

# Cosmic Inflation in a nutshell

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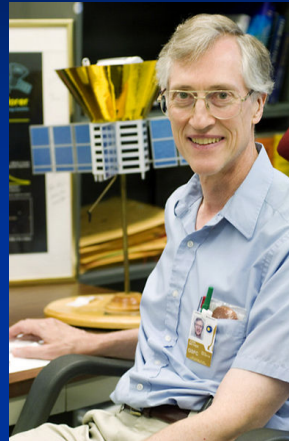
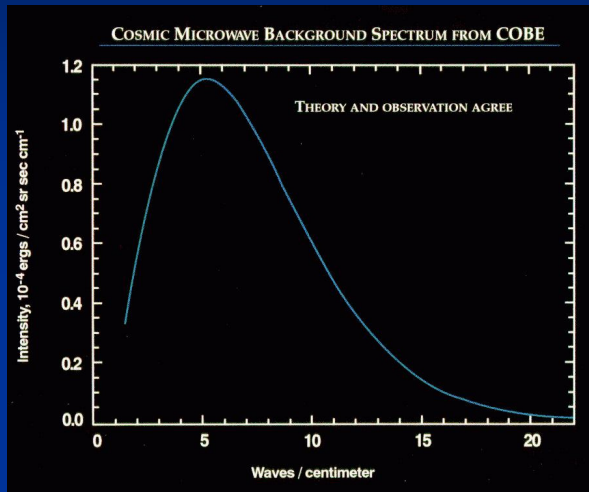
Seminar at NTHU on 05/13



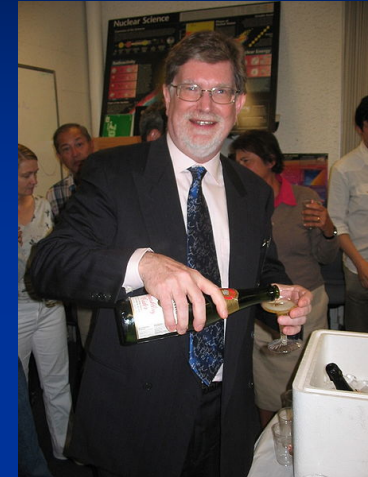
# Purpose of this talk

- What is cosmic inflation? What are people doing when they work on inflation?
- What are the important observables in the frontier of cosmology?
- Some of my works.

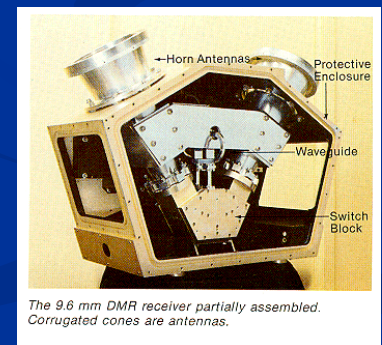
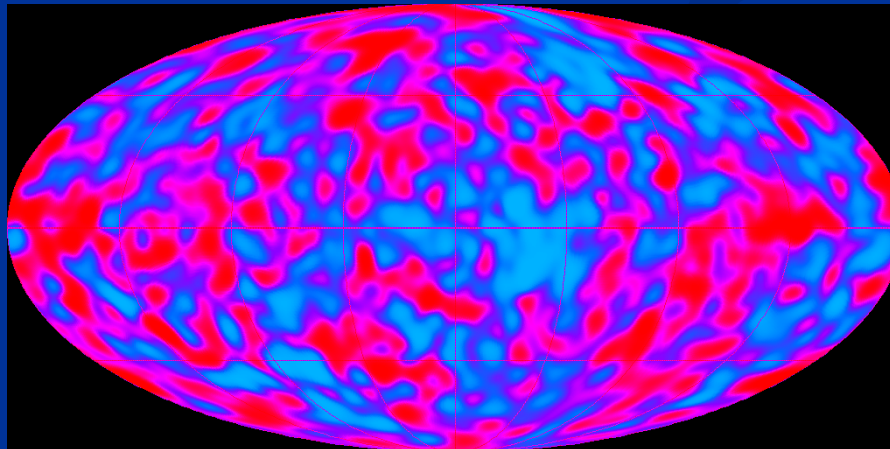
# Nobel prize in Physics 2006



John Mather



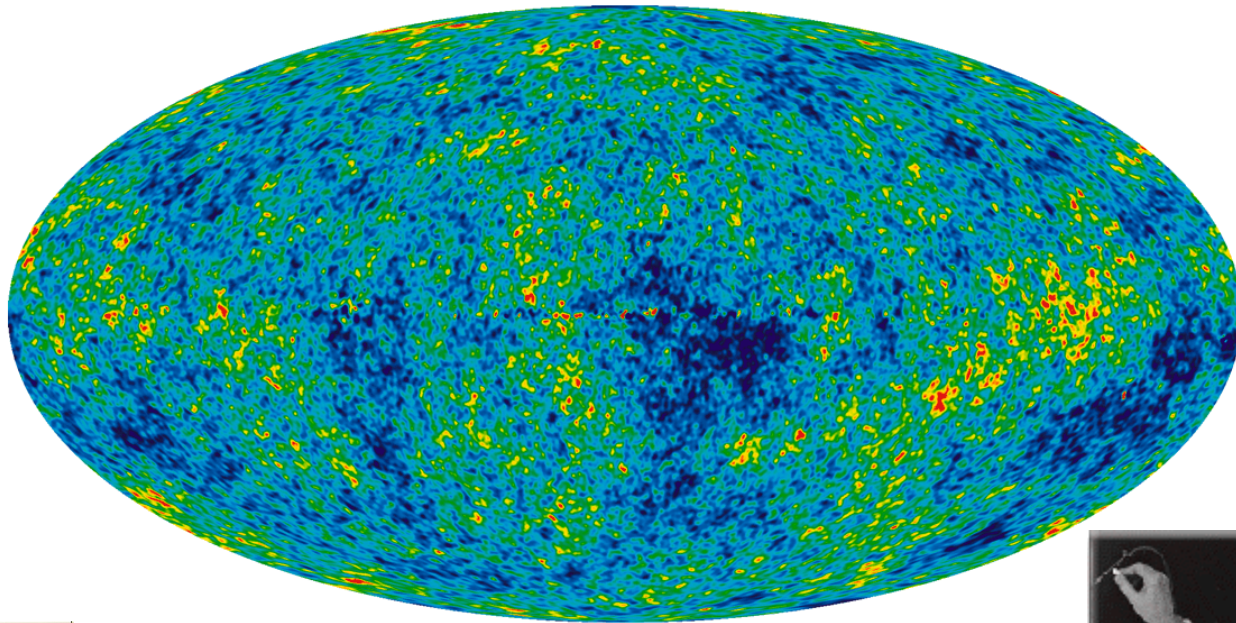
George Smoot



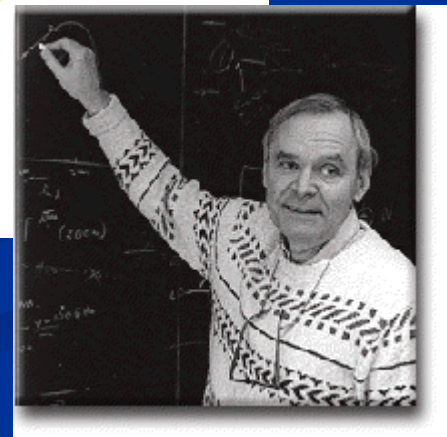
COBE FIRAS

COBE DMR

# The CMB (WMAP 7-year)

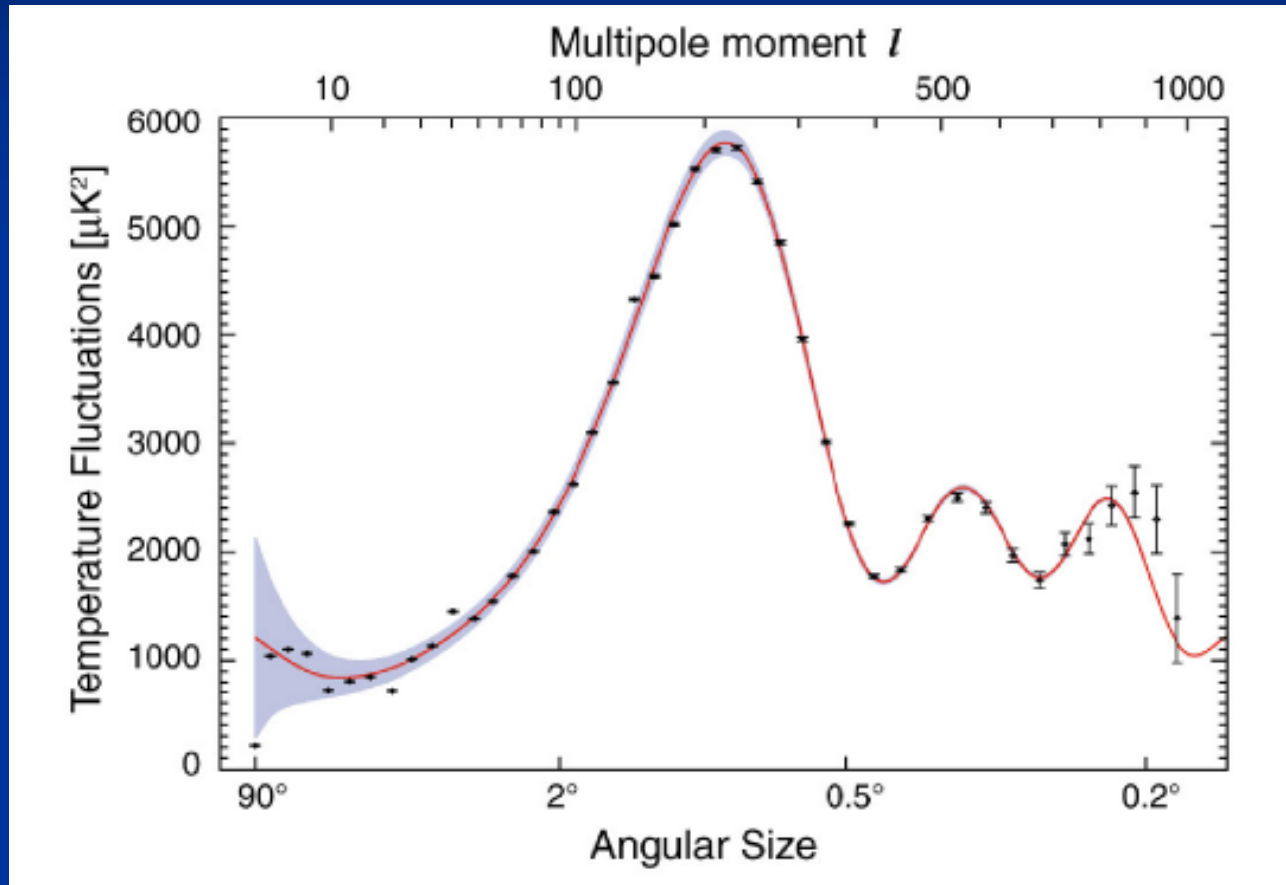


TV noise

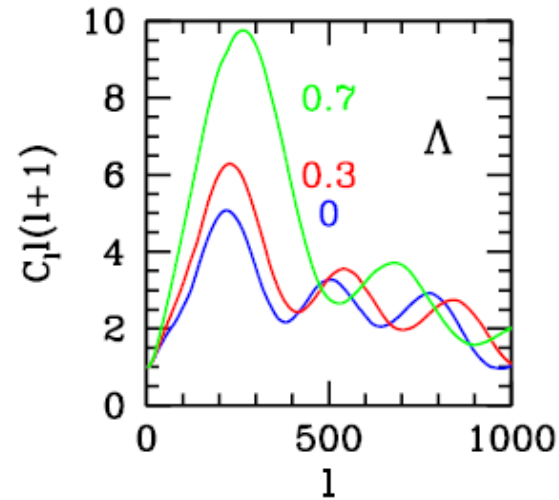
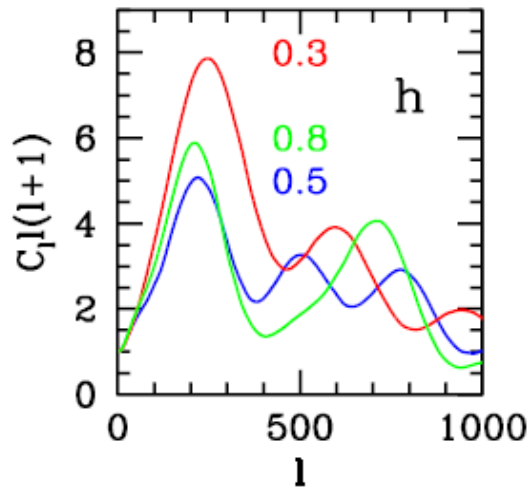
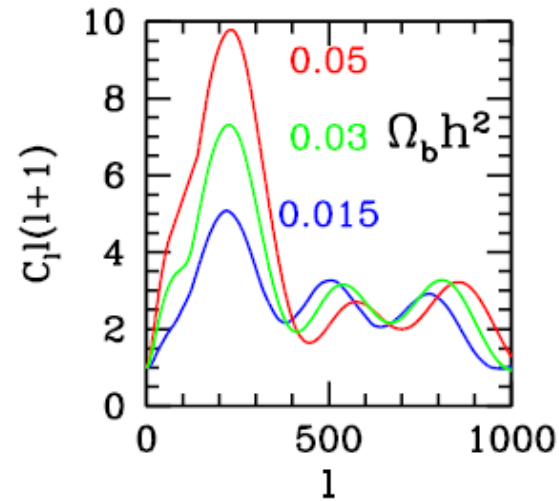
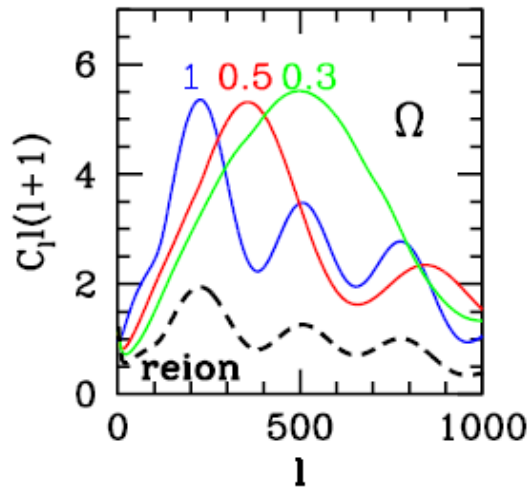


D. Wilkinson

# CMB Anisotropy (“see the sound”)







Jungman G, Kamionkowski M, Kosowsky A, Spergel DN (1996)

# Cosmology

## Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],$$

## The Einstein Equations

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu},$$

## The Friedmann Equations

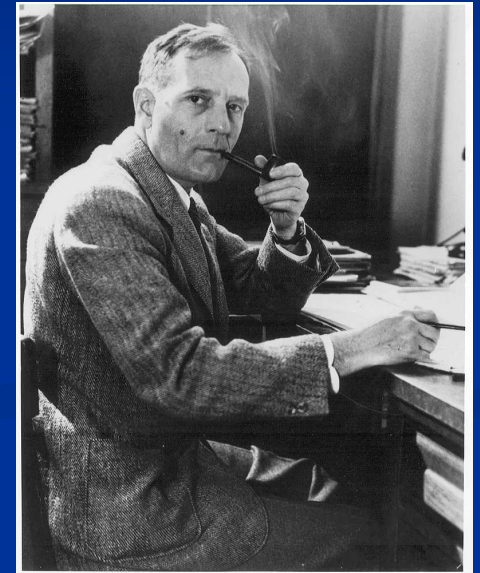
$$H^2 = \frac{8\pi G_N}{3} \rho - \frac{K}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} (\rho + 3P),$$

$$H = \dot{a}/a$$

Hubble parameter

$$\begin{aligned} \text{Radiation : } & a \propto t^{1/2}, \quad \rho \propto a^{-4}, \\ \text{Dust : } & a \propto t^{2/3}, \quad \rho \propto a^{-3}. \end{aligned}$$



E. Hubble

# Cosmic Inflation



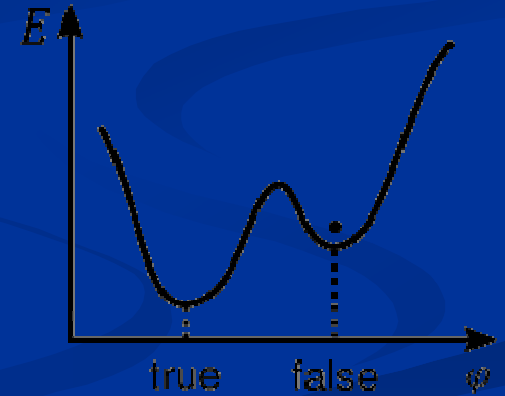
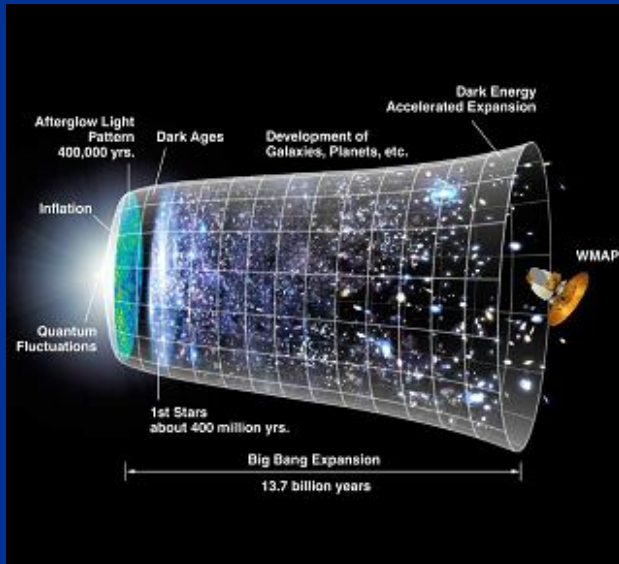
Alan Guth (1981)



CML and K. Sato



CML and A. Starobinsky





# Why do we need inflation?

- Flatness problem, horizon problem, unwanted relics problem..... in old big bang theory.
- Inflation can provide primordial density perturbations in the early Universe which are the seeds for the Large-Scale Structure (LSS) in the distribution of galaxies and the dark matter and for the Cosmic Microwave Background (CMB) temperature anisotropies.

“...the standard big bang theory says nothing about what banged, why it banged, or what happened before it banged. The inflationary universe is a theory of the “bang” of the big bang.” – Alan Guth (1997).

# Old Inflation

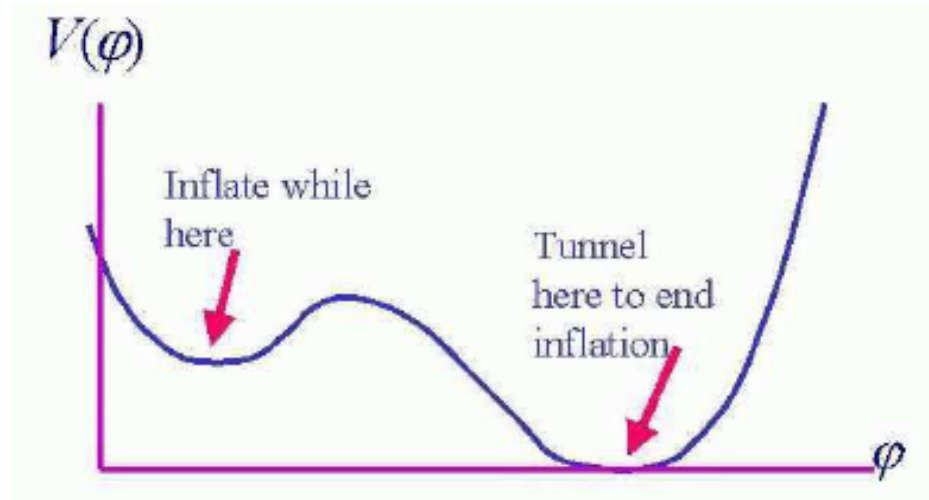


Figure 5: In the original version of inflation proposed by Guth, the potential dominated state was achieved in a local minimum of the inflaton potential. Long periods of inflation were easily produced because the timescale for inflation was set by a tunneling process. However, these models had a “graceful exit problem” because the tunneling process produced a very inhomogeneous energy distribution.

Picture from [Astro-ph/0007247](https://arxiv.org/abs/astro-ph/0007247)

# New Inflation

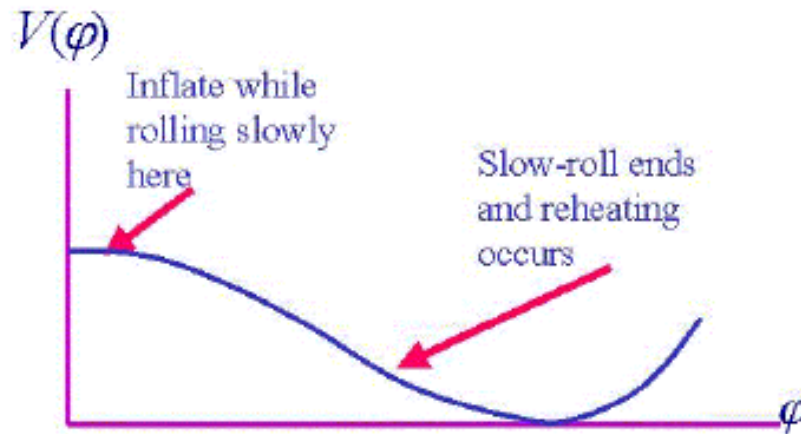


Figure 6: In “new” inflation a classical instability ends inflation. This allows for a graceful exit, but makes it more of a challenge to get sufficiently long inflation times.

# Basic Equations

## Scalar field in FRW

$$S = \int d^4x \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right],$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2 \varphi}{a^2} + V'(\varphi) = 0,$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0.$$

(homogeneous)

$$T_{\mu\nu} = -2 \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} + g_{\mu\nu} \mathcal{L} = \partial_\mu \varphi \partial_\nu \varphi + g_{\mu\nu} \left[ -\frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi) \right].$$

$$\rho_\varphi = \frac{\dot{\varphi}^2}{2} + V(\varphi)$$

$$P_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi).$$

# Basic Equations

## Slow roll parameters

$$\epsilon(\phi) \equiv \frac{M_{\text{P}}^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1,$$
$$|\eta(\phi)| \equiv M_{\text{P}}^2 \left| \frac{V''}{V} \right| \ll 1.$$

## Number of e-folds

$$N \equiv \ln \frac{a(t_{\text{end}})}{a(t)} = \int_t^{t_{\text{end}}} H dt \approx \frac{1}{M_{\text{P}}^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi,$$



# From small to large and from quantum to classical

$$|\delta\varphi(x)| \sim \frac{H(\varphi)}{2\pi}.$$

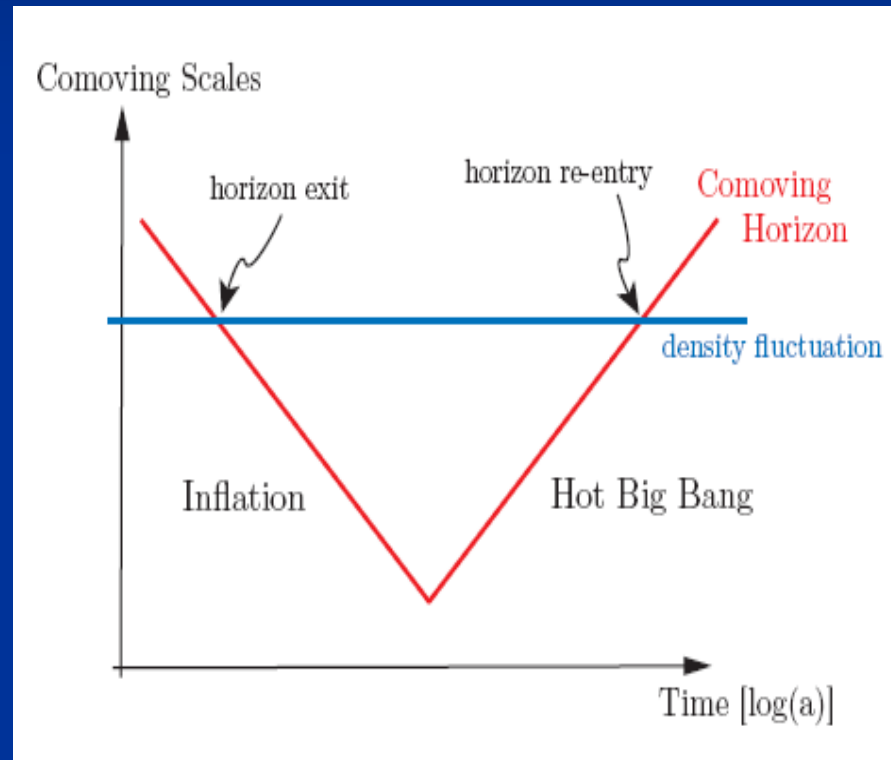
This can be compared with  
Hawking temperature:

$$T_H = \frac{H}{2\pi}$$



CML and S. Hawking

$$N=50\sim 60$$



# Cosmological perturbation and the spectrum

$$\begin{aligned} ds^2 &= a^2(\tau)[-(1 + 2\Phi)d\tau^2 + (1 - 2\Psi)\gamma_{ij}dx^i dx^j] \\ &= -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j. \end{aligned}$$

Longitudinal gauge

$$\mathcal{R} = \Phi - \frac{H^2}{\dot{H}} \left( \Phi + \frac{\dot{\Phi}}{H} \right)$$

Comoving curvature perturbation (in the long-wavelength limit)

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2} \langle |\mathcal{R}|^2 \rangle = \frac{k^3}{2\pi^2} \frac{H^2}{\dot{\phi}_0^2} |\delta\phi_k|^2.$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{24\pi^2 M_{\text{P}}^4} \frac{V}{\epsilon}.$$

$$\mathcal{P}_{\mathcal{R}}^{1/2} \simeq 5 \times 10^{-5},$$

**CMB normalization**

# The spectral index

$$n_S - 1 \equiv \frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k} = 2\eta - 6\epsilon,$$

1001.4538

## SEVEN-YEAR WILKINSON MICROWAVE ANISOTROPY PROBE (WMAP<sup>1</sup>) OBSERVATIONS: COSMOLOGICAL INTERPRETATION

E. KOMATSU<sup>2</sup>, K. M. SMITH<sup>3</sup>, J. DUNKLEY<sup>4</sup>, C. L. BENNETT<sup>5</sup>, B. GOLD<sup>5</sup>, G. HINSHAW<sup>6</sup>, N. JAROSIK<sup>7</sup>, D. LARSON<sup>5</sup>, M. R. NOLTA<sup>8</sup>, L. PAGE<sup>7</sup>, D. N. SPERGEL<sup>3,9</sup>, M. HALPERN<sup>10</sup>, R. S. HILL<sup>11</sup>, A. KOGUT<sup>6</sup>, M. LIMON<sup>12</sup>, S. S. MEYER<sup>13</sup>, N. ODEGARD<sup>6</sup>, G. S. TUCKER<sup>14</sup>, J. L. WEILAND<sup>11</sup>, E. WOLLACK<sup>6</sup>, AND E. L. WRIGHT<sup>15</sup>

*Submitted to the Astrophysical Journal Supplement Series*

### ABSTRACT

The combination of 7-year data from *WMAP* and improved astrophysical data rigorously tests the standard cosmological model and places new constraints on its basic parameters and extensions. By combining the *WMAP* data with the latest distance measurements from the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies (Percival et al. 2009) and the Hubble constant ( $H_0$ ) measurement (Riess et al. 2009), we determine the parameters of the simplest 6-parameter  $\Lambda$ CDM model. The power-law index of the primordial power spectrum is  $n_s = 0.963 \pm 0.012$  (68% CL) for this data combination, a measurement that excludes the Harrison-Zeldovich-Peebles spectrum by more than  $3\sigma$ . The other parameters, including those beyond the minimal set, are also consistent with, and improved from, the 5-year results. We find no convincing deviations from the minimal model. The 7-

# Tensor to scalar ratio (Primordial Gravity Waves)

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{2M_{\text{P}}^2\epsilon} \left(\frac{H_*}{2\pi}\right)^2 \left(\frac{k}{aH_*}\right)^{n_{\mathcal{R}}-1},$$

$$\mathcal{P}_{\mathcal{T}}(k) = \frac{k^3}{2\pi^2} \langle h_{ij}^* h^{ij} \rangle = \frac{8}{M_{\text{P}}^2} \left(\frac{H_*}{2\pi}\right)^2 \left(\frac{k}{aH_*}\right)^{n_{\mathcal{T}}},$$

$$r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon.$$

The **holy grail** of cosmology obserbation and smoking gun of inflation!

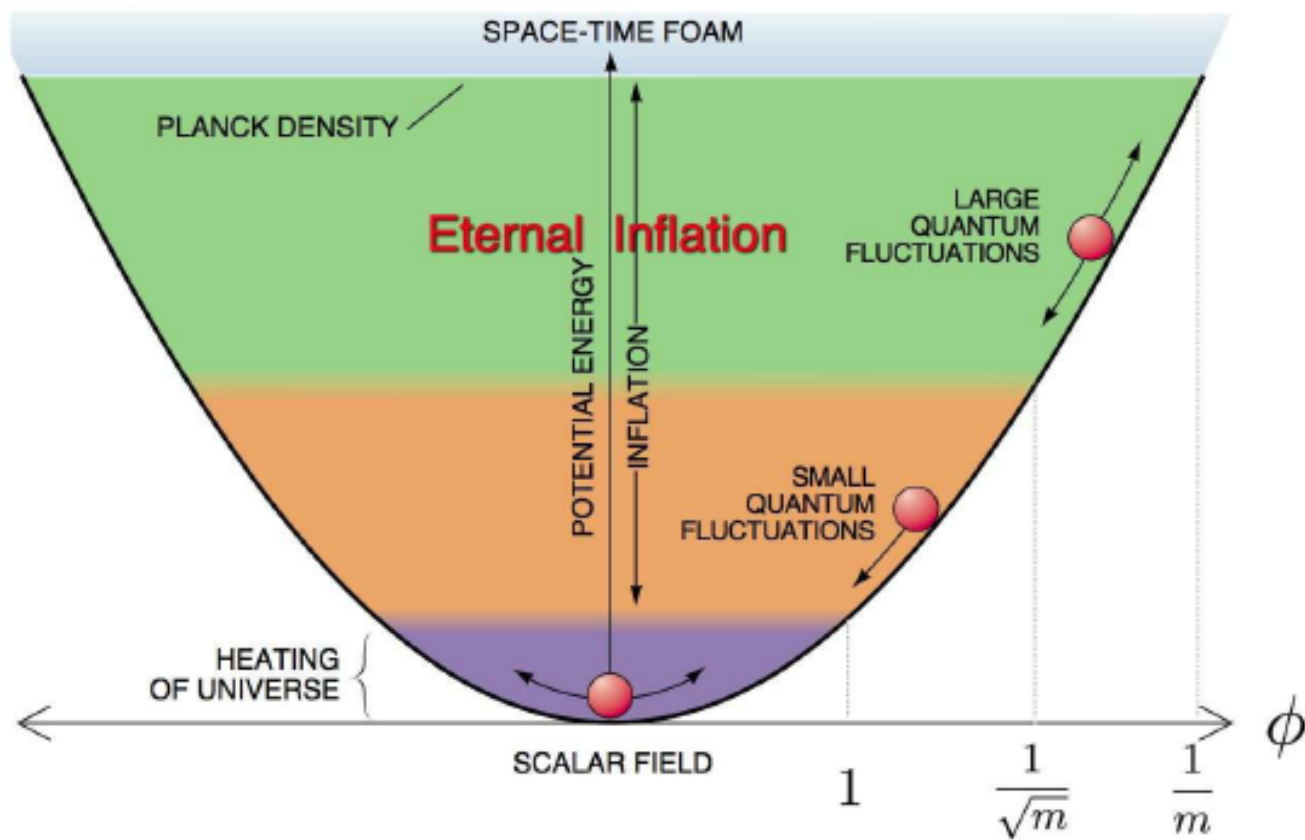
Current bound:  $r < 0.2$





# Chaotic Inflation

$$V(\phi) = \frac{m^2}{2}\phi^2$$

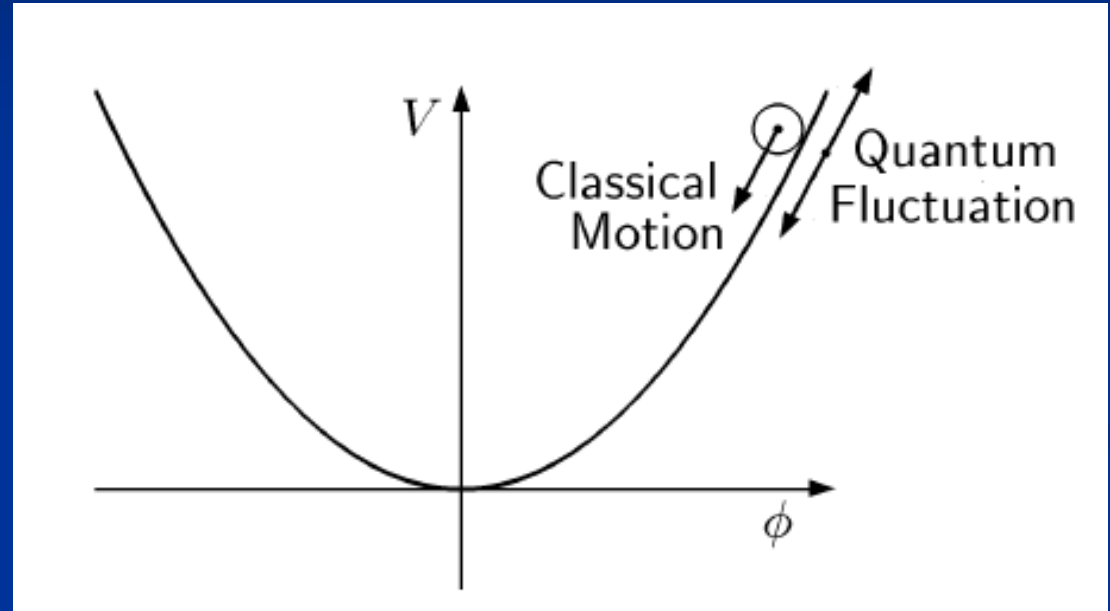


# Eternal inflation

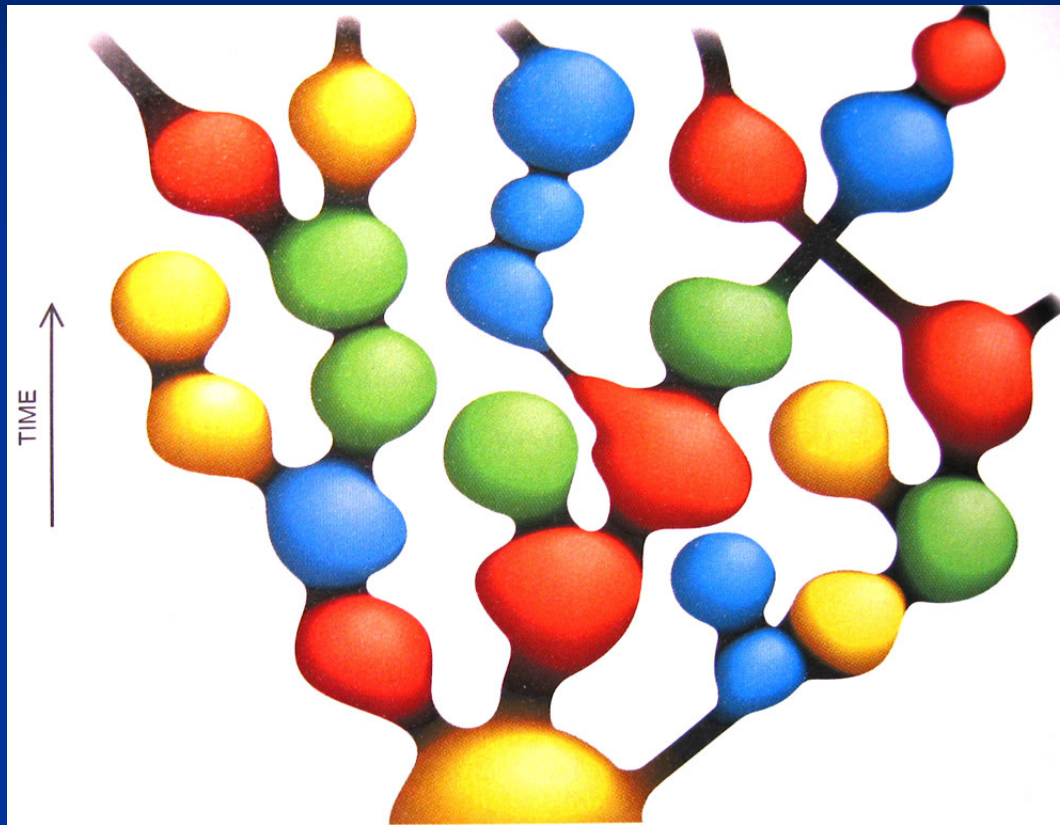
$$\Delta\phi_{\text{qu}} = \frac{H}{2\pi} .$$

$$\Delta\phi_{\text{cl}} = \frac{M_{\text{P}} m}{\sqrt{12\pi}} H^{-1} = \frac{1}{4\pi} \frac{M_{\text{P}}^2}{\phi} .$$

$$\Delta\phi_{\text{qu}}(\phi^*) = \Delta\phi_{\text{cl}}(\phi^*) ,$$

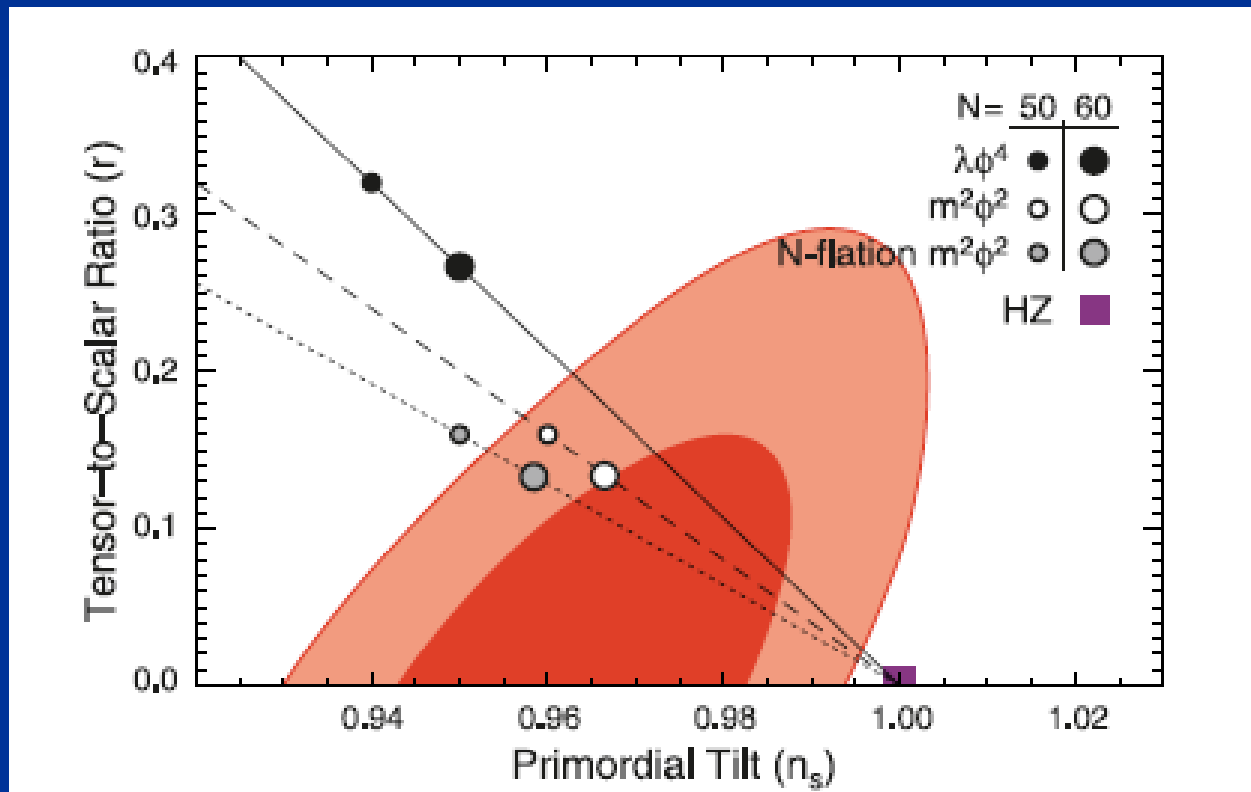


# Eternal Inflation

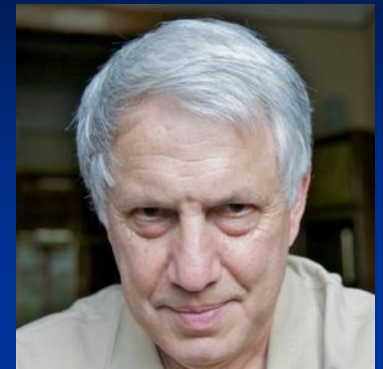


**SELF-REPRODUCING COSMOS** appears as an extended branching of inflationary bubbles. Changes in color represent “mutations” in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.

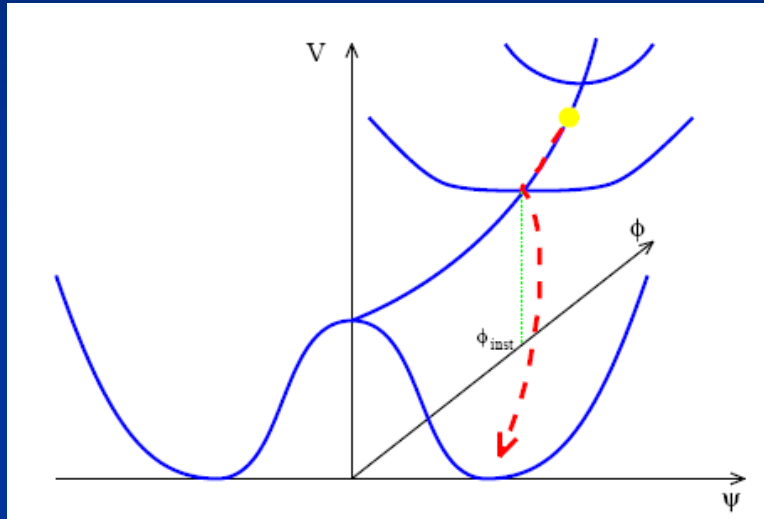
# Experimental test of (chaotic) inflation



# Hybrid Inflation



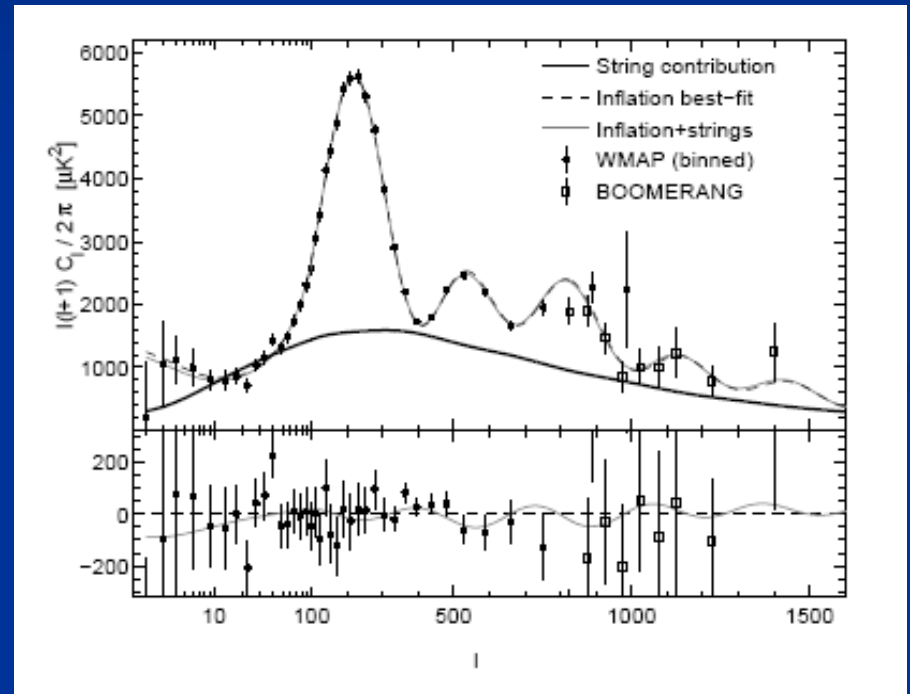
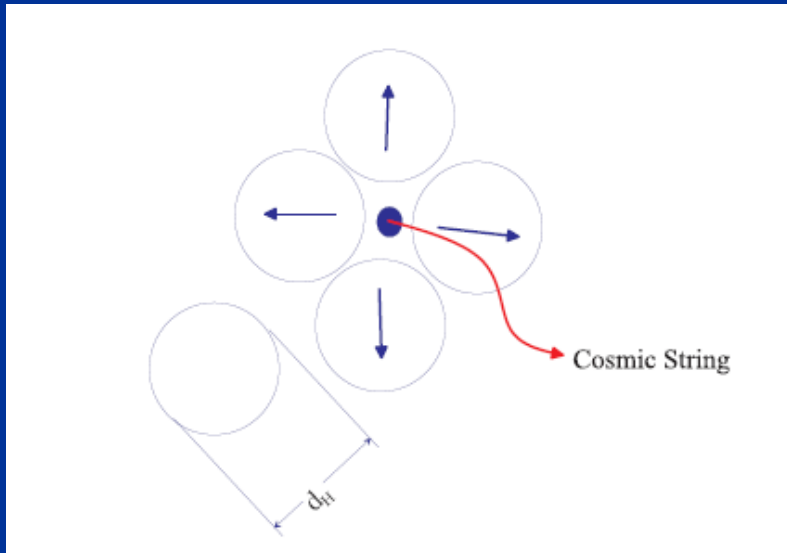
A. Linde



$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2 .$$



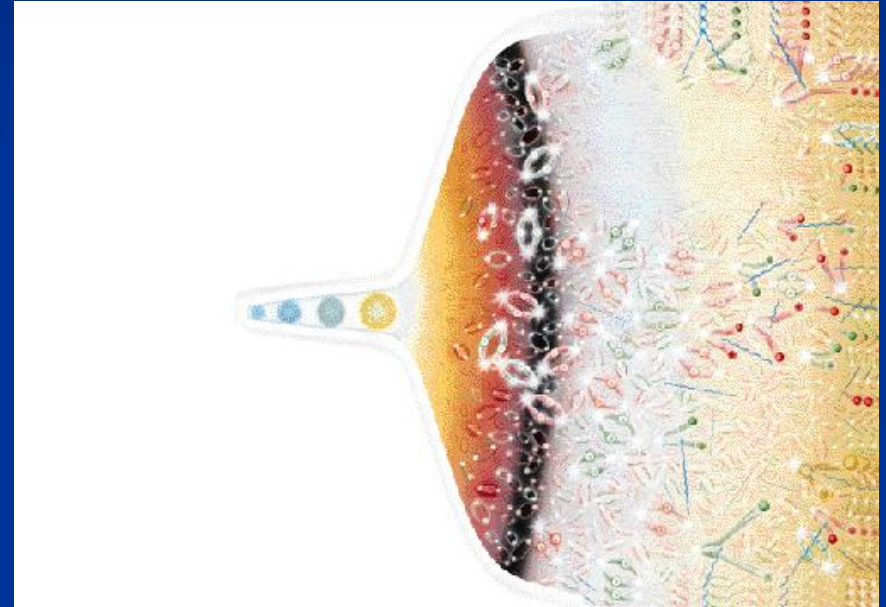
# Cosmic strings



# Reheating: the “hot big bang” in modern version

$$\rho(t_r) \simeq \frac{3\Gamma^2 M_p^2}{8\pi} \simeq \frac{\pi^2 N(T_r)}{30} T_r^4 .$$

$$T_r \simeq 0.2\sqrt{\Gamma M_p} .$$



# Hilltop Inflation

L. Boubekeur and D. H. Lyth hep-ph/0502047

$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \dots = V_0 \left( 1 - \frac{1}{2}|\eta_0| \left( \frac{\phi}{M_{\text{P}}} \right)^2 + \dots \right),$$

More hilltop Inflation models 0707.3826 [hep-ph]

$$\begin{aligned} V(\phi) &= V_0 \pm \frac{1}{2}m^2\phi^2 - \lambda \frac{\phi^p}{M_{\text{P}}^{p-4}} + \dots \\ &\equiv V_0 \left( 1 + \frac{1}{2}\eta_0 \frac{\phi^2}{M_{\text{P}}^2} \right) - \lambda \frac{\phi^p}{M_{\text{P}}^{p-4}} + \dots, \end{aligned}$$



Hill Top - the home of Beatrix Potter

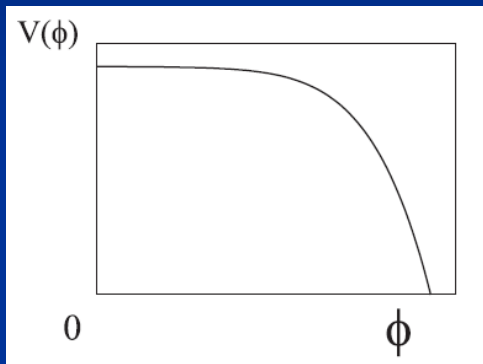


CML,

D. Lyth,

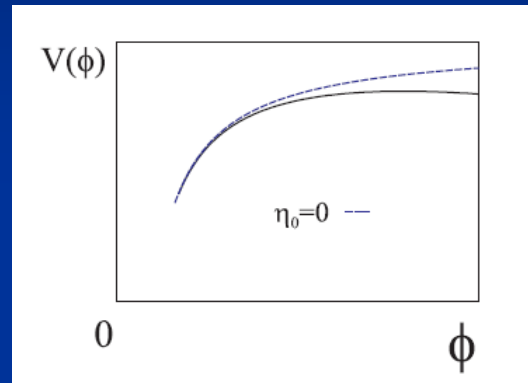
K. Kohri

# Three classes of Hilltop Inflation



$$\eta_0 \leq 0 \text{ and } p > 2$$

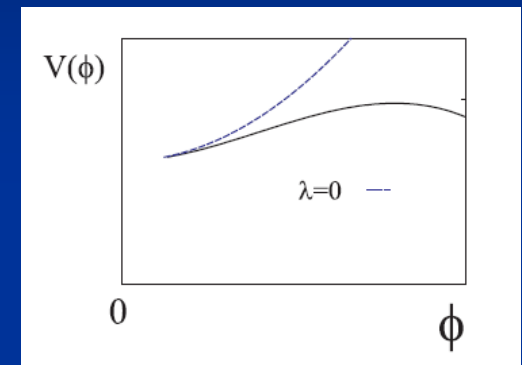
R invariance New Inflation



$$\eta_0 < 0 \text{ and } p < 0$$

(Hilltop) D-term Inflation

(Hilltop) F-term Inflation



$$\eta_0 > 0 \text{ and } p > 2$$

(Hilltop) Supernatural Inflation

# Hilltop Inflation

Analytical solutions for the spectrum and the spectral index

$$\mathcal{P}_\zeta = \frac{1}{12\pi^2} \left( \frac{V_0}{M_{\text{P}}^4} \right)^{\frac{p-4}{p-2}} e^{-2\eta_0 N}$$

$$\times \frac{\left[ p\lambda(e^{(p-2)\eta_0 N} - 1) + \eta_0 x \right]^{\frac{2p-2}{p-2}}}{\eta_0^{\frac{2p-2}{p-2}} (\eta_0 x - p\lambda)^2}$$

$$n - 1 = 2\eta_0 \left[ 1 - \frac{\lambda p(p-1)e^{(p-2)\eta_0 N}}{\eta_0 x + p\lambda(e^{(p-2)\eta_0 N} - 1)} \right]$$

$$\left( \frac{\phi}{M_{\text{P}}} \right)^{p-2} = \left( \frac{V_0}{M_{\text{P}}^4} \right) \frac{\eta_0 e^{(p-2)\eta_0 N}}{\eta_0 x + p\lambda(e^{(p-2)\eta_0 N} - 1)}$$

$$x \equiv \left( \frac{V_0}{M_{\text{P}}^4} \right) \left( \frac{M_{\text{P}}}{\phi_{\text{end}}} \right)^{p-2},$$



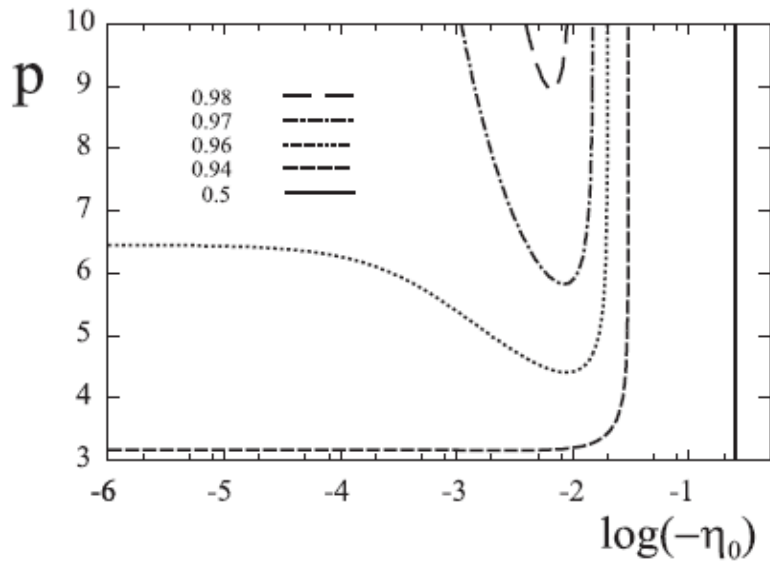


FIG. 4:  $p \geq 3$ , contours of  $n$  in the  $\log(-\eta_0)$ - $p$  plane.

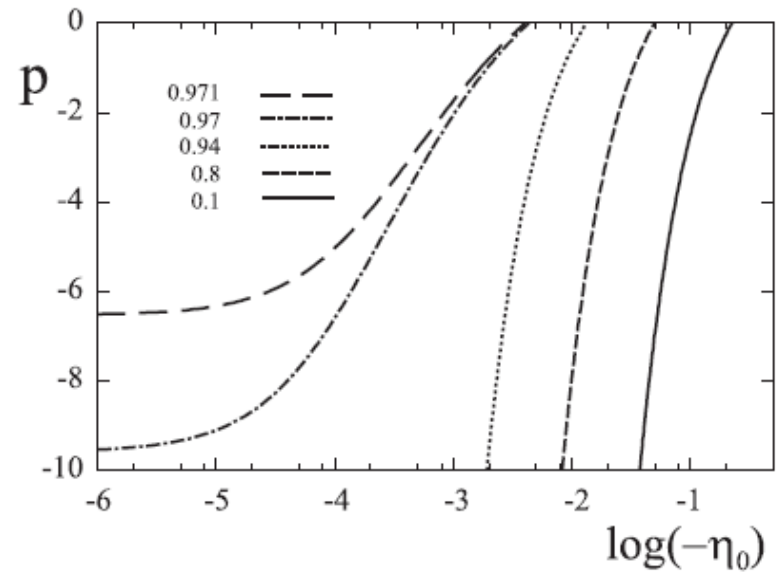


FIG. 5:  $p < 0$ , contours of  $n$  in the  $\log(-\eta_0)$ - $p$  plane.

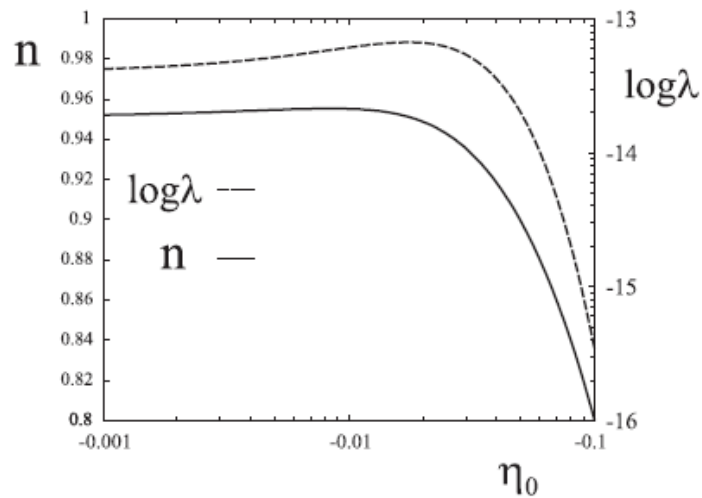


FIG. 7: Model 1,  $p=4$ ,  $n$  and  $\log \lambda$  versus  $\eta_0$ .

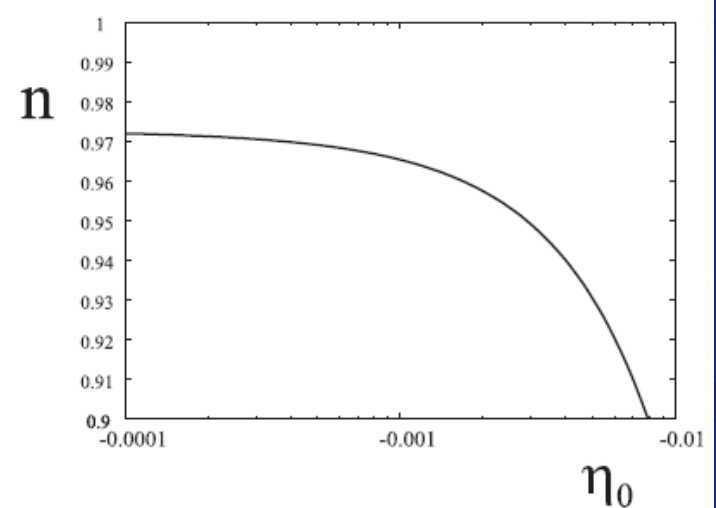
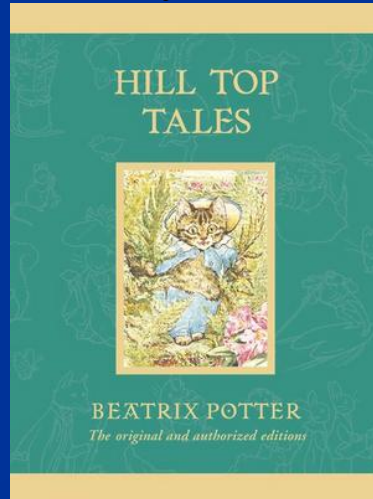


FIG. 13: Model 2,  $p=-4$ ,  $n$  versus  $\eta_0$ .

# Benefits of Hilltop Inflation

- Produce “(topological) Eternal Inflation”
- Produce the spectral index required from CMB observation.
- Reduce the inflation scale. (Solve the cosmic string problem of hybrid inflation.)





# Non-Gaussianity

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL}[\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle],$$

$$f_{NL}^{\text{local}} = 32 \pm 21 \text{ (68\% CL)}.$$

The 95% limit is  $-10 < f_{NL}^{\text{local}} < 74$ .

From WMAP 7-year

# Methods to generate NG

- Curvaton
- Multi-field Inflation
- Inhomogeneous reheating
- Generate curvature perturbation at the end of inflation
- etc.

# Important observables

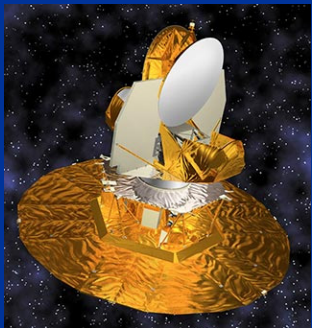
- The spectral index
- The tensor to scalar ratio
- Non-Gaussianity
- etc.

# Planck Satellite:

$$\Delta n_s \leq 0.01$$

$$f_{NL} < 5$$

$$r < 0.01$$



WMAP



PLANCK May 14, 2009

# Epilog

- The very early universe is “an accelerator for poor people”.---Zeldovich 1970
- “...but the richest man’s as well.”---Linde
- “Now is the time to be a cosmologist.”---Mark Kamionkowski 0706.2986[astro-ph]
- “Our universe is an ultimate test of fundamental physics.”---Renata Kallosh hep-th/0702059