Effects of a pre-inflation radiation-dominated epoch to CMB anisotropy

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Scalar field fluctuations in Schwarzschild-de Sitter space-time

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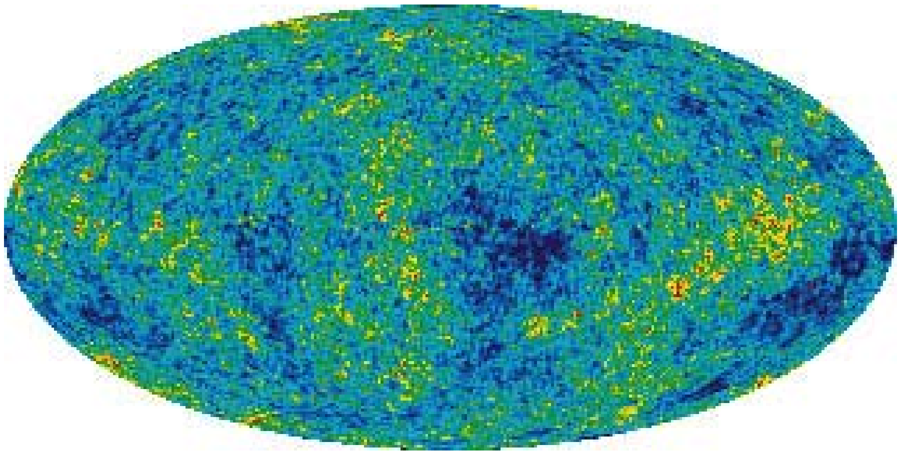
Effects of a pre-inflation radiation-dominated epoch to CMB anisotropy

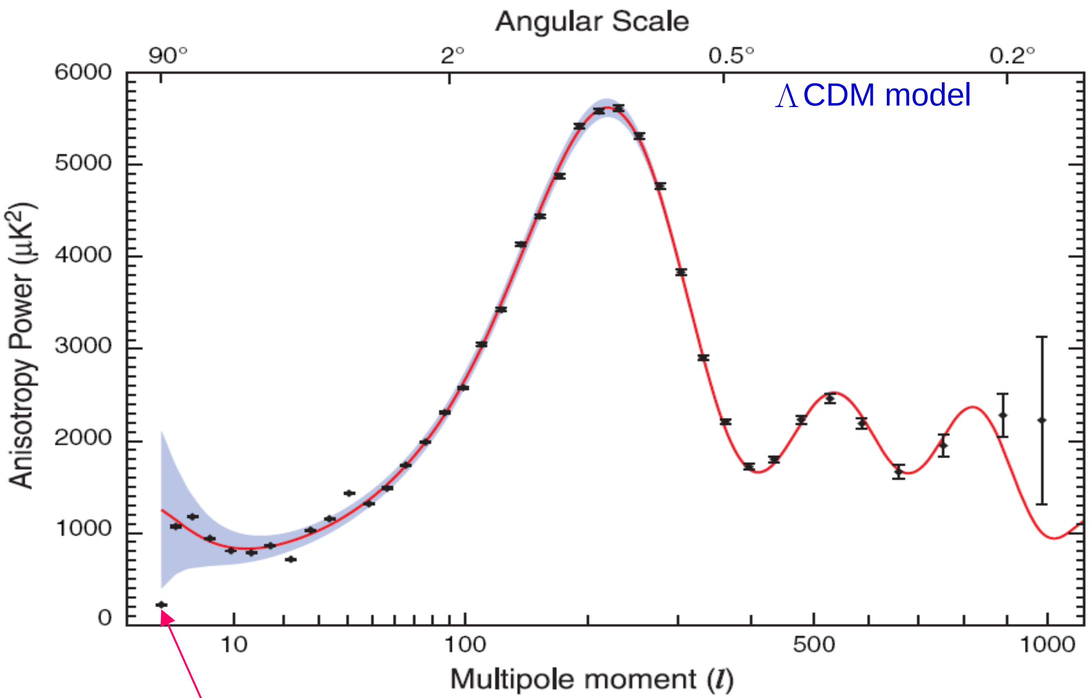
Outline

- Motivation: at ℓ=2 WMAP data & ΛCDM model is not consistent
- Our Model: we assume a pre-inflation radiation-dominated phase before inflation
- Numerical results
- Conclusion

WMAP (Wilkinson Microwave Anisotropy Probe)

about 2.73k

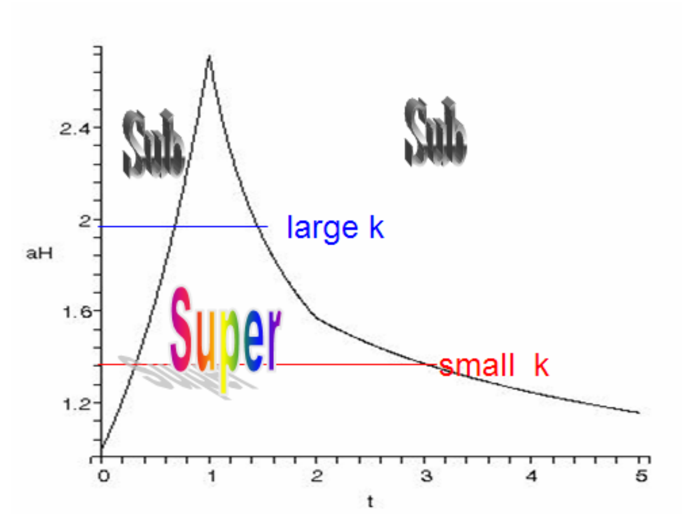




deviation

The key feature of inflation

$a=e^{Ht}$,
$a=t^{1/2}$,
$a=t^{2/3}$



Modes were out and then entered our horizon again

K=aH

horizon crossing condition

Equation for inflation fluctuation

$$\varphi(t,\vec{y}) = \overline{\varphi} + \phi(t,\vec{y})$$

$$\frac{\delta T}{T} \propto \phi(t,\vec{y})$$

$$\Box\phi(t,\vec{y}) = 0$$

$$ds^2 = g_{\mu\nu}dy^\mu dy^\nu = dt^2 - a^2(t)d\vec{y}^2$$

$$\boxed{\ddot{\phi}_k(t) + 3\frac{\dot{a}}{a}\dot{\phi}_k(t) + \left(\frac{k^2}{a^2}\right)\phi_k(t) = 0}$$

$$\dot{\phi}_k(t) \equiv \frac{d\phi_k(t)}{dt}$$

Λ CDM model

$$a(t) \propto e^{Ht}$$

only inflation phase

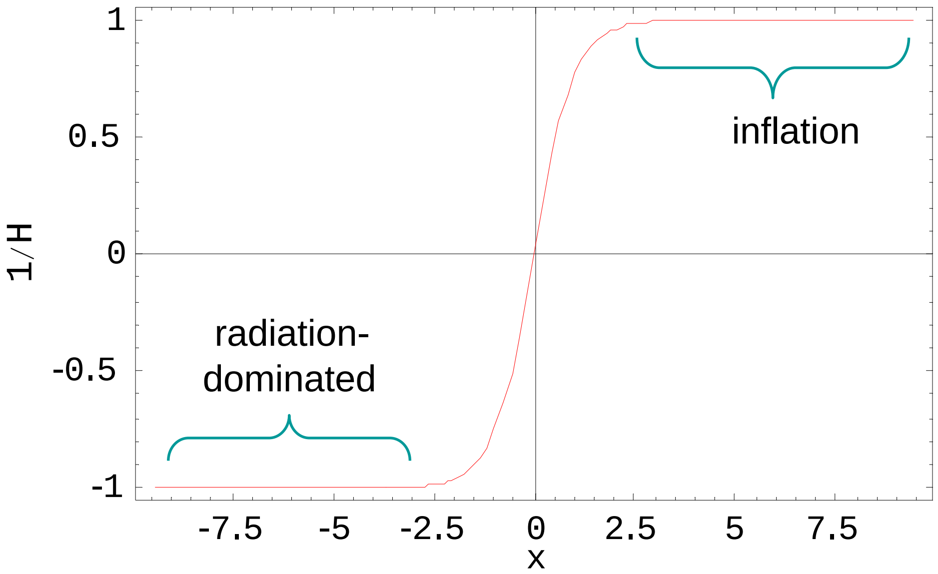
Our model

$$\rho = 3M_G^4 \left(\frac{A}{a^4} + B \right)$$

A ~ radiation component

B ~ vacuum energy

$$a(t) = \left(\frac{A}{B} \right)^{\frac{1}{4}} \left[\sinh \left(2\sqrt{B}\Lambda t \right) \right]^{\frac{1}{2}}$$



$$H \equiv \left(\frac{1}{a} \frac{da}{dt} \right)$$

Transition point for A=1 B=1 case

$a(t)$:

$$\Lambda t \ll \frac{1}{2} B^{-\frac{1}{2}} \quad , \quad a(t) \sim \sqrt{2} A^{\frac{1}{4}} (\Lambda t)^{\frac{1}{2}}$$

radiation

$$\Lambda t \gg \frac{1}{2} B^{-\frac{1}{2}} \quad , \quad a(t) \sim 2^{-\frac{1}{2}} \left(\frac{A}{B} \right)^{\frac{1}{4}} e^{\sqrt{B}\Lambda t}$$

inflation

$$\frac{d}{dt} = \frac{da}{dt}\frac{d}{da} \quad , \quad A/a_c^4 = B \quad , \quad x \equiv a - a_c \quad , \quad \rho = 3M_G^4\left(\frac{A}{a^4} + B\right)$$

$$\left[B(x+a_c)^4 + A\right]\phi_k''(x) + \left[4B(x+a_c)^3 + \frac{2A}{x+a_c}\right]\phi_k'(x) + k^2\phi_k(x) = 0 \quad , \quad \phi_k'(x) \equiv \frac{d\phi_k(x)}{dx}$$

Initial condition

radiation-dominated when a is small

$$\phi_k(x) = \frac{1}{a}\frac{1}{\sqrt{2k}}e^{ika/\sqrt{A}}$$

$$\phi_k'(x) = \left[-\frac{1}{\sqrt{2k}a^2} + \frac{i\sqrt{k}}{\sqrt{2A}a}\right]e^{ika/\sqrt{A}}$$

$$P_k^{1/2} \propto k^{3/2}\phi_k$$

$$k = aH$$

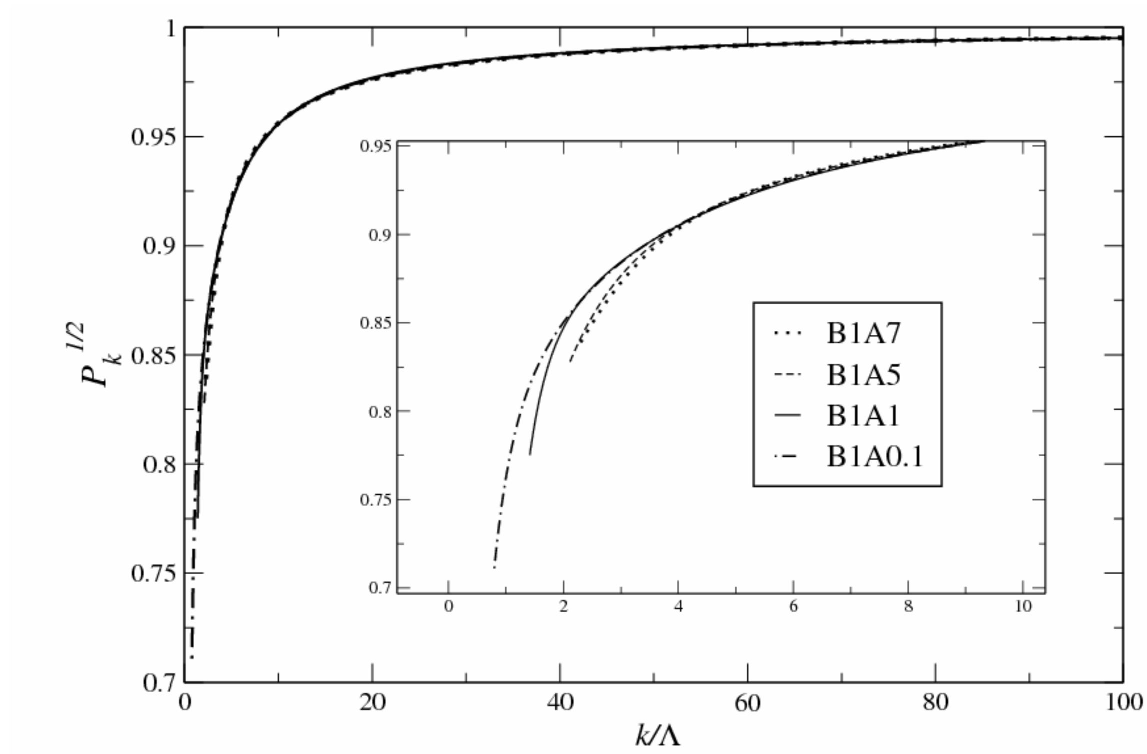
the horizon crossing k mode

$$P_k^{1/2} \propto k^{3/2}\phi_k$$

The power spectrum is a smooth curve…

A ~ radiation component

B ~ vacuum energy



But…

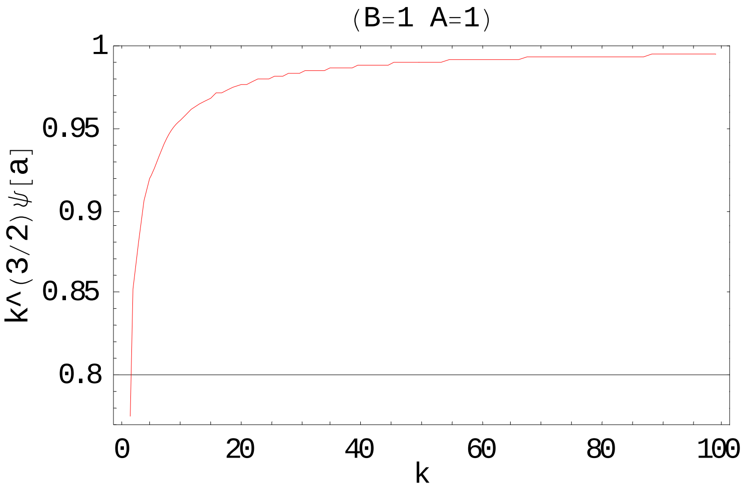
A smooth power spectrum is obtained under two conditions

(1) a(t) is a smooth transition function

(2) initial a(t) can't be too close to the phase transition point

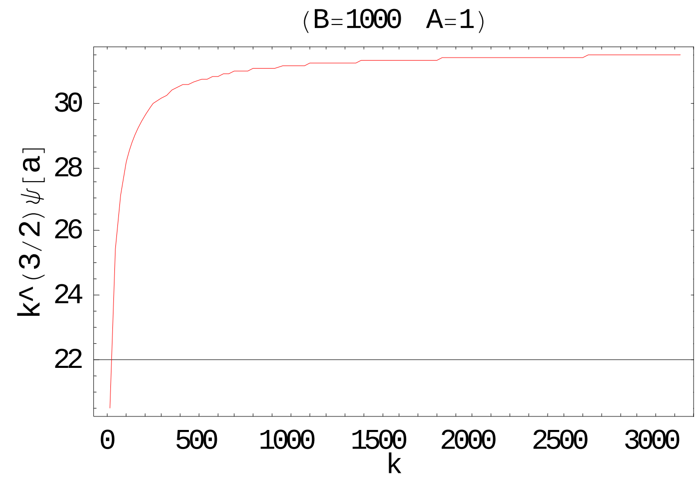
or

it will lead to oscillations on power spectrum ai=0.0001



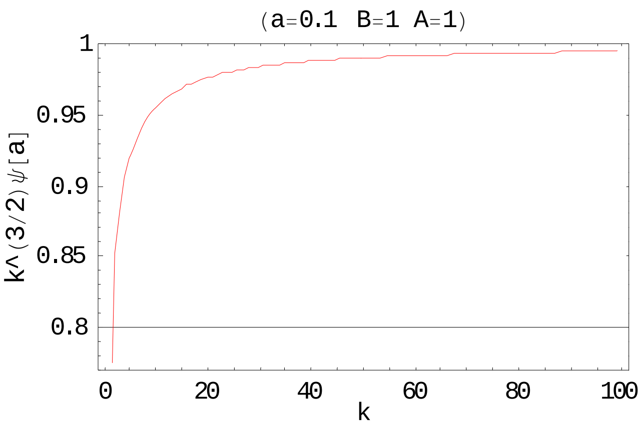
abrupt slope

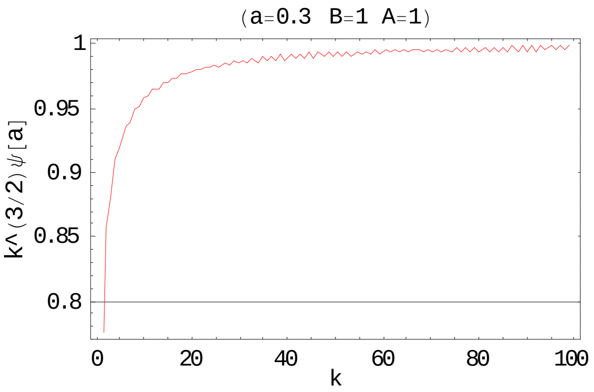
$a_i=0.0001<<a_t$

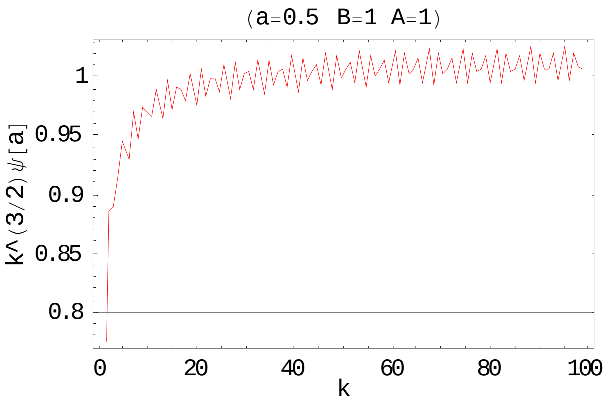


Initial a is too close to phase transition point

$a_t=(A/B)^{1/4}$







$a_t=1$

The choice of initial a is very important !

Numerical results

B=1 A=7, 5, 1 and 0.1

A ~ radiation component

B ~ vacuum energy

z ~ duration of inflation

FIG. 2 (a) B1A7

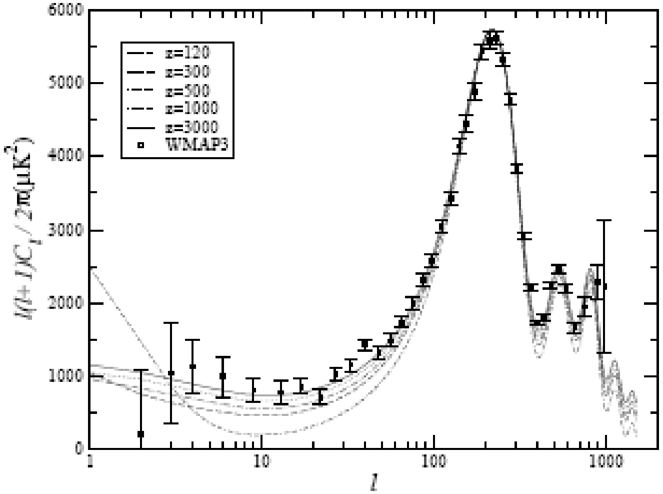


FIG. 2 (b) B1A5

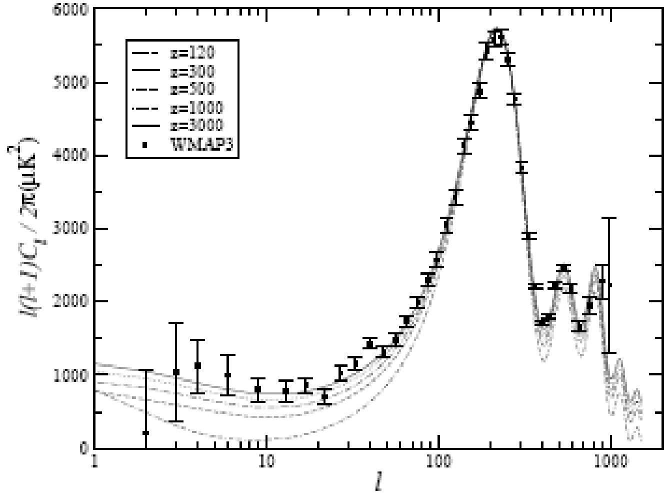


FIG. 2 (c) B1A1

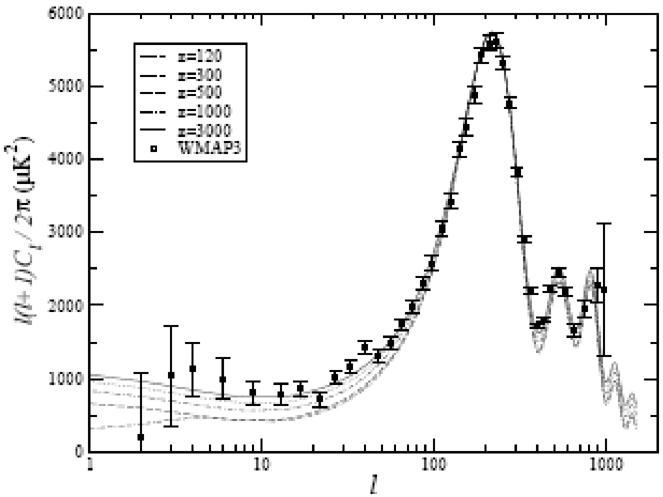
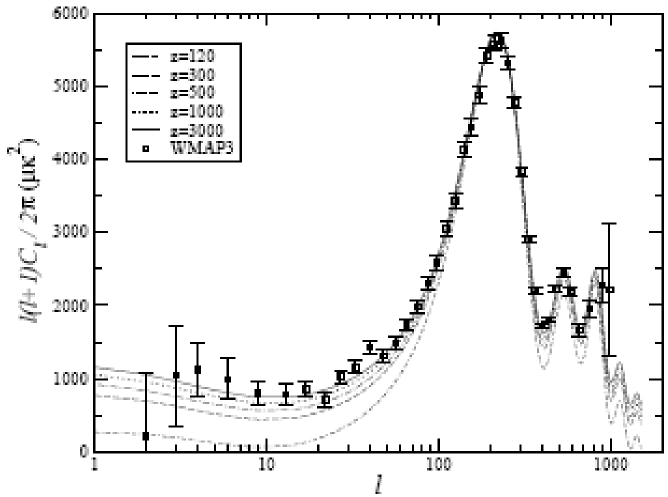
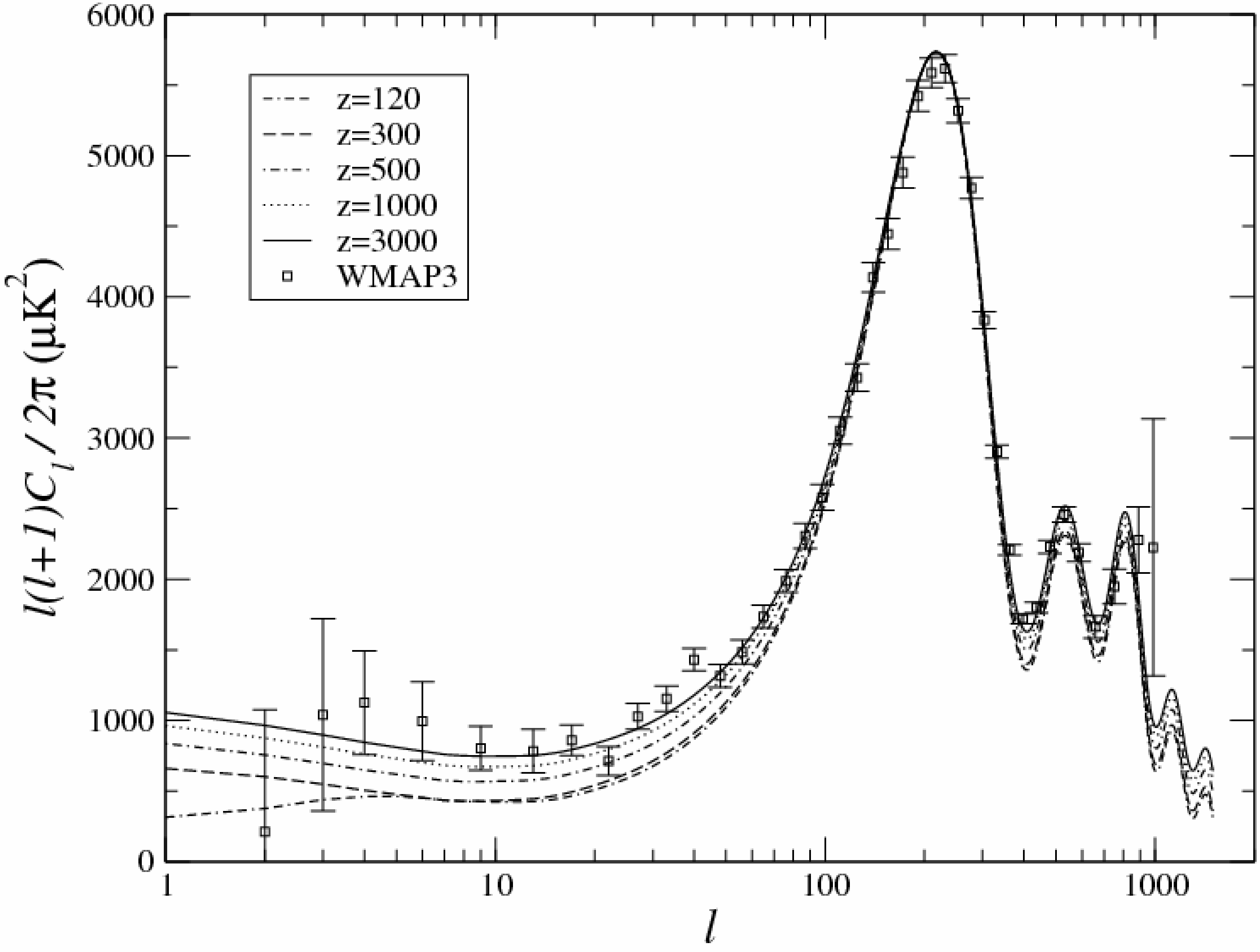


FIG. 2 (d) B1A0.1



B=1 and A=1

FIG. 2 (c) B1A1



B=0.1 A=7, 5, 1 and 0.1

FIG. 3 (a) B0.1A7

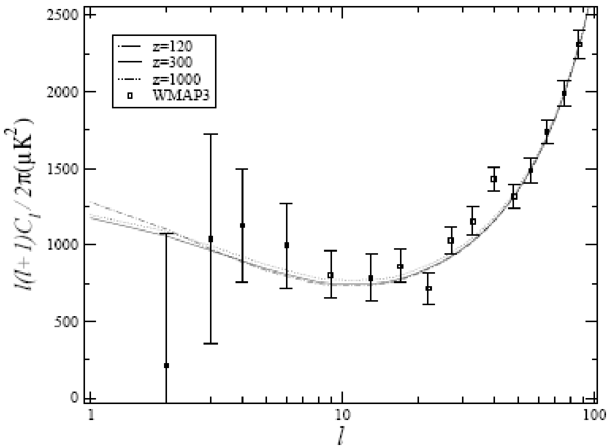


FIG. 3 (b) B0.1A5

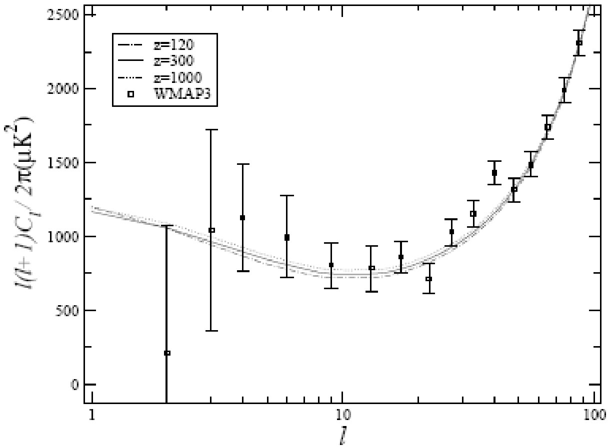


FIG. 3 (c) B0.1A1

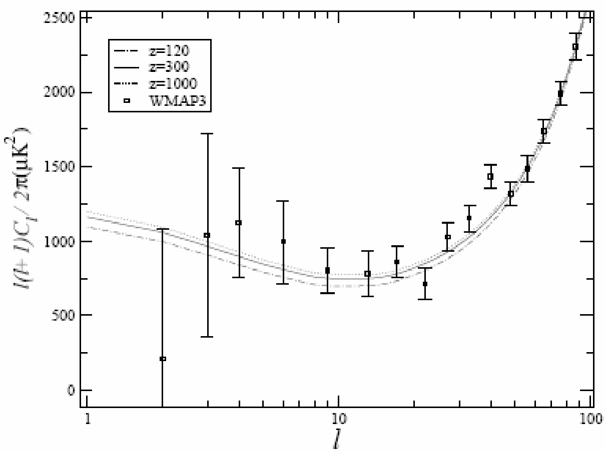
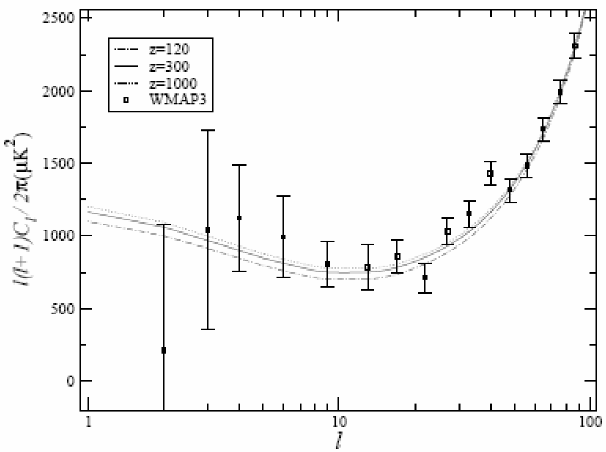


FIG. 3 (d) B0.1A0.1



Using WMAP3 data to the chi-square fitting of the CMB anisotropy spectrum

	$z=120$	$z=1000$	$z=3000$	$z=10000$	$z=30000$
$\chi^2(B1A7)$	997	103	57.91	49.27	47.70
$\chi^2(B1A5)$	1299	102	57.43	49.16	47.67
$\chi^2(B1A1)$	3548	98.37	56.12	48.87	47.59
$\chi^2(B1A0.1)$	1574	99	56.54	48.96	47.61

$N_z \simeq 10$ e-folds

Note the chi-square fitting for ΛCDM model is 47.09 in WMAP3 data

$$N = N_z + N_{cmb} \simeq 70 \text{ e-folds}$$

Using WMAP1 data to the chi-square fitting of the CMB anisotropy spectrum

	$z=120$	$z=1000$	$z=3000$	$z=10000$	$z=30000$
$\chi^2(B1A7)$	639.50	85.45	65.93	64.47	64.72
$\chi^2(B1A5)$	822.23	84.83	65.81	64.48	64.74
$\chi^2(B1A1)$	2238	82.90	65.44	64.50	64.77
$\chi^2(B1A0.1)$	992	83.51	65.55	64.49	64.75

$N_z \simeq 9$

Note the chi-square fitting for ΛCDM model is 64.99 in WMAP1 data

$$N = N_z + N_{cmb} \simeq 69 \text{ e-folds}$$

Using WMAP3 data to the chi-square fitting of the CMB anisotropy spectrum

	$z=120$	$z=1000$	$z=3000$	$z=10^4$	$z=10^5$
$\chi^2(B0.1A7)$	59	49.83	47.97	47.43	47.11
$\chi^2(B0.1A5)$	67.3	49.66	47.89	47.32	47.11
$\chi^2(B0.1A1)$	80.08	49.08	47.67	47.23	47.09
$\chi^2(B0.1A0.1)$	77.98	48.63	47.49	47.2	47.08

$\rightarrow N_z \simeq 11.5$

Note the chi-square fitting for ΛCDM model is 47.09 in WMAP3 data

$$N = N_z + N_{cmb} \simeq 71.5 \text{ e-folds}$$

Using WMAP1 data to the chi-square fitting of the CMB anisotropy spectrum

	$z=120$	$z=1000$	$z=3000$	$z=10^4$	$z=10^5$
$\chi^2(B0.1A7)$	67.13	64.49	64.66	64.87	64.99
$\chi^2(B0.1A5)$	69.68	64.49	64.68	64.87	64.98
$\chi^2(B0.1A1)$	74.51	64.50	64.75	64.91	64.99
$\chi^2(B0.1A0.1)$	73.54	64.51	64.80	64.95	64.99

$\rightarrow N_z \simeq 7$

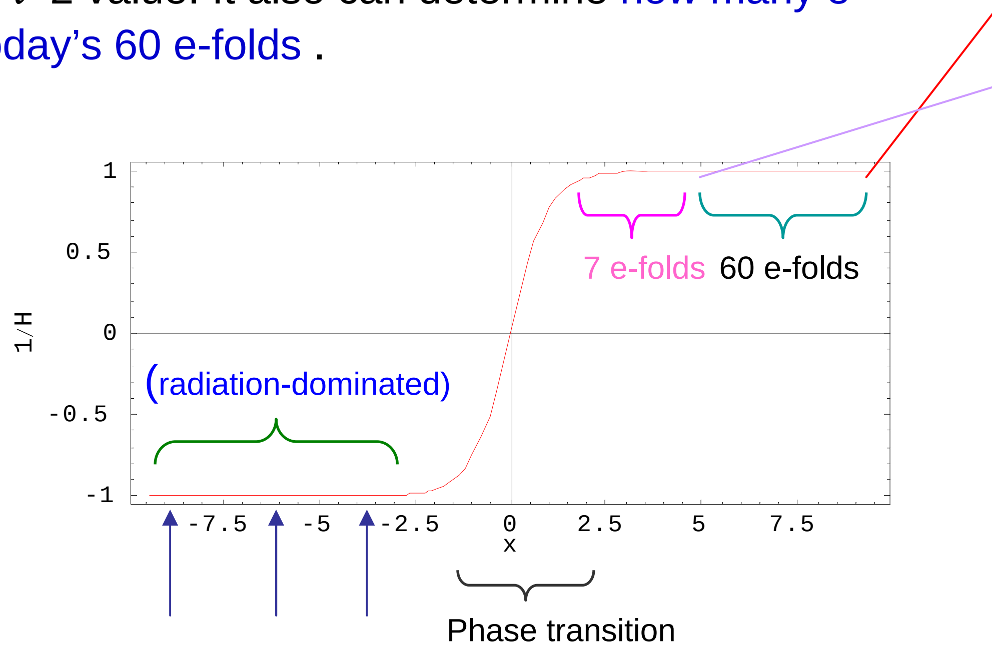
Note the chi-square fitting for ΛCDM model is 64.99 in WMAP1 data

$$N = N_z + N_{cmb} \simeq 67$$

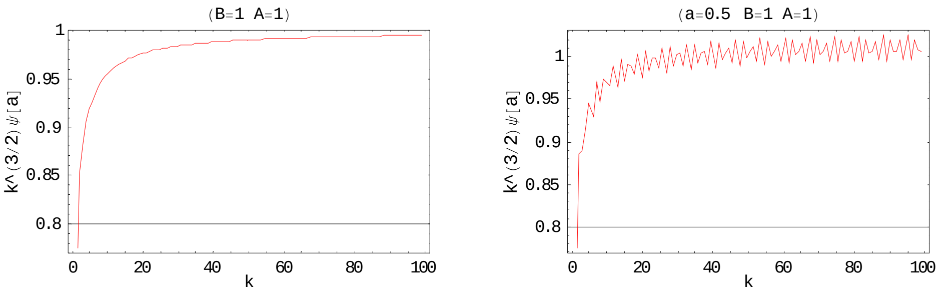
Conclusions :

(1) By adjusting the values of A(radiation component), B(vacuum energy) and z(duration of inflation) , it can lead to a low ℓ=2 value. It also can determine how many e-folds between phase transition point and today's 60 e-folds .

(2) One year WMAP data prefers our model slightly and the pre-inflation radiation-dominated phase was taking place as 67 e-folds from the end of inflation .



(3) For choosing initial value a(t) , If the initial value is too close to the transition point , it will lead to an oscillating power spectrum which is due to an improper choice of initial a(t) . But if the initial a is in the proper region (radiation-dominated region) then the power spectrum is almost the same and smooth. It's initial condition insensitive. Therefore, the oscillation is from a improper choice of initial a !



The value of A and B …

- We choose B=1 and B=0.1 with A=7,5,1,0.1 to do the simulation . The value of B is from the constraint

$$V^{\frac{1}{4}} = 0.0265\epsilon^{\frac{1}{4}} M_{Pl}$$

Where the slow-roll parameter is

$$\epsilon \equiv (M_{Pl}^2/2)(V'/V)^2 < 0.033$$

The vacuum energy that drives inflation is given by

$$V_0 = 3M_G^4 B$$

with $V_0 \simeq V$, and $M_G = 2.1 \times 10^{16}\ \mathrm{GeV}$

then $\boxed{B < 1}$

No observational constraint for the value of A , it can from the particle physics model you choose .

Therefore , the following simulations are (1) B=1 & A=7,5,1,0.1 and (2) B=0.1 & A=7,5,1,0.1

Quantitative comparison …

The chi-square fitting is the following

$$\chi^2 = \sum_{i,j}(D_i^b - T_i^b)C_{i,j}^{-1}(D_j^b - T_j^b)$$

The measured ith band power

Theoretical value

The width of the error bar in measurement

We use this chi-square fitting method to do a comparison .

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e-folding

The value of z correspond to an exponential expansion with the number of e-folding from the start of inflation given by

$$N_z = \ln(a_k/a_c) \simeq \ln z$$

The total e-folding is : $N = N_z + N_{cmb}$

From observation that $N_{cmb} \simeq 60$ e-folds

Scalar field fluctuations in Schwarzschild-de Sitter space-time

arXiv:0905.2041

Outline

- Motivation: Still some deviations and "axis of evil" !?
- Our Model: we consider the inhomogenous background during inflation, like a black hole in the space-time
- Numerical results
- Conclusion

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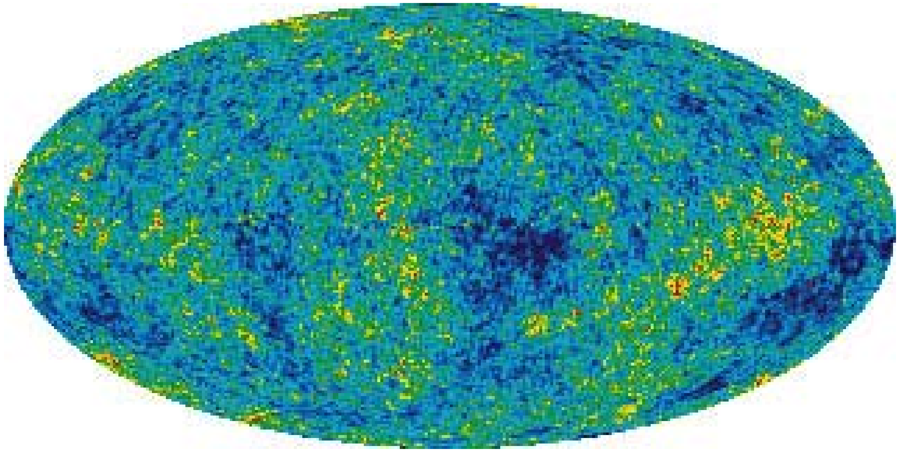
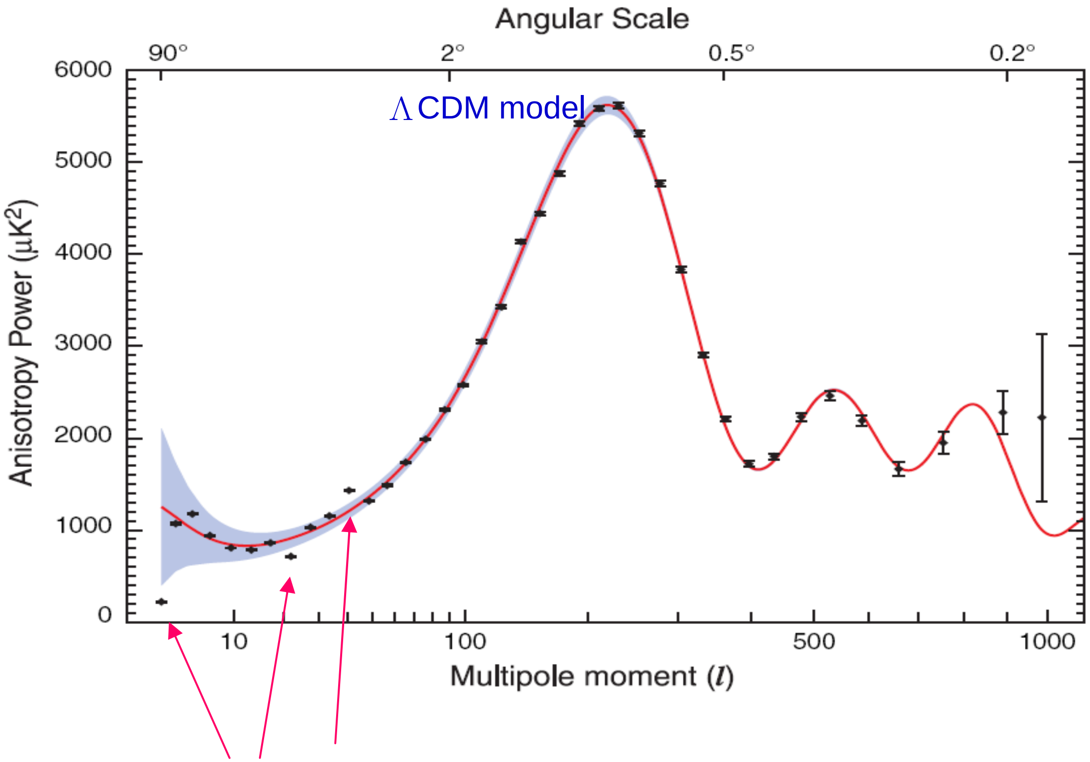
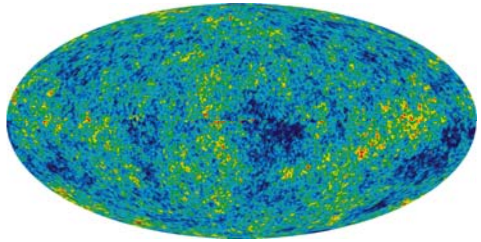




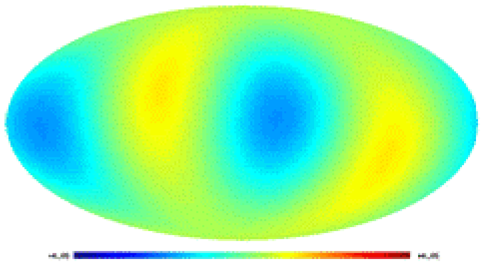
still some deviation

Land & Magueijo 05

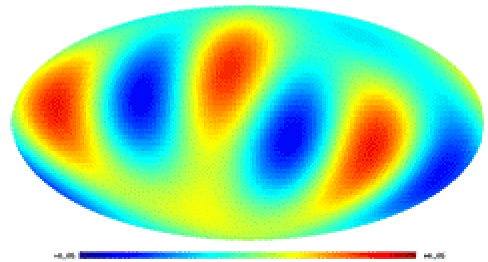
WMAP3
CMB sky map



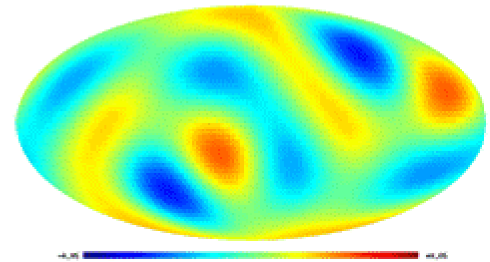
$$\delta T(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$



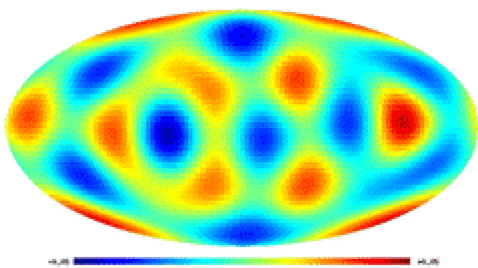
$\ell = 2$



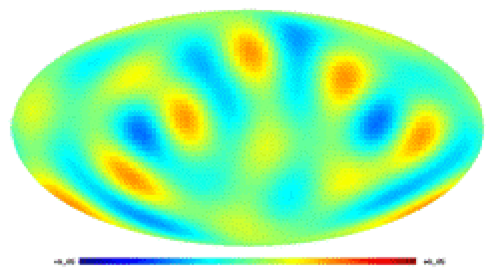
$\ell = 3$



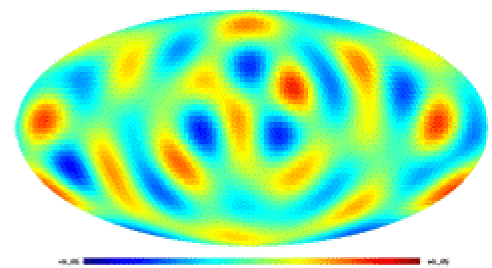
$\ell = 4$



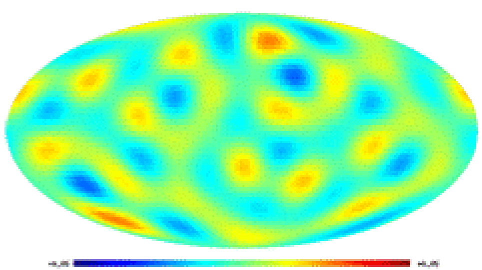
$\ell = 5$



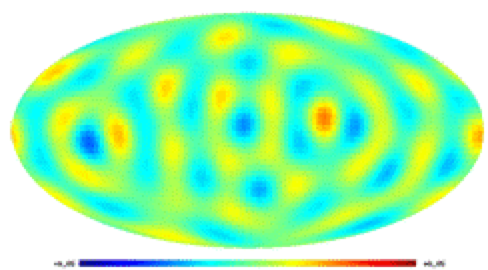
$\ell = 6$



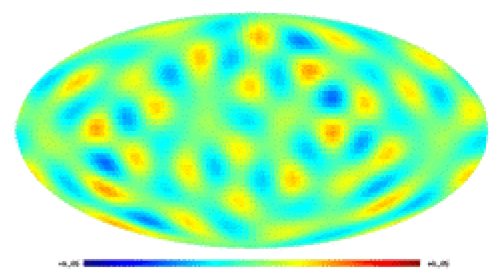
$\ell = 7$



$\ell = 8$



$\ell = 9$



$\ell = 10$

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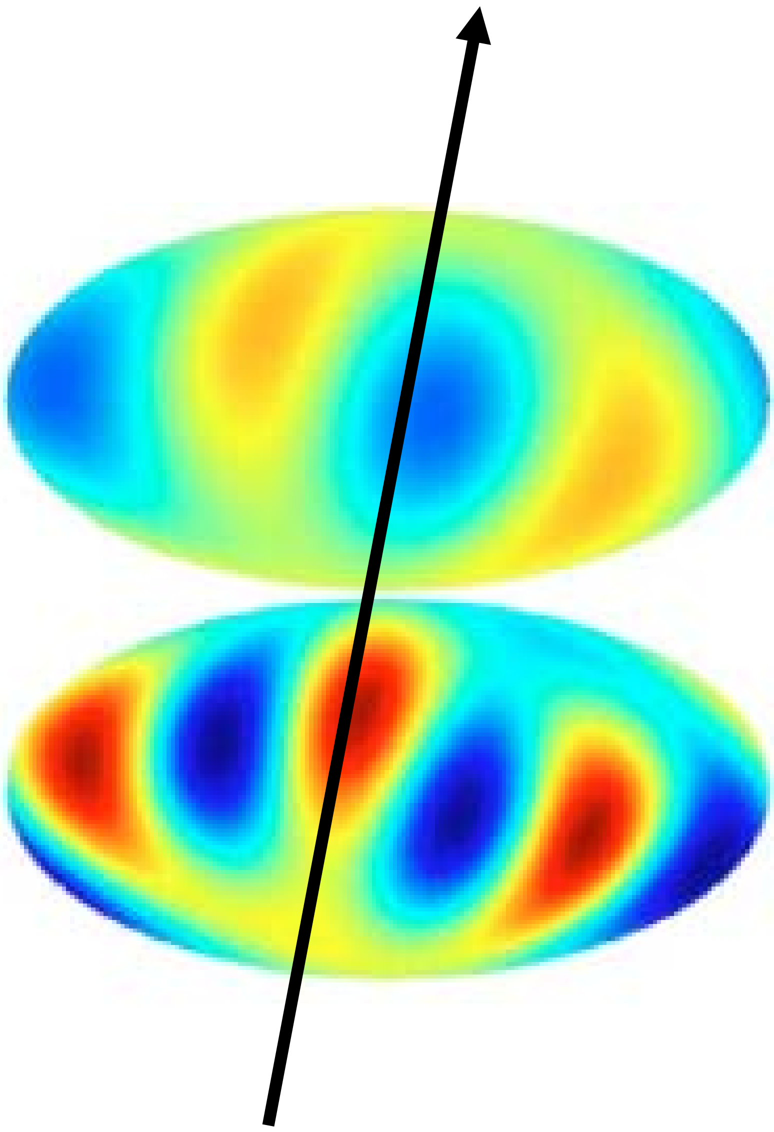
“Axis of Evil” ? $(l,b)\sim(-110°,60°)$



l=2, quadrupole

l=3, octopole

(1) Some deviations still exist (2) Axis of evil ?

Our model : an inhomogenous background, a black-hole in it ! arXiv:0905.2041

Equation for inflation fluctuation

$$\varphi(t,\vec{y}) = \overline{\varphi} + \phi(t,\vec{y})$$

$$\frac{\delta T}{T} \propto \phi(t,\vec{y})$$

$$\Box\phi(t,\vec{y}) = 0$$

$$ds^2 = g_{\mu\nu}dy^\mu dy^\nu = dt^2 - a^2(t)d\vec{y}^2 \quad , \quad a(t) = e^{Ht}$$

$$\ddot{\phi}_k(t) + 3\frac{\dot{a}}{a}\dot{\phi}_k(t) + \left(\frac{k^2}{a^2}\right)\phi_k(t) = 0$$

$$\dot{\phi}_k(t) \equiv \frac{d\phi_k(t)}{dt}$$

Inflaton quantum fluctuation : $\phi_k(t)$

$$P_k^{1/2} \propto k^{3/2}\phi_k$$

ΛCDM model

homogenous space-time

$$ds^2 = dt^2 - a^2(t)d\vec{y}^2$$

Our model

Schwarzschild-de Sitter (SdS) space-time

$$ds^2 = -\left(1 - \frac{2GM}{r} - H^2r^2\right)dt^2 + \left(1 - \frac{2GM}{r} - H^2r^2\right)^{-1}dr^2 + r^2d\Omega^2$$

static coordinate

a black hole locates at the origin !

we have to use planar coordinate

$$ds^2 = -f(r,\tau)d\tau^2 + h(r,\tau)(dr^2 + r^2d\Omega^2),\quad \begin{cases} f(r,\tau) = a^2(\tau)\left[1 - \dfrac{GM}{2a(\tau)r}\right]^2\left[1 + \dfrac{GM}{2a(\tau)r}\right]^{-2} \\ h(r,\tau) = a^2(\tau)\left[1 + \dfrac{GM}{2a(\tau)r}\right]^4 \\ a(\tau) = -1/(H\tau)\ ,\ d\tau = a^{-1}(\tau)dt \end{cases}$$

conformal time

$$\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = 0$$

evaporation time for a Schwarzschild black hole with mass M :

$$t_{\rm ev} = 5120\pi G^2 M^3$$

and, evaporation time scale longer than that of inflation needs

$$H t_{\rm ev} > 1$$

which implies that

$$\frac{M}{M_{\rm Pl}} > 3.96 \times 10^{-2} \left(\frac{M_{\rm Pl}}{H}\right)^{\frac{1}{3}}$$

$$8\pi G = \frac{1}{M_{pl}^2}$$

The process :

(1) Spherical symmetric $\phi(x) = \int_0^\infty dk \sum_{lm} \varphi_{klm}(x),\ \varphi_{klm}(x) = k^2 j_l(kr) \varphi_{kl}(\tau) Y_{lm}(\theta, \phi).$

(2) quantization

$$\hat{\phi}(x) = \int_0^\infty dk \sum_{lm} \left[\hat{a}_{klm} \varphi_{klm}(x) + \hat{a}^\dagger_{klm} \varphi^*_{klm}(x) \right], \quad \begin{array}{l} [\hat{a}_{klm}, \hat{a}_{k'l'm'}] = [\hat{a}^\dagger_{klm}, \hat{a}^\dagger_{k'l'm'}] = 0, \\ [\hat{a}_{klm}, \hat{a}^\dagger_{k'l'm'}] = \delta(k-k') \delta_{ll'} \delta_{mm'}. \end{array}$$

$$\hat{a}_{klm} |0\rangle = 0$$

(3) Using a perturbative approach to calculate the function $\varphi_{kl}(\tau)$

and assume the quantity $\epsilon \equiv GMH$ so that we can expand f and h in powers of ϵ

also write $\varphi_l = \varphi_l^{(0)} + \varphi_l^{(1)} + \varphi_l^{(2)} + \cdots$ as an expansion in powers of ϵ

then solve the equation order by order.

Then we compute the two-point correlation function $\langle 0 | \hat{\phi}(x) \hat{\phi}(x') | 0 \rangle = \int_0^\infty dk \sum_{lm} \varphi_{klm}(x) \varphi^*_{klm}(x')$

$$\text{As } x' \to x, \quad \langle 0 | \hat{\phi}^2(x) | 0 \rangle = \int_0^\infty \frac{dk}{k} \sum_l \frac{2l+1}{4\pi} k^5 j_l^2(kr) |\varphi_{kl}(\tau)|^2$$

the spectrum function

$$P_{kl}(\tau) = \frac{k^5}{4\pi} |\varphi_{kl}(\tau)|^2$$

A. Zeroth order

$$\partial_\tau^2 \varphi_l^{(0)} - \frac{2}{\tau}\partial_\tau \varphi_l^{(0)} - \partial_r^2 \varphi_l^{(0)} - \frac{2}{r}\partial_r \varphi_l^{(0)} + \frac{l(l+1)}{r^2}\varphi_l^{(0)} = 0$$

Bessel transform $\varphi_l^{(0)}(r,\tau) = \int_0^\infty dk k^2 j_l(kr) \varphi_{kl}^{(0)}(\tau)$

then we have $\partial_\tau^2 \varphi_{kl}^{(0)}(\tau) - \frac{2}{\tau}\partial_\tau \varphi_{kl}^{(0)}(\tau) + k^2 \varphi_{kl}^{(0)}(\tau) = 0$,

and the solution is

$$\varphi_{kl}^{(0)}(\tau) = C_1(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(1)}(-k\tau) + C_2(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(2)}(-k\tau)$$

To choose Bunch-Davies vacuum, we take $C_1 = -\frac{H}{k^2\sqrt{2k}}$, and $C_2 = 0$.

The solution is $\varphi_{kl}^{(0)}(\tau) = -\frac{H\tau}{k\sqrt{\pi k}}\left(1 - \frac{i}{k\tau}\right) e^{-ik\tau}$,

and $P_{kl}^{(0)}(\tau) = \frac{k^5}{4\pi}\left|\varphi_{kl}^{(0)}(\tau)\right|^2 = \frac{H^2}{4\pi^2}(1 + k^2\tau^2)$

As $\tau \to 0$, $P_{kl}^{(0)}(\tau) \to H^2/(4\pi^2)$.

scale-invariant power spectrum of de-Sitter quantum fluctuation

B. First order

$$\partial_\tau^2\varphi_l^{(1)} - \frac{2}{\tau}\partial_\tau\varphi_l^{(1)} - \partial_r^2\varphi_l^{(1)} - \frac{2}{r}\partial_r\varphi_l^{(1)} + \frac{l(l+1)}{r^2}\varphi_l^{(1)} = J_1 \ , \quad J_1(r,\tau) \ = \ \frac{4\epsilon\tau}{r}\left(\partial_\tau^2\varphi_l^{(0)} - \frac{1}{\tau}\partial_\tau\varphi_l^{(0)}\right)$$

Use Green's function $G(r,\tau;r',\tau')$

$$\partial_\tau^2 G - \frac{2}{\tau}\partial_\tau G - \partial_r^2 G - \frac{2}{r}\partial_r G + \frac{l(l+1)}{r^2}G = \frac{\delta(r-r')\delta(\tau-\tau')}{r^2}$$

completeness property of spherical Bessel function

$$\int_0^\infty dk k^2 \left[\sqrt{\frac{2}{\pi}} r j_l(kr)\right]\left[\sqrt{\frac{2}{\pi}} r' j_l(kr')\right] = \delta(r-r'),$$

and taking

$$G_l(r,\tau;r',\tau') = \int_0^\infty dk k^2 g_k(\tau,\tau') j_l(kr) j_l(kr') \ ,$$

the equation becomes

$$\partial_\tau^2 g_k - \frac{2}{\tau}\partial_\tau g_k + k^2 g_k = \frac{2}{\pi}\delta(\tau-\tau') \ .$$

For the retarded Green's function, $g_k = 0$ for $\tau' > \tau > \tau_i$, where τ_i denotes an initial time when the source begins to operate. For $0 > \tau > \tau'$,

$$g_k(\tau,\tau') \ = \ \frac{i}{2\tau'^2 k^3}\Big[(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(1)}(-k\tau)(-k\tau')^{\frac{3}{2}} H_{\frac{3}{2}}^{(2)}(-k\tau') - (-k\tau')^{\frac{3}{2}} H_{\frac{3}{2}}^{(1)}(-k\tau')(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(2)}(-k\tau)\Big] \ .$$

With this retarded Green's function, the first order $\varphi_l^{(1)}(r,\tau)$ can be expressed as

$$\varphi_l^{(1)}(r,\tau) = \int_0^\infty dk k^2 j_l(kr)\varphi_{kl}^{(1)}(\tau) = \int_0^\infty dr' r'^2 \int_{\tau_i}^0 d\tau' G(r,\tau;r',\tau') J_1(r',\tau').$$

Hence, we find that

$$\varphi_{kl}^{(1)}(\tau) = \frac{2i\epsilon H}{\sqrt{\pi}k^3}\int_0^\infty dk' k'^2 (k'^{\frac{1}{2}}) \int_0^\infty dr' r' j_l(kr') j_l(k'r') \int_{\tau_i}^{\tau} d\tau' e^{-ik'\tau'} \times \left[(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(1)}(-k\tau)(-k\tau')^{\frac{3}{2}} H_{\frac{3}{2}}^{(2)}(-k\tau') - (-k\tau')^{\frac{3}{2}} H_{\frac{3}{2}}^{(1)}(-k\tau')(-k\tau)^{\frac{3}{2}} H_{\frac{3}{2}}^{(2)}(-k\tau)\right]$$

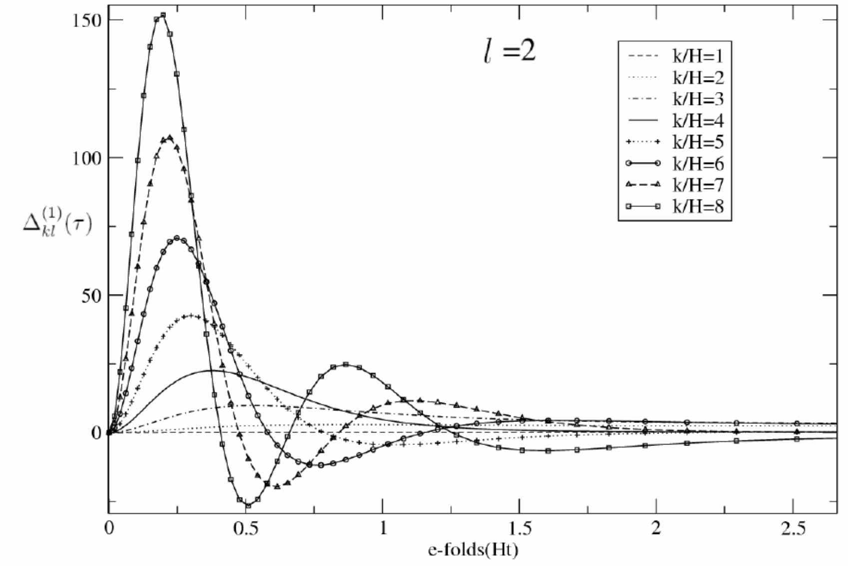
It is useful to rewrite $\varphi_{kl}^{(1)}(\tau)$ as $\varphi_{kl}^{(1)}(\tau) = \epsilon\left[\alpha_{kl}(\tau)\varphi_{kl}^{(0)}(\tau) + \beta_{kl}(\tau)\varphi_{kl}^{(0)*}(\tau)\right]$

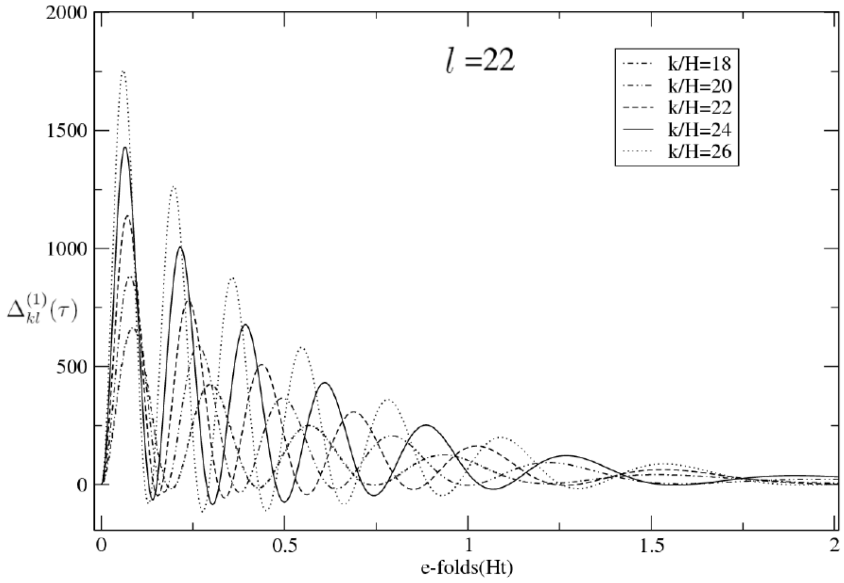
$$\left\{\begin{aligned}
\alpha_{kl}(\tau) &= \frac{-2i\Gamma(l+1)}{\Gamma(l+\frac{3}{2})\Gamma(\frac{1}{2})}\left[\int_0^1 dk' k'^{l+\frac{5}{2}} F\left(l+1,\frac{1}{2};l+\frac{3}{2};k'^2\right)\int_{k\tau_i}^{k\tau} d\tau' e^{-ik'\tau'} e^{i\tau'}(\tau'+i)\right. \\
&\left. + \int_1^\infty dk' k'^{-l+\frac{1}{2}} F\left(l+1,\frac{1}{2};l+\frac{3}{2};\frac{1}{k'^2}\right)\int_{k\tau_i}^{k\tau} d\tau' e^{-ik'\tau'} e^{i\tau'}(\tau'+i)\right] \\
\beta_{kl}(\tau) &= \frac{2i\Gamma(l+1)}{\Gamma(l+\frac{3}{2})\Gamma(\frac{1}{2})}\left[\int_0^1 dk' k'^{l+\frac{5}{2}} F\left(l+1,\frac{1}{2};l+\frac{3}{2};k'^2\right)\int_{k\tau_i}^{k\tau} d\tau' e^{-ik'\tau'} e^{-i\tau'}(\tau'-i)\right. \\
&\left. + \int_1^\infty dk' k'^{-l+\frac{1}{2}} F\left(l+1,\frac{1}{2};l+\frac{3}{2};\frac{1}{k'^2}\right)\int_{k\tau_i}^{k\tau} d\tau' e^{-ik'\tau'} e^{-i\tau'}(\tau'-i)\right].
\end{aligned}\right.$$

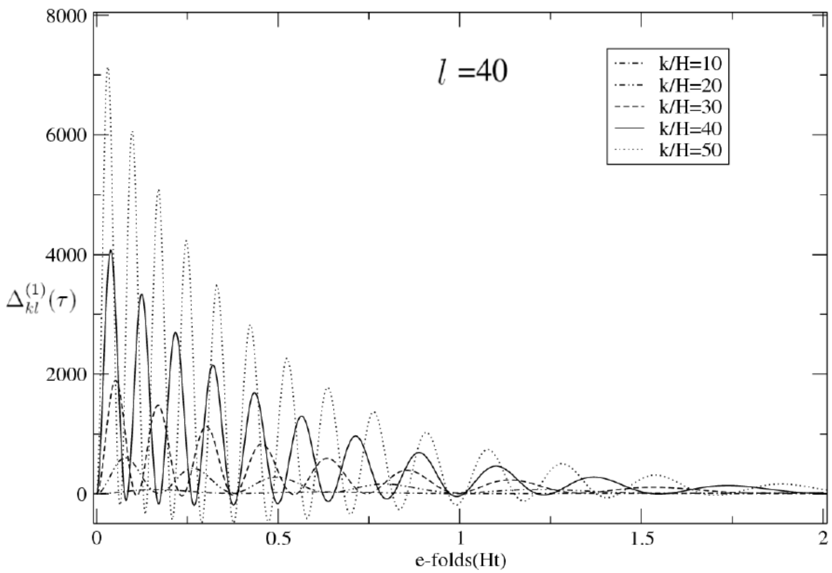
The first order spectrum function is given by

$$P_{kl}^{(1)}(\tau) = \frac{H^2}{4\pi^2}\epsilon\Delta_{kl}^{(1)}(\tau) \quad , \quad \Delta_{kl}^{(1)}(\tau) = \frac{2\pi k^5}{\epsilon H^2}\mathrm{Re}\left[\varphi_{kl}^{(1)}(\tau)\varphi_{kl}^{(0)*}(\tau)\right]$$

Numerical results



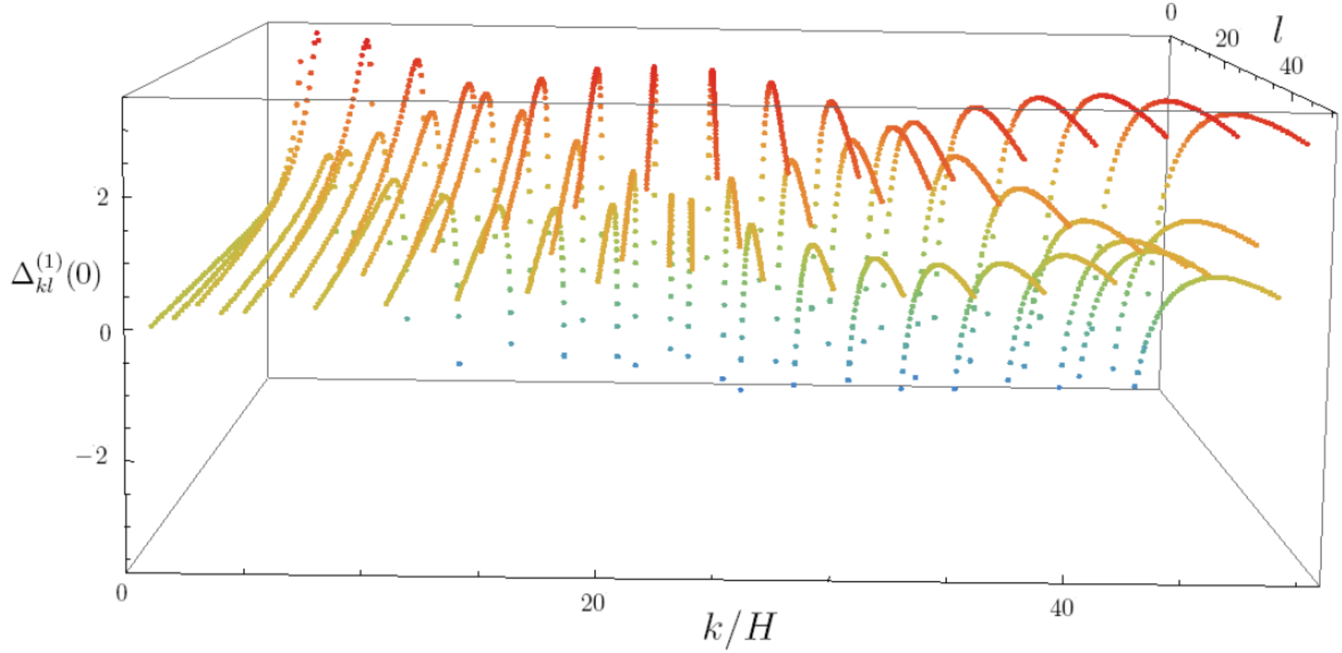


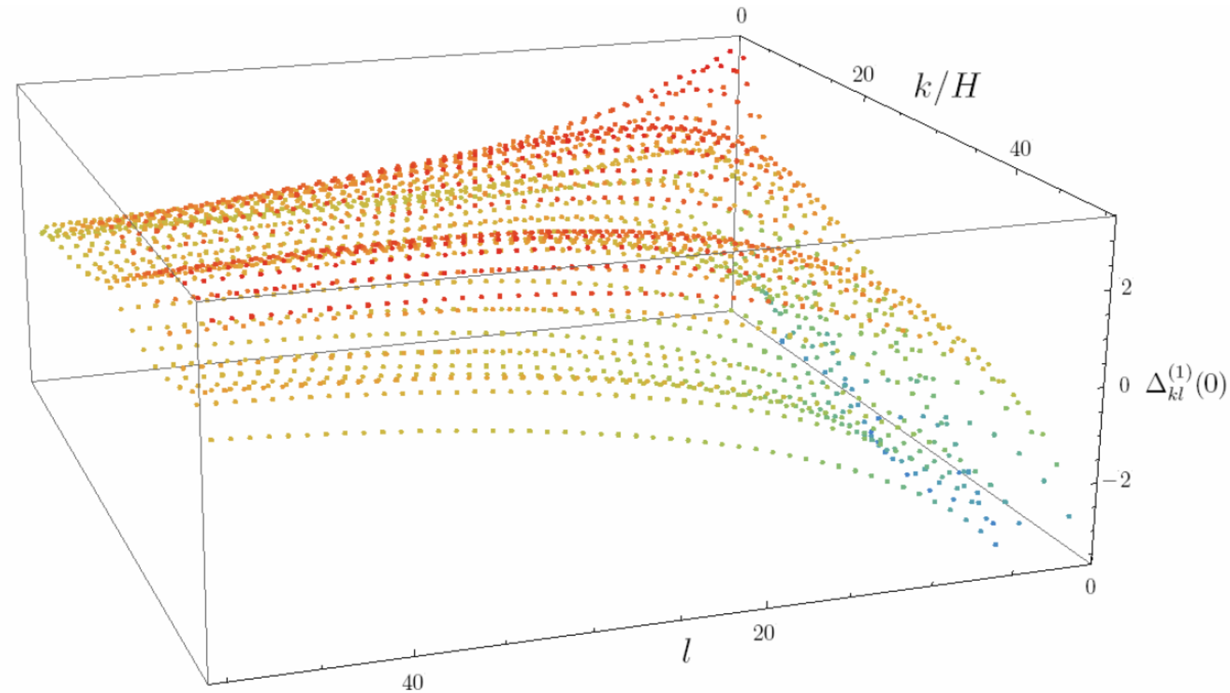


A k-mode $\Delta_{kl}^{(1)}(\tau)$ oscillates when the mode is still sub-horizon. Once the mode crosses out the horizon, $\Delta_{kl}^{(1)}(\tau)$ stops oscillating and gradually approaches a constant value.

3D plots

The asymptotic value $\Delta^{(1)}_{kl}(0)$



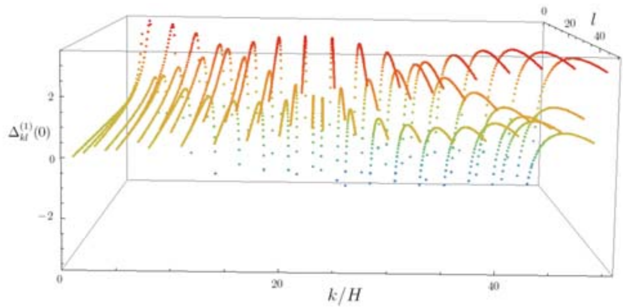


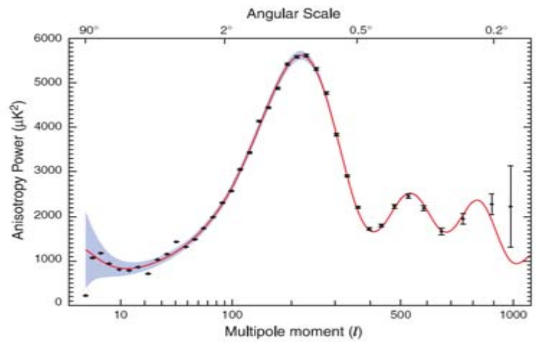
$\Delta^{(1)}_{kl}(0)$ is suppressed in low l and low k regions.
Comparing to the zero-order de Sitter power spectrum, it is roughly decreased by the expansion parameter ϵ .

Conclusion:

(1) We obtained $\Delta_{kl}^{(1)}(\tau)$ up to first-order perturbation results in the case that the black hole located at the origin of the coordinates, for expansion parameter satisfied $\epsilon \equiv GMH < 1$.

(2) The suppressed power of $\Delta_{kl}^{(1)}(\tau)$ in low l and low k regions could give rise to a suppression of the large –scale CMB anisotropy.





(3) The effect to the CMB, a detailed calculation is on going, including the black hole locates somewhere else in the Universe.