IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Supersymmetric Cosmology
-problems and prospects-

5 Nov. 2009 @NTHU

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Plan of Talk

0. Why inflation?

1. Gravitino problem [thermal production]

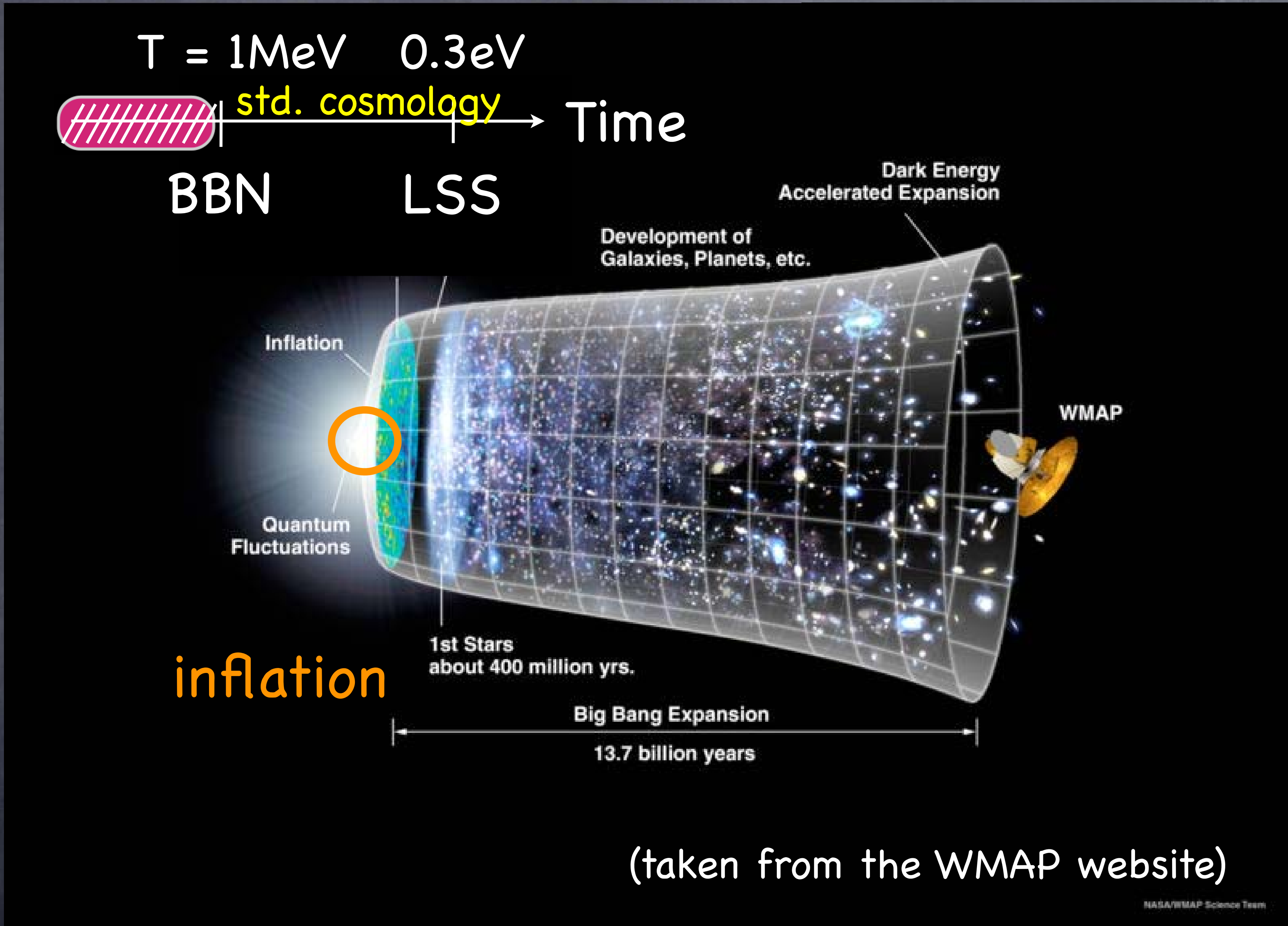
2. New gravitino problem [non-thermal]

3. Constraints on inflation models

4. Conclusions

0. Why inflation?

History of the universe

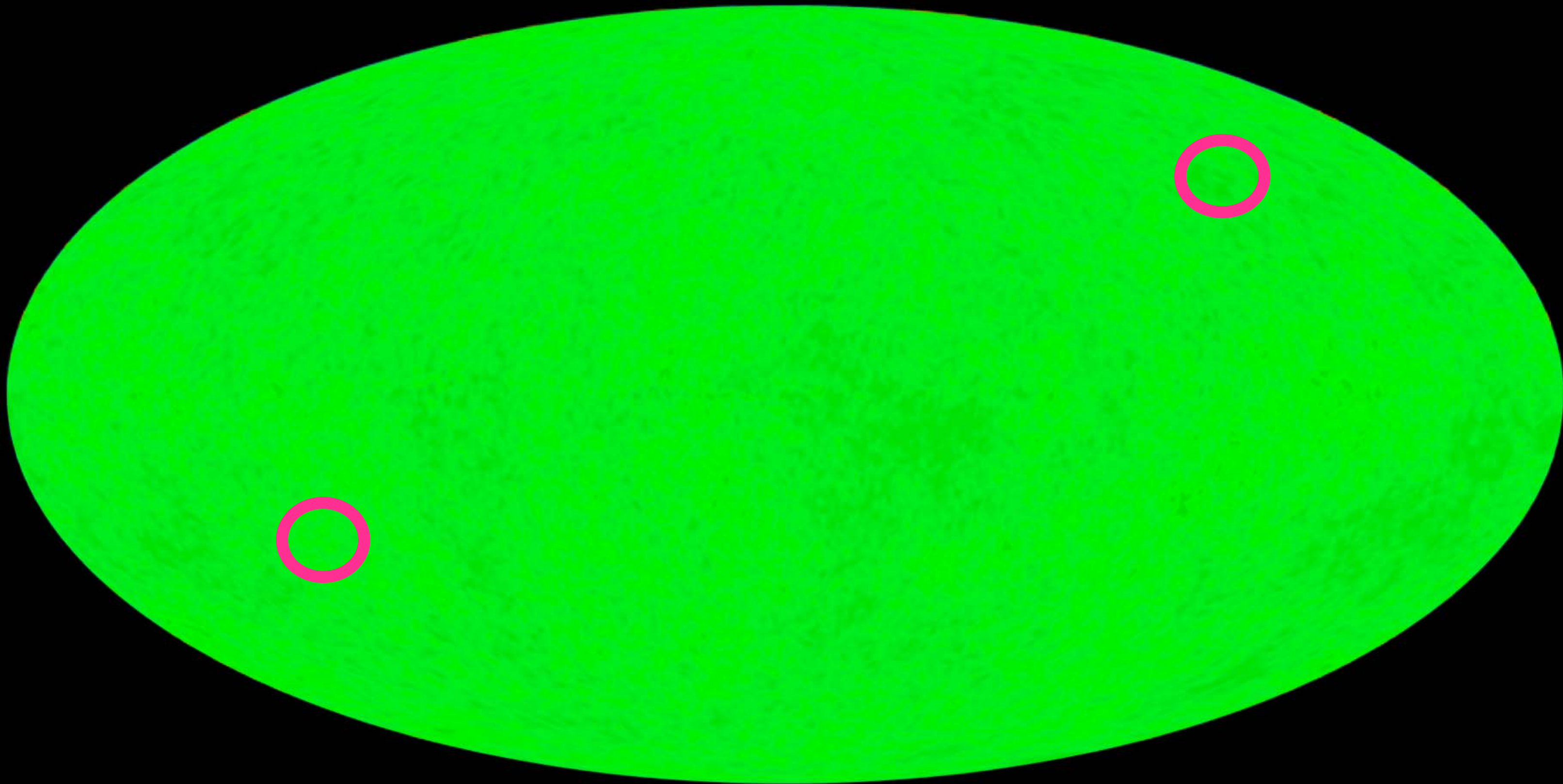


The std. big bang cosmology was firmly established by the following observations.

- Hubble's law (expanding universe)
- Big Bang Nucleosynthesis (BBN)
- Cosmic Microwave Background (CMB)

Why inflation?

Horizon problem



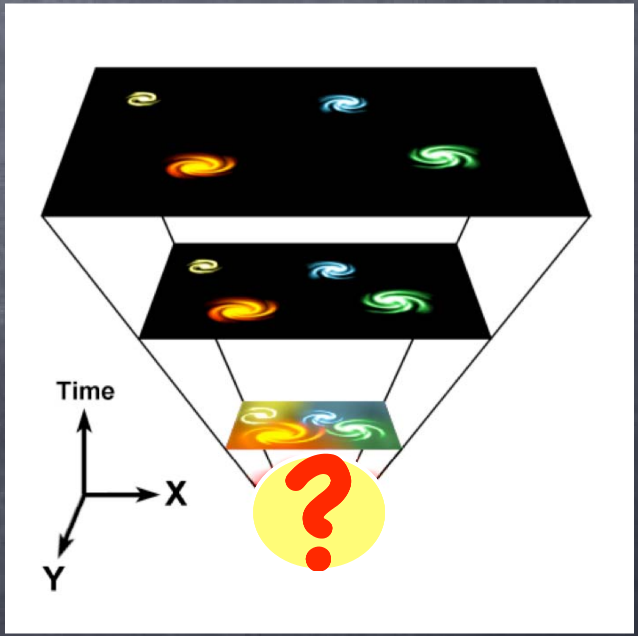
Why do the CMB photons from different directions have almost the same temperature??

Other problems:

flatness problem, unwanted relics (e.g., monopole, gravitino), the origin of the density perturbation...

Those problems are related with the initial condition of the Universe.

How far can we extrapolate the std. cosmology back in time?

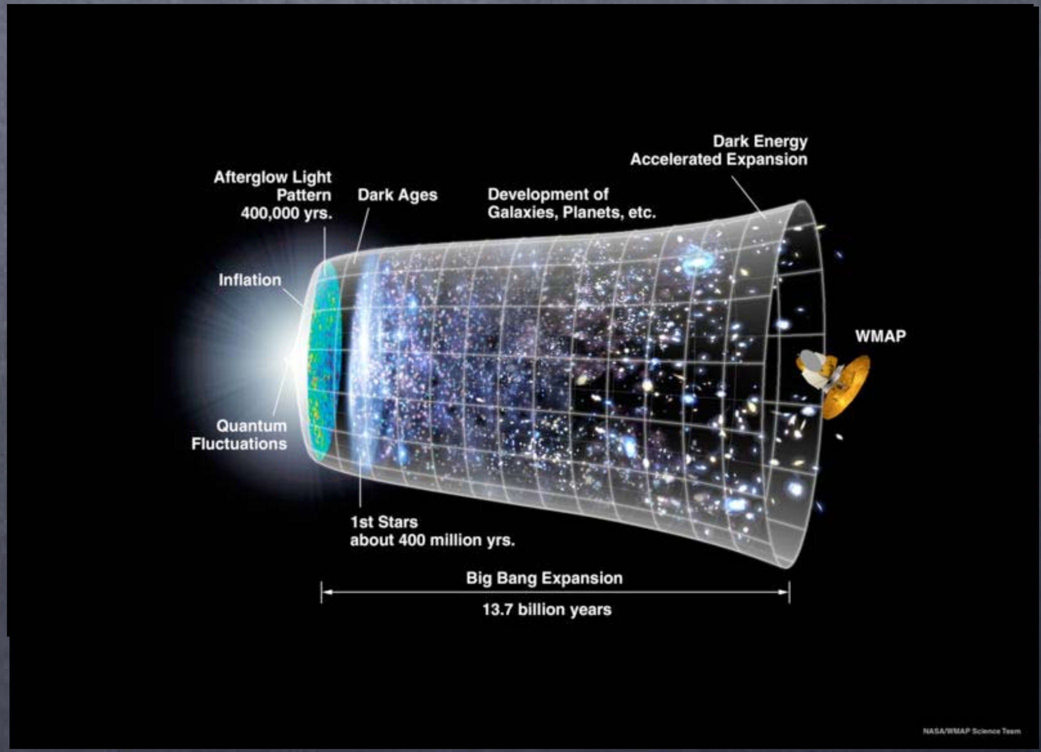


Inflation

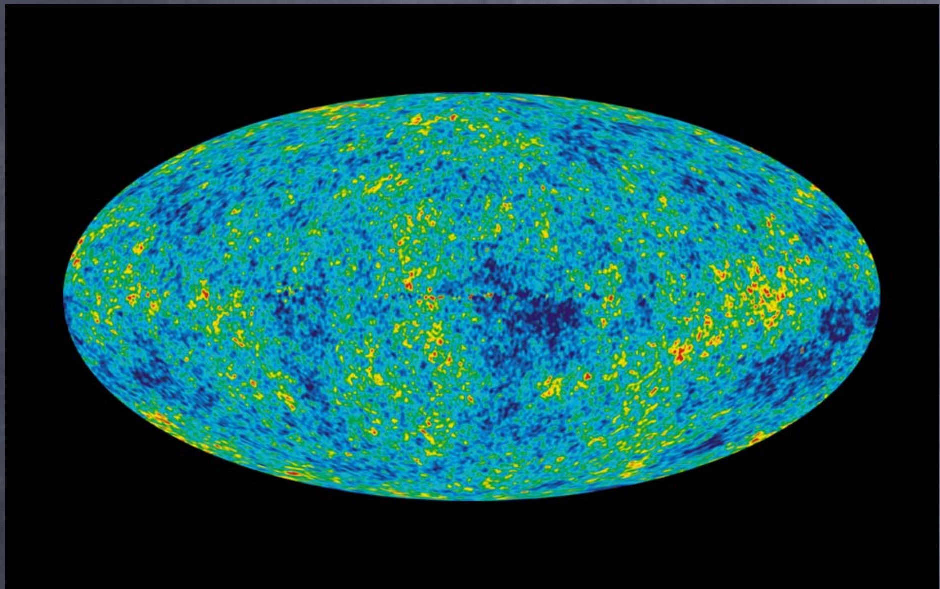
: a phase of the exponential expansion.

⇒ solves the horizon, flatness and unwanted-relic problems.

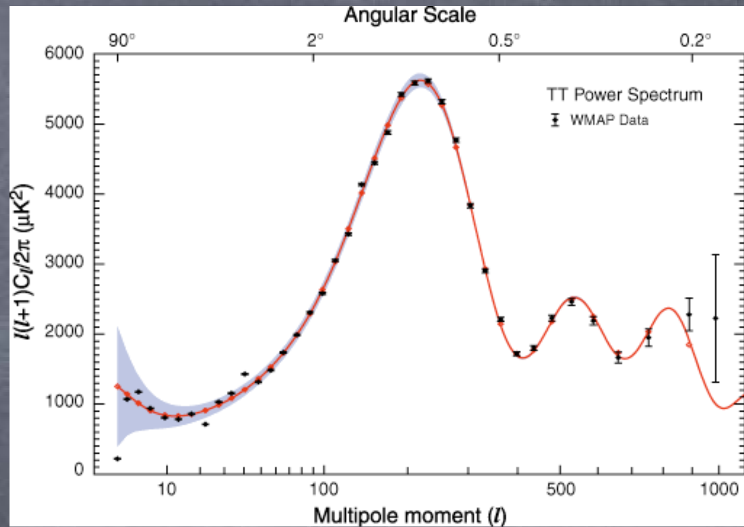
- Slow-roll inflation explains the origin of the density fluctuations.



- Inflation is now strongly supported by observations such as WMAP.



Power-law LCDM model fits the WMAP data quite well.

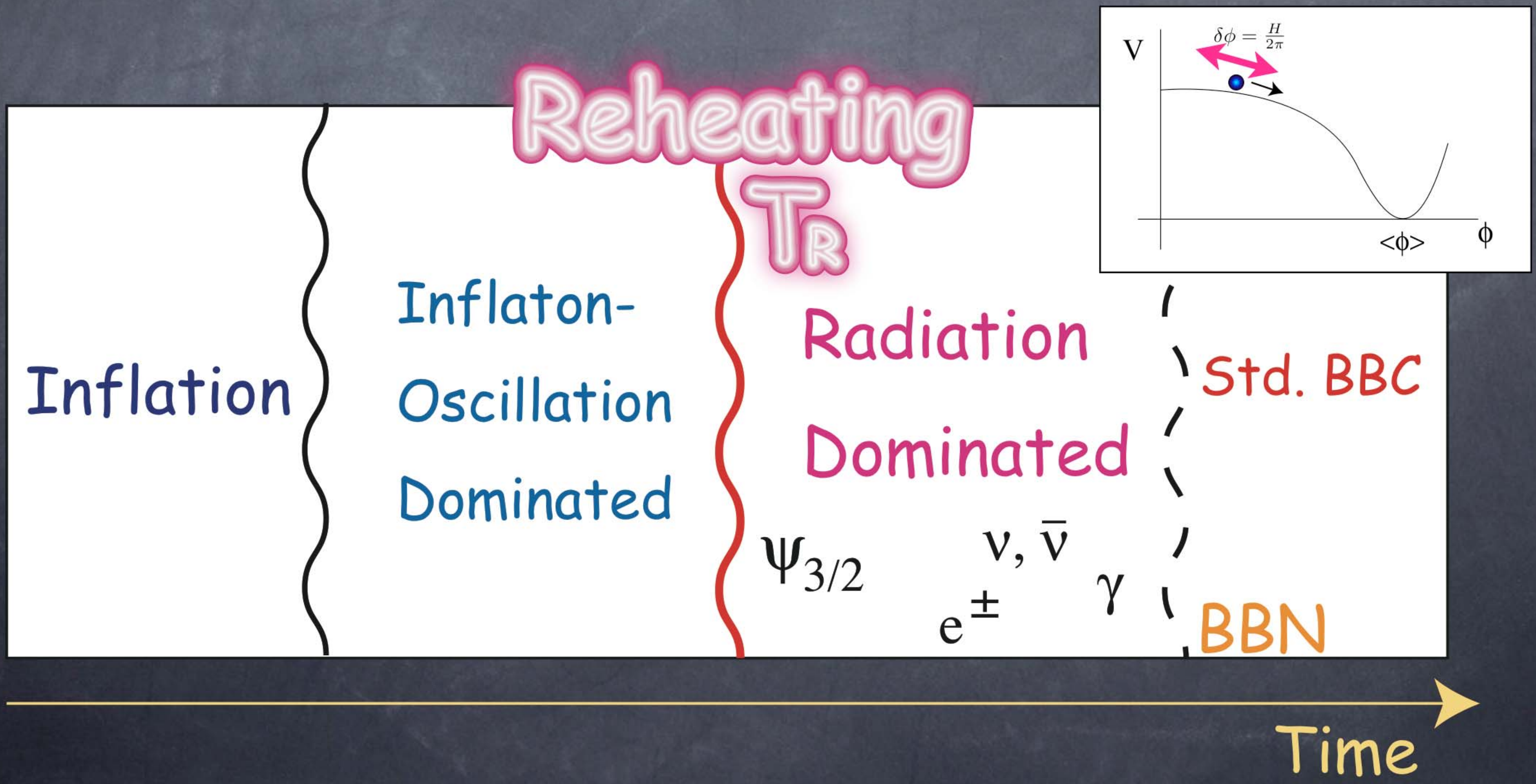


CMB temperature anisotropy contains information on the inflation!!!

1. Gravitino Problem
(Thermal production)

Thermal history after inflation

- Inflaton-decay reheats the universe.
- Reheating contains information on the inflaton.



Reheating

Γ_ϕ : the total decay rate of the inflaton.

The reheating occurs when $H \sim \Gamma_\phi$,
and the reheating temperature is

$$H \sim \Gamma_\phi \sim \frac{T_R^2}{M_P}$$

May be insufficient to describe reheating.

T_R is related to the total decay rate of the inflaton.

Gravitino

- Superpartner of the graviton.
- It becomes massive by eating the goldstino when the local SUSY is spontaneously broken. (super-Higgs mechanism)
- Interactions are very weak, and suppressed by M_p or $F \sim m_{3/2} M_P$

Cosmologically important due to its longevity

Gravitino production

- Thermal production

$$Y_{3/2}^{(\mathrm{th})} = Y_{3/2}^{(\mathrm{th})}(T_R)$$

- Non-thermal production

$$Y_{3/2}^{(\mathrm{nt})} = Y_{3/2}^{(\mathrm{nt})}(T_R, m_\phi, \langle\phi\rangle)$$

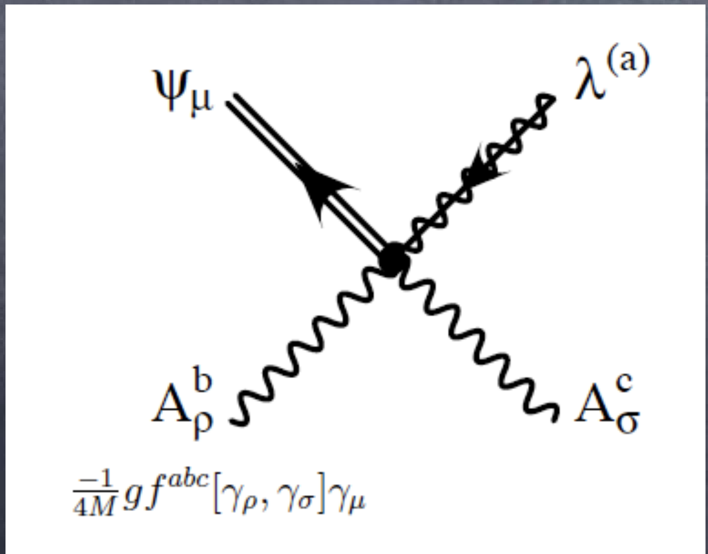
Inflaton parameters (mass, vev)

Gravitino problem

(from thermal scatterings) Weinberg 82, Krauss 83

For high T_R, many gravitinos are abundantly produced by particle scatterings, leading to cosmological difficulties.

e.g.)



Moroi `95

Boltzmann equation

$$\dot{n}_{3/2} + 3H n_{3/2} = \langle \sigma v \rangle \, n_{\rm rad}^2$$

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \sim \frac{\frac{1}{M_P^2} T_R^6}{\frac{T_R^2}{M_P} \cdot g_* T_R^3}$$

$$\sim 0.01 \frac{T_R}{M_P}$$

Gravitino abundance (from thermal scattering)

Moroi, Murayama, Yamaguchi 93,
Bolz, Brandenburg, Buchmueller 01; Pradler, Steffen 06

$$Y_{3/2}^{(\mathrm{th})} \simeq 2.3 \times 10^{-14} \times T_{\mathrm{R}}^{(8)} \left[1 + 0.015 \ln T_{\mathrm{R}}^{(8)} - 0.0009 \ln^2 T_{\mathrm{R}}^{(8)}\right]$$
$$+1.5 \times 10^{-14} \times \left(\frac{m_{1/2}}{m_{3/2}}\right)^2 T_{\mathrm{R}}^{(8)} \left[1 - 0.037 \ln T_{\mathrm{R}}^{(8)} + 0.0009 \ln^2 T_{\mathrm{R}}^{(8)}\right],$$

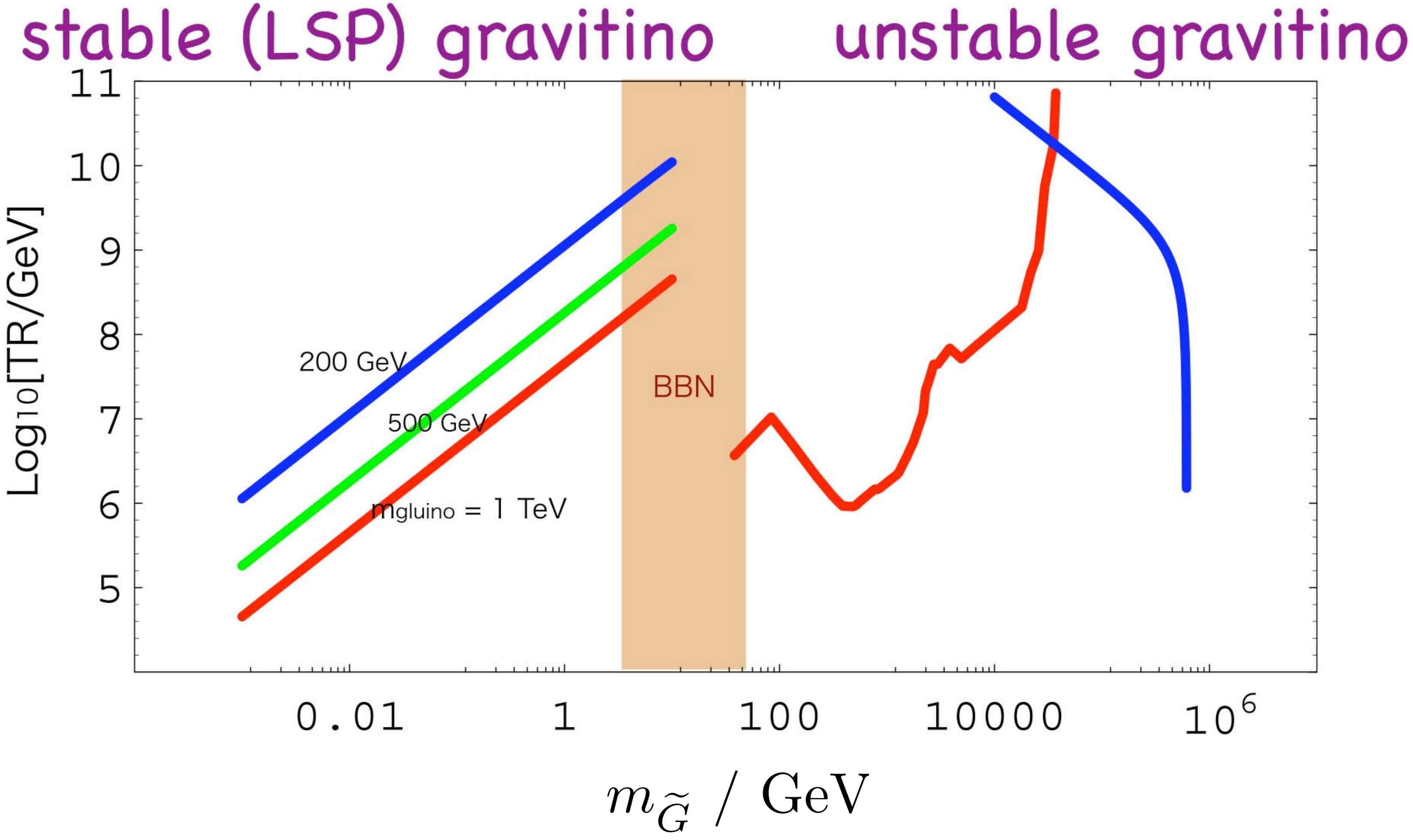
Kawasaki et al, 0804.3745

Note that the abundance of gravitinos produced by thermal scatterings depends only on the reheating temperature.

$$Y_{3/2}^{(\mathrm{th})} = Y_{3/2}^{(\mathrm{th})}(T_R)$$

This is NOT the case with non-thermally produced gravitinos.

Gravitino Problems

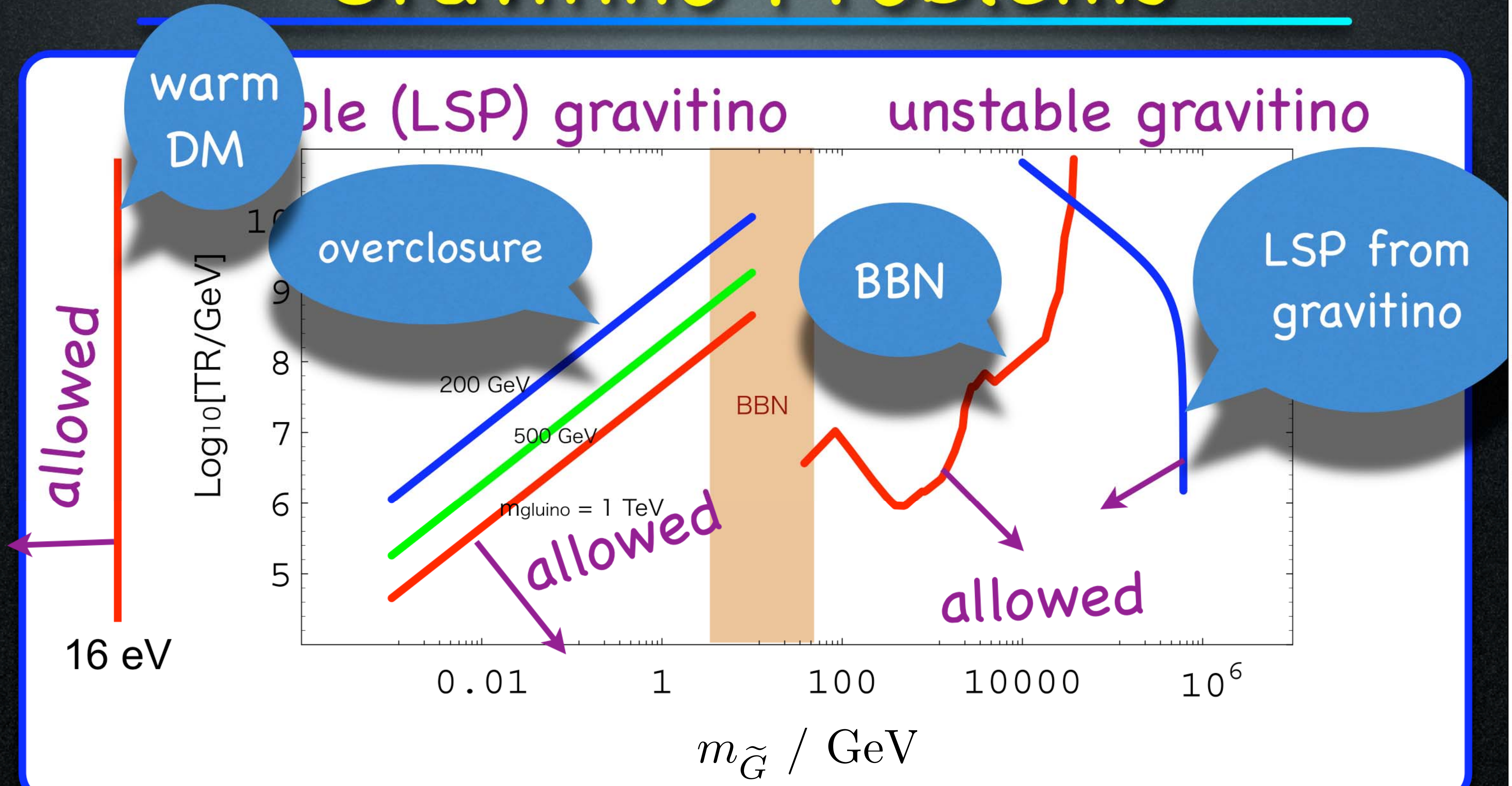


(NOTE: precise line positions in this figure may be out-dated.)

courtesy of K. Hamaguchi

Sorry, I drop references.

Gravitino Problems



(NOTE: precise line positions in this figure may be out-dated.)

courtesy of K. Hamaguchi

Sorry, I drop references.

2. Gravitino Problem (Non-thermal production)

Gravitino production

- Thermal production

$$Y_{3/2}^{(\mathrm{th})} = Y_{3/2}^{(\mathrm{th})}(T_R)$$

- Non-thermal production

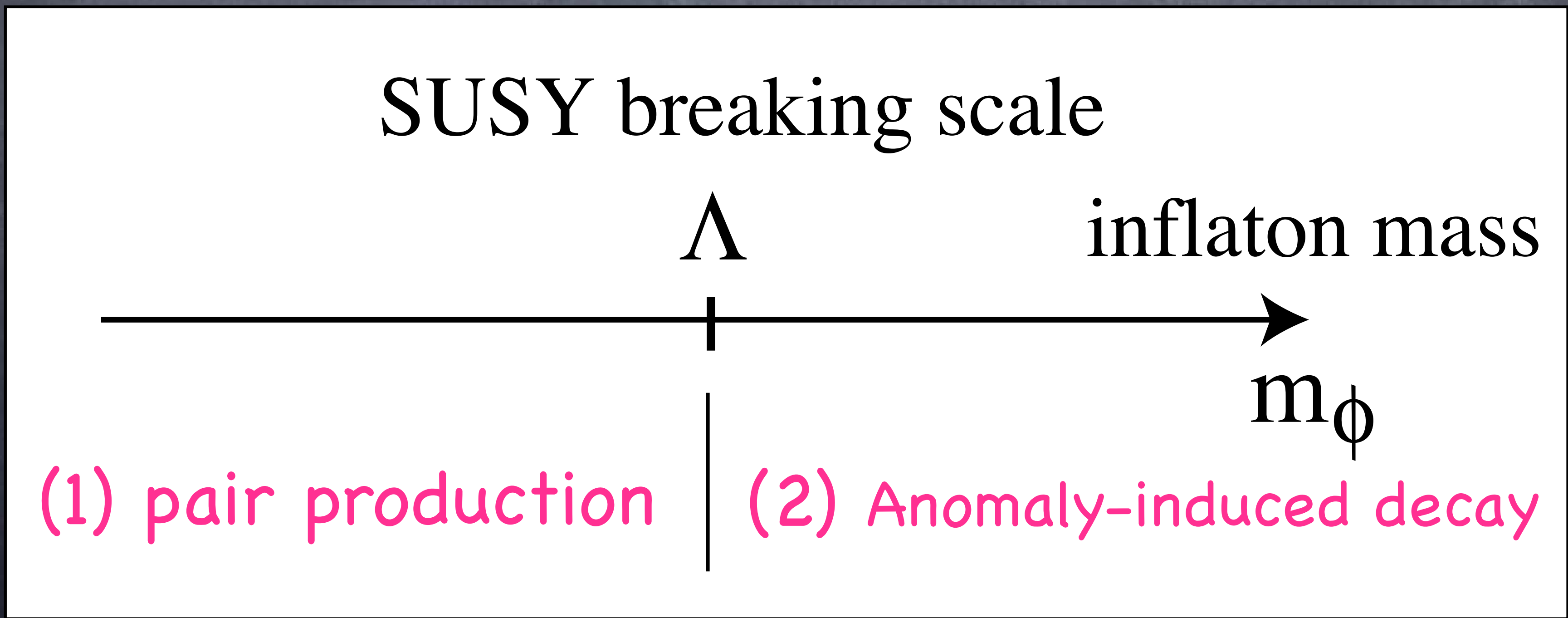
$$Y_{3/2}^{(\mathrm{nt})} = Y_{3/2}^{(\mathrm{nt})}(T_R, m_\phi, \langle\phi\rangle)$$

Inflaton parameters (mass, vev)

Non-thermal Gravitino Production:

(1) Gravitino pair production (direct) :
(induced by the mixing between ϕ and z)

(2) Anomaly-induced decay (indirect) :
(decay into the hidden gauge sector)



2.1. Gravitino Pair-Production

Gravitino Pair-Production

Kawasaki, F.T. and Yanagida, hep-ph/0603265, 0605297
Asaka, Nakamura and Yamaguchi, hep-ph/0604132

- Relevant interactions:

$$e^{-1}\mathcal{L} = -\frac{1}{8}\epsilon^{\mu\nu\rho\sigma}\left(G_\phi \partial_\rho \hat{\phi} + G_z \partial_\rho z - \text{h.c.}\right)\bar{\psi}_\mu \gamma_\nu \psi_\sigma$$

$$-\frac{1}{8}e^{G/2}\left(G_\phi \hat{\phi} + G_z z + \text{h.c.}\right)\bar{\psi}_\mu \left[\gamma^\mu, \gamma^\nu\right]\psi_\nu,$$

ϕ : inflaton field

z : SUSY breaking field, w/ $G^z G_z \simeq 3$

$$G \equiv K + \ln|W|^2$$

Taking account of the mixings,

$$G_\phi \sim \langle\phi\rangle \frac{m_{3/2}}{m_\phi} \quad \text{for} \quad m_\phi < m_z$$

Gravitino Pair Production Rate:

$$\Gamma_{3/2} \simeq \frac{|G_\phi|^2}{288\pi} \frac{m_\phi^5}{m_{3/2}^2 M_P^2} \simeq \frac{1}{32\pi} \left(\frac{\langle \phi \rangle}{M_P} \right)^2 \frac{m_\phi^3}{M_P^2}$$

Endo, Hamaguchi and F.T., hep-ph/0602061
Nakamura and Yamaguchi, hep-ph/0602081

for $m_\phi < m_z$

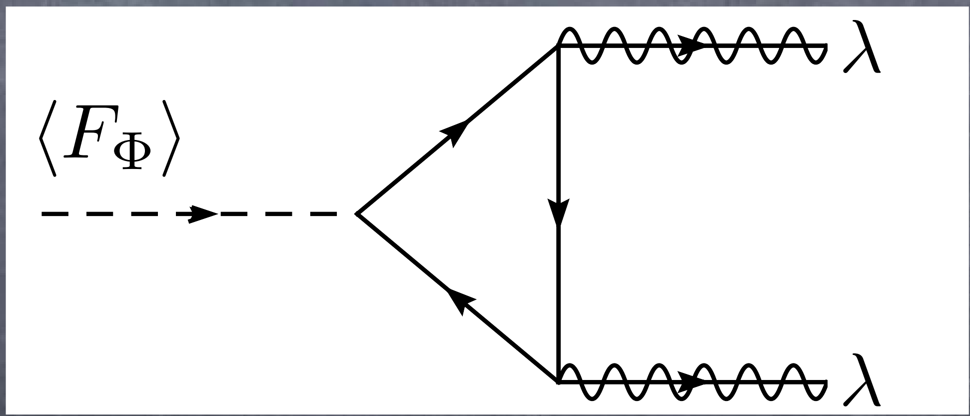
- Gravitino pair production is effective for low-scale inflation models.
- Gravitino abundance is inversely proportional to the reheating temperature! [later]

2.2. Anomaly-induced decay

Anomaly-induced decay

cf. anomaly-mediated SUSY breaking:

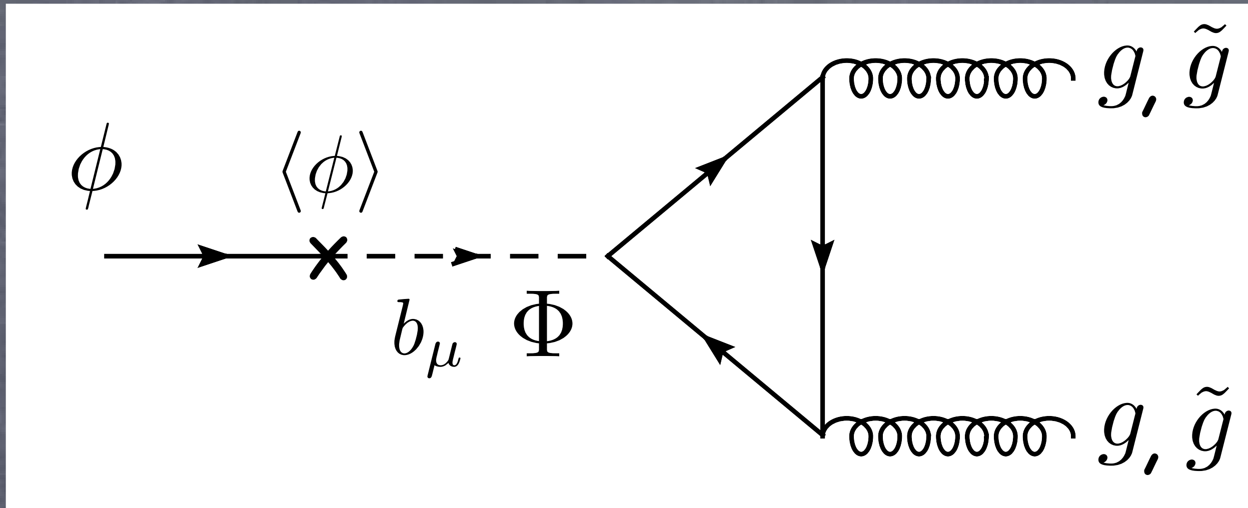
The SW anomaly mediates the effects of the SUSY breaking to the visible sector.



Φ contains scalar auxiliary field in gravity supermultiplet.

$$\Rightarrow \quad m_\lambda = -\frac{g^2}{16\pi^2} b_0 m_{3/2}$$

In a similar way, the anomaly couples the inflaton to any gauge sectors!



$$\mathcal{L} = \frac{g^2}{64\pi^2} X_G \, \phi(F_{mn}F^{mn} - iF_{mn}\tilde{F}^{mn}) - \frac{g^2}{32\pi^2} X_G m_\phi \phi^* \lambda\lambda + \text{h.c.},$$

$$X_G = (T_G - T_R)K_\phi + \frac{2T_R}{d_R}(\log\det K|''_R)_{,\phi},$$

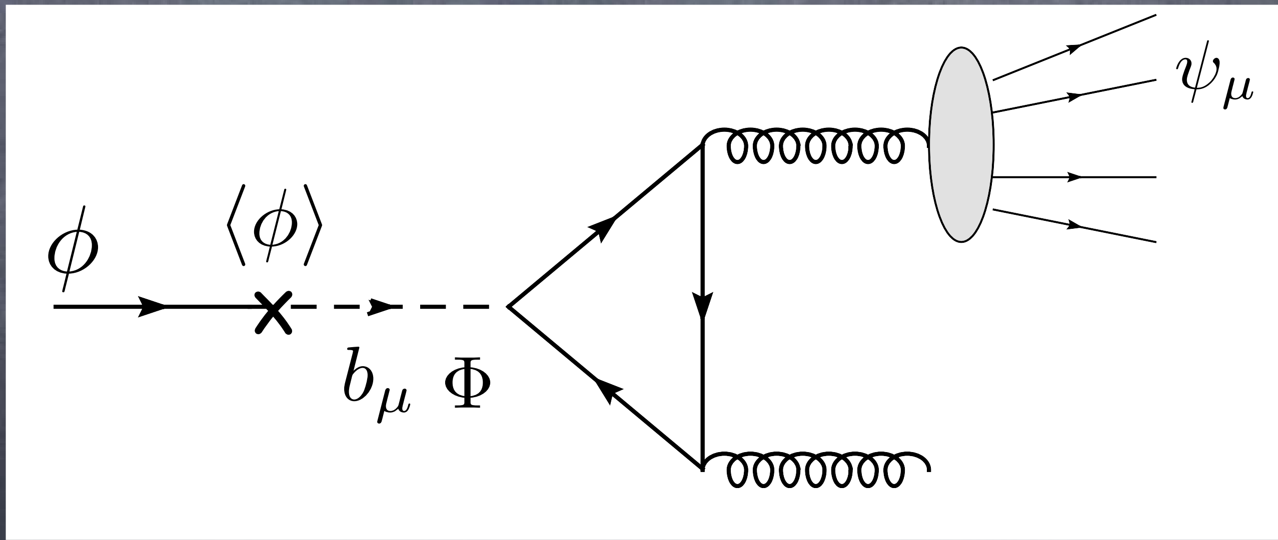
$$\Gamma^{(\text{anomaly})} \simeq \frac{N_g \alpha^2}{256\pi^3} |X_G|^2 m_\phi^3,$$

Decay into SUSY breaking sector

Endo, F.T, Yanagida hep-ph/0701042

- (Through Yukawa interactions at tree level)
- Through anomalies in SUGRA (at one-loop)

$$\Gamma_{\rm DSB} = \frac{N_g^{(h)}\alpha_h^2}{256\pi^3}(T_G^{(h)} - T_R^{(h)})^2 \left(\frac{\langle\phi\rangle}{M_P}\right)^2 \frac{m_\phi^3}{M_P^2}$$



The gravitinos are produced from the hidden hadron decay.

Summary on the gravitino production rates:

$$\Gamma_{3/2} \simeq \begin{cases} \dfrac{1}{32\pi}\left(\dfrac{\langle\phi\rangle}{M_P}\right)^2 \dfrac{m_\phi^3}{M_P^2}, & \text{for} \quad m_\phi < \Lambda \\ \dfrac{\alpha^2}{256\pi^3}\left(\dfrac{\langle\phi\rangle}{M_P}\right)^2 \dfrac{m_\phi^3}{M_P^2}, & \text{for} \quad m_\phi > \Lambda \end{cases}$$

3. Cosmological Constraints

Gravitino Abundance:

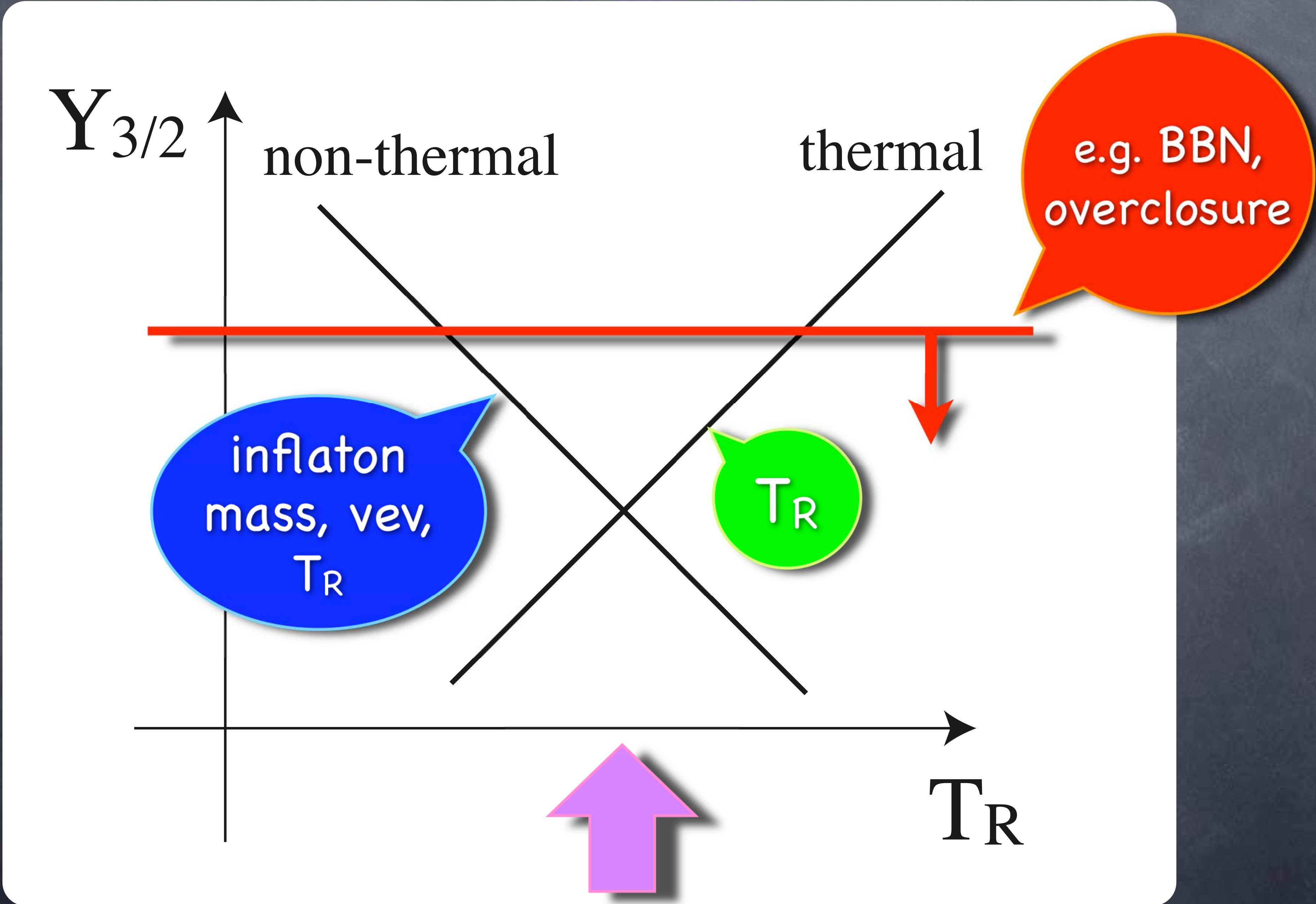
$$Y_{3/2} \simeq 2 \frac{\Gamma_{3/2}}{\Gamma_{\rm total}} \frac{3}{4} \frac{T_R}{m_\phi},$$

$$\sim 10^{-14} \left(\frac{g_*}{200}\right)^{-\frac{1}{2}} \left(\frac{T_R}{10^6 {\rm GeV}}\right)^{-1}$$

$$\times \left(\frac{\langle\phi\rangle}{10^{15} {\rm GeV}}\right)^2 \left(\frac{m_\phi}{10^{10} {\rm GeV}}\right)^2$$

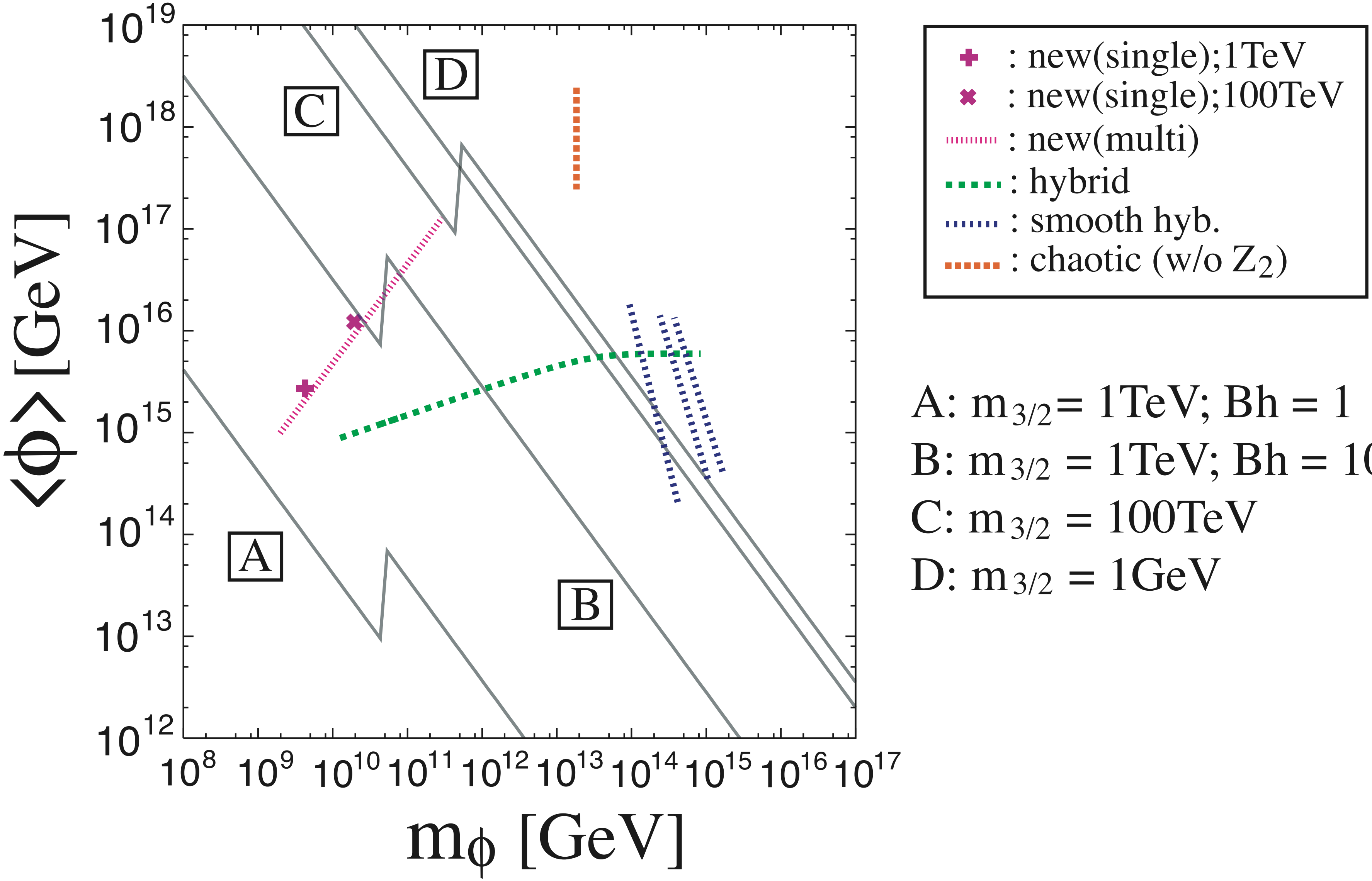
Note: $\Gamma_{\rm total} \sim \frac{T_R^2}{M_P}$

Gravitino Abundance



Conservative

Constraints on the inflation models;

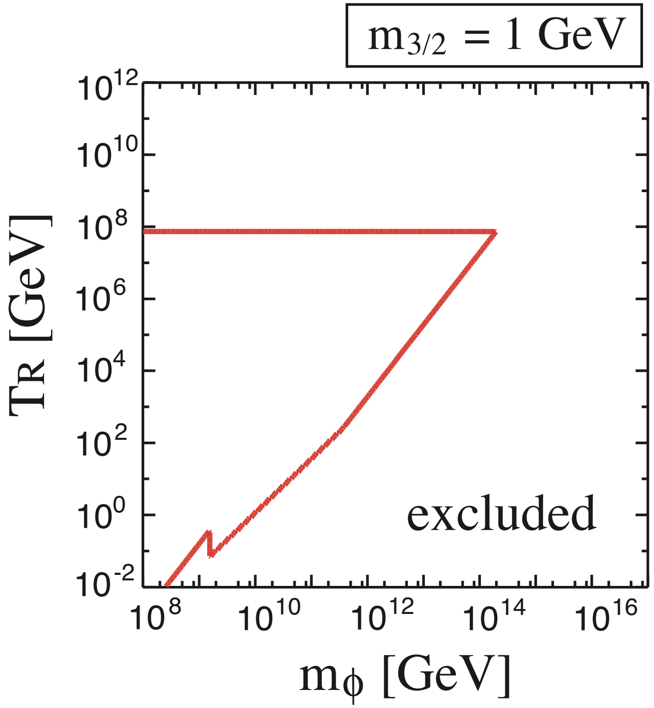


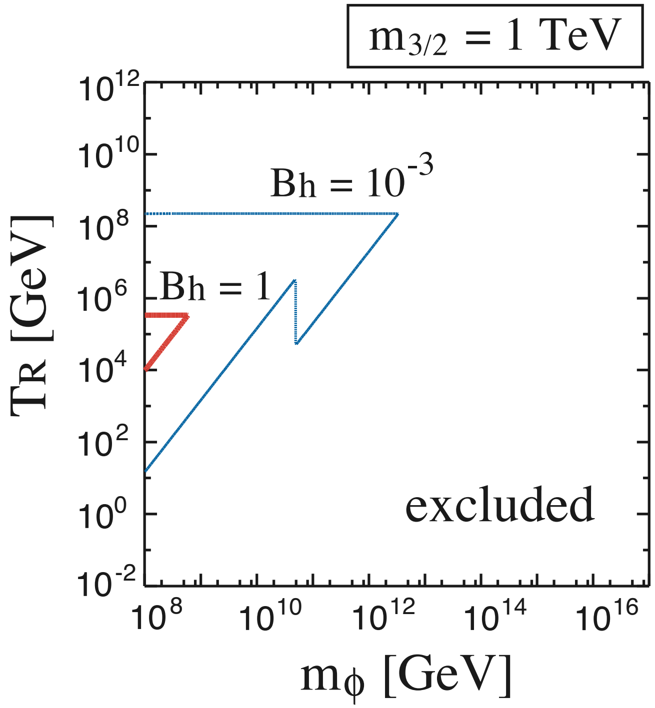
A: $m_{3/2}$= 1TeV; Bh = 1

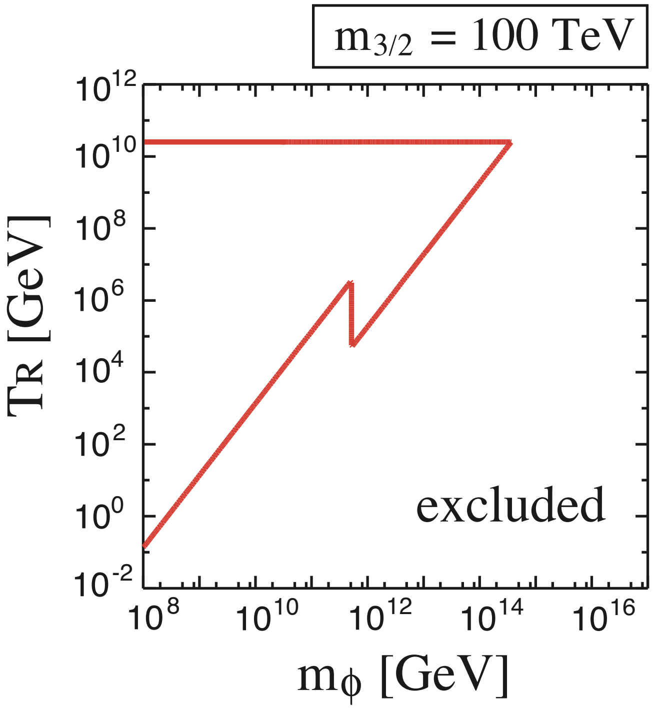
B: $m_{3/2}$ = 1TeV; Bh = 10^{-3}

C: $m_{3/2}$ = 100TeV

D: $m_{3/2}$ = 1GeV

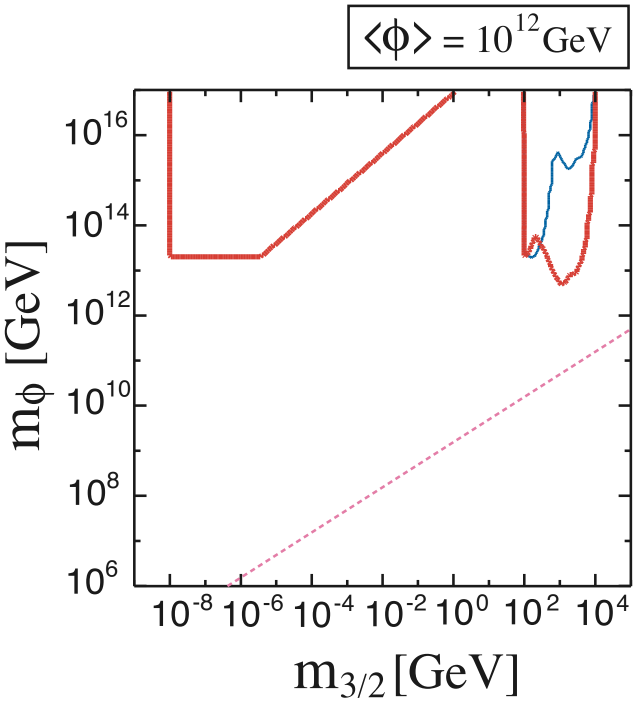


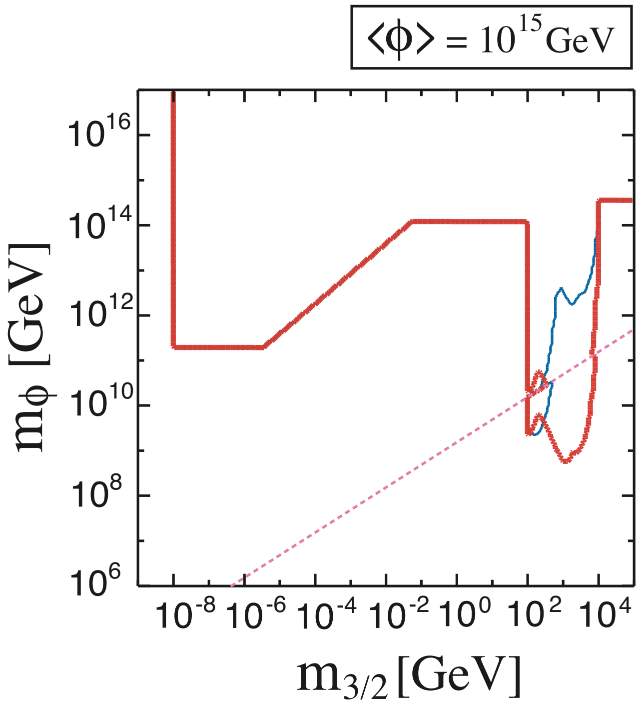


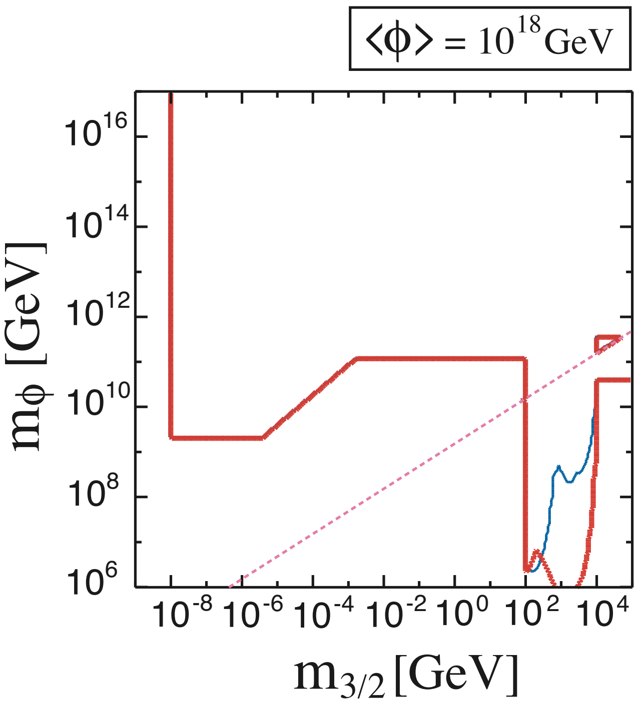


T_R cannot be arbitrarily low!

$$\langle \phi \rangle = 10^{15}\,\mathrm{GeV}$$







Smaller mass and vev are favored, if T_R is same.

Solutions:

(i) Postulate a symmetry on the inflaton.

e.g.) chaotic inflation

$$V = \frac{1}{2}m^2\phi^2 \text{ w/ } \phi \leftrightarrow -\phi$$

(ii) AMSB, GMSB

cosmological constraints are relaxed.

(iii) late-time entropy production

3-2. Cosmological Constraints (decay in to the SM sector)

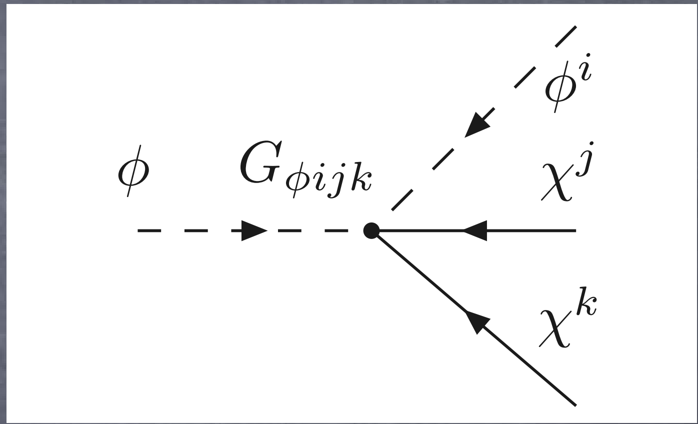
Spontaneous decay at tree level

$$W = Y\,\phi^i\phi^j\phi^k$$

$$\mathcal{L} = -\frac{1}{2}e^{G/2}G_{\phi ijk}\,\hat{\phi}\,\phi^i\chi^j\chi^k + \text{h.c.}.$$

$$G_{\phi ijk} = -\frac{W_\phi}{W}\frac{W_{ijk}}{W} + \frac{W_{\phi ijk}}{W}$$

$$\simeq K_\phi\frac{W_{ijk}}{W} + \frac{W_{\phi ijk}}{W},$$



- Decay Rate through the Yukawa coupling:

$$\Gamma_Y = \frac{C^{(3)}_{ijk}}{256\pi^3}m_\phi^3 \qquad C^{(3)}_{ijk} \equiv e^{G/2}G_{\phi ijk}$$

The inflaton decays into the visible sector through the top Yukawa coupling:

$$W = Y_t T Q H_u,$$

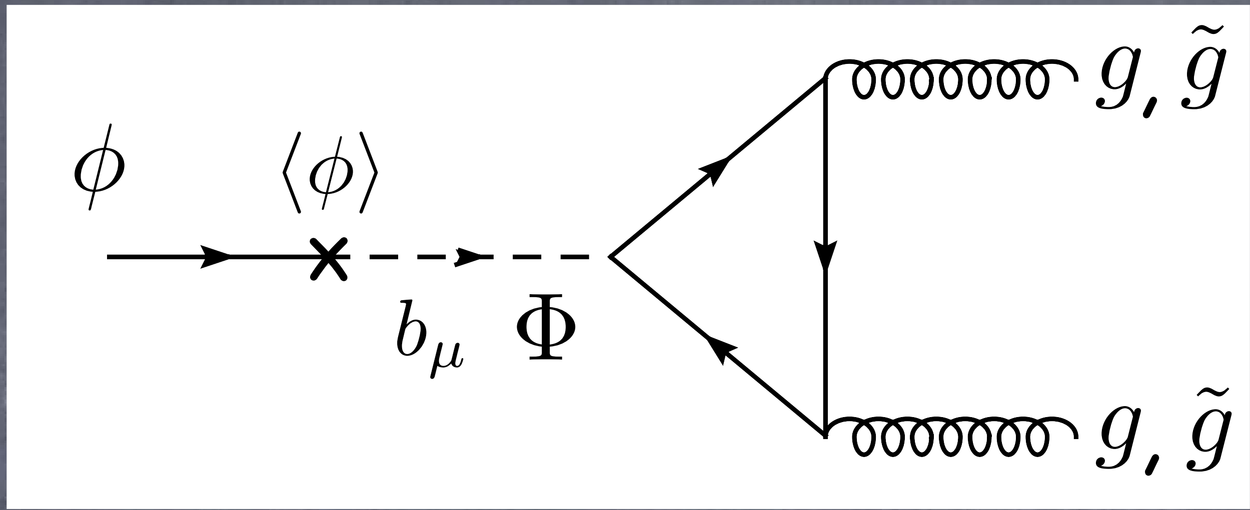
The decay rate is given by

$$\Gamma_T = \frac{3}{128\pi^3} |Y_t|^2 \left(\frac{\langle \phi \rangle}{M_P} \right)^2 \frac{m_\phi^3}{M_P^2},$$

(minimal Kahler is assumed)

The reheating temperature is bounded below!!

e.g.) inflaton decay through SU(3)c interactions;

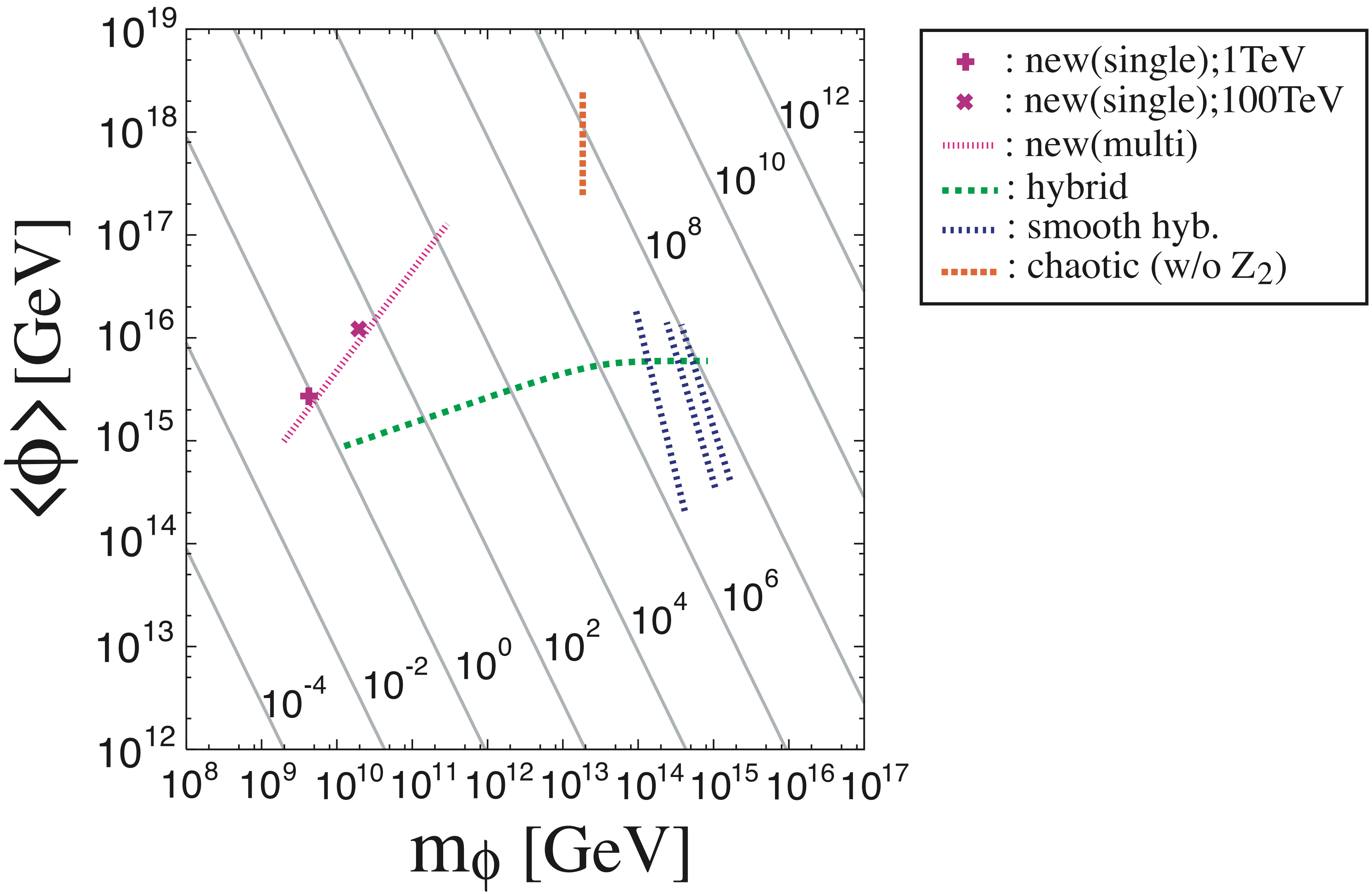


$$\Gamma_{\mathrm{SU(3)}} \simeq \frac{9}{32\pi^3}\alpha_s^2 \frac{\langle\phi\rangle^2}{M_P^2}\frac{m_\phi^3}{M_P^2},$$

(minimal Kahler is assumed)

The decay rate is smaller by one order of magnitude than that through the top Yukawa coupling.

Lower limit on the reheating temperature:

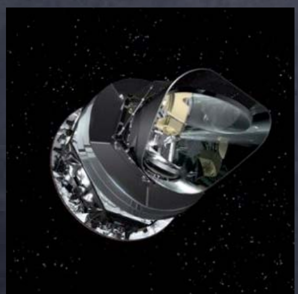


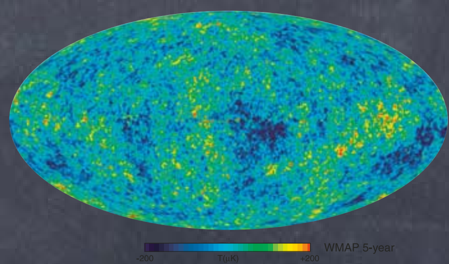
4. Conclusion

We have discovered that gravitinos are generically produced by the inflaton decay.

The abundance of the non-thermally produced gravitinos depends on the inflaton parameters, which enables us to distinguish inflation models.

Finding SUSY mediation mechanism is important for cosmology, in particular, inflation models !!





Back-up Slides

Potential minimization

$$V = e^G \left(G^i G_i - 3 \right)$$

Differentiating V w.r.t. ϕ

$$\Rightarrow \quad G^\phi \nabla_\phi G_\phi + G^z \nabla_\phi G_z + G_\phi = 0$$

$$\nabla_\phi G_\phi \sim \frac{W_{\phi\phi}}{W} \sim \frac{m_\phi}{m_{3/2}} \gg 1$$

$$\nabla_\phi G_z \sim \frac{W_\phi}{W} \frac{W_z}{W} \sim \langle \phi \rangle$$

$$\Rightarrow \quad G_\phi \sim \langle \phi \rangle \frac{m_{3/2}}{m_\phi}$$

Mass Matrix in SUGRA

$$V = e^G \left(G^i G_i - 3 \right)$$

$$M^2_{ij*} = \frac{\partial^2 V}{\partial \varphi^i \partial \varphi^{\dagger j}} = e^G \left(\nabla_i G_k \nabla_{j*} G^k - R_{ij^* k\ell^*} G^k G^{\ell *} + g_{ij^*} \right),$$

$$M^2_{ij} = M^2_{ji} = \frac{\partial^2 V}{\partial \varphi^i \partial \varphi^j} = e^G \left(\nabla_i G_j + \nabla_j G_i + G^k \nabla_i \nabla_j G_k \right),$$

$$\nabla_\phi G_\phi \sim \frac{W_{\phi\phi}}{W} \sim \frac{m_\phi}{m_{3/2}} \gg 1$$

$$\nabla_\phi G_z \sim \frac{W_\phi}{W} \frac{W_z}{W} \sim \langle \phi \rangle$$

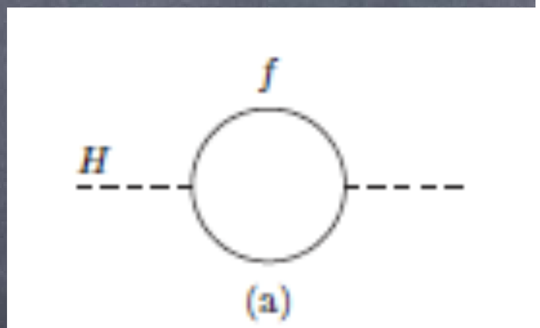
$\Rightarrow$ $M^2_{\phi \bar{z}} \neq 0$

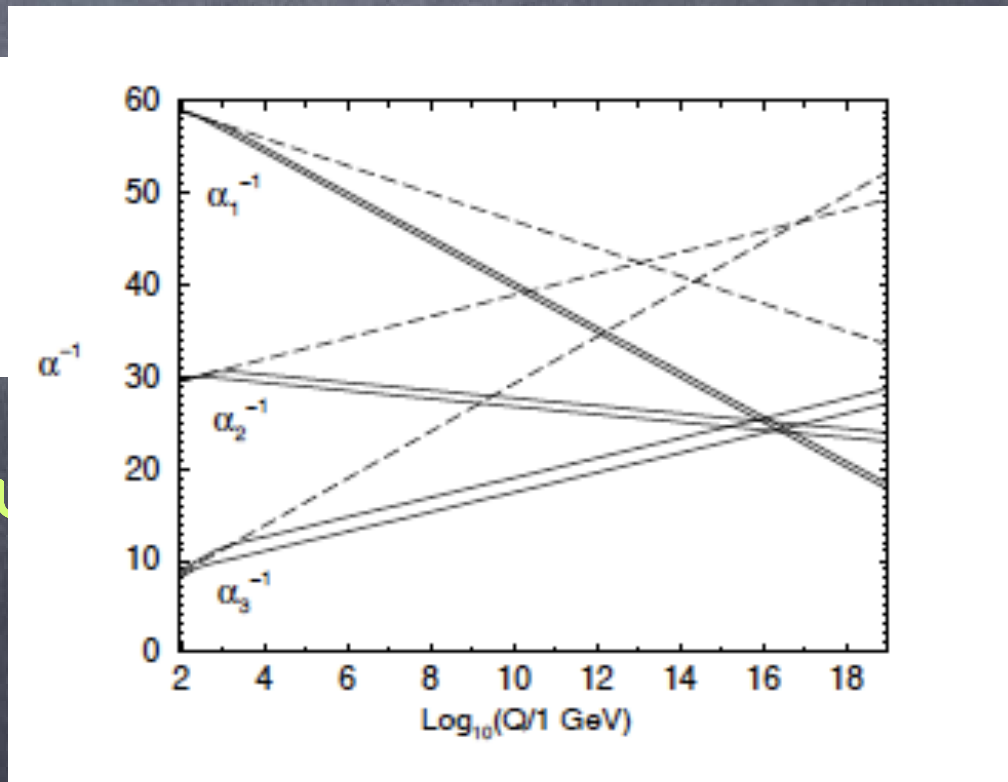
2. Inflation Models in Supergravity

Why Supersymmetry?

Boson ⟷ Fermion

- A solution to the gauge hierarchy problem:
  SUSY stabilizes the electroweak scale against the radiative corrections.





- The gauge coupling u

- Relevant interactions for inflation in supergravity:

$K(\phi, \phi^*)$: Kahler potential $\quad K = |\phi|^2 + \cdots$

$W(\phi)$: Superpotential

$$e^{-1}\mathcal{L} = -g_{ij^*}\partial_\mu\phi_i\partial^\mu\phi_j^* - V(\phi)$$

$$V = e^K\left(D_iWg^{ij^*}(D_jW)^* - 3|W|^2\right) + V_D$$

$$g_{ij^*} \equiv \frac{\partial^2 K}{\partial\phi_i\partial\phi_j^*} \qquad D_iW = W_i + K_iW$$

Eta-problem?

- The slow-roll parameters must be small enough for the inflation to last long enough.

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \left(\frac{V''}{V}\right)$$

- In supergravity, eta tends to be of order unity.

$$V = e^K \left(D_i W g^{ij^*} (D_j W)^* - 3|W|^2 \right) + V_D$$

Fine-tuning (one part in hundred) is needed, but not so severe...

Supersymmetric inflation models

- New inflation model (single & multi-fields)
- Hybrid inflation model (F-term)
- Smooth Hybrid inflation model
- Chaotic inflation model
- etc.

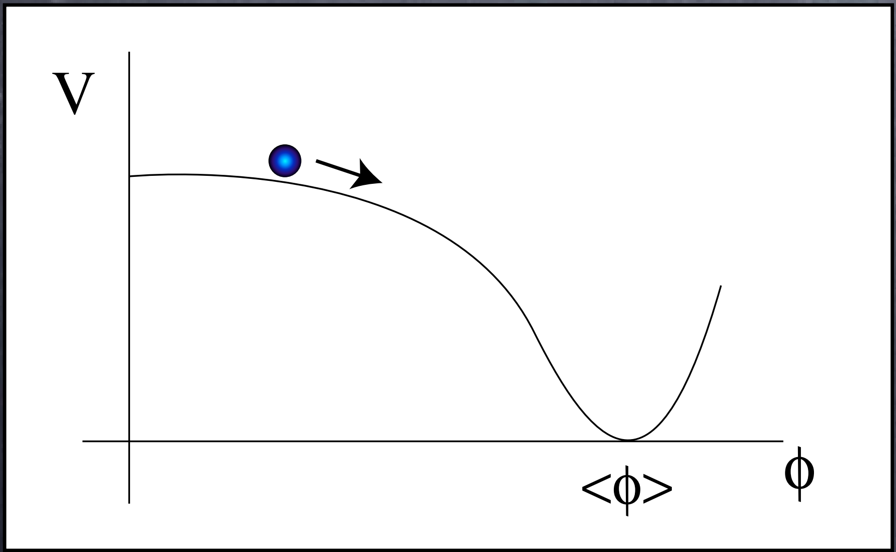
New inflation model

$$
\begin{aligned}
K(\phi,\phi^\dagger) &= |\phi|^2 + \frac{k}{4}|\phi|^4, \\
W(\phi) &= v^2\phi - \frac{g}{n+1}\phi^{n+1}.
\end{aligned}
$$

Izawa and Yanagida ,`97

Generic superpotential under discrete Z_{2n} R-symmetry.

$$
V(\varphi) \simeq v^4 - \frac{k}{2}v^4\varphi^2 - \frac{g}{2^{\frac{n}{2}-1}}v^2\varphi^n + \frac{g^2}{2^n}\varphi^{2n}
$$



$$
\langle\phi\rangle \simeq (v^2/g)^{1/n}
$$

$$
m_\phi \simeq nv^2/\langle\phi\rangle
$$

The spectral index

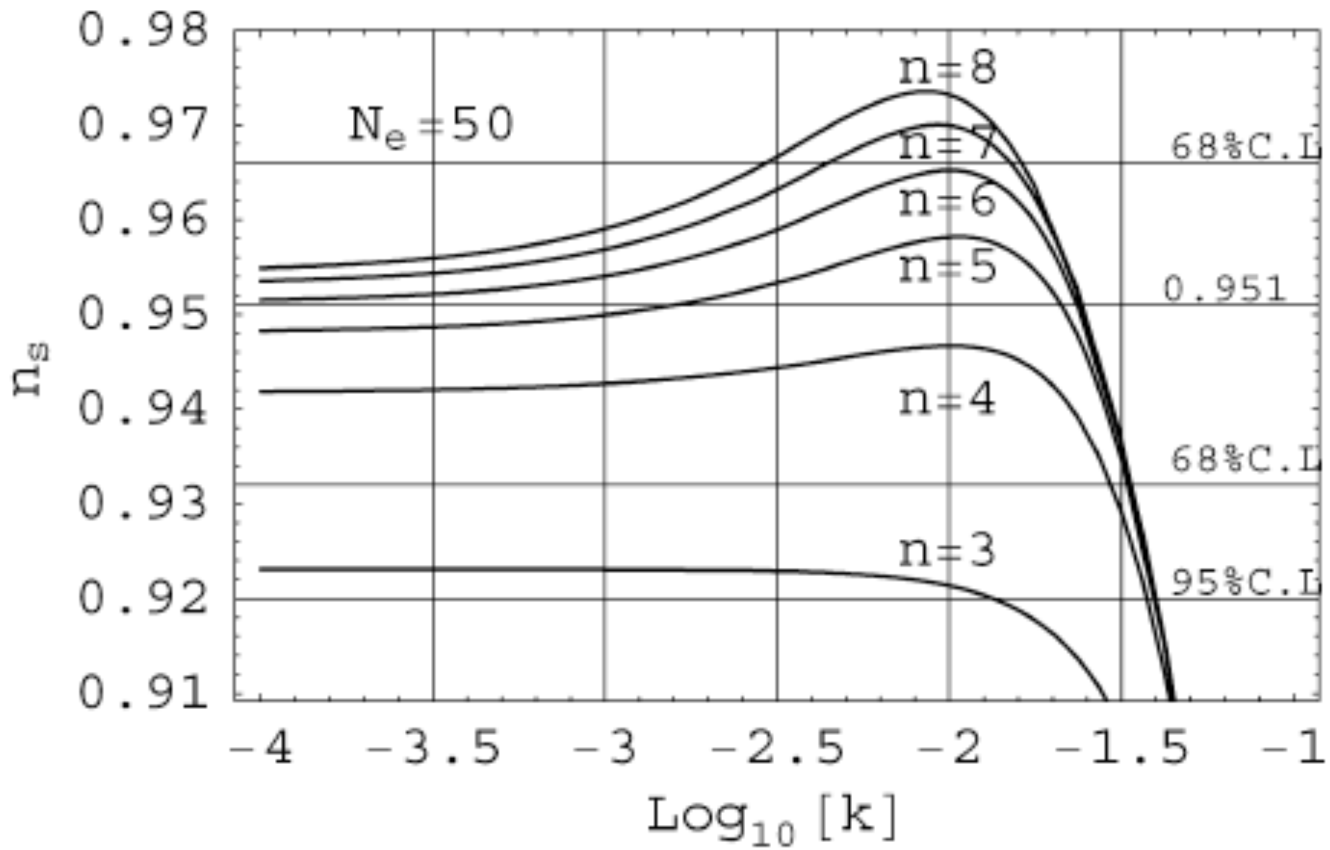
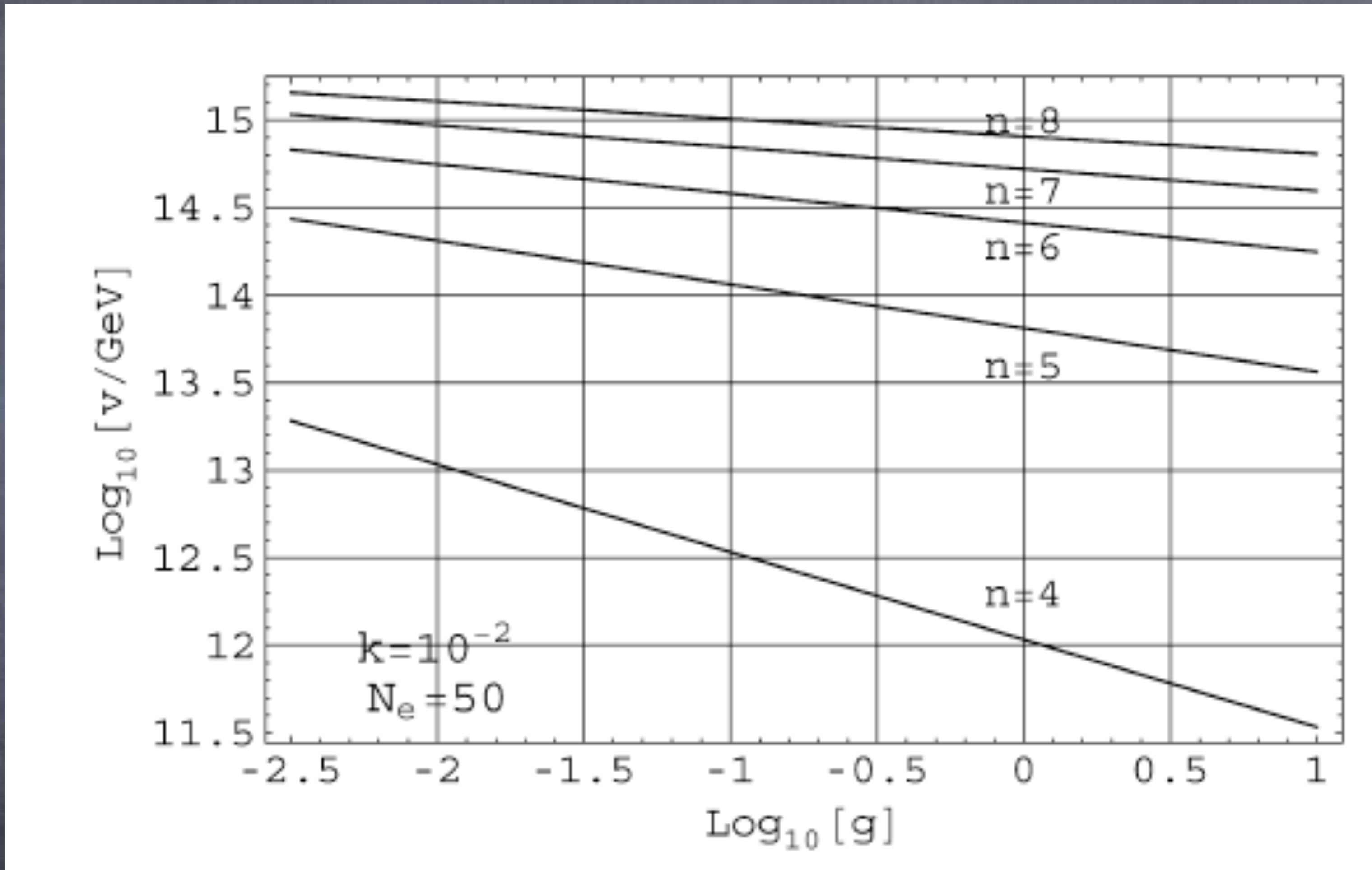


Fig. 1. The k dependence of the spectral index n_s for $n = 3$–8 and $N_e = 50$. The horizontal grid lines correspond to the result of WMAP three year data [10]. For $k \gtrsim 10^{-2}$, $n_s \simeq 1 - 2k$, and for $k = 0$, $n_s \simeq (N_e(n-2) - (n-1))/(N_e(n-2) + (n-1))$.

Ibe, Shinbara, Yanagida,`06

The inflation scale

Ibe, Shinbara, Yanagida,`06

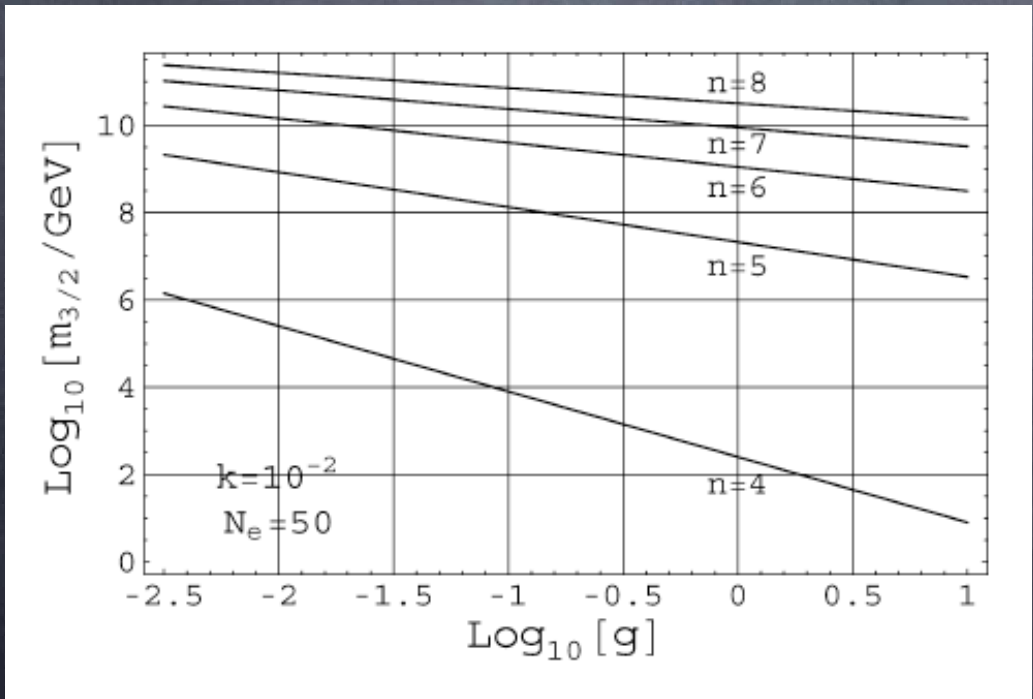


$H_{inf} = 10^6 - 10^8$GeV for n=4.

The gravitino mass is related to the inflaton parameters in this model.

$$\Lambda_{SUSY}^4 - 3|W(\phi_0)|^2 \simeq 0.$$

$$m_{3/2} = W(\phi_0) \simeq \frac{nv^2}{n+1}\left(\frac{v^2}{g}\right)^{\frac{1}{n}}$$



The single-field new inflation model favors a heavy gravitino.

Multi-field new inflation

Asaka, Hamaguchi, Kawasaki,Yanagida ,`97
Senoguz, Shafi, `04

- The same dynamics can be realized by the following Kahler and superpotentials:

$$W = \chi(v^2 - g\phi^n)$$

$$K = |\phi|^2 + |\chi|^2 + \frac{\kappa_1}{4}|\phi|^4 + \kappa_2|\phi|^2|\chi|^2 + \frac{\kappa_3}{4}|\chi|^4 + \cdots$$

$$\Rightarrow \quad V(\varphi) \simeq v^4 - \frac{k}{2}v^4\varphi^2 - \frac{g}{2^{\frac{n}{2}-1}}v^2\varphi^n \qquad k \equiv \kappa_2 - 1$$

Also, the gravitino mass is not related to the inflaton parameters in this model.

Hybrid inflation model

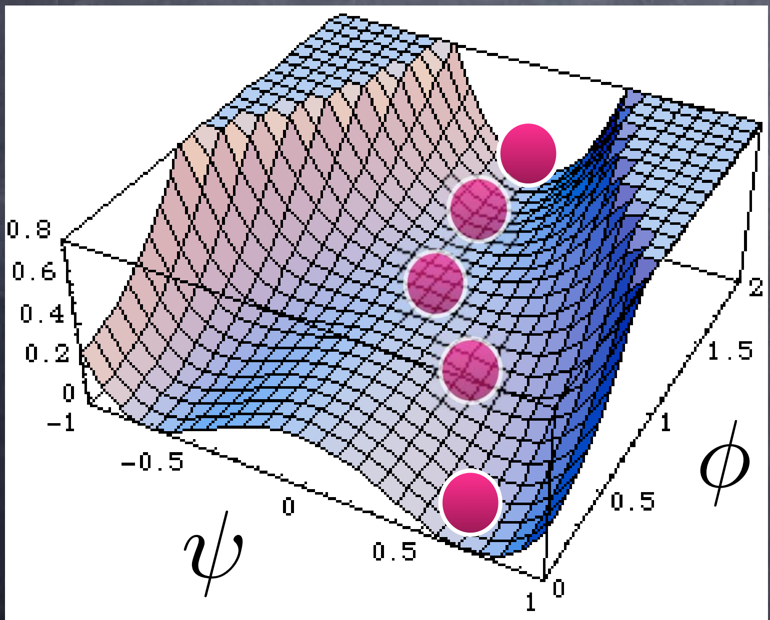
Copeland et al, Dvali et al, `94 Linde and Riotto, `97

$$W(\phi, \psi, \tilde{\psi}) = \phi(\mu^2 - \lambda \tilde{\psi} \psi),$$ w/ minimal Kahler

R-charge: $\phi(+2), \; \psi \, \tilde{\psi}(0)$

U(1) gauge: $\phi(0), \; \psi(1), \; \tilde{\psi}(-1)$

For $|\phi| \gg \mu/\sqrt{\lambda}$ $\quad \langle \psi \rangle = \langle \tilde{\psi} \rangle = 0$



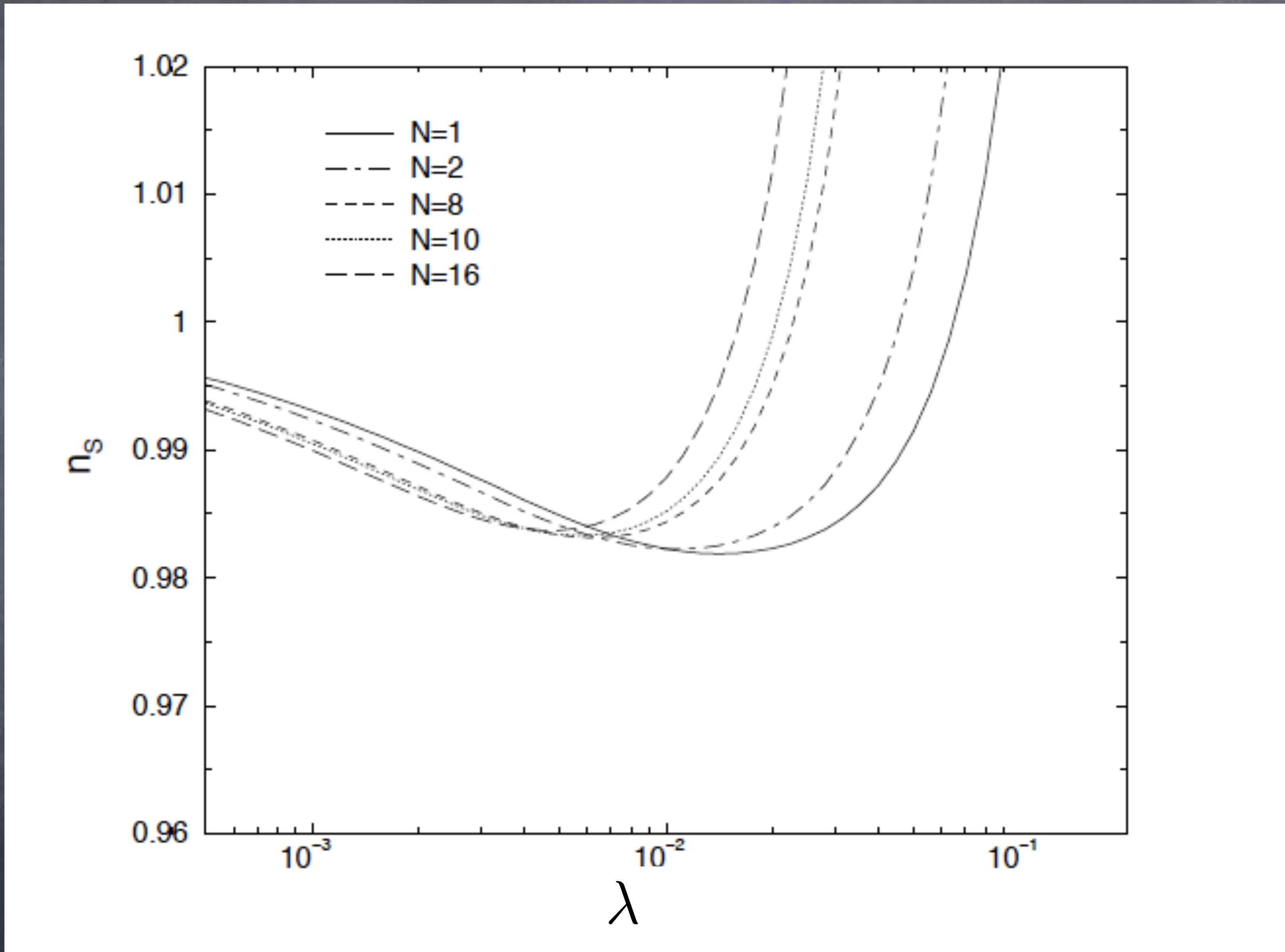
$$V(\varphi) \simeq \mu^4 \left(1 + \frac{\lambda^2}{8\pi^2} \ln \left(\frac{\varphi}{\varphi_c} \right) + \cdots \right)$$

The global minimum is located at

$$\langle \phi \rangle = 0$$

$$\langle \psi \rangle = \langle \tilde{\psi} \rangle = \mu/\sqrt{\lambda}$$

Scalar spectral index: $n_s \simeq 0.98 - 1.0$



Bastro-Gil, King, Shafi, `06

Smooth hybrid inflation

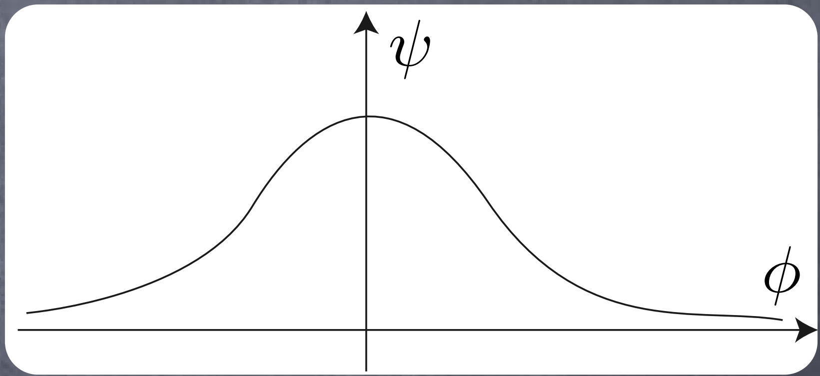
Lazarides et al `95

$$W(\phi, \psi, \tilde{\psi}) = \phi \left(\mu^2 - \frac{(\tilde{\psi}\psi)^n}{M^{2n-2}} \right).$$

Global minimum is located at

$$\langle \phi \rangle = 0$$

$$\langle \psi \rangle = \langle \tilde{\psi} \rangle = (\mu M^{n-1})^{1/n}$$



The dynamics is similar to hybrid inflation, but n_s is slightly smaller.

$$n_s \simeq 0.967 - 0.97$$

Chaotic Inflation

Kawasaki, Yamaguchi and Yanagida ,`00

$$V = e^K \left(D_i W g^{ij^*} (D_j W)^* - 3|W|^2 \right) + V_D$$

The exponential pre-factor was the obstacle for the chaotic inflation in supergravity.

Let us impose a shift symmetry on K:

$$\phi \longrightarrow \phi + iA$$

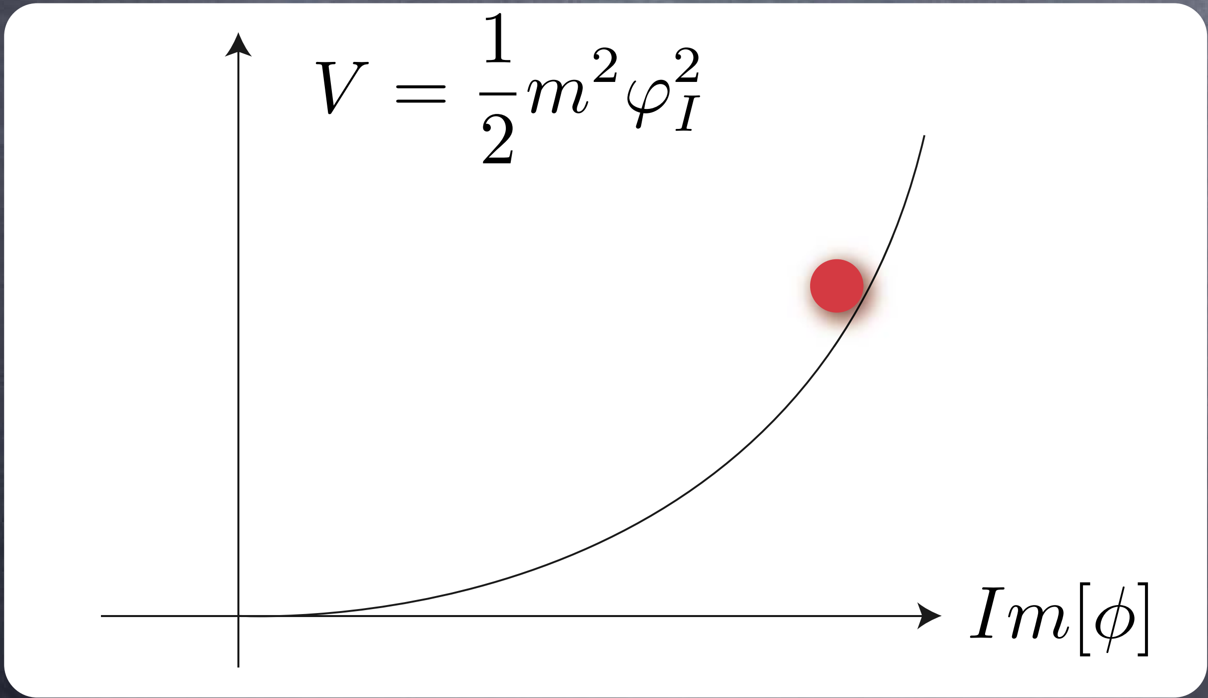
$$K(\phi + \phi^\dagger) = c\,(\phi + \phi^\dagger) + \frac{1}{2}(\phi + \phi^\dagger)^2 + \cdots$$

Imaginary component does not appear in the exponent!

If the symmetry is broken by the following W,

$$W = m\phi\psi$$

$$V \simeq \frac{1}{2}m^2\varphi^2$$



WMAP normalization:

$$m = 2 \times 10^{13}\,\mathrm{GeV}$$

$$n_s \simeq 0.96$$

Summary of Inflation Models