Introduction to electroweak phase transition

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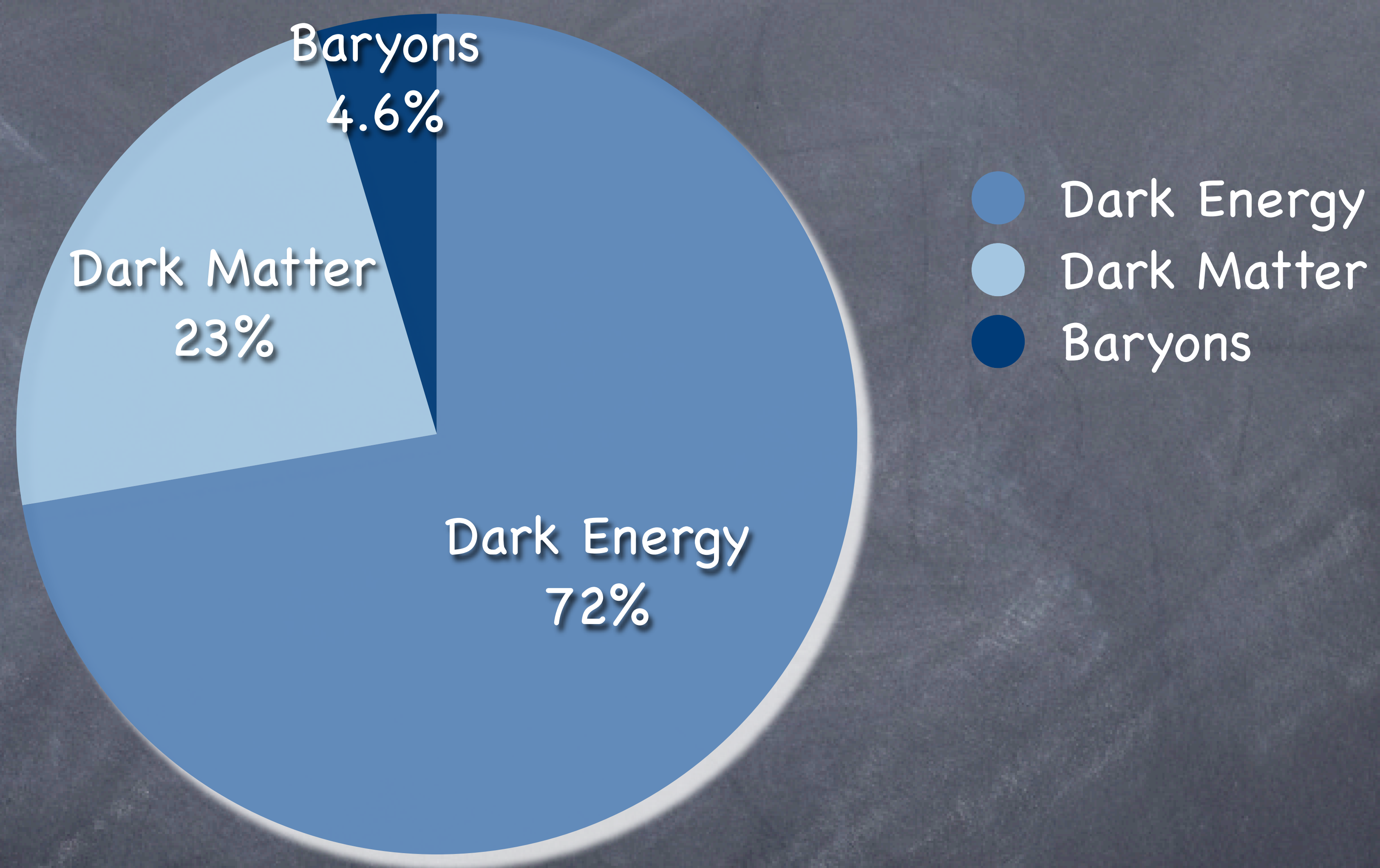
March 12, 2009@NTHU

Outline

- Overview
- Effective potential
- Sphaleron
- Electroweak phase transition (EWPT)
  SM, MSSM
- Updated analysis of EWPT in the MSSM
- Summary

Motivation

- Energy budget of the Universe

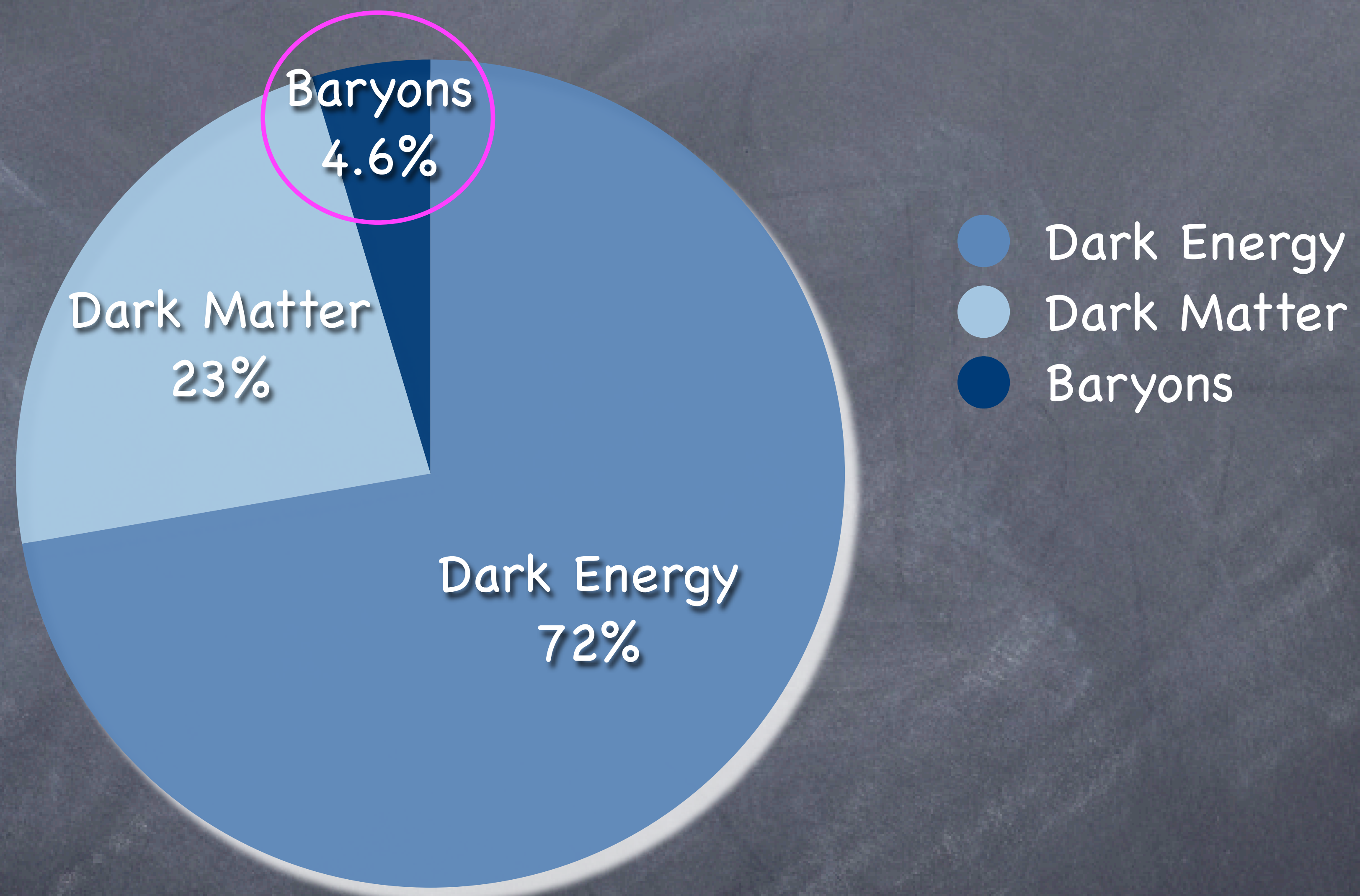


- 95% of the Universe is made of dark object.
- It should be stressed that there remains a mystery in the visible sector as well.

Where did antibaryons go?

Motivation

- Energy budget of the Universe



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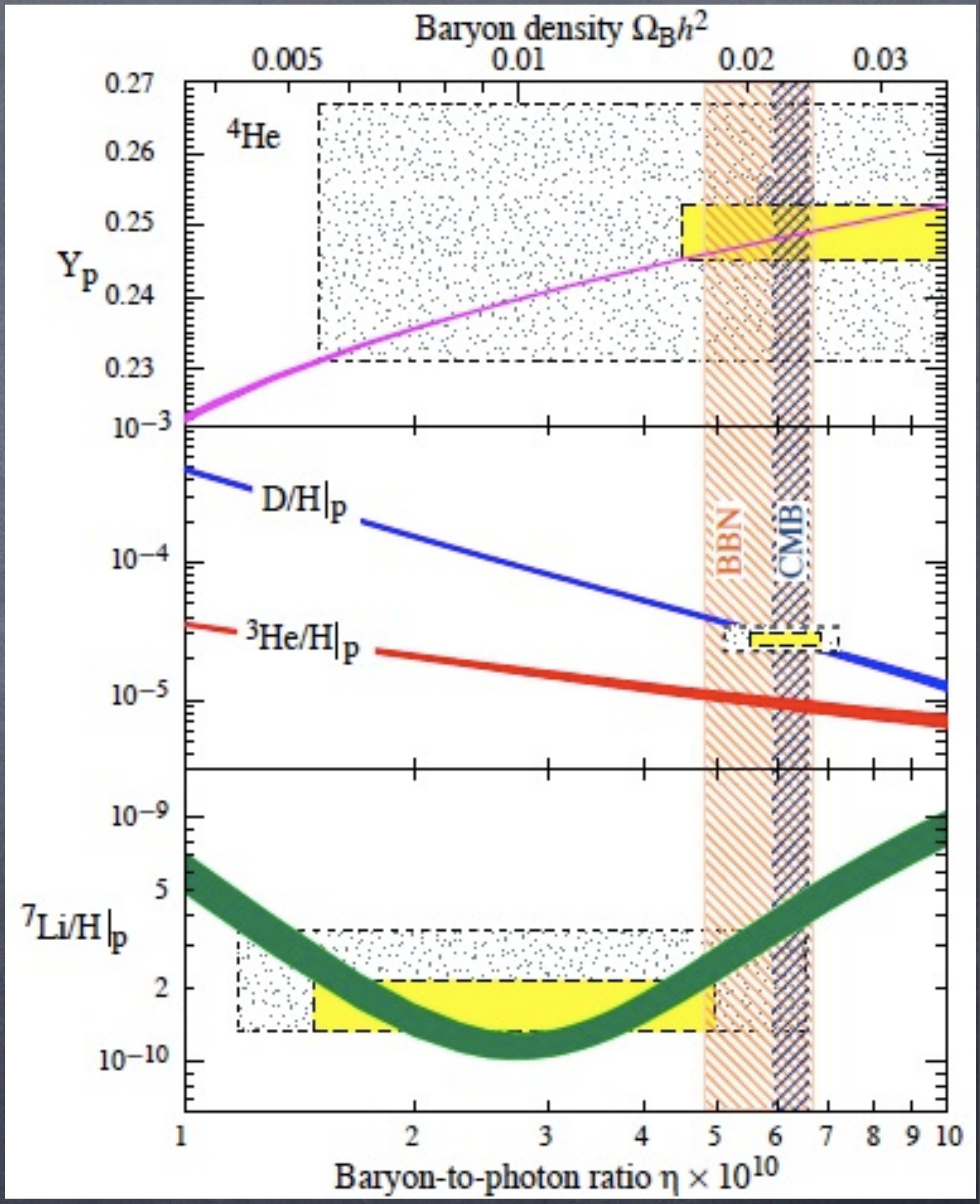
Where did antibaryons go?

BAU

Baryon Asymmetry of the Universe (BAU)

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma}$$
$$= (4.7 - 6.5) \times 10^{-10} \ (95\%\text{C.L.})$$

- If the BAU is generated before T=1 MeV, the light element abundances can be explained by the standard Big-Bang cosmology.



[PDG '08]

Question:

How did the BAU arise dynamically?

Sakharov's criteria

To get the BAU from initially baryon symmetric Universe, the following conditions must be satisfied. [Sakharov, '67]

(1) Baryon number (B) violation
(2) C and CP violation
(3) out of equilibrium

(1) is trivial.

w/o (2), namely, if C and CP symmetries exist,

$$[\rho(t), \mathcal{O}] = 0, \ (\mathcal{O} = C, CP), \ i\hbar \frac{\partial \rho(t)}{\partial t} + [\rho(t), H] = 0,$$

$$\begin{aligned} \langle n_B \rangle &= \mathrm{tr}(\rho_B n_B) = \mathrm{tr}(\rho_B n_B \mathcal{O}^{-1} \mathcal{O}) \\ &= \mathrm{tr}(\rho_B \mathcal{O} n_B \mathcal{O}^{-1}) = -\mathrm{tr}(\rho_B n_B) \end{aligned}$$

$$\therefore \ \boxed{\langle n_B \rangle = 0} \qquad (CBC^{-1} = -B, \ (CP)B(CP)^{-1} = -B)$$

B is vector-like

w/o (3), namely, if the B violating process is in equilibrium, one would get

$$n_b = n_{\bar{b}} \quad \Rightarrow \quad \langle n_B \rangle = 0$$

[N.B.] The masses of particle and antiparticle are assumed to be the same. ($\because$ CPT theorem)

Two possibilities

(1) B-L generation above the electroweak (EW) scale
    Leptogenesis, GUTs, Affleck-Dine etc

(2) B generation during the EW phase transition (PT)
    EW baryogenesis (BG)

(2) is directly linked to EW Physics.

→ It is testable at colliders

EW baryogenesis

- B violation: sphaleron process
- C violation: chiral gauge interaction
- CP violation: Kobayashi-Maskawa (KM) phase and other sources in the BSM.
- out of equilibrium: 1st order PT with expanding bubble wall

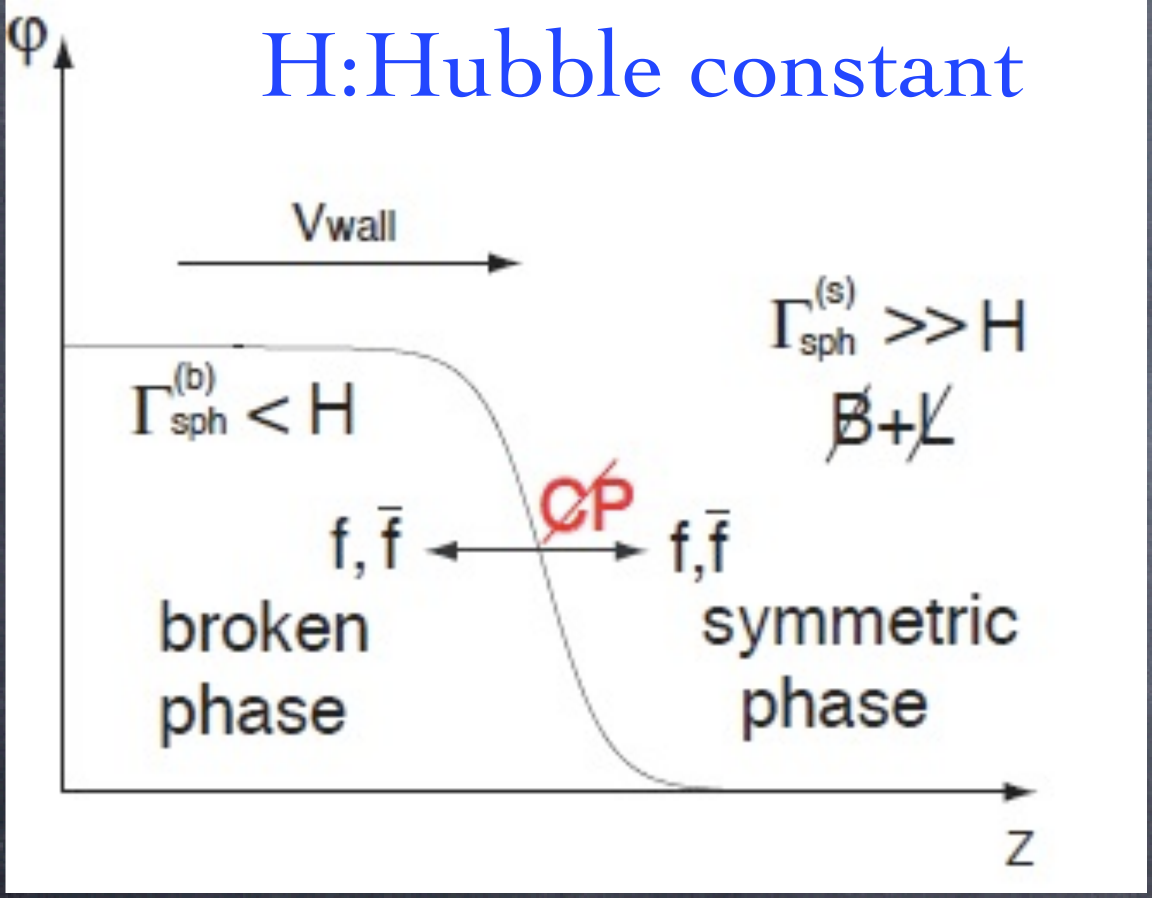
The SM has the problems with the last 2 conditions:

- The KM phase is too small to generate the BAU.
- The PT is not 1st order for the viable Higgs mass. (>114.4 GeV)

The SM failed to explain the BAU

Mechanism of EWBG

[Kuzmin, Rubakov, Shaposhnikov, PLB155,36 ('85)]



- Due to CP violation, asymmetry of particle number densities at the bubble wall occur.
- they diffuse into symmetric phase.
- Left-handed particle number densities are converted into B via sphaleron process.
- Sphaleron process is decoupled after the PT.
- B is frozen.

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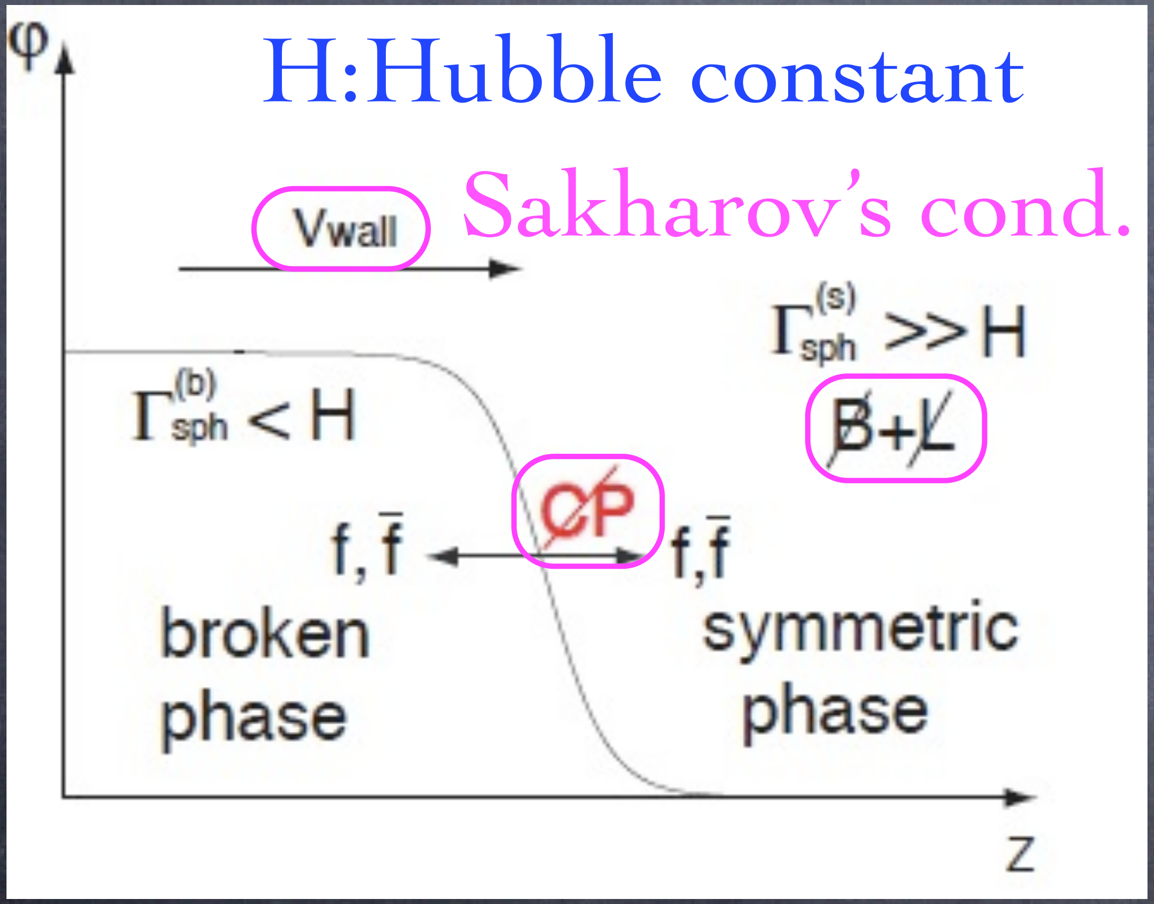

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Effective potential

Effective potential T=0

The effective potential is defined by

$$V_{\text{eff}}(\varphi) = -\Gamma[\varphi(x) = \varphi] / \int d^4x$$

1-loop effective potential:

$$V_1(\varphi) = -\frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \ln\det\Big[iG^{-1}(p;\varphi)\Big].$$

e.g. scalar loop: After a Wick rotation,

$$V_1(\varphi) = \frac{1}{2}\mu^\epsilon \int \frac{d^D p_E}{(2\pi)^D} \ln\Big(p_E^2 + m^2(\varphi)\Big), \quad D = 4 - \epsilon$$

After performing the integration, we can get

$$V_1(\varphi) = \frac{m^4}{64\pi^2}\bigg(-\frac{2}{\epsilon} - \ln 4\pi + \gamma_E + \ln\frac{m^2}{\mu^2} - \frac{3}{2} + \mathcal{O}(\epsilon) + \cdots\bigg).$$

MS-bar scheme:

$$V_1(\varphi) = \frac{m^4}{64\pi^2}\bigg(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\bigg).$$

μ: renormalization scale

Effective potential at finite T

Imaginary time formalism:

$$\int \frac{d^4k}{(2\pi)^4} \to iT \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} (\cdots)\Big|_{k^0=i\omega_n} \qquad \omega_n = \begin{cases} 2n\pi T & \text{(boson)} \\ (2n+1)\pi T & \text{(fermion)} \end{cases}$$

1-loop effective potential:

$$V_1(\varphi, T) = \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \ln(w_n^2 + w^2), \qquad \omega = \sqrt{\boldsymbol{k}^2 + m^2}$$

Frequency sum:

$$\sum_{n=-\infty}^{\infty} \frac{z}{z^2 + 4\pi^2 n^2} = \frac{1}{2} + \frac{1}{e^z - 1}, \qquad \text{(boson)}$$

$$\sum_{n=-\infty}^{\infty} \frac{z}{z^2 + (2n+1)^2 \pi^2} = \frac{1}{2} - \frac{1}{e^z + 1} \qquad \text{(fermion)}$$

$$
\begin{aligned}
V_1(\varphi, T) &= \int \frac{d^3k}{(2\pi)^3}\left[\frac{w}{2} + T\ln\left(1 \mp e^{-w/T}\right)\right] \\
&= \boxed{V_1(\varphi)} + \frac{T^4}{2\pi^2} I_{B,F}(a^2), \quad a^2 = m^2(\varphi)/T^2.
\end{aligned}
$$

T=0

where

$$
I_{B,F}(a^2) = \int_0^\infty dx\ x^2 \ln\left(1 \mp e^{-\sqrt{x^2+a^2}}\right),
$$

[N.B.]

Since the divergences appear only in the 1st term (T=0 part), the counter terms at T=0 are enough.

High T expansion

For $a=m/T<<1$, $I_{B,F}$ can be expanded in powers of a.

boson:

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{32}\left(\log\frac{a^2}{\alpha_B} - \frac{3}{2}\right) + \mathcal{O}(a^6)$$

fermion:

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{32}\left(\log\frac{a^2}{\alpha_F} - \frac{3}{2}\right) + \mathcal{O}(a^6)$$

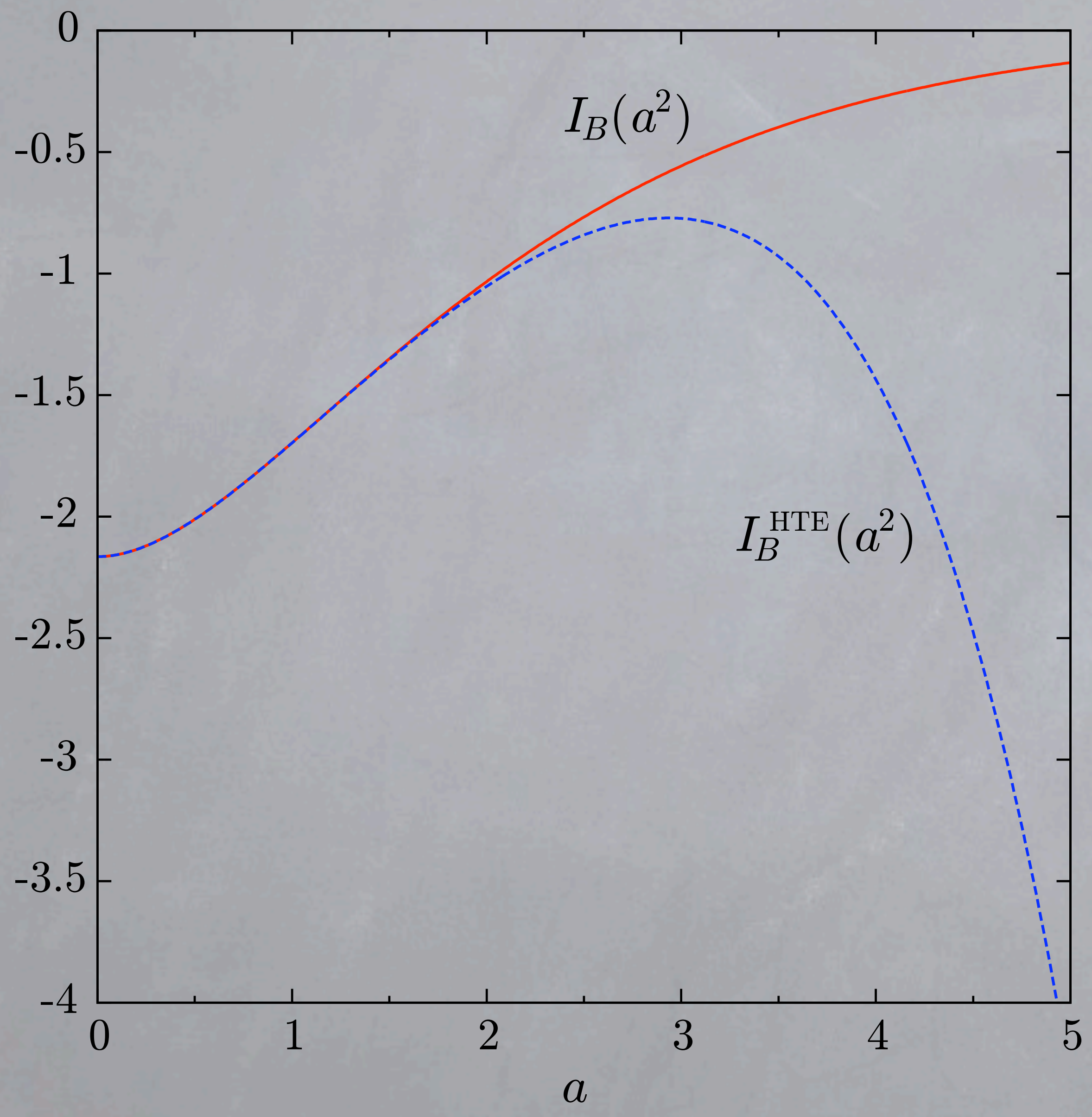
$$\log\alpha_B = 2\log 4\pi - 2\gamma_E \simeq 3.91, \quad \log\alpha_F = 2\log\pi - 2\gamma_E \simeq 1.14,$$

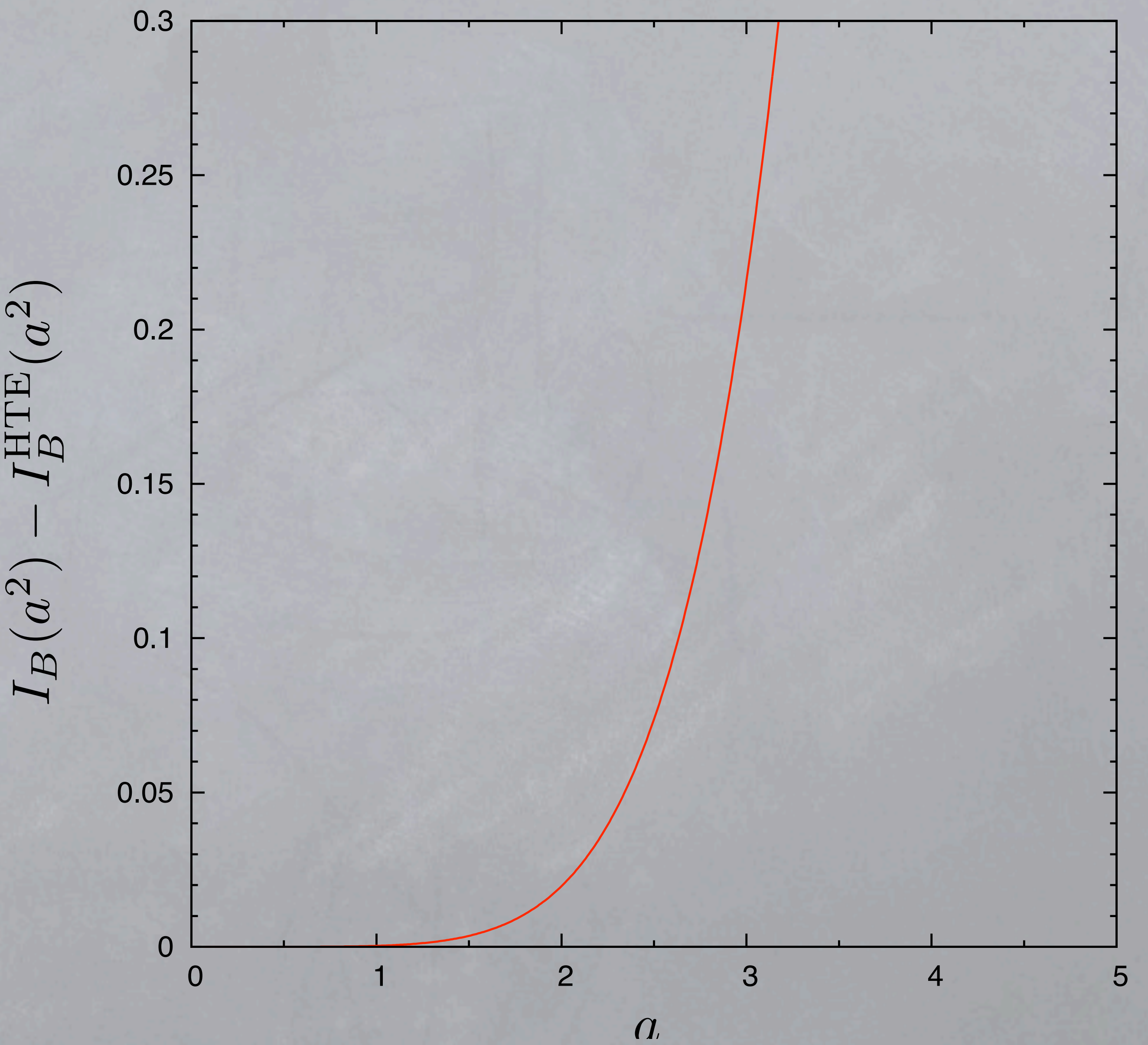
Euler's constant: $\gamma_E \simeq 0.577$

The bosonic loop gives a cubic term a^3 which comes from a zero frequency mode.

Validity of HTE

Boson case

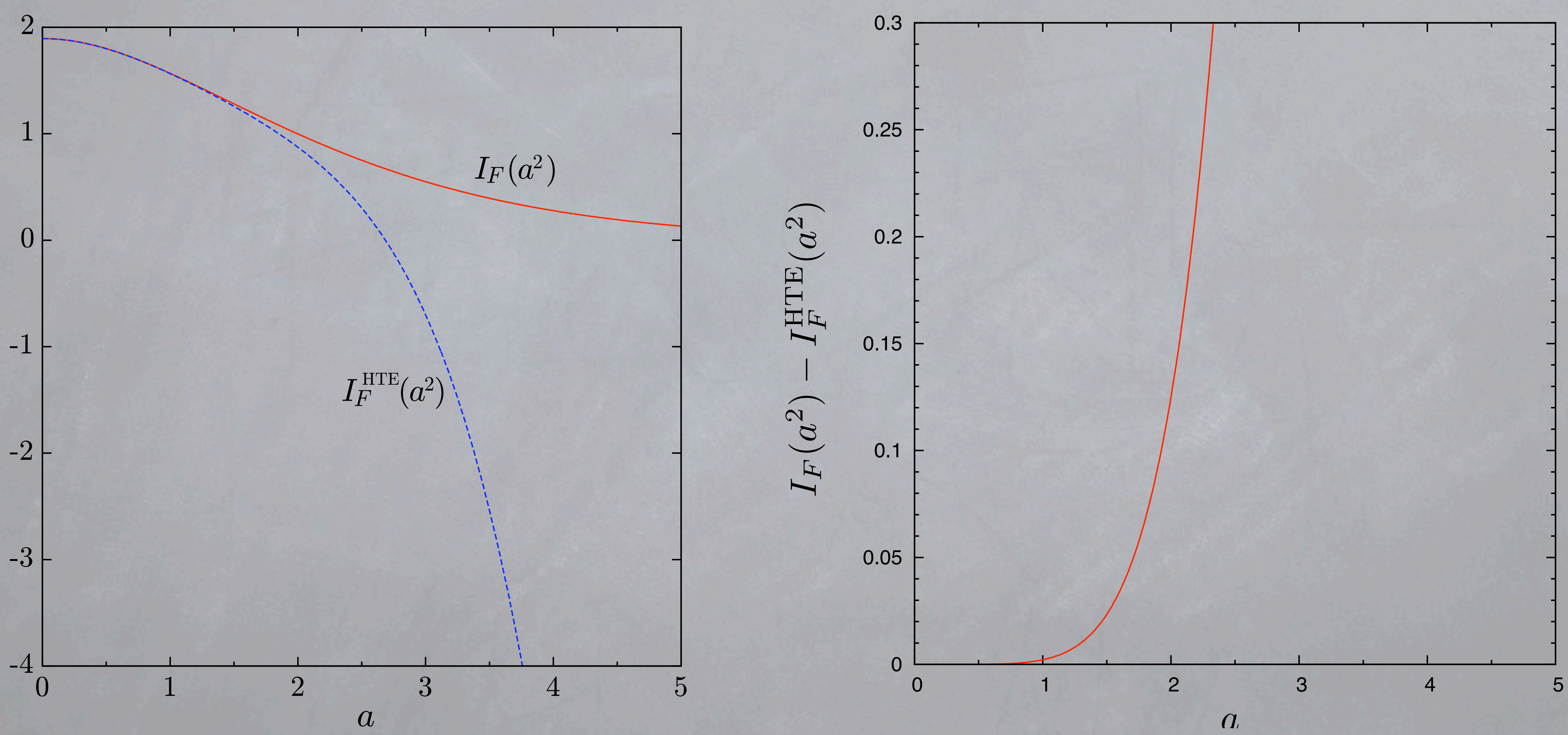




$$|I_B(a^2) - I_B^{\mathrm{HTE}}(a^2)| < 0.05 \text{ for } a \lesssim 2.3$$

Validity of HTE

Fermion case



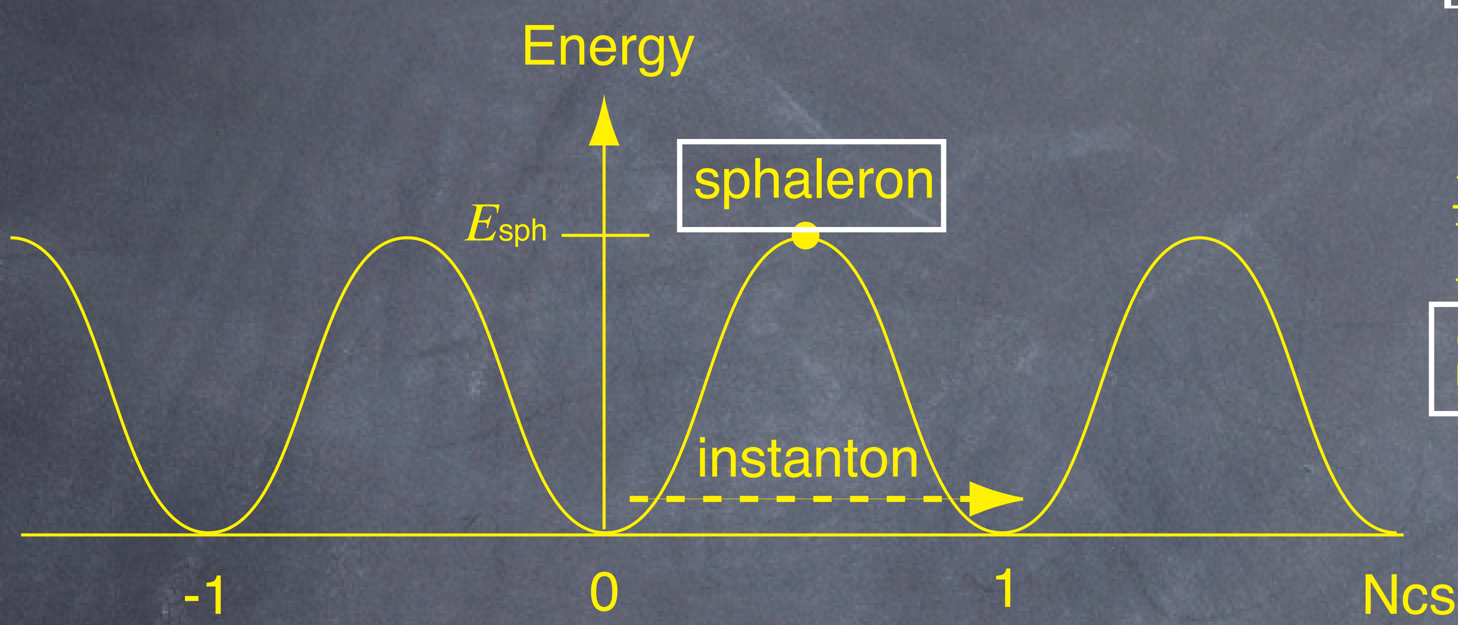
$$|I_F(a^2) - I_F^{\mathrm{HTE}}(a^2)| < 0.05 \text{ for } a \lesssim 1.7$$

Sphaleron

Sphaleron

- A static saddle point solution with finite energy of the gauge-Higgs system.

[N.S. Manton, PRD28 ('83) 2019]



$\Delta B \neq 0$

Instanton: quantum tunneling

Sphaleron: thermal fluctuation

$$\Delta B = 3\Delta N_{CS}$$

$$N_{CS} = \frac{g_2^2}{32\pi^2}\int d^3x\, \epsilon_{ijk}\mathrm{Tr}\left[F_{ij}A_k - \frac{2}{3}g_2 A_i A_j A_k\right]$$

Transition rates:

$$\Gamma^{(b)}_{\rm sph} \simeq (\alpha_W T)^4 e^{-E_{\rm sph}/T}, \quad \text{(broken phase)}$$

$$\Gamma^{(s)}_{\rm sph} \simeq (\alpha_W T)^4, \quad \text{(symmetric phase)}, \quad \alpha_W = g_2^2/4\pi$$

B violation is active at finite T but is suppressed at T=0.

⇒ no proton decay problem

Sphaleron decoupling condition:

To preserve the generated B after the PT,

$$\frac{\Gamma^{(b)}_{\rm sph}}{T_c^3} < H(T_c) \quad \Rightarrow \quad \boxed{\frac{v_c}{T_c} \gtrsim 1.}$$

Hubble parameter:
$H(T) \simeq 1.66\sqrt{g_*}T^2/m_{\rm P}$

Sphaleron solution for SU(2) gauge-Higgs system

Ansatz:

$$\begin{aligned} A_i(\mu, r, \theta, \phi) &= -\frac{i}{g_2} f(r)\partial_i U(\mu, \theta, \phi)U^{-1}(\mu, \theta, \phi), \\ \Phi(\mu, r, \theta, \phi) &= \frac{v}{\sqrt{2}}\left[(1-h(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h(r)U(\mu, \theta, \phi)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]. \end{aligned}$$

Energy functional:

$$E_{\rm sph} = \frac{4\pi v}{g_2}\int_0^\infty d\xi \left[4\left(\frac{df}{d\xi}\right)^2 + \frac{8}{\xi^2}(f-f^2)^2 + \frac{\xi^2}{2}\left(\frac{dh}{d\xi}\right)^2 + h^2(1-f)^2 + \frac{\lambda}{4g_2^2}\xi^2(h^2-1)^2\right].$$

$$\xi = g_2 v r$$

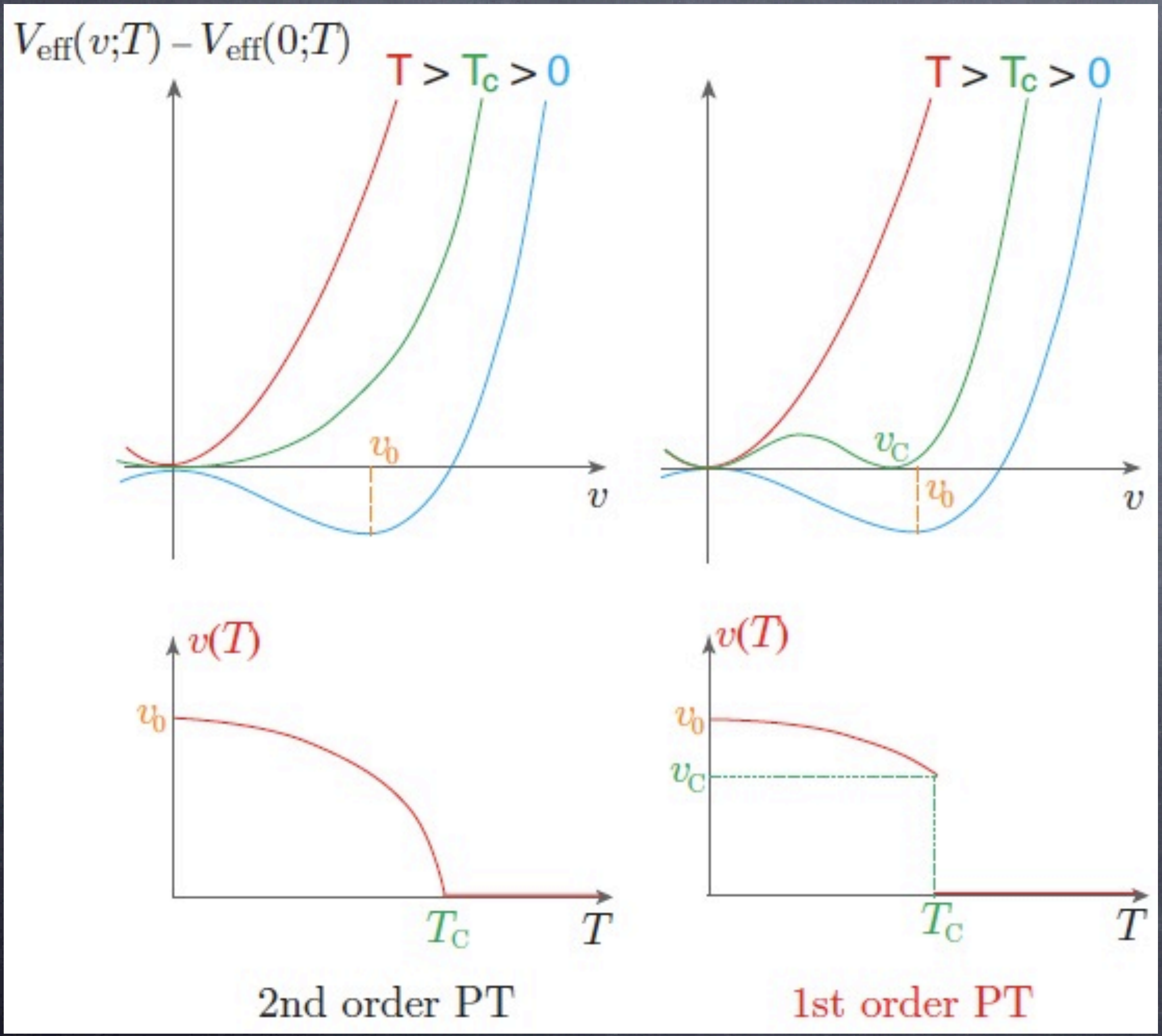
Equation of motion:

$$\begin{aligned} \frac{d^2}{d\xi^2}f(\xi) &= \frac{2}{\xi^2}f(\xi)(1-f(\xi))(1-2f(\xi)) - \frac{1}{4}h^2(\xi)(1-f(\xi)), \\ \frac{d}{d\xi}\left(\xi^2\frac{dh(\xi)}{d\xi}\right) &= 2h(\xi)(1-f(\xi))^2 + \frac{\lambda}{g_2^2}(h^2(\xi)-1)h(\xi). \end{aligned}$$

PT in the SM

Order of the PT

This is what the 1st and 2nd order PTs look like.



- order parameter = Higgs VEV
- EWBG requires 1st order PT

$$v_C \equiv \lim_{T \uparrow T_C} v \neq 0$$

Higgs potential

$$
\begin{aligned}
V_{\rm eff}(\varphi) &= V_0(\varphi) + \Delta V(\varphi) + \Delta V^{\rm c.t.} \\
&= V_0(\varphi) + \Delta_g V(\varphi) + \Delta_t V(\varphi) + \Delta_t V(\varphi, T) + \Delta V^{\rm c.t.},
\end{aligned}
$$

Tree: $V_0(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$

1-loop:

$$
\begin{aligned}
\Delta_g V(\varphi) &= 2\cdot 3F\Big(m_W^2(\varphi)\Big) + 3F\Big(m_Z^2(\varphi)\Big), \\
\Delta_t V(\varphi) &= -4\cdot 3F\Big(m_t^2(\varphi)\Big), \quad F\Big(m^2(\varphi)\Big) = \frac{m^4(\varphi)}{64\pi^2}\left(\ln\frac{m^2(\varphi)}{M_{\rm ren}^2} - \frac{3}{2}\right)
\end{aligned}
$$

1-loop finite T:

$$
\Delta V(\varphi, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z} n_i I_B(a_i^2) + n_t I_F(a_t^2)\right],
$$

Renormalization conditions:

$$
\left.\frac{\partial(\Delta V(\varphi) + \Delta V^{\rm c.t.})}{\partial\varphi}\right|_{\varphi=v_0} = 0, \quad \text{(vacuum)}
$$

$$
\left.\frac{\partial^2(\Delta V(\varphi) + \Delta V^{\rm c.t.})}{\partial\varphi^2}\right|_{\varphi=v_0} = 0, \quad \text{(Higgs mass)}
$$

Higgs potential (cont)

If we use the high T expansion,

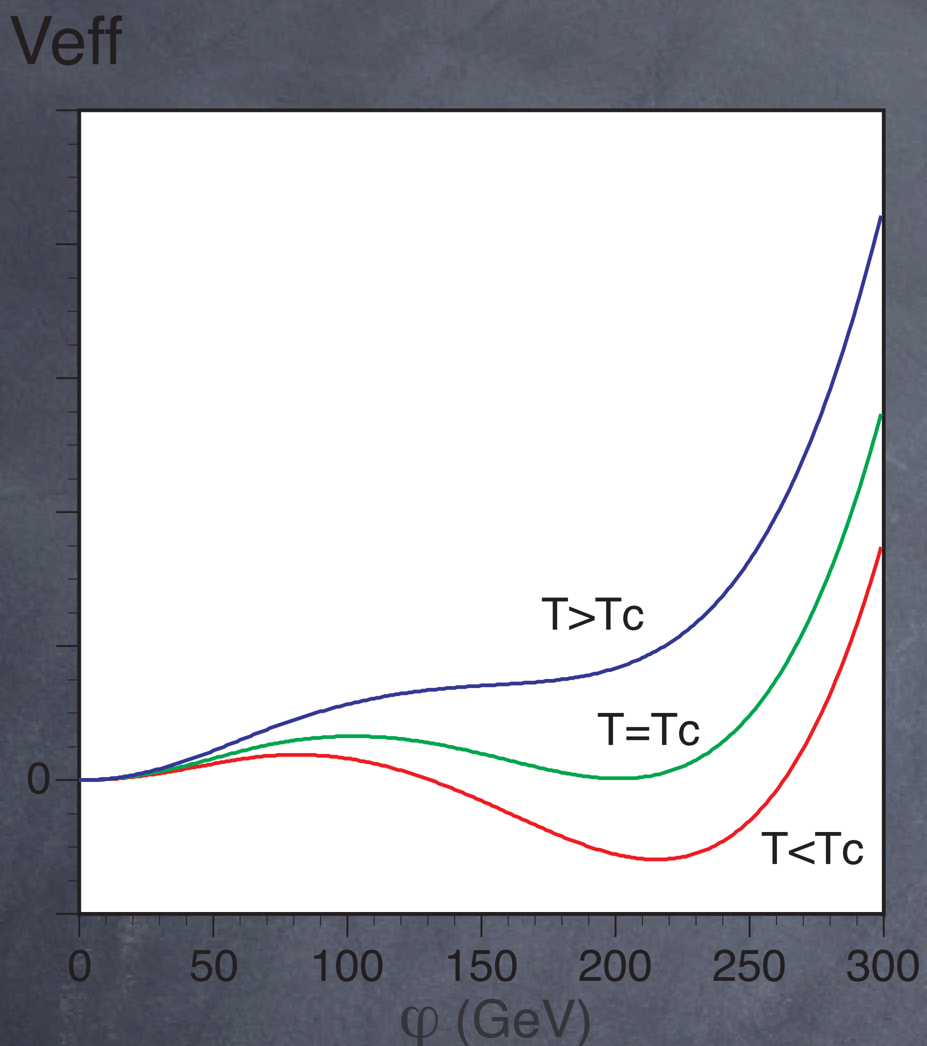
$$V_{\rm eff}(\varphi, T) \simeq D(T^2 - T_0^2)\varphi^2 - ET|\varphi|^3 + \frac{\lambda_T}{4}\varphi^4 + \cdots,$$

$$\begin{aligned}
T_0^2 &= \frac{1}{D}\left(\frac{1}{4}m_h^2 - 2Bv_0^2\right), \\
B &= \frac{3}{64\pi^2 v_0^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right), \\
D &= \frac{1}{8v_0^2}\left(2m_W^2 + m_Z^2 + 2m_t^2\right), \\
E &= \frac{1}{4\pi v_0^3}\left(2m_W^3 + m_Z^3\right) \sim 10^{-2}, \\
\lambda_T &= \frac{m_h^2}{2v_0^2}\left[1 - \frac{3}{8\pi^2 v_0^2 m_h^2}\left\{2m_W^4 \log\frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log\frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log\frac{m_t^2}{\alpha_F T^2}\right\}\right],
\end{aligned}$$

The critical temperature T_c is given by

$$T_c^2 = \frac{T_0^2}{1 - E^2/(\lambda_{T_c} D)}.$$

At T_c



The potential has two degenerate minima at

$$\varphi = 0, \quad \varphi_c = \frac{2ET_c}{\lambda_{T_c}}.$$

Sphaleron decoupling condition:

$$\frac{\varphi_c}{T_c} = \frac{2E}{\lambda_{T_c}} \gtrsim 1$$

Since $\lambda_{T_c} \sim m_h^2/2v_0^2$,

$$\mathrm{m}_h \lesssim 48 \text{ GeV}.$$

This upper bound has been excluded by the LEP data.

[N.B.] Higgs mass (λ) $\nearrow$ strength of the PT $\searrow$

$E \nearrow$ strength of the PT $\nearrow$

What is the minimally required value of E for m_h=114.4 GeV?

Minimal value of E

From the sphaleron decoupling condition, $\dfrac{\varphi_c}{T_c} = \dfrac{2E}{\lambda_{T_c}} > 1$

$$E_{\rm min} > \frac{m_h^2}{4v_0^2} \simeq 0.054, \quad \text{for } m_h = 114.4 \text{ GeV}$$

SM contributions:

$$E_{\rm SM} = \frac{1}{4\pi v_0^3}\left(2m_W^3 + m_Z^3\right) \simeq 0.01$$

Note: The origin of E is the zero frequency modes of the bosonic loops.

To have a strong 1st order PT, the extra bosonic degrees of freedom are needed.

Caveat

"Bosons do not always play a role."

Suppose that the bosonic particle whose mass is given by

$$M^2 = m^2 + g^2\varphi^2, \quad m^2 : \text{gauge invariant mass}$$
$$g : \text{coupling constant}$$

For $m^2 \ll g^2\varphi^2$

$$V_{\text{eff}} \ni -g^3\varphi^3 T \quad \Rightarrow \quad \text{strengthen the 1st order PT}$$

The loop effect is large.

For $m^2 \gg g^2\varphi^2$ No $(-g^3\varphi^3 T)$ term in V_{eff}

The loop effect is vanishing.

Requirements: 1. large coupling g, 2. small m^2.

PT in the MSSM

Stop loop effect

[Carena, Quiros, Wagner, PLB380 ('96) 81]

The LEP bounds on m_H and ρ-parameter constraints demand

$$m_{\tilde{q}}^2 \gg m_{\tilde{t}_R}^2, X_t^2, \quad X_t = A_t - \mu/\tan\beta.$$

- Stop masses:

$$\bar{m}_{\tilde{t}_1}^2 = m_{\tilde{t}_R}^2 + D_{\tilde{t}_R}^2 + \frac{y_t^2 \sin^2\beta}{2}\left(1 - \frac{|X_t|^2}{m_{\tilde{q}}^2}\right) v^2,$$

$$\bar{m}_{\tilde{t}_2}^2 = m_{\tilde{q}}^2 + D_{\tilde{t}_L}^2 + \frac{y_t^2 \sin^2\beta}{2}\left(1 + \frac{|X_t|^2}{m_{\tilde{q}}^2}\right) v^2 \simeq m_{\tilde{q}}^2\,,$$

soft SUSY breaking masses: $m_{\tilde{q}}^2, m_{\tilde{t}_R}^2, \quad D_{\tilde{t}_{L,R}}^2 \sim \mathcal{O}(g^2)$

To have a large loop effect, $m_{\tilde{t}_R}^2$ should be small.

$m_{\tilde{t}_R}^2 = 0$ gives $\boxed{m_{\tilde{t}_1} < m_t}$

Furthermore

$X_t = 0$ (no-mixing) maximizes the loop effect

Stop loop effect (cont)

Using the High T expansion,

- Coefficient of cubic term in $V_{eff}(T)$

$$V_{\text{eff}} \ni -(E_{\text{SM}} + E_{\tilde{t}_1})Tv^3 \qquad E_{\tilde{t}_1} \simeq +\frac{y_t^3 \sin^3\beta}{4\sqrt{2}\pi}\left(1 - \frac{|X_t|^2}{m_{\tilde{q}}^2}\right)^{3/2}.$$

$$E_{\tilde{t}_1} \simeq 0.054, \text{ for } X_t = 0$$

Therefore $E = (E_{\text{SM}} + E_{\tilde{t}_1}) > E_{\min} = 0.054$

Such a light stop can play a role in strengthening the 1st order PT.

As I wrote before, to have a strong 1st order PT

Requirements: 1. large coupling g, 2. small m^2.

$\rightarrow$ top Yukawa y_t $\qquad \rightarrow m^2_{\tilde{t}_R} = 0$

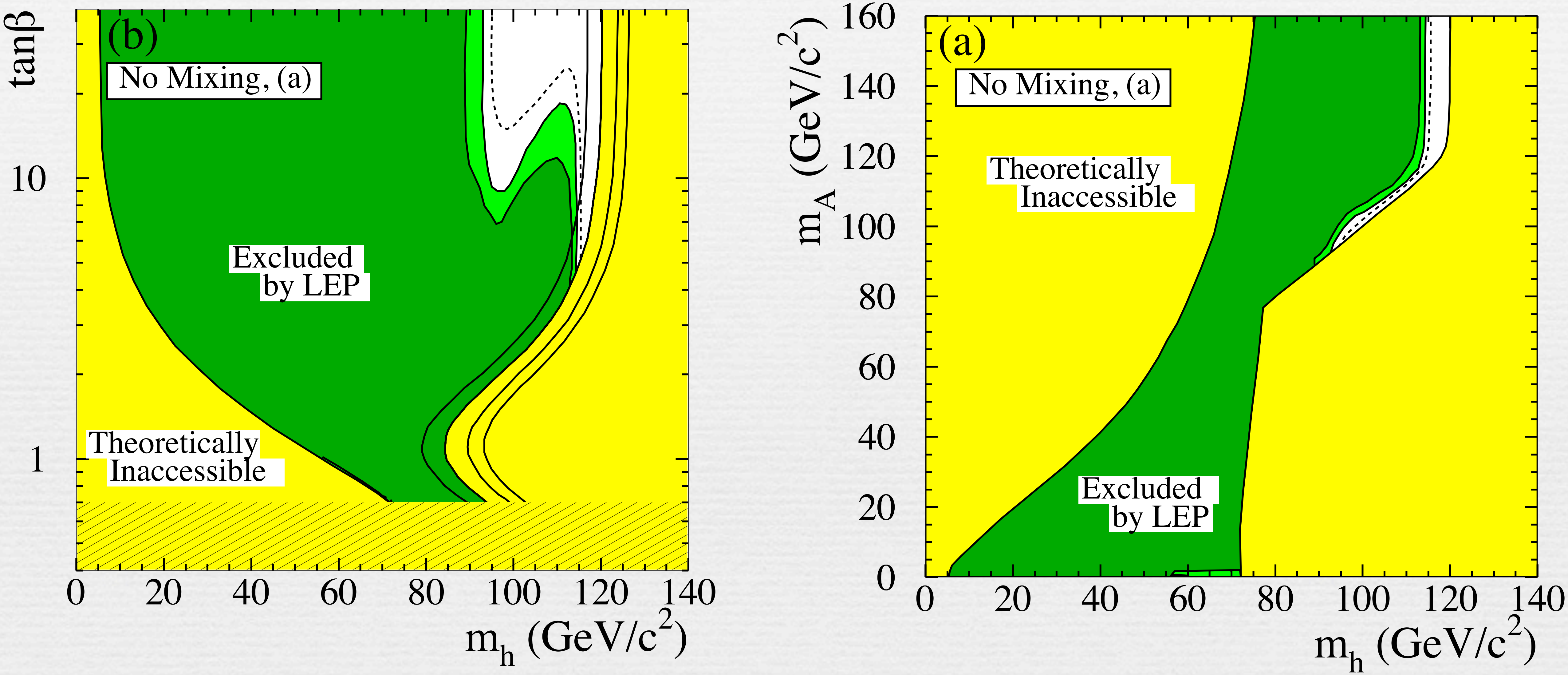
and large statistical factor $n_{\tilde{t}} = N_C \times 2 = 6$

Updated analysis

[K. Funakubo (Saga U.), E.S.]

Tension in the MSSM BG

- The LEP data put a strong constrains on the light Higgs boson.



- There is a tension between the LEP data and the sphaleron decoupling in the MSSM.

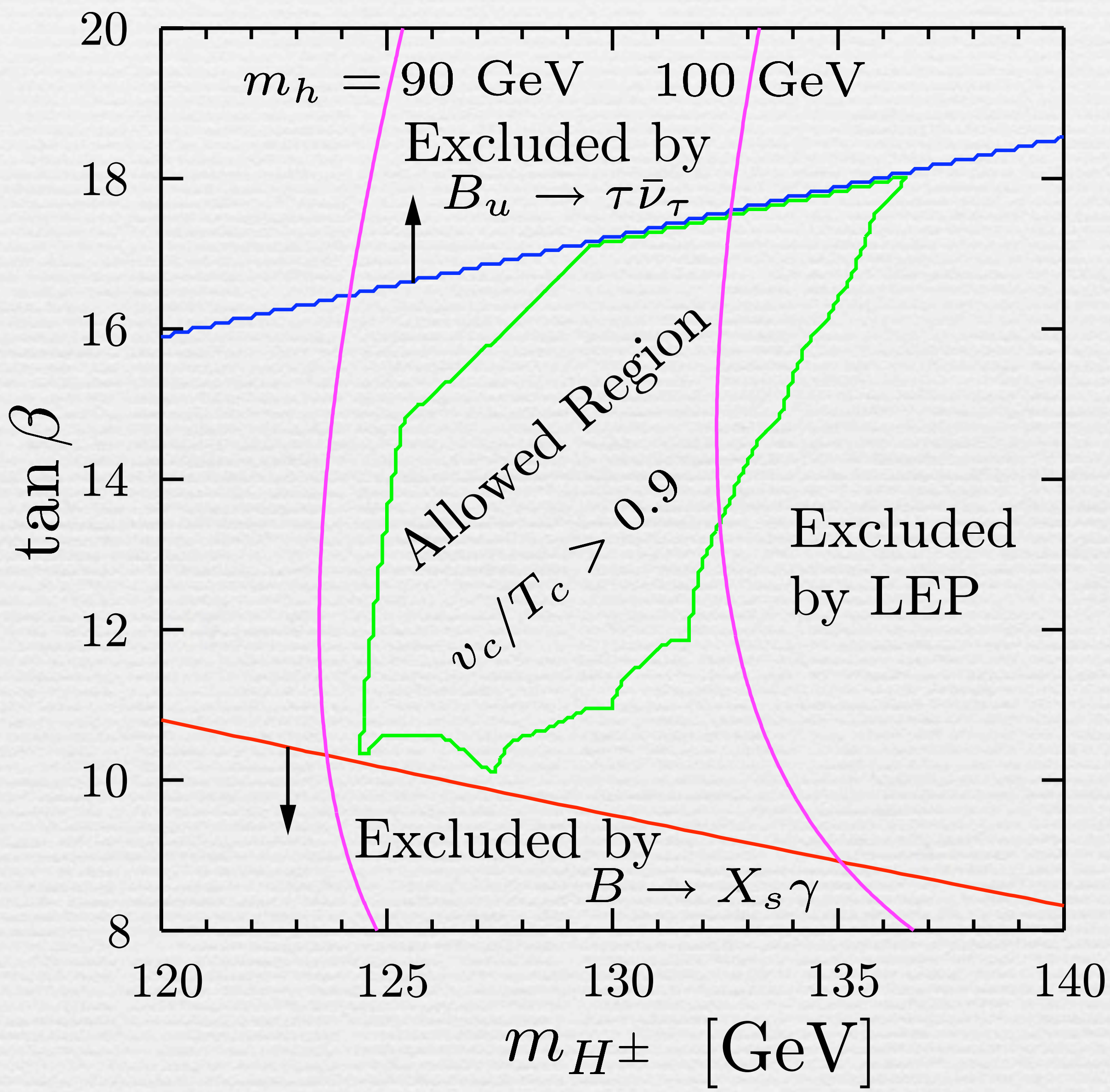
↓

More precise analysis of the sphaleron decoupling is needed.

Allowed region

- The allowed region is highly constrained by the experimental data.

$m_{\tilde{q}} = 1200$ GeV, $m_{\tilde{t}_R} \simeq 0$, $A_t = A_b = -300$ GeV.



Maximal v/T:

$\tan\beta = 10.1$, $m_{H^\pm} = 127.4$ GeV

$$\frac{v_C}{T_C} = \frac{107.10 \text{ GeV}}{116.27 \text{ GeV}} = 0.92$$

↓

The sphaleron process is not decoupled at Tc.

Loophole: supercooling

⇒ The PT begins to proceed with bubble wall at below Tc.

We need to know the dynamics of bubble wall.

Critical bubble

critical bubble = static solution which is unstable against variation of radius.

Higgs fields: $$\Phi_d = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_d \\ 0 \end{pmatrix}, \quad \Phi_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_u \end{pmatrix},$$

Energy functional:

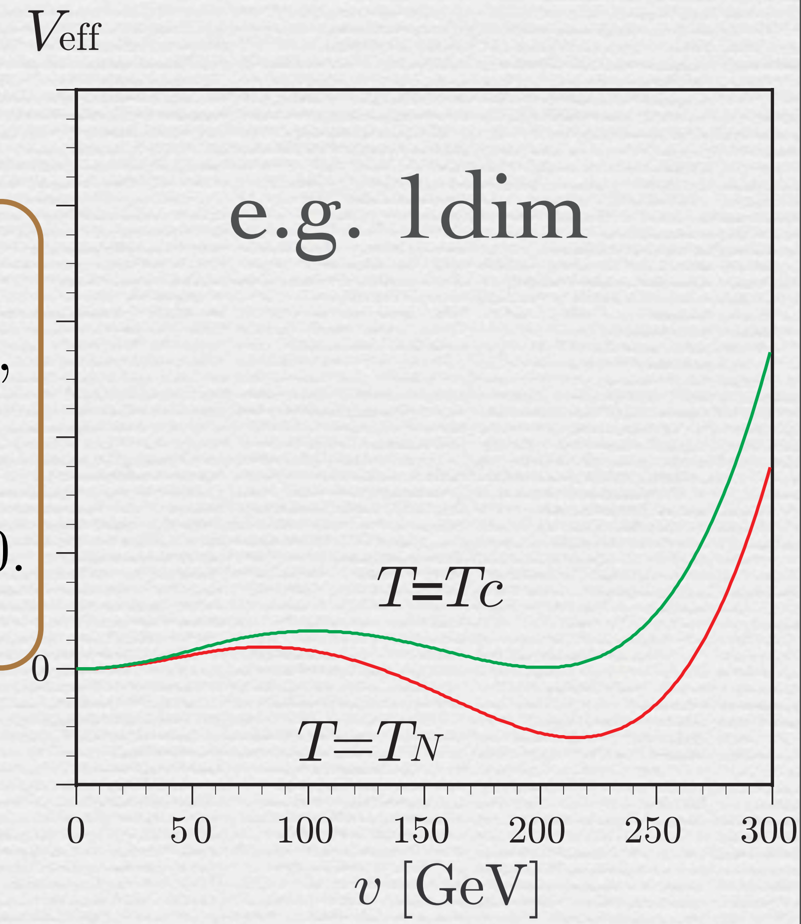
$$E = 4\pi \int_0^\infty dr\ r^2 \left[\frac{1}{2}\left\{\left(\frac{d\rho_d}{dr}\right)^2 + \left(\frac{d\rho_u}{dr}\right)^2\right\} + V_{\rm eff}(\rho_d, \rho_u; T)\right] \qquad r = \sqrt{\boldsymbol{x}}$$

Equation of motion:

b.c.

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_d}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial \rho_d} = 0, \qquad \lim_{r\to\infty}\rho_d(r) = 0, \quad \lim_{r\to\infty}\rho_u(r) = 0,$$

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\rho_u}{dr}\right) + \frac{\partial V_{\rm eff}}{\partial \rho_u} = 0. \qquad \left.\frac{d\rho_d(r)}{dr}\right|_{r=0} = 0, \quad \left.\frac{d\rho_u(r)}{dr}\right|_{r=0} = 0.$$



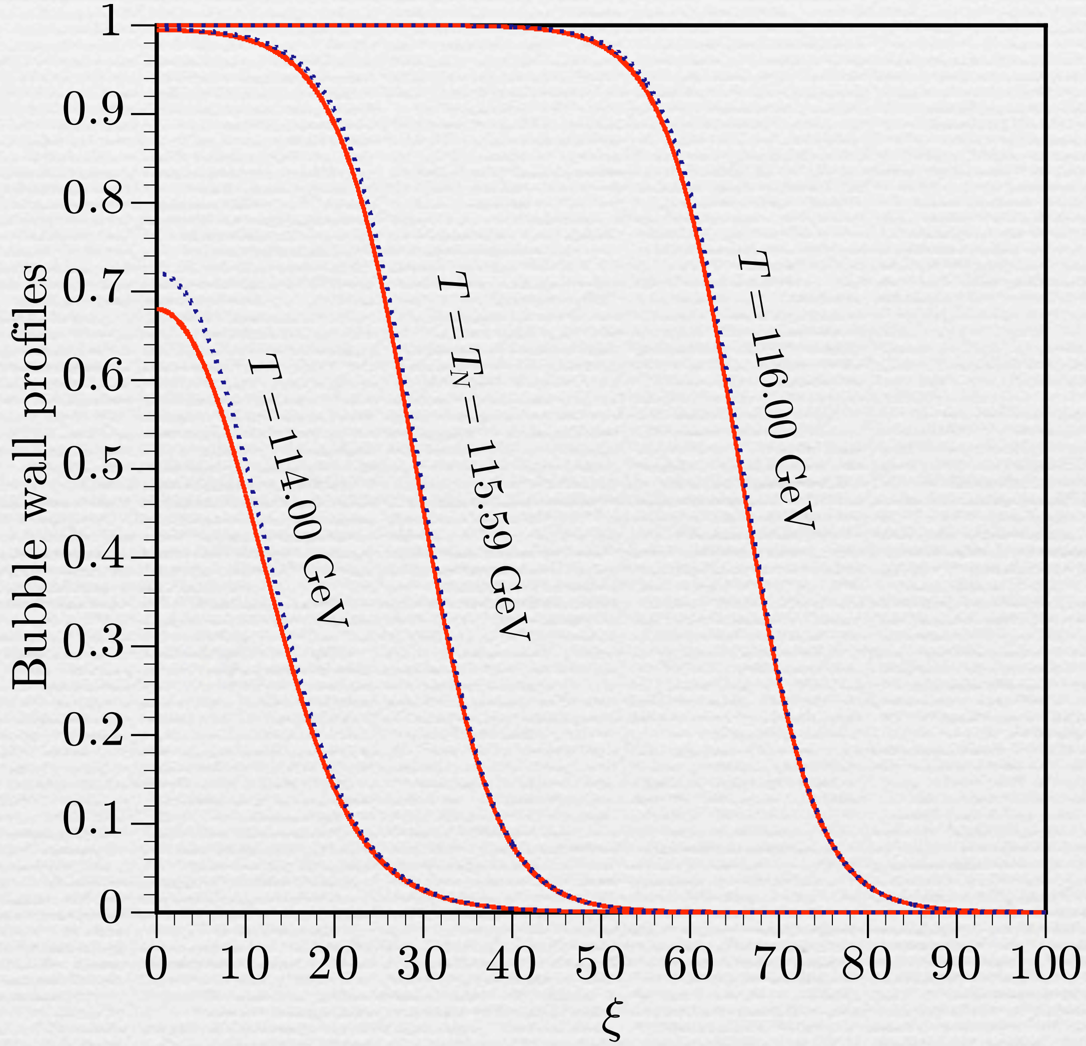
Bubbles can be nucleated at below Tc.

Bubble nucleation

Nucleation rate: $\Gamma_N(T) \simeq T^4 \left(\frac{E_{\rm cb}(T)}{2\pi T}\right)^{3/2} e^{-E_{\rm cb}(T)/T}$

[A.D. Linde, NPB216 ('82) 421]

Nucleation T: $\Gamma_N(T_N)H(T_N)^{-3} = H(T_N)$



$\xi = vr,\ h_1(\xi) = \frac{\rho_d(r)}{v\cos\beta},\ h_2(\xi) = \frac{\rho_u(r)}{v\sin\beta}$

Numerical results: (preliminary)

$$\frac{v_N}{T_N} = \frac{116.73}{115.59} = 1.01$$

10% enhancement! But,

Sphaleron decoupling cond.@T_N:

$\mathcal{E} = 1.77,\ \mathcal{N}_{\rm tr} = 6.65,\ \mathcal{N}_{\rm rot} = 12.27$

$$\frac{v}{T} > 1.35$$

- The sphaleron process is not decoupled at TN either.

More examples

$A_b = A_t = -300$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$\mu = 100$ GeV, $M_2 = 500$ GeV,

$m_{\tilde{q}}$ (GeV)	1200	1300	1400	1500
$\tan\beta$	10.11	9.87	9.75	9.57
$m_{H^\pm}$ (GeV)	127.30	127.40	127.40	127.40
v_C/T_C	$\frac{107.095}{116.274} = 0.921$	$\frac{107.512}{116.496} = 0.923$	$\frac{107.768}{116.770} = 0.923$	$\frac{107.914}{117.045} = 0.922$
$\tan\beta_C$	13.812	13.640	13.606	13.465
v_N/T_N	$\frac{116.726}{115.585} = 1.010$	$\frac{117.155}{115.798} = 1.012$	$\frac{117.403}{116.068} = 1.012$	$\frac{117.530}{116.340} = 1.010$
$\tan\beta_N$	13.684	13.503	13.462	13.317
$E_{\rm cb}/(4\pi v_0)$	5.623	5.633	5.646	5.659
$E_{\rm cb}/T_N$	150.386	150.379	150.369	150.360
$E_{\rm sph}/(4\pi v_0/g_2)$	1.7686	1.7695	1.7704	1.7711
$\mathcal{N}_{\rm tr}$	6.6522	6.6576	6.6623	6.6666
$\mathcal{N}_{\rm rot}$	12.266	12.253	12.241	12.230
$v_N/T_N >$	1.345	1.344	1.344	1.343

Typically, $v_N/T_N > 1.34$ is needed for sphaleron decoupling.

More examples

$A_b = A_t = -300$ GeV, $m_{\tilde{t}_R} = 10^{-4}$ GeV, $m_{\tilde{b}_R} = 1000$ GeV,
$\mu = 100$ GeV, $M_2 = 500$ GeV,

$m_{\tilde{q}}$ (GeV)	1200	1300	1400	1500
$\tan\beta$	10.11	9.87	9.75	9.57
$m_{H^\pm}$ (GeV)	127.30	127.40	127.40	127.40
v_C/T_C	$\frac{107.095}{116.274} = 0.921$	$\frac{107.512}{116.496} = 0.923$	$\frac{107.768}{116.770} = 0.923$	$\frac{107.914}{117.045} = 0.922$
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Typically, $v_N/T_N > 1.34$ is needed for sphaleron decoupling.

Is MSSM BG dead?

It looks almost dead, but there might be a way out.

- T_N ⇒ onset of the PT. We should know a temperature at which the PT ends. The sphaleron decoupling condition should be imposed at such a temperature.

- Higher order (2-loop) contributions must be taken into account. [J.R. Espinosa, NPB475, ('06) 273]

⇒ The sphaleron decoupling cond. might be relaxed.

- The potential can be extended in such a way that stop also has a nontrivial VEV. (Color-Charge-Breaking vacuum)
  ⇒ MSSM BG is viable. [Canena et al, NPB812,('09) 243]
  [N.B.] EW vacuum: metastable, CCB vacuum: global minimum

If the refined sphaleron decoupling cond. is used, is it still viable?

Summary

- We have studied EW bayogenesis focusing on the EWPT.
- In the SM, the PT is not 1st order for the viable Higgs mass. $m_H > 114.4\ \mathrm{GeV}$
- In the MSSM, the 1st order PT can be strengthen by the light stop. $m_{\tilde{t}_1} < m_t$
- However, it is found that the sphaleron process is not decoupled at both T_c and T_N.
- More refined analysis is needed to reach a convincing conclusion for successful EWBG in the MSSM.

Review papers

- A.G. Cohen, D.B. Kaplan, A.E. Nelson, hep-ph/9302210
- M. Quiros, Helv.Phys.Acta 67 ('94)
- V.A. Rubakov, M.E. Shaposhnikov, hep-ph/9603208
- K. Funakubo, hep-ph/9608358
- M. Trodden, hep-ph/9803479
- A. Riotto, hep-ph/9807454
- W. Bernreuther, hep-ph/0205279