^a(x+\widehat{\nu}L) = \mathrm{e}^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(x)(\Omega_\nu)_{\beta\alpha}^\dagger.$

G.Parisi, 1983.

Twisted Polyakov Loop scheme
The scheme

G.M.de Divitiis *et al.*, 1994

- Polyakov loops in the twisted direction:

  $P_1(y,z,t) = \mathrm{Tr}\langle \Pi_j U_1(j,y,z,t)\Omega_1 \mathrm{e}^{2iy\pi/3L}\rangle$ (gauge and translation invariant).

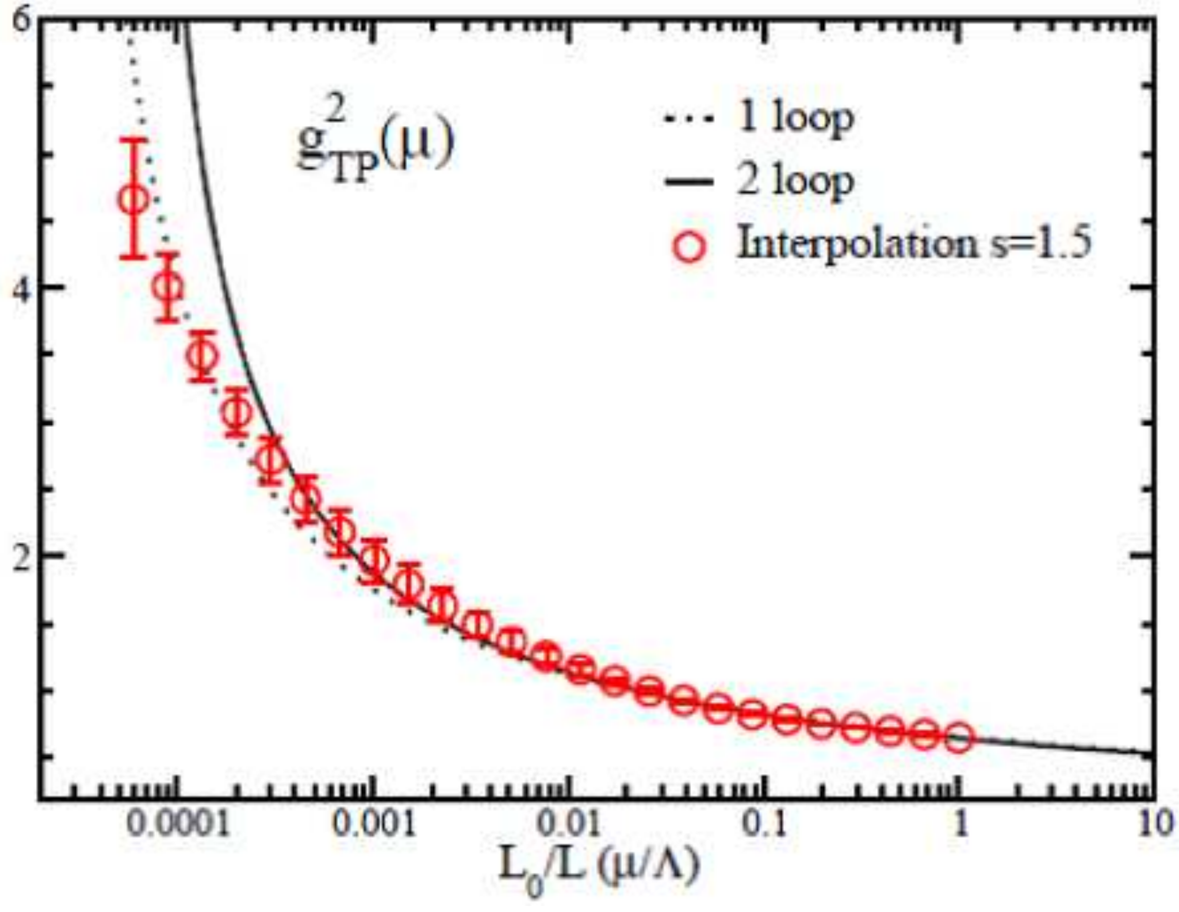
- The coupling constant

$$g^2_{\mathrm{TP}}(L) = \frac{1}{k}\frac{\langle \sum y,z P_1(y,z,L/2)P_1^*(0,0,0)\rangle}{\langle \sum x,y P_3(x,y,L/2)P_3^*(0,0,0)\rangle},$$

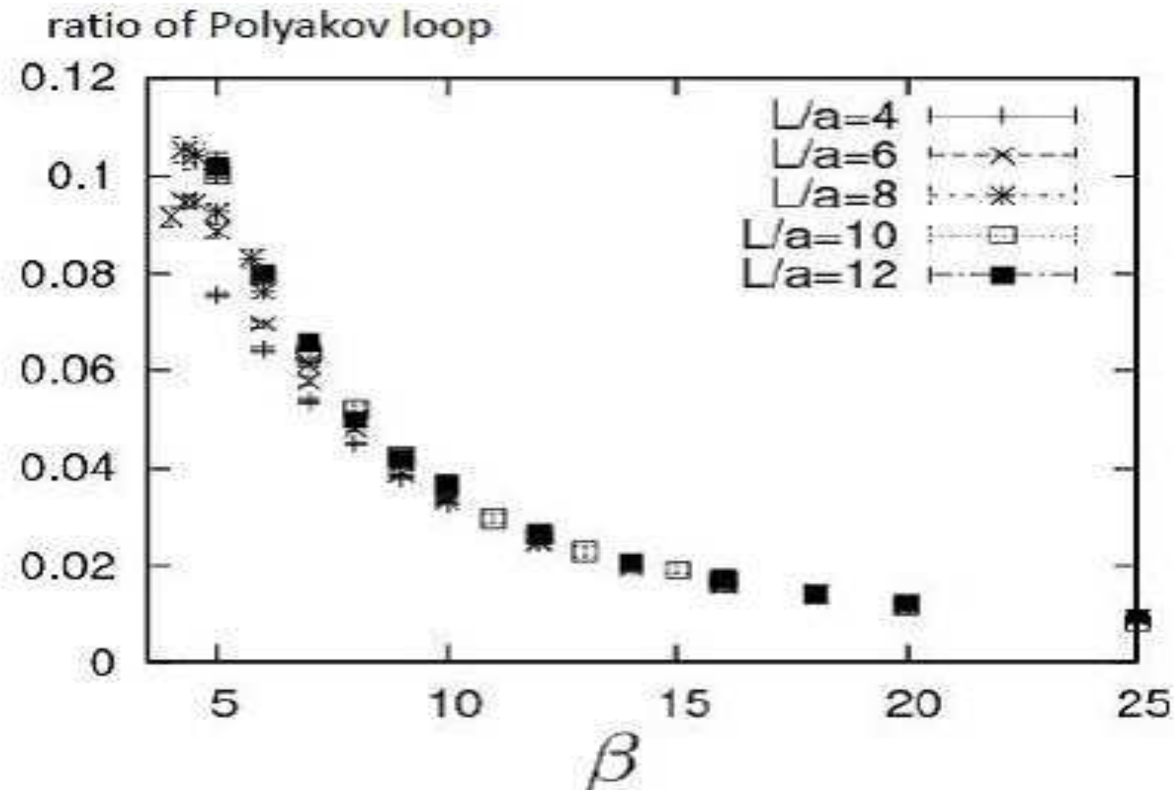
where $k = \frac{1}{24\pi^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{n^2+(1/3)^2} \sim 0.031847$

- Special feature: At $L \to \infty$, $g^2_{\mathrm{TP}} \to \frac{1}{k} \sim 32$ if there is no IRFP.

Twisted Polyakov Loop scheme
Result for pure-YM

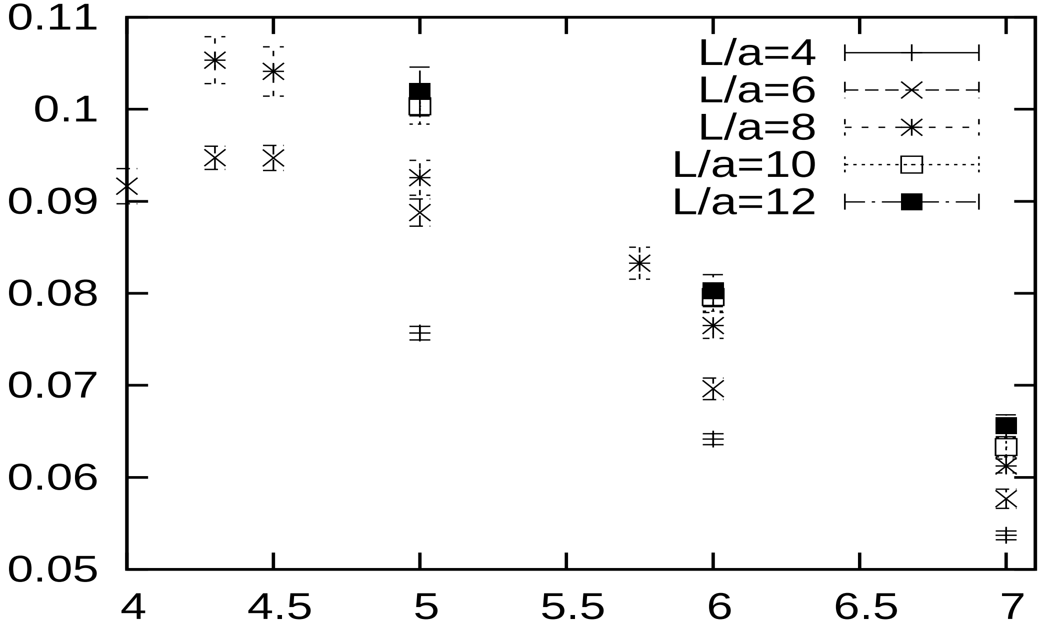


Twisted Polyakov Loop scheme
$N_f = 12$ simulation with staggered fermion



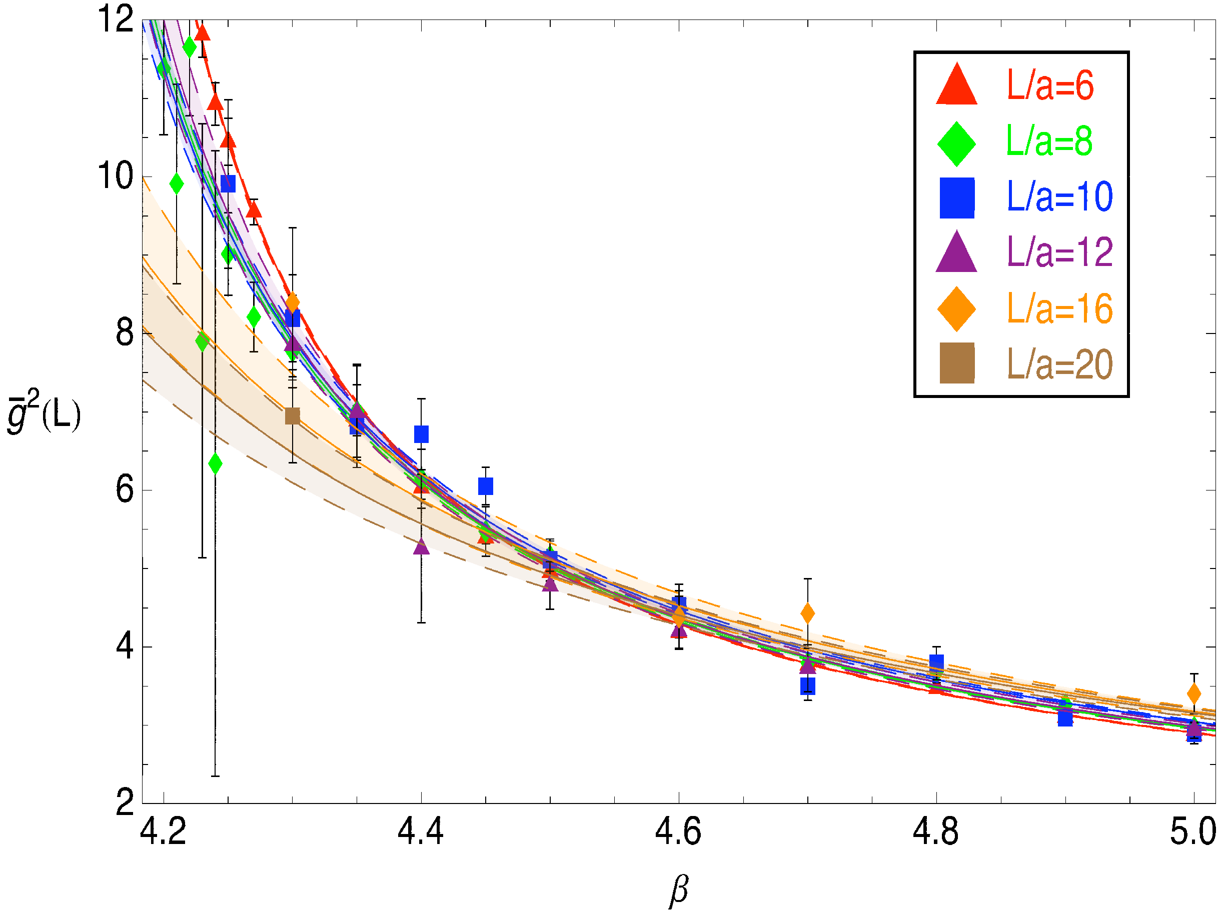
- $N_f = N_t \times N_s = 4 \times 3 = 12$.
- $s = 1 \Rightarrow L = 4, 6, 8, 10;$ $s = 1.5 \Rightarrow L = 6, 9, 12, 15.$

Twisted Polyakov Loop scheme
$N_f = 12$ simulation at low$-\beta$



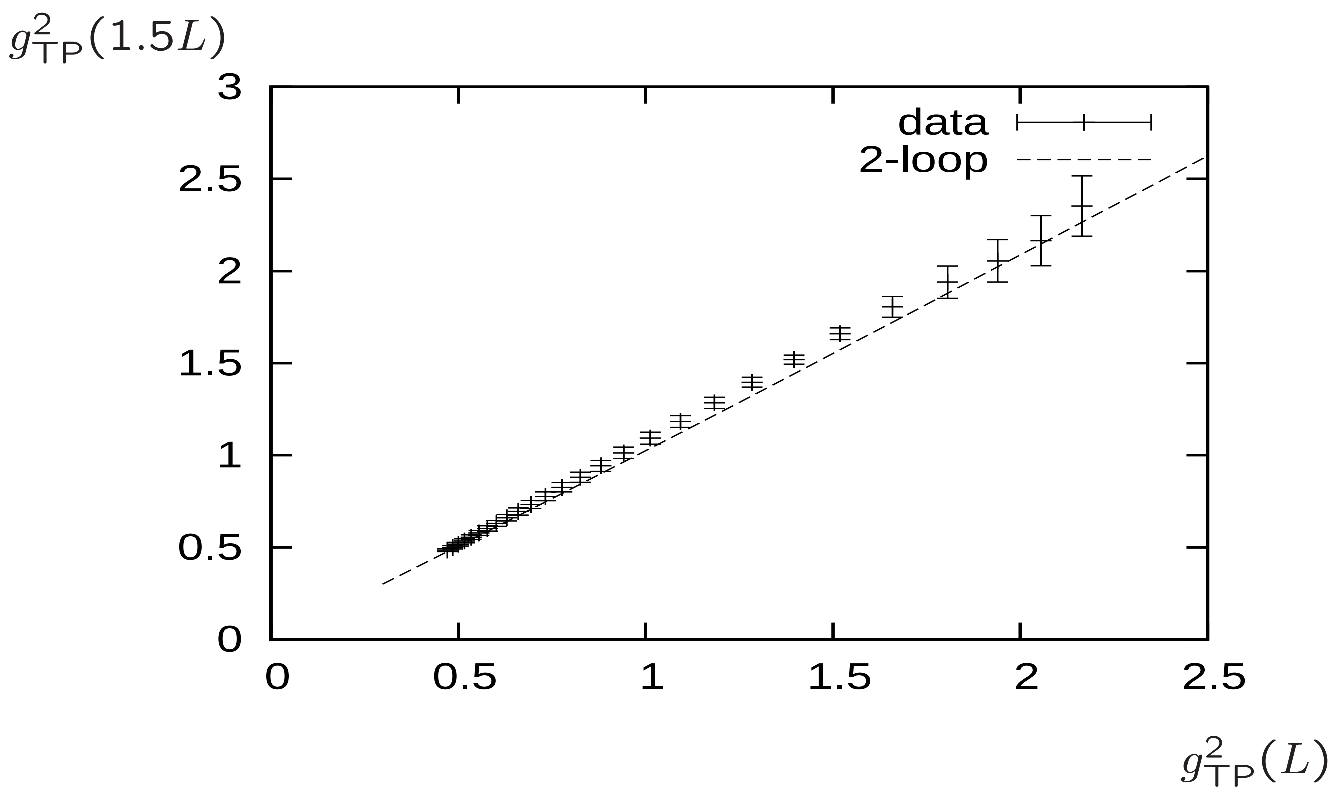
Very different from the SF-scheme.....

Twisted Polyakov Loop scheme
Compare with the SF scheme



T.Appelquist, G.Fleming and E.Neil, PRD79, 076010 (2009).

Twisted Polyakov Loop scheme
Step-scaling behaviour



Twisted Polyakov Loop scheme
Obtaining the β function

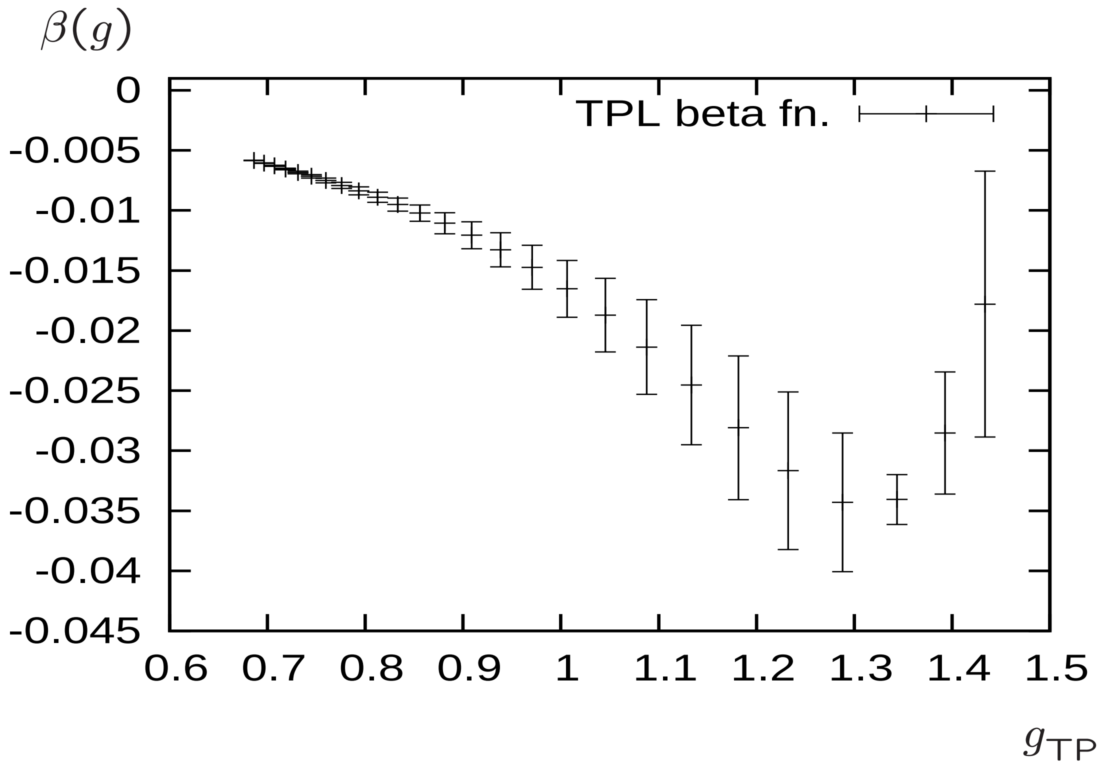
1. $g^2(1.5L) = g^2(L) + s_0\ g^4(L) + s_1\ g^6(L) + s_2\ g^8(L) + s_3\ g^{10}(L)$,

   where we use perturbative results for s_0 and s_1, then fit s_2 and s_3.

2. Then the value of the β function at each step is

$$\beta(g(1.5L)) = \beta(g(L)) \left(\frac{g(L)}{g(1.5L)}\right) \frac{\partial g^2(1.5L)}{\partial g^2(L)}.$$

Twisted Polyakov Loop scheme
Result for β function



Concluding remarks and outlook

- We have designed a new non-perturbative Wilson Loop scheme for calculating the running coupling constant.
- We have demonstrated it is valid through numerical tests in pure-YM.
- Using the Twisted Polyakov Loop scheme, we have seen evidence (preliminary!) that $N_f = 12$ QCD contains a non-trivial IR fixed point.
- Other studies under way.

It is a new lattice research avenue.