古典和量子隱形斗篷簡介

(Introduction to Classical and Quantum Invisibility Cloaks)

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17 Deceomber 2010

Outlines

1. Introduction to the EM invisibility cloak
2. How to design an EM invisibility cloak
3. Cloaking of matter waves
4. A more general formulation of quantum cloaking
5. Controlling spinor fields
6. Cloaking the spinor fields
7. Applications

1. Introduction to Invisibility Cloak of EM Waves

Controlling Electromagnetic Fields

J. B. Pendry,[1*] D. Schurig,[2] D. R. Smith[2]

Using the freedom of design that metamaterials provide, we show how electromagnetic fields can be redirected at will and propose a design strategy. The conserved fields—electric displacement field **D**, magnetic induction field **B**, and Poynting vector **B**—are all displaced in a consistent manner. A simple illustration is given of the cloaking of a proscribed volume of space to exclude completely all electromagnetic fields. Our work has relevance to exotic lens design and to the cloaking of objects from electromagnetic fields.

To exploit electromagnetism, we use materials to control and direct the fields: a glass lens in a camera to produce an image, a metal cage to screen sensitive equipment, "blackbodies" of various forms to prevent unwanted reflections. With homogeneous materials, optical design is largely a matter of choosing the interface between two materials. For example, the lens of a camera is optimized by altering its shape so as to minimize geometrical aberrations. Electromagnetically inhomogeneous materials offer a different approach to control light; the introduction of specific gradients in the refractive index of a material can be used to form lenses and other optical elements, although the types and ranges of such gradients tend to be limited.

A new class of electromagnetic materials (*1*, *2*) is currently under study: metamaterials, which owe their properties to subwavelength details of structure rather than to their chemical composition, can be designed to have properties difficult or impossible to find in nature. We show how the design flexibility of metamaterials can be used to achieve new electromagnetic devices and how metamaterials enable a new paradigm for the design of electromagnetic structures at all frequencies from optical down to DC.

Progress in the design of metamaterials has been impressive. A negative index of refraction (*3*) is an example of a material property that does not exist in nature but has been enabled by using metamaterial concepts. As a result, negative refraction has been much studied in recent years (*4*), and realizations have been reported at both GHz and optical frequencies (*5*–*8*). Novel magnetic properties have also been reported over a wide spectrum of frequencies. Further information on the design and construction of metamaterials may be found in (*9*–*13*). In fact, it is now conceivable that a material can be constructed whose permittivity and permeability values may be designed to vary independently and arbitrarily throughout a material, taking positive or negative values as desired.

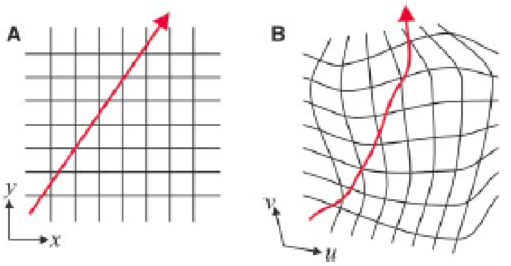

Fig. 1. (**A**) A field line in free space with the background Cartesian coordinate grid shown. (**B**) The distorted field line with the background coordinates distorted in the same fashion. The field in question may be the electric displacement or magnetic induction fields **D** or **B**, or the Poynting vector **S**, which is equivalent to a ray of light.

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物理 Physics

操控电磁场

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周 磊†，黄学勤†，郝加明† 译

摘要 我们在此展示如何利用特异材料所提供的设计自由度来随心所欲地改变电磁场的方向，并提出了一个设计方案。电位移矢量 $\boldsymbol{D}$ 、磁感应强度 $\boldsymbol{B}$ 和坡印亭矢量 $\boldsymbol{S}$ 等守恒场在一个统一的方式下被置换。拿隐形作为一个简单的例证，人们可以将一个特定区域内的电磁场完全排除在外。我们的工作与特异棱镜的设计及如何使物体在电磁场中隐形相关。

当我们利用各类电磁现象时，经常会采用一些特定的材料去控制和改变电磁场：比如用照相机中的透镜成像，用金属笼子屏蔽敏感的电磁设备，采用各种各样的“黑体”来防止不必要的反射等。均匀材料的光学设计很大程度上就是选择两种材料的界面。例如，我们通过改变照相镜头的形状使其得到优化，从而使几何相差达到极小。电磁非均匀材料提供了控制光的另一种方法；人们利用具有梯度变化折射率系数的材料来制作透镜及其他光学元件，尽管这些梯度变化的形式和范围受到一定的限制。

作为一类新型电磁材料，特异材料正成为科学研究的热点之一[1,2]。这种材料可以被设计成具有一些自然界中很难或不可能存在的奇异性质，而这些性质起源于特异材料的亚波长结构细节，而非材料本身的化学成分。我们将展示，如何运用特异材料设计的灵活性来构造新的电磁器件，以及如何利用特异材料来设计适用于从光频到直流电的任意频率的电磁结构。

特异材料的设计已取得了重大的进展。负的折射率系数[3]就是一个很好的例子，它本身在自然材料中不存在，但是可以在特异材料中实现。因而负折射现象近年来被广泛研究[4]，在微波乃至光频段都有相关报道[5~8]。在更宽的频谱内，许多新颖的磁学特性也相继被发现。关于设计和制造特异材料的更多信息读者可以参阅参考文献 [9] ～ [13]。事实上，人们设想可以构造出这样一种材料，其任意一点的介电常数和磁导率可以相互独立且随意地变化，并且可以根据需要取正值或负值。

我们如果能这样前所未有地控制材料特性并合成所需要的非均匀的复合材料，我们即拥有了一个设计电磁器件的强大方法。譬如，如果能获得合适的特异材料，我们将展示如何随意支配电位移矢量 $\boldsymbol{D}$，

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J. B. Pendry, D. Schurig, and D.R. Smith, Science 312, 1780 (2006).

Optical Conformal Mapping

Ulf Leonhardt

An invisibility device should guide light around an object as if nothing were there, regardless of where the light comes from. Ideal invisibility devices are impossible, owing to the wave nature of light. This study develops a general recipe for the design of media that create perfect invisibility within the accuracy of geometrical optics. The imperfections of invisibility can be made arbitrarily small to hide objects that are much larger than the wavelength. With the use of modern metamaterials, practical demonstrations of such devices may be possible. The method developed here can also be applied to escape detection by other electromagnetic waves or sound.

According to Fermat's principle (*1*), light rays take the shortest optical paths in dielectric media, where the refractive index n integrated along the ray trajectory defines the path length. When n is spatially varying, the shortest optical paths are not straight lines, but are curved. This light bending is the cause of many optical illusions. Imagine a situation where a medium guides light around a hole in it. Suppose that all parallel bundles of incident rays are bent around the hole and recombined in precisely the same direction as they entered the medium. An

U. Leonhardt, Science 312, 1777 (2006).

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we have agreed that it would be greatly beneficial for CAS and AAAS to undertake joint activities. For our first initiative, we determined to publish a selection of papers from Science translated into Chinese.

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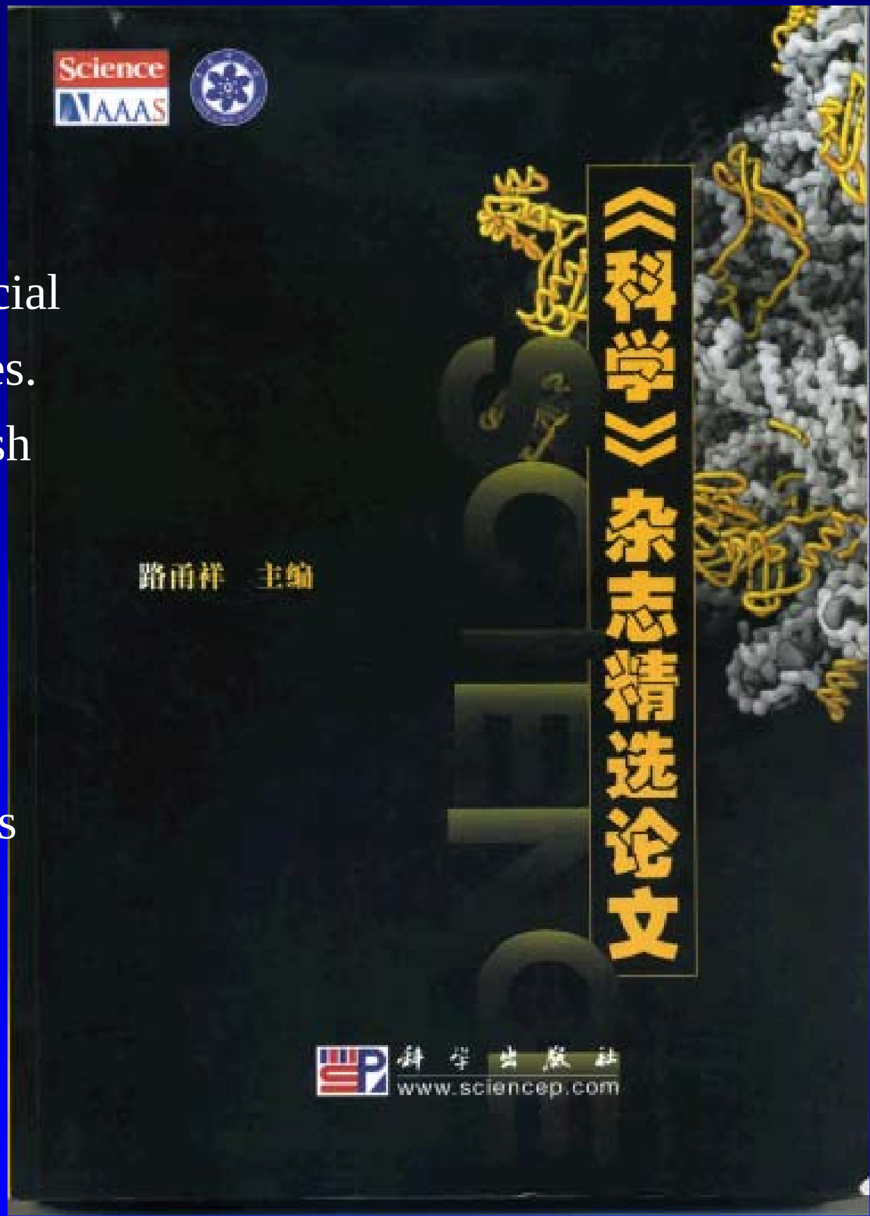

Contents

Metamaterial Electromagnetic Cloak at Microwave Frequencies

D. Schurig,[1] J. J. Mock,[1] B. J. Justice,[1] S. A. Cummer,[1] J. B. Pendry,[2] A. F. Starr,[3] D. R. Smith[1*]

A recently published theory has suggested that a cloak of invisibility is in principle possible, at least over a narrow frequency band. We describe here the first practical realization of such a cloak; in our demonstration, a copper cylinder was "hidden" inside a cloak constructed according to the previous theoretical prescription. The cloak was constructed with the use of artificially structured metamaterials, designed for operation over a band of microwave frequencies. The cloak decreased scattering from the hidden object while at the same time reducing its shadow, so that the cloak and object combined began to resemble empty space.

D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr, and D.R. Smith, Science 314, 977 (2006).

LETTERS

Optical cloaking with metamaterials

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Artificially structured metamaterials have enabled unprecedented flexibility in manipulating electromagnetic waves and producing new functionalities, including the cloak of invisibility based on coordinate transformation[1–3]. Unlike other cloaking approaches[4–6], which are typically limited to subwavelength objects, the transformation method allows the design of cloaking devices that render a macroscopic object invisible. In addition, the design is not sensitive to the object that is being cloaked. The first experimental demonstration of such a cloak at microwave frequencies was recently reported[7]. We note, however, that that design[7] cannot be implemented for an optical cloak, which is certainly of particular interest because optical frequencies are where the word 'invisibility' is conventionally defined. Here we present the design of a non-magnetic cloak operating at optical frequencies. The principle and structure of the proposed cylindrical cloak are analysed, and the general recipe for the implementation of such a device is provided.

REPORTS

Broadband Ground-Plane Cloak

R. Liu,[1]* C. Ji,[2]* J. J. Mock,[1] J. Y. Chin,[3] T. J. Cui,[3]† D. R. Smith[1]†

The possibility of cloaking an object from detection by electromagnetic waves has recently become a topic of considerable interest. The design of a cloak uses transformation optics, in which a conformal coordinate transformation is applied to Maxwell's equations to obtain a spatially distributed set of constitutive parameters that define the cloak. Here, we present an experimental realization of a cloak design that conceals a perturbation on a flat conducting plane, under which an object can be hidden. To match the complex spatial distribution of the required constitutive parameters, we constructed a metamaterial consisting of thousands of elements, the geometry of each element determined by an automated design process. The ground-plane cloak can be realized with the use of nonresonant metamaterial elements, resulting in a structure having a broad operational bandwidth (covering the range of 13 to 16 gigahertz in our experiment) and exhibiting extremely low loss. Our experimental results indicate that this type of cloak should scale well toward optical wavelengths.

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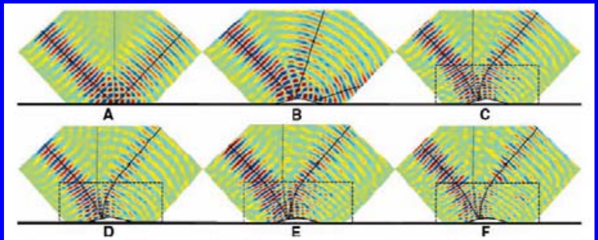
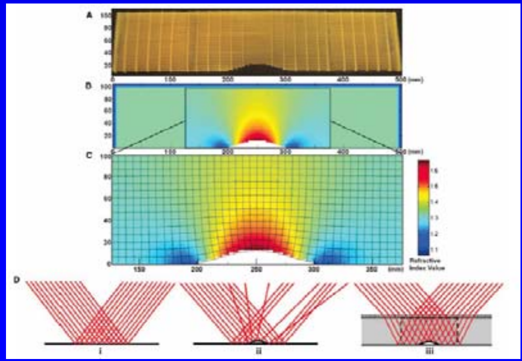


Fig. 3. Measured field mapping (E-field) of the ground, perturbation, and ground-plane cloaked perturbation. The rays display the wave propagation direction, and the dashed line indicates the normal of the ground in the case of free space and that of the ground-plane cloak in the case of the transformed space. **(A)** Collimated beam incident on the ground plane at 14 GHz. **(B)** Collimated beam incident on the perturbation at 14 GHz (control). **(C)** Collimated beam incident on the ground-plane cloaked perturbation at 14 GHz. **(D)** Collimated beam incident on the ground-plane cloaked perturbation at 13 GHz. **(E)** Collimated beam incident on the ground-plane cloaked perturbation at 15 GHz **(F)** Collimated beam incident on the ground-plane cloaked perturbation at 16 GHz.



Three-dimensional photonic metamaterials at optical frequencies

NA LIU[1], HONGCANG GUO[1], LIWEI FU[1], STEFAN KAISER[2], HEINZ SCHWEIZER[1] AND HARALD GIESSEN[1]*
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Metamaterials are artificially structured media with unit cells much smaller than the wavelength of light. They have proved to possess novel electromagnetic properties, such as negative magnetic permeability and negative refractive index[1–3]. This enables applications such as negative refraction[4], superlensing[5] and invisibility cloaking[6]. Although the physical properties can already be demonstrated in two-dimensional (2D) metamaterials, the practical applications require 3D bulk-like structures[4–6]. This prerequisite has been achieved in the gigahertz range for microwave applications owing to the ease of fabrication by simply stacking printed circuit boards[4,6]. In the optical domain, such an elegant method has been the missing building block towards the realization of 3D metamaterials. Here, we present a general method to manufacture 3D optical (infrared) metamaterials using a layer-by-layer technique[7–9]. Specifically, we introduce a fabrication process involving planarization, lateral alignment and stacking. We demonstrate stacked metamaterials, investigate the interaction between adjacent stacked layers and analyse the optical properties of stacked metamaterials with respect to an increasing number of layers.

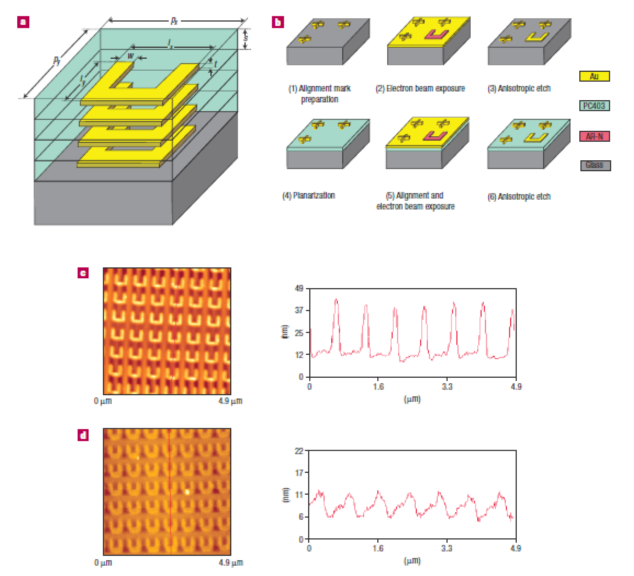

Figure 1 Structure geometry and fabrication procedures. a, Schematic diagram of the structure with definitions of the geometrical parameters: $l_x = 430$ nm, $l_y = 380$ nm, $w = 80$ nm, $t = 20$ nm, $s = 70$ nm, $p_x = 700$ nm and $p_y = 700$ nm. **b**, Processing scheme. **c**,**d**, Atomic force microscopy images and cross-section profiles of gold SRRs before (**c**) and after (**d**) planarization. Before planarization (**c**), the peak-to-valley height is approximately 27 nm. It is slightly larger than the original gold thickness of the SRRs, 20 nm, owing to the presence of a thin layer of AR-N resist after etching. After planarization (**d**) with a 70 nm PC403, the peak-to-valley height is reduced to ~5 nm.

Broadband Invisibility by Non-Euclidean Cloaking

Ulf Leonhardt[1,2]* and Tomáš Tyc[2,3]

Invisibility and negative refraction are both applications of transformation optics where the material of a device performs a coordinate transformation for electromagnetic fields. The device creates the illusion that light propagates through empty flat space, whereas in physical space, light is bent around a hidden interior or seems to run backward in space or time. All of the previous proposals for invisibility require materials with extreme properties. Here we show that transformation optics of a curved, non-Euclidean space (such as the surface of a virtual sphere) relax these requirements and can lead to invisibility in a broad band of the spectrum.

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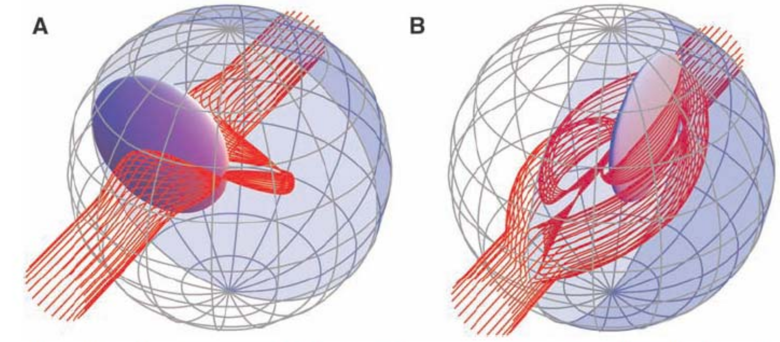


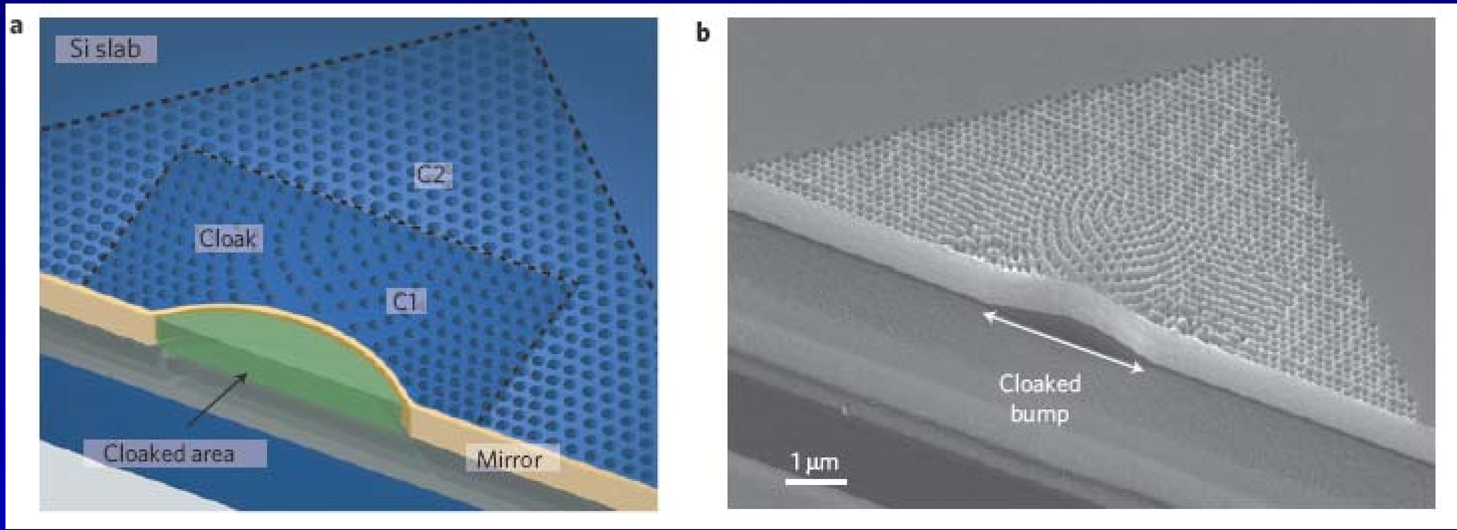
Fig. 3. 3D cloaking. One can extrapolate the ideas illustrated in Fig. 2 to 3D space, replacing the plane by flat space and the sphere by a hypersphere. The lentil-shaped object indicates the hidden interior of the device, and the partly shaded grid denotes the boundary of the invisibility device. For better contrast, light rays are shown in red. **(A)** Rays are bent around the invisible region. **(B)** In three dimensions, some rays turn out to perform two loops in hyperspace that appear in physical space as light wrapped around the invisible interior.

An optical cloak made of dielectrics

Jason Valentine[1]*, Jensen Li[1]*, Thomas Zentgraf[1]*, Guy Bartal[1] and Xiang Zhang[1,2]†

Invisibility devices have captured the human imagination for many years. Recent theories have proposed schemes for cloaking devices using transformation optics and conformal mapping[1–4]. Metamaterials[5,6], with spatially tailored properties, have provided the necessary medium by enabling precise control over the flow of electromagnetic waves. Using metamaterials, the first microwave cloaking has been achieved[7] but the realization of cloaking at optical frequencies, a key step towards achieving actual invisibility, has remained elusive. Here, we report the first experimental demonstration of optical cloaking. The optical 'carpet' cloak is designed using quasi-conformal mapping to conceal an object that is placed under a curved reflecting surface by imitating the reflection of a flat surface. The cloak consists only of isotropic dielectric materials, which enables broadband and low-loss invisibility at a wavelength range of 1,400–1,800 nm.

Nature Mater. 8, 568 (2009).



The carpet cloak design that transforms a mirror with a bump into a virtually mirror.

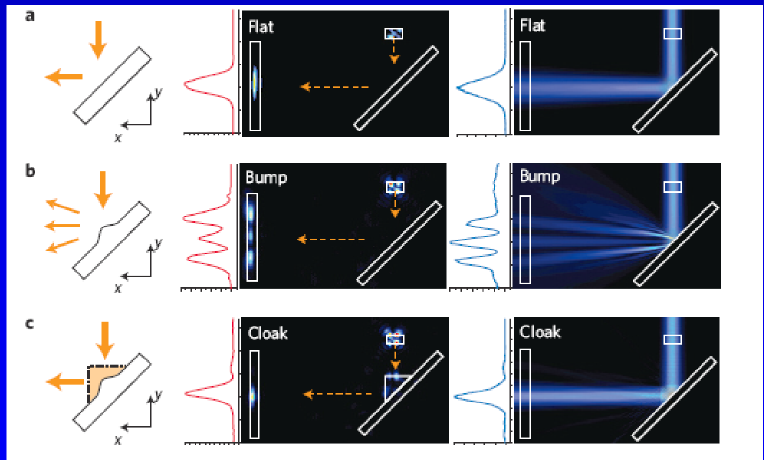


Figure 3 | Optical carpet cloaking at a wavelength of 1,540 nm. a–c, The results for a Gaussian beam reflected from a flat surface (**a**), a curved (without a cloak) surface (**b**) and the same curved reflecting surface with a cloak (**c**). The left column shows the schematic diagrams. The middle column shows the optical microscope images and normalized intensity along the output grating position. The curved surface scatters the incident beam into three separate lobes, whereas the cloaked curved surface maintains the original profile, similar to reflection from a flat surface. The experimental intensity profile agrees well with the intensity profile ($|E_z|^2$) obtained from 2D simulations, which is plotted next to the spatial field magnitude ($|E_z|$) in the right column.

Three-Dimensional Invisibility Cloak at Optical Wavelengths

Tolga Ergin,[1,2*†] Nicolas Stenger,[1,2*] Patrice Brenner,[2] John B. Pendry,[3] Martin Wegener[1,2,4]

We have designed and realized a three-dimensional invisibility-cloaking structure operating at optical wavelengths based on transformation optics. Our blueprint uses a woodpile photonic crystal with a tailored polymer filling fraction to hide a bump in a gold reflector. We fabricated structures and controls by direct laser writing and characterized them by simultaneous high–numerical-aperture, far-field optical microscopy and spectroscopy. A cloaking operation with a large bandwidth of unpolarized light from 1.4 to 2.7 micrometers in wavelength is demonstrated for viewing angles up to 60°.

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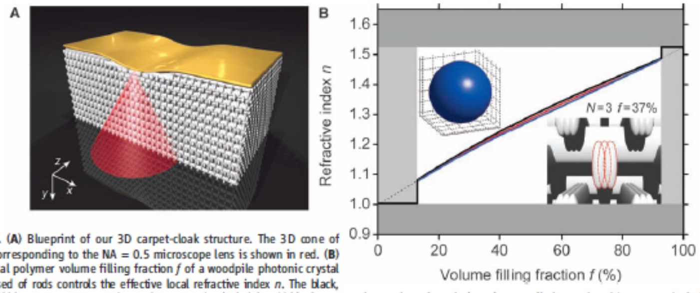
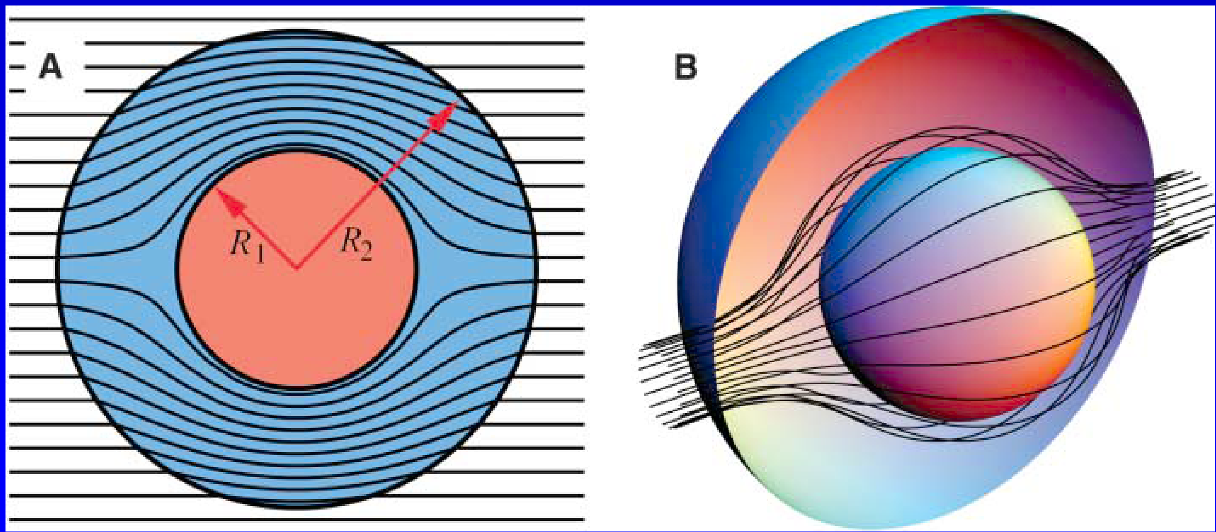


Fig. 1. (**A**) Blueprint of our 3D carpet-cloak structure. The 3D cone of light corresponding to the NA = 0.5 microscope lens is shown in red. (**B**) The local polymer volume filling fraction f of a woodpile photonic crystal composed of rods controls the effective local refractive index n. The black, red, and blue curves correspond to rod aspect ratios (= height/width) of 1, 2, and 3, respectively. The polymer needs to be connected, as do the air voids. These conditions impose lower and upper bounds on f, shown here for a rod aspect ratio of 2. The range of accessible values of n is restricted to be within 1.00 (air) and 1.52 (bulk polymer). The filling fraction is actually controlled by the number of voxels ($N \in \{0,3,4,5,6\}$) that each rod is composed of. The case of $N = 3$ is illustrated by the inset in the lower right corner. For this case and for a vacuum wavelength of 2.4 μm and a rod spacing of 0.8 μm, the upper left inset depicts the nearly spherical isofrequency surface in wave-vector space.

2. How to design an invisibility cloak of EM waves

Invisibility cloaks for classical waves that can hide an object from the detection of the probe waves and is externally invisible were proposed in 2006.

J. B. Pendry *et al*., Science 312, 1780 (2006);
U. Leonhardt, Science 312, 1777 (2006).



Maxwell equations are expressed as

$$F_{\alpha\beta,\mu} + F_{\beta\mu,\alpha} + F_{\mu\alpha,\beta} = 0, \qquad (1)$$

$$G_{,\alpha}^{\ \ \alpha\beta} = J^{\beta}, \qquad (2)$$

where $$G^{\alpha\beta} = \begin{pmatrix} 0 & -cD_1 & -cD_2 & -cD_3 \\ cD_1 & 0 & -H_3 & -H_2 \\ cD_2 & -H_3 & 0 & -H_1 \\ cD_3 & -H_2 & -H_1 & 0 \end{pmatrix} = \frac{1}{2} C^{\alpha\beta\mu\nu} F_{\mu\nu} \qquad (3)$$

with $C^{\alpha\beta\mu\nu}$ being the constitutive tensor representing the propeties of the medium, including the permittivity, and permeability.

Under coordinate transformation, $C^{\alpha\beta\mu\nu}$ tansforms as

$$C^{\alpha'\beta'\mu'\nu'} = |\det(\Lambda_{\alpha}^{\alpha'})|^{-1} \Lambda_{\alpha}^{\alpha'} \Lambda_{\beta}^{\alpha'} \Lambda_{\mu}^{\alpha'} \Lambda_{\nu}^{\alpha'} C^{\alpha\beta\mu\nu}, \text{ where } \Lambda_{\alpha}^{\alpha'} = \frac{\partial x^{\alpha'}}{\partial x^{\alpha}}. \qquad (4)$$

Specifically, the permitivity and permeability transform as

$$\varepsilon^{i'j'} = |\det \Lambda_{i}^{i'}|^{-1} \Lambda_{i}^{i'} \Lambda_{j}^{j'} \varepsilon^{ij} \qquad (5a)$$

$$\mu^{i'j'} = |\det \Lambda_{i}^{i'}|^{-1} \Lambda_{i}^{i'} \Lambda_{j}^{j'} \mu^{ij} \qquad (5b)$$

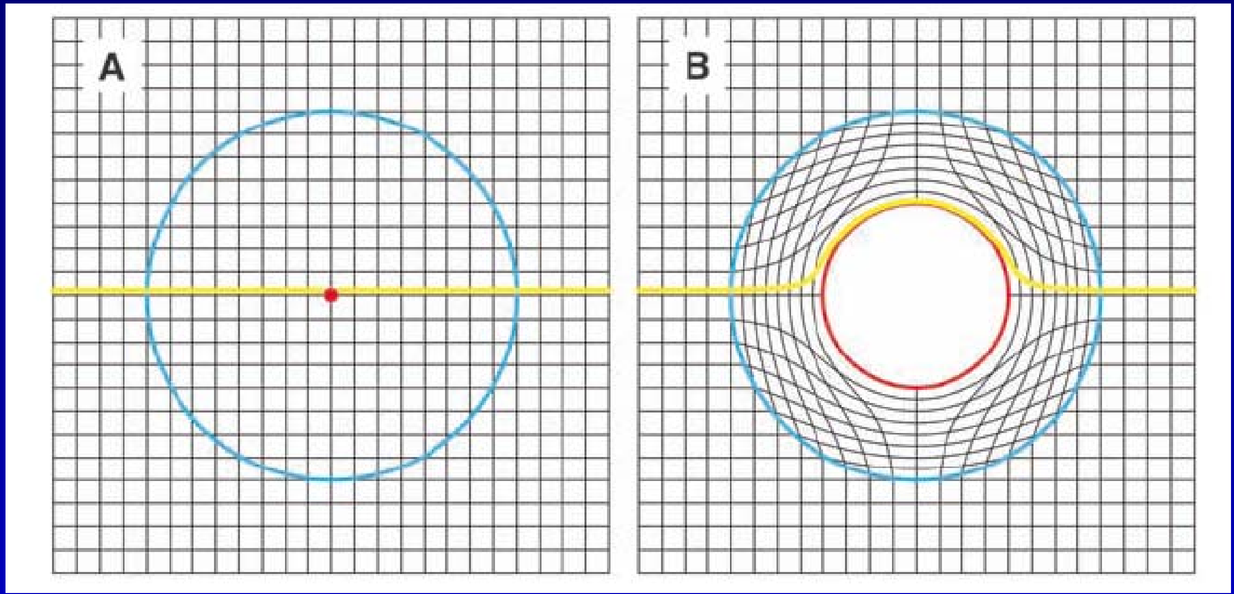
Maxwell's equations, Eqs (1) and (2), together with the medium specified by Eq. (5) describe a single electromagnetic behavior, but this behavior can be interpreted in two ways (see D. Schurig *et al*., Optics Express 14, 9794 (2006)).

First (the traditional interpretation): the material propety tensors that appear on the left and the right hand sides of (5) represent the same material properties, but in different spaces. The components in the transformed space are different from those in the original space, such as $(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}) \neq (\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{\varphi\varphi})$.

$$\varepsilon^{i'j'} = |\det \Lambda_i^{i'}|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \varepsilon^{ij},$$
$$\mu^{i'j'} = |\det \Lambda_i^{i'}|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \mu^{ij} \quad (5)$$

Second: an alternative interpretation is that the material property tensors on the left and right hand side of (5) represent different material porpeties. Both sets of tensor components are interpreted as components in a flat Cartesian space. The form invariance of Maxwell's equations insures that both interpretations lead to the same electromagnetic behavior. Pentry refer to this second view as the materials interpretation.

Based on the second interpretation, (5) are the primary tools for the transfomation design method of invisibility cloak.



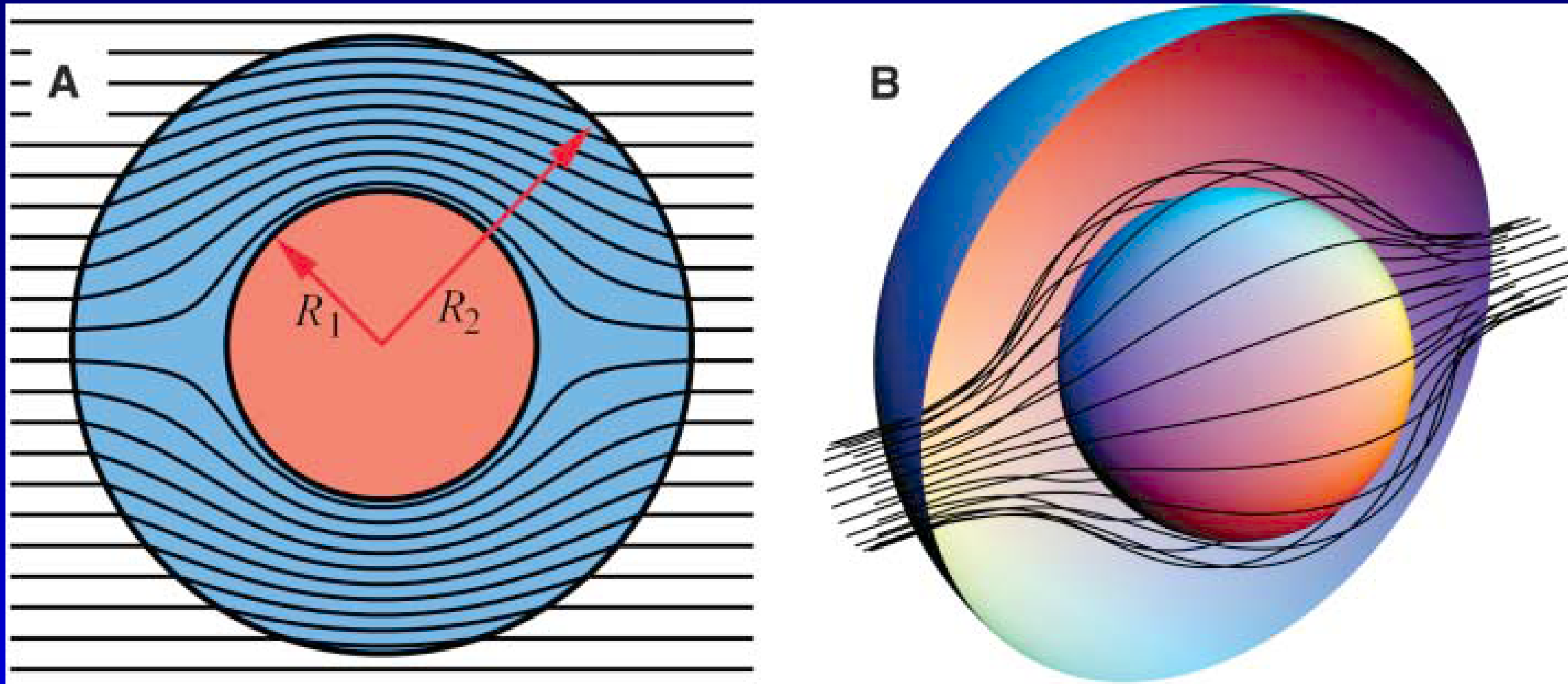
U. Leonhardt, and T. Tyc, Science 323, 110 (2009).

The cloaking device performs a curvilinear coordinate transformation from the flat Euclidean space (A) to physical space (B).

(i.e. the permitivity ε and permibility μ are transformed from the values of free space to $\varepsilon^{i'j'}$ and $\mu^{i'j'}$ that bend the straight coordinate lines to the curved coordinate lines).

Because the curved coordinate lines of physical space are transformations of straight lines. Physical space is Euclidean as well.

However, the device creates the illusion such that light feels it propagates through the flat space. The interior of cloak can hide an object from the detection of light and is externally invisible.



J. B. Pendry, D. Schurig, and D.R. Smith, Science 312, 1780 (2006).

操控电磁场

J. B. Pendry[1*], D. Schurig[2], D. R. Smith[2]
周　磊[†]，黄学勤[†]，郝加明[†]　译

摘要　我们在此展示如何利用特异材料所提供的设计自由度来随心所欲地改变电磁场的方向，并提出了一个设计方案。电位移矢量 D 、磁感应强度 B 和坡印亭矢量 S 等守恒场在一个统一的方式下被置换。拿隐形作为一个简单的例证，人们可以将一个特定区域内的电磁场完全排除在外。我们的工作与特异棱镜的设计及如何使物体在电磁场中隐形相关。

The experimental confimation

Metamaterial Electromagnetic Cloak at Microwave Frequencies

D. Schurig,[1] J. J. Mock,[1] B. J. Justice,[1] S. A. Cummer,[1] J. B. Pendry,[2] A. F. Starr,[3] D. R. Smith[1]*

A recently published theory has suggested that a cloak of invisibility is in principle possible, at least over a narrow frequency band. We describe here the first practical realization of such a cloak; in our demonstration, a copper cylinder was "hidden" inside a cloak constructed according to the previous theoretical prescription. The cloak was constructed with the use of artificially structured metamaterials, designed for operation over a band of microwave frequencies. The cloak decreased scattering from the hidden object while at the same time reducing its shadow, so that the cloak and object combined began to resemble empty space.

D. Schurig, J.J. Mock, B.J. Justice, S.A. Cummer, J.B. Pendry, A.F. Starr, and D.R. Smith, Science 314, 977 (2006).

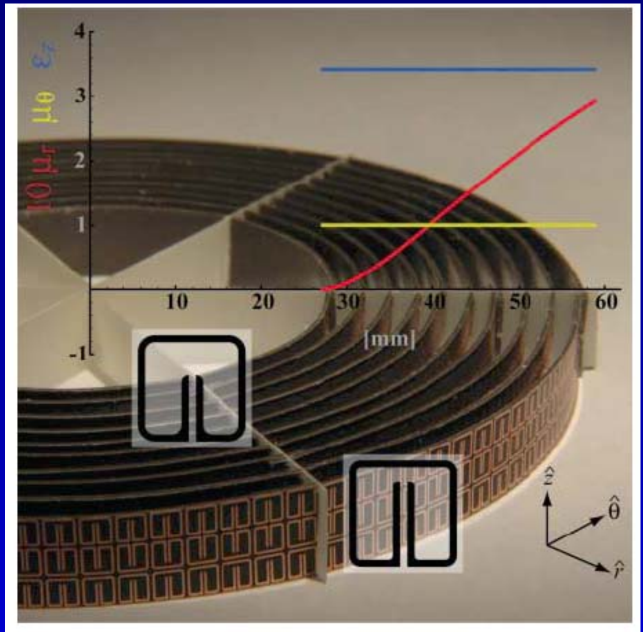


Fig. 1. 2D microwave cloaking structure (background image) with a plot of the material parameters that are implemented. μ_r (red line) is multiplied by a factor of 10 for clarity. μ_θ (green line) has the constant value 1. ε_z (blue line) has the constant value 3.423. The SRRs of cylinder 1 (inner) and cylinder 10 (outer) are shown in expanded schematic form (transparent square insets).

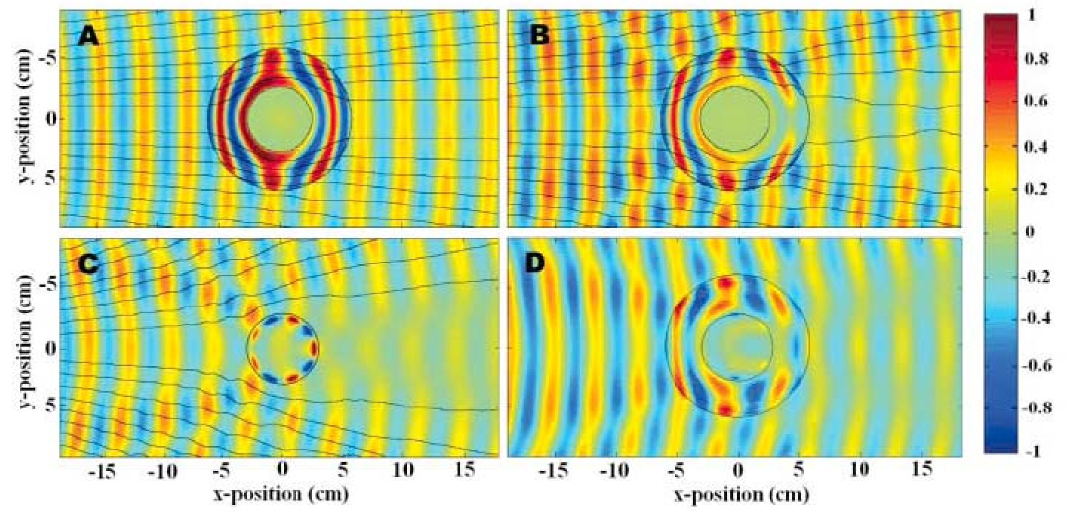


Fig. 4. Snapshots of time-dependent, steady-state electric field patterns, with stream lines [black lines in (A to C)] indicating the direction of power flow (i.e., the Poynting vector). The cloak lies in the annular region between the black circles and surrounds a conducting Cu cylinder at the inner radius. The fields shown are (**A**) the simulation of the cloak with the exact material properties, (**B**) the simulation of the cloak with the reduced material properties, (**C**) the experimental measurement of the bare conducting cylinder, and (**D**) the experimental measurement of the cloaked conducting cylinder. Animations of the simulations and the measurements (movies S1 to S5) show details of the field propagation characteristics within the cloak that cannot be inferred from these static frames. The right-hand scale indicates the instantaneous value of the field.

D. Schurig *et al.*, Science 314, 977 (2006).

Ideal theoretical Parameters

$$\varepsilon_r = \mu_r = \frac{r-a}{r}, \quad \varepsilon_\varphi = \mu_\varphi = \frac{r}{r-a}$$

$$\varepsilon_z = \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}$$

Parameters for real experiment

$$\mu_r = \left(\frac{r-a}{r}\right)^2, \quad \mu_\varphi = 1$$

$$\varepsilon_z = \left(\frac{b}{b-a}\right)^2$$

The recent experimental report

Three-Dimensional Invisibility Cloak at Optical Wavelengths

Tolga Ergin,[1,2*†] Nicolas Stenger,[1,2*] Patrice Brenner,[2] John B. Pendry,[3] Martin Wegener[1,2,4]

We have designed and realized a three-dimensional invisibility-cloaking structure operating at optical wavelengths based on transformation optics. Our blueprint uses a woodpile photonic crystal with a tailored polymer filling fraction to hide a bump in a gold reflector. We fabricated structures and controls by direct laser writing and characterized them by simultaneous high–numerical-aperture, far-field optical microscopy and spectroscopy. A cloaking operation with a large bandwidth of unpolarized light from 1.4 to 2.7 micrometers in wavelength is demonstrated for viewing angles up to 60°.

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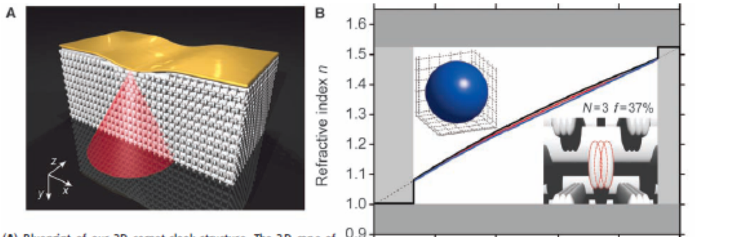

Fig. 1. (**A**) Blueprint of our 3D carpet-cloak structure. The 3D cone of light corresponding to the NA = 0.5 microscope lens is shown in red. (**B**) The local polymer volume filling fraction f of a woodpile photonic crystal composed of rods controls the effective local refractive index n. The black, red, and blue curves correspond to rod aspect ratios (= height/width) of 1, 2, and 3, respectively. The polymer needs to be connected, as do the air voids. These conditions impose lower and upper bounds on f, shown here for a rod aspect ratio of 2. The range of accessible values of n is restricted to be within 1.00 (air) and 1.52 (bulk polymer). The filling fraction is actually controlled by the number of voxels ($N \in \{0,3,4,5,6\}$) that each rod is composed of. The case of $N = 3$ is illustrated by the inset in the lower right corner. For this case and for a vacuum wavelength of 2.4 µm and a rod spacing of 0.8 µm, the upper left inset depicts the nearly spherical isofrequency surface in wave-vector space.

3. Cloaking of Matter Waves

PRL 100, 123002 (2008) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2008

Cloaking of Matter Waves

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Invariant transformation for quantum mechanical systems is proposed. A cloaking of matter wave can be realized at given energy by designing the potential and effective mass of the matter waves in the cloaking region. The general conditions required for such a cloaking are determined and confirmed by both the wave and particle (classical) approaches. We show that it may be possible to construct such a cloaking system for cold atoms using optical lattices.

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independent Schrödinger equation is written as

$$-\frac{\hbar^2}{2}\vec{\nabla}\cdot(\hat{m}^{*-1}\vec{\nabla}\psi) + V\psi = E\psi, \qquad (1)$$

where the spatially dependent and anisotropic effective mass $\hat{m}^* = m_0\hat{m}$ is generally a tensor (m_0 is the mass in free space), and $V(\vec{r})$ is a "macroscopic" potential. For instance, for electrons in a crystal with slowly varying composition, $V(\vec{r}) = E_b(\vec{r}) + U(\vec{r})$, where $E_b(\vec{r})$ is the energy of the local band edge and $U(\vec{r})$ is a slowly varying external potential [15]. The above equation can also be rewritten as two first-order differential equations

$$\vec{u} = \hat{m}^{-1}\vec{\nabla}\psi, \qquad -\frac{\hbar^2}{2m_0}\vec{\nabla}\cdot\vec{u} = (E - V)\psi. \qquad (2)$$

Utilizing the form Eq. (2), we consider an invariant coordinate transformation $(x_1, x_2, x_3) \to (q_1, q_2, q_3)$, by assuming both coordinate bases to be orthogonal. It is straightforward to show that divergence of vector $\vec{u}$ and gradient of the wave function ψ in the old coordinate frame are related to those in the new coordinates by

$$\vec{\nabla}_x\psi = \hat{h}^{-1}\vec{\nabla}_q\psi, \qquad \vec{\nabla}_x\cdot\vec{u} = \frac{1}{|\det(\hat{h})|}\vec{\nabla}_q\cdot\vec{v}, \qquad (3)$$

where $h_i = |\partial\vec{x}/\partial q_i|$ are the Lamé coefficients, $\hat{h}_{ij} = h_i\delta_{ij}$ (δ_{ij} is the Kronecker delta), and we define a new vector $\vec{v} = |\det(\hat{h})|\hat{h}^{-1}\vec{u}$. Combining Eq. (2) and (3) we thus obtain the Schrödinger equations in the new coordinate system,

$$-\frac{\hbar^2}{2m_0}\nabla_q\cdot\vec{v} = \det|\hat{h}|(E - V)\psi$$
$$\vec{v} = \det|\hat{h}|(\hat{h}\,\hat{m}\,\hat{h})^{-1}\vec{\nabla}_q\psi. \qquad (4)$$

Clearly, Eqs. (4) are mathematically equivalent to the Eq. (2), under the following transformations:

$$\hat{m}' = \frac{\hat{h}\,\hat{m}\,\hat{h}}{\det|\hat{h}|}, \qquad V' = E + |\det(\hat{h})|(V - E) \qquad (5)$$

of the potential and effective mass, respectively. Those

Recently, Zhang *et al*. [Phys. Rev. Lett. 100, 123002 (2008)] explore the possibility of cloaking matter waves.

They show that if the effective mass of a quantum particle and the potential acting on it are controlled as follows:

$$M_{rr} = g'(r)\left(\frac{r}{g(r)}\right)^2 M_0, \quad M_{\theta\theta} = M_{\varphi\varphi} = \frac{1}{g'(r)} M_0,$$

$$V(r,E) = \left[1 - \left(\frac{g}{r}\right)^2 g'(r)\right] E,$$

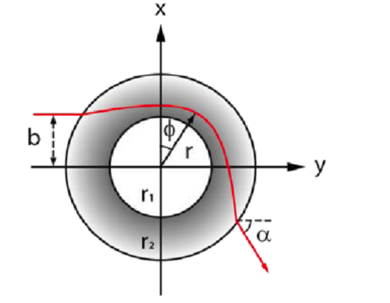

FIG. 1 (color online). Schematic of a spherical cloaking system with interaction potential and effective mass tuned in the shell region between r_1 and r_2. An impinging particle travels along $+y$ direction and has an initial momentum P_0, and impact parameter b. After passing throughout the system it leaves at an angle α.

where M_{rr} is the rr componenent of effective mass, and $g(r)$ is a monotonic radial scaling function with $g(r_1) = 0$ and $g(r_2) = b$ with r_1 (r_2) being the inner (outer) radius of quantum cloak, the matter wave of the particle can indeed be cloaked.

A quantum cloak with appropriate medium parameters can thus be designed. A beam of matter wave incident on the cloak will be guided along the cloaking shell without any distortion and loss when it returns to the original propagation direction,

i.e. the invisibility is at the quantum level!.

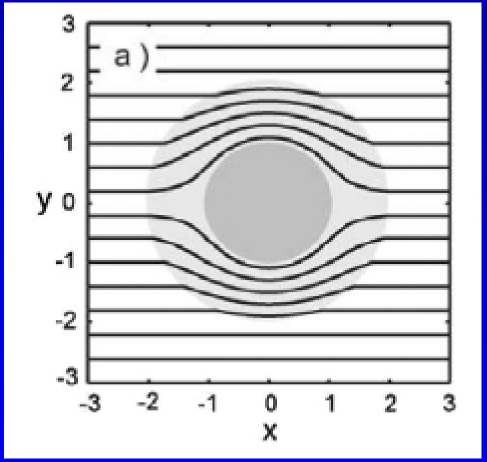

FIG. 2. The trajectories of particles passing through a cloaking system with $r_2 = 2r_1$

Zhang *et al*. [Phys. Rev. Lett. 100, 123002 (2008)]

Such a quantum cloak with radial scaling property may be achieved through the design of media as proposed by Zhang with BEC and Greenleaf *et al*. [Phys. Rev. Lett. 101, 220404 (2008)] with the aggregation of many ring-shaped potential barriers to mimic the cloaking condition.

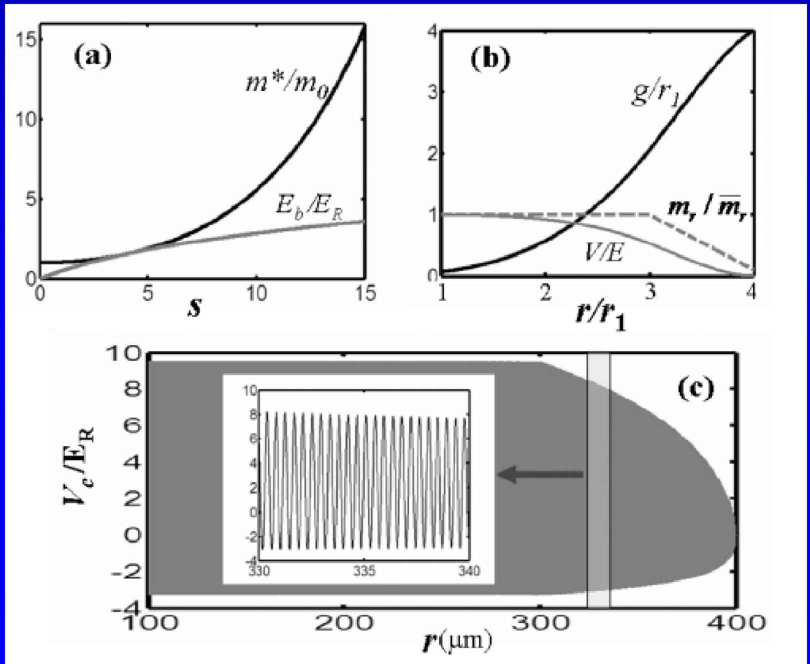

FIG. 3. (a) The dependence of effective mass and band edge energy on the amplitude of the optical lattice potential s (b) The plot of the radial scaling function g (black), effective mass along radial direction m_r (gray dashed) and potential V (gray) for a reduced cloaking design with $r_2 = 4r_1$, $\bar{m} = 10m_0$ and $\delta = r_1$. (c) The combined optical and magnetic potential profile to achieve the proposed cloaking system for $r_1 = 100$ μm, $r_2 = 400$ μm and $\delta = 100$ μm. Inset is a magnified view of the shadowed area.

Zhang *et al*. [Phys. Rev. Lett. 100, 123002 (2008)]

Please allow me to tell you a story.
I consulted my schoolmate, Professor Pi-gang Luan, about the recent development of invisibility cloak about the end of 2008.

4. A more general formulation of quantum cloaking

independent Schrödinger equation is written as

$$-\frac{\hbar^2}{2}\vec{\nabla}\cdot(\hat{m}^{*-1}\vec{\nabla}\psi)+V\psi=E\psi, \qquad (1)$$

where the spatially dependent and anisotropic effective mass $\hat{m}^*=m_0\hat{m}$ is generally a tensor (m_0 is the mass in free space), and $V(\vec{r})$ is a "macroscopic" potential. For instance, for electrons in a crystal with slowly varying composition, $V(\vec{r})=E_b(\vec{r})+U(\vec{r})$, where $E_b(\vec{r})$ is the energy of the local band edge and $U(\vec{r})$ is a slowly varying external potential [15]. The above equation can also be rewritten as two first-order differential equations

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of the potential and effective mass, respectively. Those

Zhang *et al*. [Phys. Rev. Lett. 100, 123002 (2008)]

$$\sqrt{g}(E-V)\Psi=\frac{1}{2}\{-\hbar^2\partial_i(M^{ij}\partial_j\Psi)+iq\hbar(\partial_i M^{ij}A_j)\Psi$$
$$+2iq\hbar M^{ij}A_j\partial_i\Psi+q^2M^{ij}A_jA_i\Psi\}, \qquad (1)$$

where $g=|g_{ij}|$ is the determinant of the generic metric tensor g_{ij}, V is the scalar potential, $M^{ij}=g^{ij}\sqrt{g}/M_0$ is the effective mass tensor with g^{ij} being the inverse of g_{ij}, q is the charge carried by the quantum particle, and A_j is the components of the vector potential $\mathbf{A}$ that describes the magnetic field. The property of a quantum cloak is completely determined by the effective mass tensor M^{ij} in the sense of material interpretation [1]. Such a property can be achieved through the design of media as proposed by Zhang with Bose-Einstein condensation and Greenleaf with potentials approach. In view of more feasible consideration, we shall study a 2D cloak and assume the potential $V=0$. For the AB effect under consid-

Phys. Rev. A 79, 051605 (R) (2009)

D.H. Lin, and P.G. Luan

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **79**, 051605(R) (2009)

Cloaking of matter waves under the global Aharonov-Bohm effect

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We discuss the Aharonov-Bohm effect of a magnetic flux for its influence on a two-dimensional quantum cloak. It is shown that the matter wave of a charged particle under the global influence of the Aharonov-Bohm effect can still be perfectly cloaked and guided by the quantum cloak. Since the presence of the global influence of a magnetic flux on charged particles is universal, the perfect cloaking and guiding nature not only provides an ideal setup to cloak an object from matter waves but also provides an ideal setup to test the global physics of charged matter waves in the presence of a bare magnetic flux.

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$$\sqrt{g}(E-V)\Psi = \frac{1}{2}\{-\hbar^2\partial_i(M^{ij}\partial_j\Psi) + iq\hbar(\partial_i M^{ij}A_j)\Psi + 2iq\hbar M^{ij}A_j\partial_i\Psi + q^2M^{ij}A_jA_i\Psi\}, \quad (1)$$

where $g=|g_{ij}|$ is the determinant of the generic metric tensor g_{ij}, V is the scalar potential, $M^{ij}=g^{ij}\sqrt{g}/M_0$ is the effective mass tensor with g^{ij} being the inverse of g_{ij}, q is the charge carried by the quantum particle, and A_j is the components of the vector potential **A** that describes the magnetic field. The property of a quantum cloak is completely determined by the effective mass tensor M^{ij} in the sense of material interpretation [1]. Such a property can be achieved through the design of media as proposed by Zhang with Bose-Einstein condensation and Greenleaf with potentials approach. In view of more feasible consideration, we shall study a 2D cloak and assume the potential $V=0$. For the AB effect under consid-

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$$\sqrt{g}(E-V)\Psi = \frac{1}{2}\left\{-\hbar^2\partial_i(M^{ij}\partial_j\Psi) + iq\hbar(\partial_i M^{ij}A_j\Psi) + 2iq\hbar M^{ij}A_j\partial_i\Psi + q^2A_iM^{ij}A_j\Psi\right\}$$

where $M^{ij} = g^{ij}\sqrt{g}/M_0$.

The Schrödinger equation of a free quantum particle in arbitrary curvilinear coordinates

$$-\frac{\hbar^2}{2M_0}\frac{1}{\sqrt{g}}\partial_i\left(\sqrt{g}\,g^{ij}\partial_j\Psi\right)=i\hbar\partial_t\Psi$$

When a charged particle interacts with a EM field, its wave function is multiplied by a Dirac phase factor.

$$\bar{\Psi}=\Psi\exp\left\{\frac{iq}{\hbar}\int A_\mu dx^\mu\right\}$$

$$\frac{1}{2M_0}\left\{-\frac{\hbar^2}{\sqrt{g}}\partial_i\left(\sqrt{g}\,g^{ij}\partial_j\bar{\Psi}\right)+\frac{i\hbar q}{\sqrt{g}}\left(\partial_i\sqrt{g}\,g^{ij}A_j\right)\bar{\Psi}+2iq\hbar g^{ij}A_j\left(\partial_i\bar{\Psi}\right)+q^2A_ig^{ij}A_j\bar{\Psi}\right\}$$

$$=i\hbar\left(\partial_t-\frac{iq}{\hbar}A_0\right)\bar{\Psi}$$

$$\sqrt{g}\,(E-V)\Psi$$

$$=\frac{1}{2}\left\{-\hbar^2\partial_i(M^{ij}\partial_j\Psi)+iq\hbar(\partial_iM^{ij}A_j\Psi)+2iq\hbar M^{ij}A_j\partial_i\Psi+q^2A_iM^{ij}A_j\Psi\right\}$$

where $M^{ij}=g^{ij}\sqrt{g}\,/\,M_0$.

4a. Quantum Cloak as an Ideal Setup for Global Quantum Effect

The cloaking ability of a 2D quantum cloak does not diminish or degrade even when the global AB effect is present, see Phys. Rev. A 79, 051605 (R) (2009).

Since the quantum cloak can guide the matter wave detouring the cloaked region, it provides us an ideal setup to manifest the global AB effect, prohibiting the matter wave from penetrating into the region of nonzero magnetic field.

Thus, the quantum cloak not only can cloak an object from matter waves but also is an ideal instrument for testing the global influence of a magnetic flux.The idea can be generalized to other kinds of nonlocal quantum effects, such as the scalar AB, AB-EPR, and the Ahoronov-Casher effects and so forth.

Aharonov-Bohm Effect

The vector potential of a magnetic flux along the z-axis

hidden in the cloaked region can be expressed as $\vec{A} = \Phi \dfrac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$,

where $\hat{i}$ and $\hat{j}$ stand for the unit vector along the x and y axis, respectively.

The associated magnetic field is confined to an infinitely thin tube,

$B_3 = \dfrac{\Phi}{2\pi} \varepsilon_{3ij} \partial_i \partial_j \varphi(\vec{x}) = \Phi \delta(\vec{x}_\perp)$ with $\vec{x}_\perp = (x, y)$.

The magnitude of the magnetic flux is $\iint dxdyB_3 = \Phi$.

Here we see the vector potential of the flux spreads out through the entire x-y plane such that the charged particles would be influenced by the potential through the minimal coupling $(\vec{p} - q\vec{A})$ in Hamiltonian even though they do not enter the region of nonzero magnetic field $\vec{B}$.

Aharonov and Bohm first expected the nonlocal influence of an infinitely thin magnetic flux on a charged particle and argued that the scattering cross section caused by the nonlocal effect (the AB effect) had the anaylitic expression $\sigma_{AB} = \dfrac{\sin^2(\pi\mu)}{2\pi k \cos^2(\varphi/2)}$, Phys. Rev. 115, 485 (1959), where $\mu = \Phi/\Phi_0$ with $\Phi_0 = h/q$ being the fundamental flux quantum.

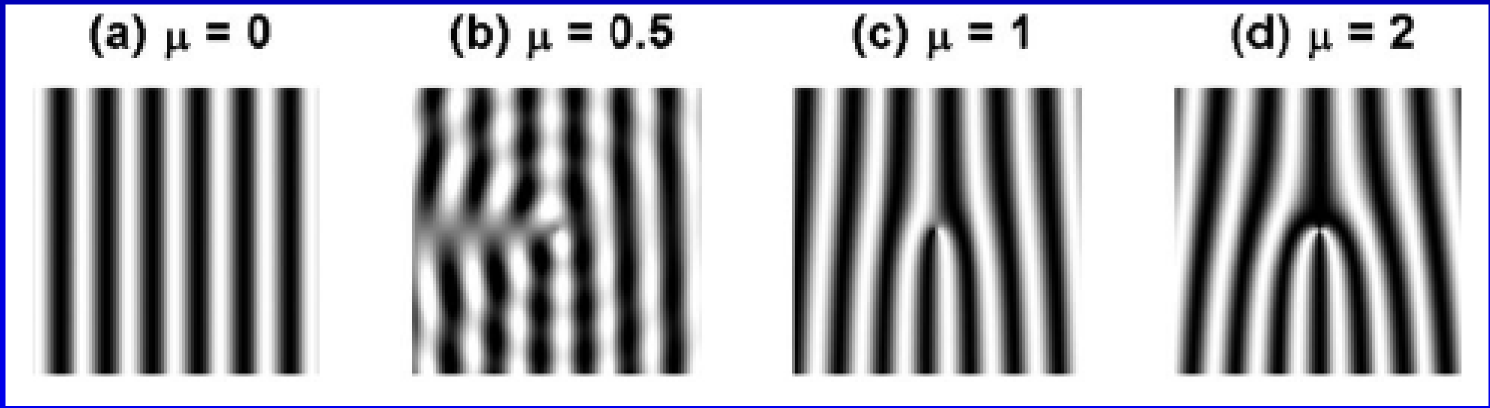

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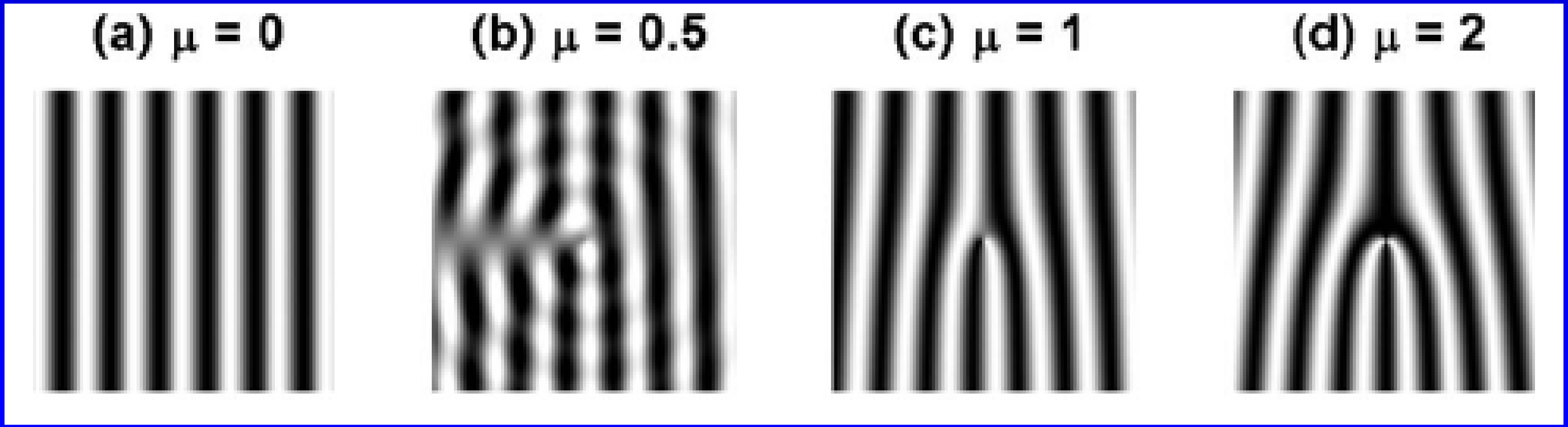
Fig. (a) to Fig. (d) show the scattering patterns of wave functions under the AB effect of a magnetic flux along the *z*-axis. The matter waves are incident from the left and leave towards the right.

The AB effect is the starting point
of studying the global effect of quantum physics.
Nevertheless, it is hard to realize such a bare flux experiment under the assumption that the matter wave does not touch the region of nonzero magnetic field.
Here, obviously, if we could have quantum cloak,
it would provides an ideal tool to check the physics caused by the nonlocal quantum interference of a bare magnetic flux.



With $\sqrt{g} = f'(\rho) f(\rho)/\rho$, $M_{\rho\rho} = M_0 \rho f'(\rho)/f(\rho)$,
$M_{\varphi\varphi} = M_0 f(\rho)/[\rho f'(\rho)]$,

where $f(\rho)$ is an arbitrary radial function,

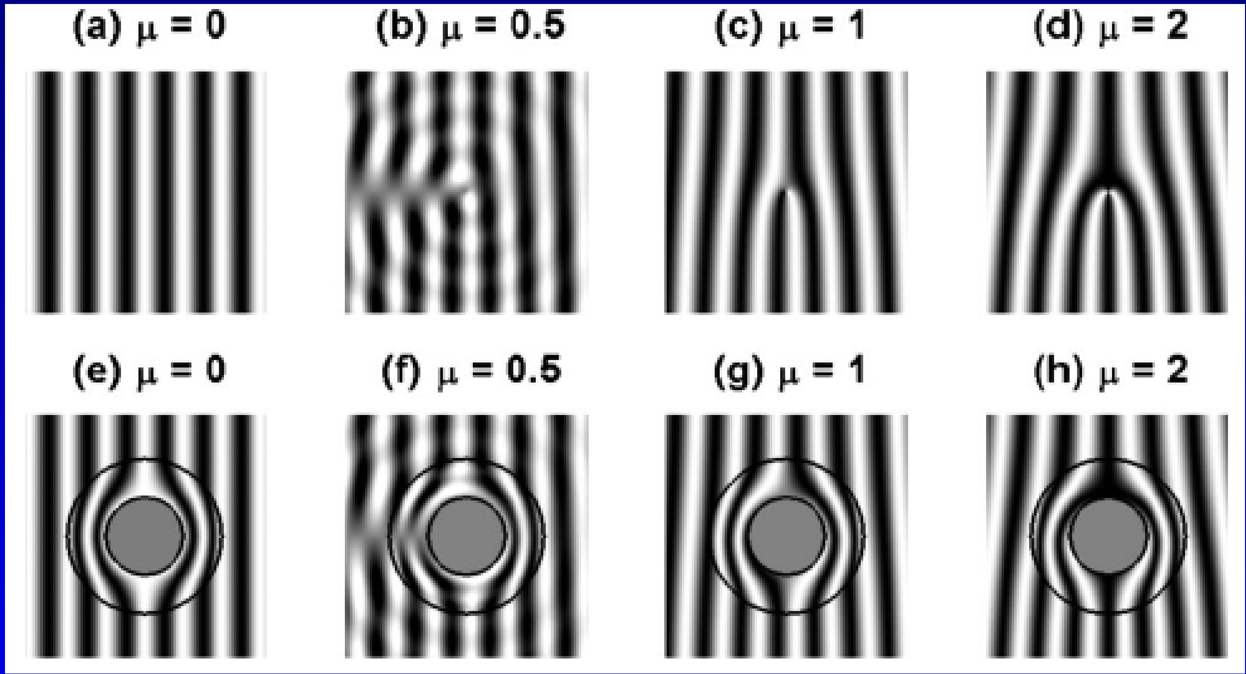
We can get the perfectly cloaking solution in the cloaking shell

$$\Psi = \sum_{\mathrm{m=-\infty}}^{\infty} (-i)^{|m-\mu|} J_{|m-\mu|}\left[k_0 f(\rho)\right] e^{im\varphi}.$$

where $\mu = \Phi/\Phi_0$ with $\Phi_0 = h/q$ being the fundamental flux quantum.

Take the linear scaling $f(\rho) = b\dfrac{(\rho - a)}{(b-a)}$.

Cloaking of Matter Waves under the Aharonov - Bohm Effect



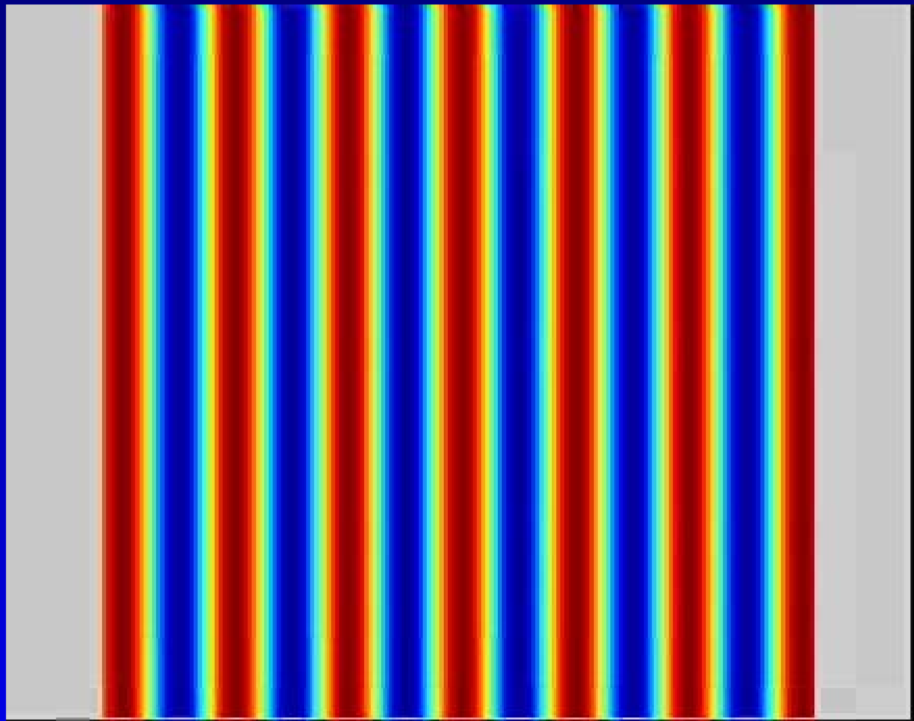
Phys. Rev. A 79, 051605 (R) (2009)

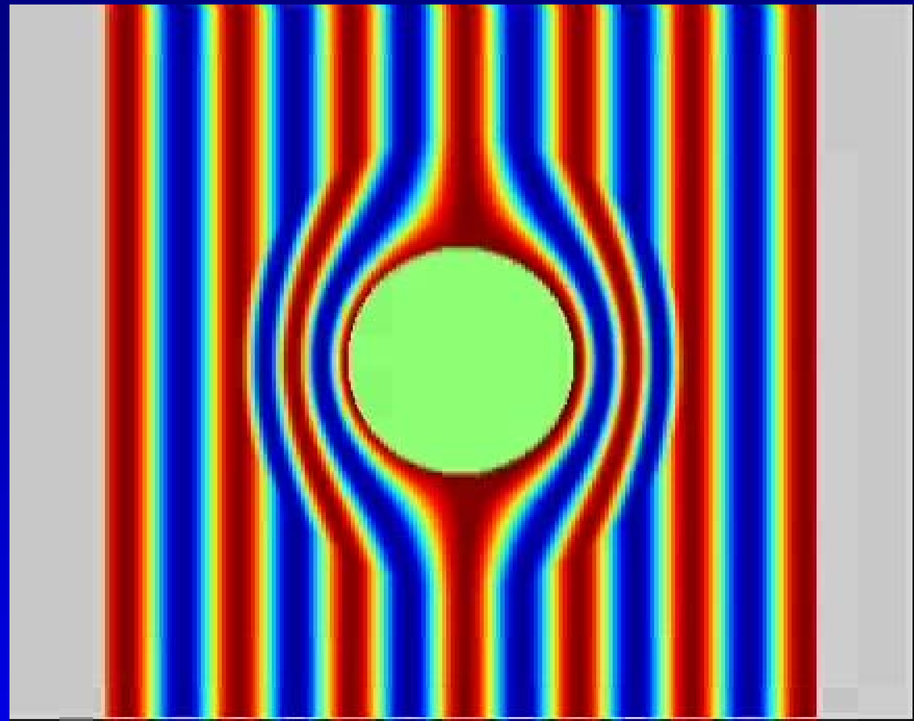
The cloaking of charged matter waves under the Aharonov-Bohm effect of a thin magnetic flux along the z-axis. In all cases, the matter waves are incident from the left and leave towards the right. Fig. (a) to Fig. (d) show the scattering patterns of wave functions under the AB effect without the quantum cloak, where $\mu=\Phi/\Phi_0$ is in unit of the fundamental flux quantum $\Phi_0=h/q$. Fig. (e) to Fig. (h) exhibit that the matter waves are perfectly cloaked and guided by the quantum cloak in the presence of the AB effect. Outside the cloak, the outgoing waves completely coincide with the patterns shown in the first row.

Fig. (a)

$$\mu = \frac{\Phi}{h/q} = 0.$$

Fig. (e)



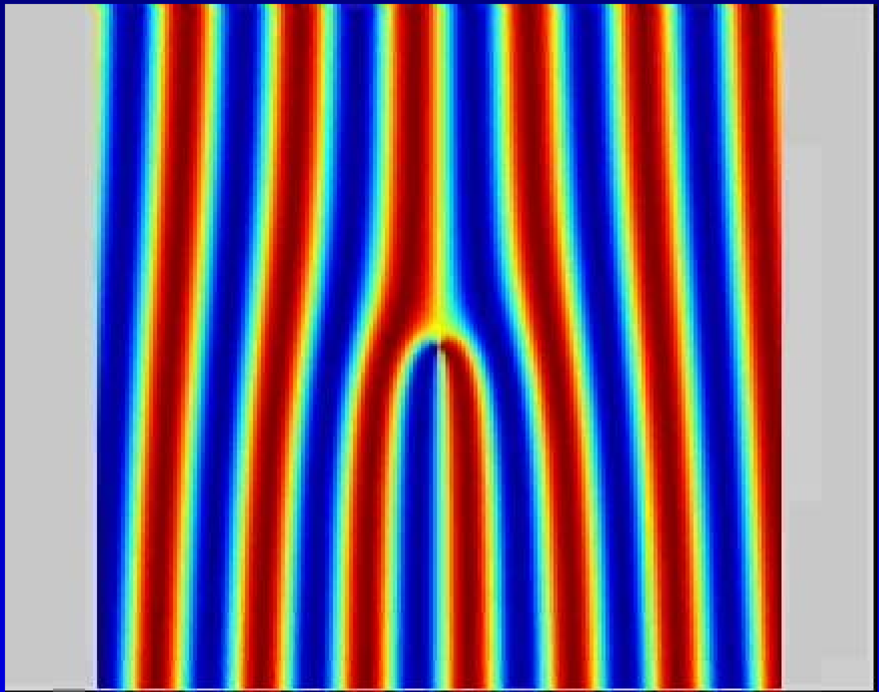


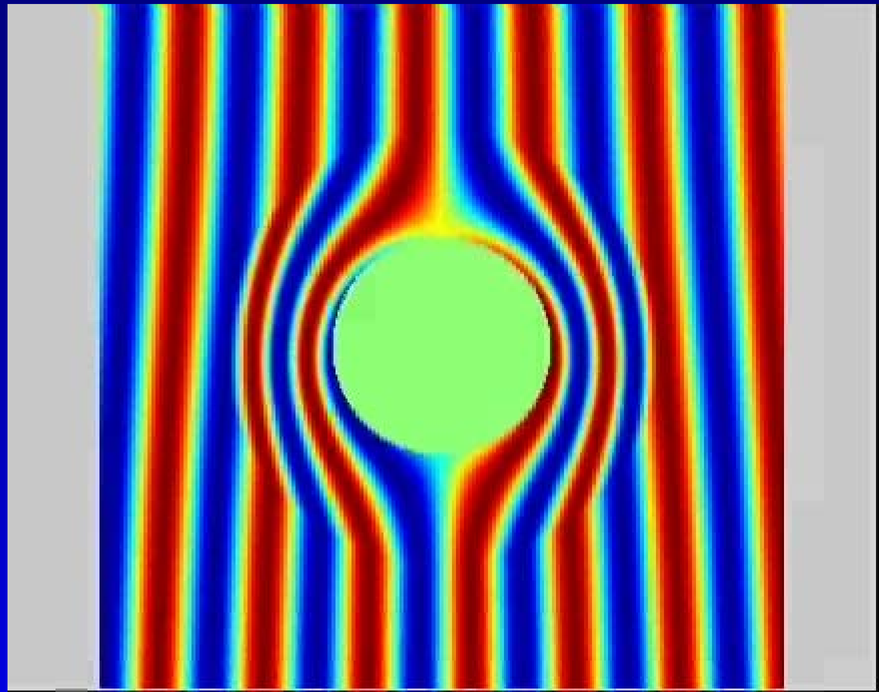
Here is the numerical simulation of cloaking of matter waves without the AB effect. Fig. (a) shows the matter waves are incident from the left and leave towards the right. Fig. (e) exhibit that the matter waves are perfectly cloaked and guided by the quantum cloak. Outside the cloak, the outgoing waves completely coincide with the patterns shown in the Fig. (a).

Fig. (b)

$\mu = \frac{\Phi}{h/q} = 1.$

Fig. (f)





The numerical simulation of cloaking of charged matter waves under the AB effect with $\mu=\Phi/\Phi_0=1$. Fig. (f) exhibit that the matter waves are perfectly cloaked and guided by the quantum cloak. Outside the cloak, the outgoing waves completely coincide with the patterns shown in the Fig. (b).

Fig. (c)

$$\mu = \frac{\Phi}{h/q} = 2.$$

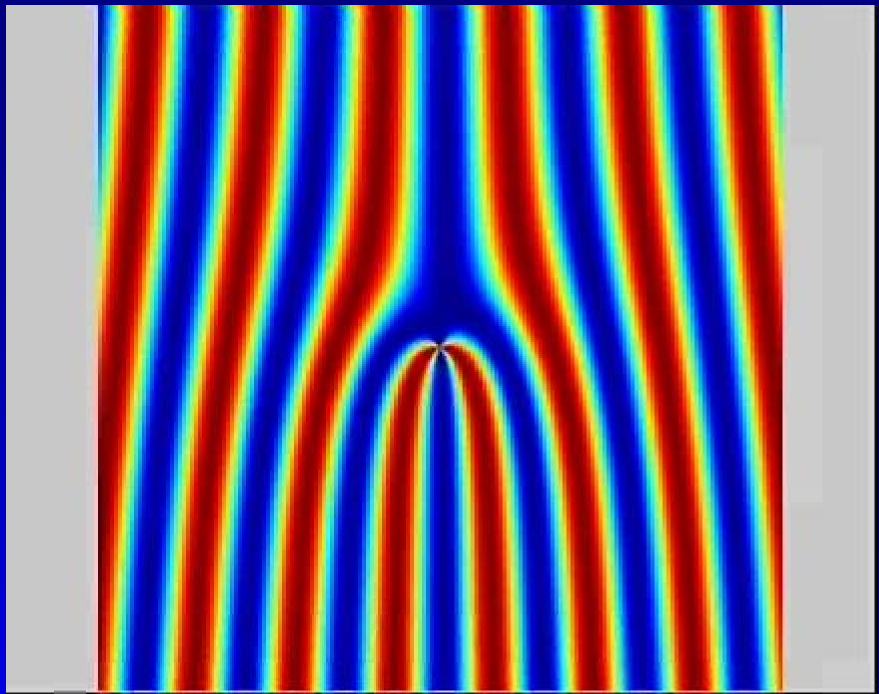
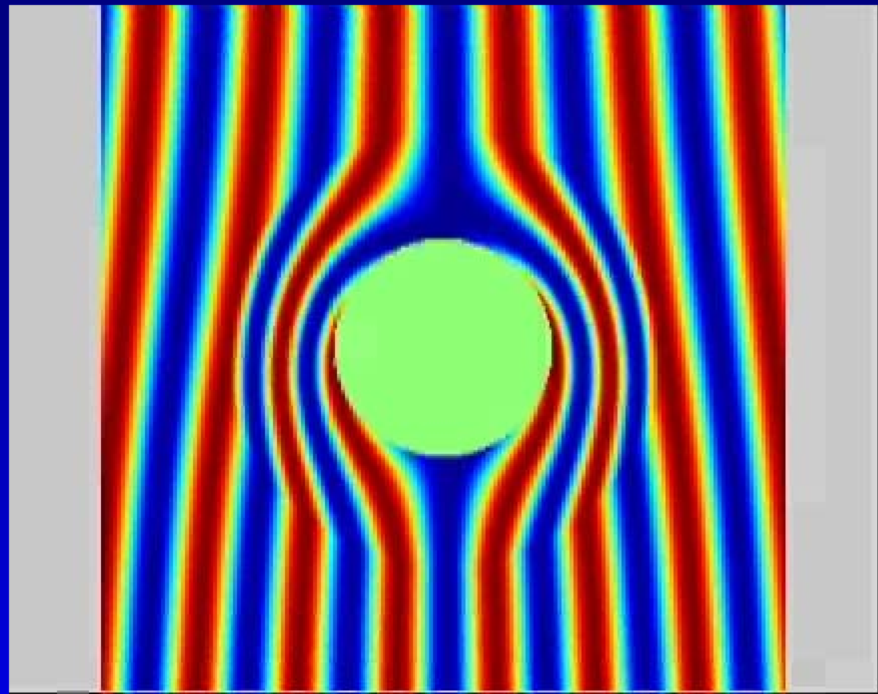


Fig. (g)

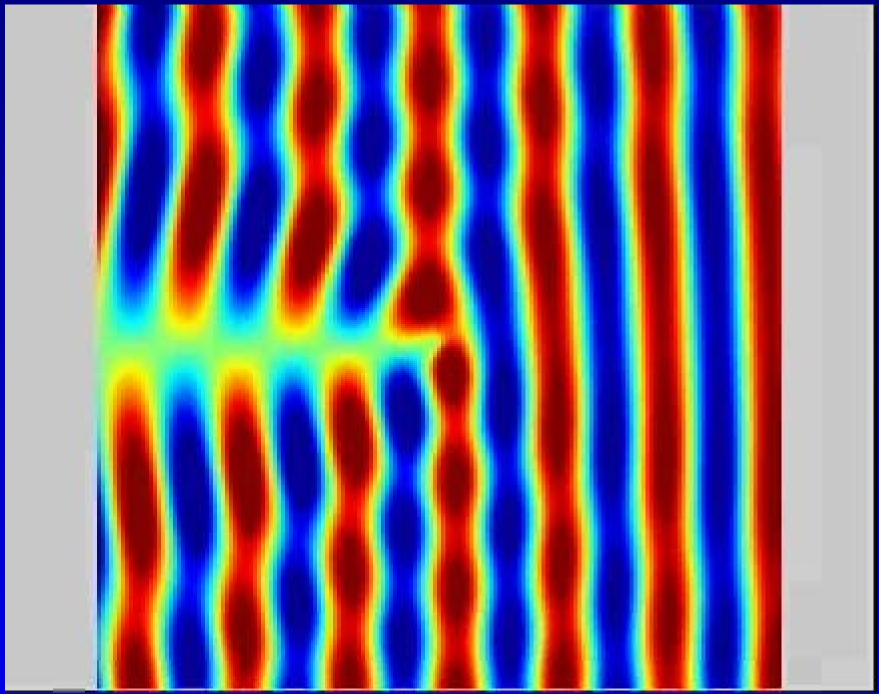


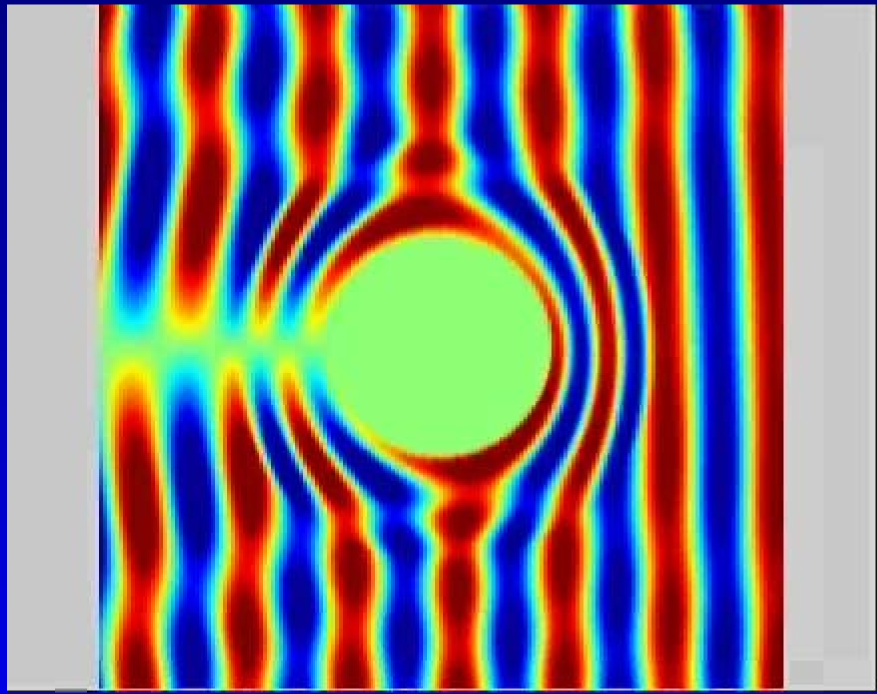
The cloaking of charged matter waves. Fig. (c) shows the matter waves are incident from the left and leave towards the right under the Aharonov-Bohm effect with μ=Φ/Φ_0=2. Fig. (g) exhibit that the matter waves are perfectly cloaked and guided by the quantum cloak. Outside the cloak, the outgoing waves completely coincide with the patterns shown in heft hand side.

Fig. (d)

$$\mu = \frac{\Phi}{h/q} = 0.5.$$

Fig. (h)

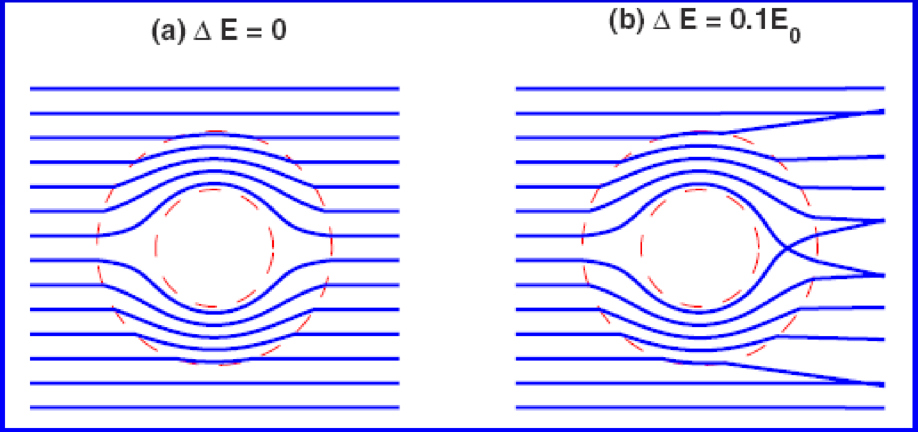




Numerical simulation for cloaking of charged matter waves under the AB effect with fractional flux quantum $\mu=\Phi/\Phi_0=0.5$. The matter waves are incident from the left. Fig. (h) exhibit the matter waves are perfectly cloaked and guided by the quantum cloak. Outside the cloak, the outgoing waves completely coincide with the patterns shown in Fig. (d).

Perfectness of Quantum Cloaking System

The quantum cloaking system is sensitive to the incident energy of particles. If the energy of particles is different from the designed cloaking energy, the matter wave will be scattered, degrading the perfectness of the cloaking system. We analyze this problem relying on the classical limit approach. Since the classical exact solution in general gives the profile of a quantum system, the classical limit affords us some useful insight on how critical the cloaking condition needs to be satisfied. Besides, in contrast with a study through the classical limit, the nonlocal quantum nature of the AB interference effect can be revealed.



The sensitivity of the quantum cloak to the incident energy. Fig. (a) shows the perfect conformal trajectories of the classical particles when they are injected with the designed cloaking energy, where $\Delta E = E - E_0$. Fig. (b) exhibits the departures of paths from the perfect trajectories when the deviation of the incident energy is $\Delta E = 0.1 E_0$.

Conclusion remark for cloaking of matter waves

So far, we have explored the global influence of the Aharonov-Bohm effect on a 2D quantum cloak. It is shown that the efficiency of the quantum cloak is unaltered even when there is a magnetic flux inside the cloaked region, that exhibits the nonlocal influence on the charged paticle.

Owing to the perfect nature of guiding matter waves, the cloak not only cloaks an object hidden in the cloaked region from the incident matter waves but also provides an ideal instrument to test the physical effects of a single bare flux.

The idea may be generalized to the other kinds of global influence of quantum effects such as the AB-EPR, AC effects, and so forth.

Two fundamental questions :

(1) **Does there exists the cloaking transformation of the most fundamental spin - 1/2 paticles?**

(2) **If it exists indeed, how to find it, and how to formulate a cloaking theory for the spinnor fields?**

5. Controlling Spinor Fields

Dirac equation of curved space-time, 4-component formulation:

$$\left\{\tilde{\gamma}^{\mu}(\partial_{\mu}-\Gamma_{\mu})+iM\right\}\Psi=0,$$

where spin connection

$$\Gamma_{\mu}=-\frac{1}{4}\tilde{\gamma}^{\alpha}(\tilde{\gamma}_{\alpha,\mu}-\tilde{\gamma}_{\lambda}\Gamma^{\lambda}_{\alpha\mu}) \text{ with } \tilde{\gamma}^{\alpha}=h^{\alpha}_{(j)}\gamma^{j}.$$

Dirac equation of curved space-time, 2-component formulation:

$$\nabla_{A\dot{B}} P^{A} + iM\bar{Q}_{\dot{B}} = 0,$$

$$\nabla_{A\dot{B}} Q^{A} + iM\bar{P}_{\dot{B}} = 0.$$

Here A, B are the spinor index of the spinor field

$$\Psi = \begin{pmatrix} P_{A} \\ \bar{Q}^{\dot{B}} \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} Q^{A}, & \bar{P}_{\dot{B}} \end{pmatrix}.$$

Dirac equation of curved space-time, spin frame formulation:

$$\nabla_{a\dot{b}} P^{a} + iM\bar{Q}_{\dot{b}} = 0,$$

$$\nabla_{a\dot{b}} Q^{a} + iM\bar{P}_{\dot{b}} = 0.$$

Here the connections between spin frame and spinor are given by

$P_a = \zeta_a^{\ A} P_A, \quad \nabla_{a\dot{b}} P^c = \zeta_a^{\ A} \zeta_b^{\ B} \zeta_C^{\ c} \nabla_{A\dot{B}} P^C,$

$\nabla_{a\dot{b}} P^c \equiv \partial_{a\dot{b}} P^c + \Gamma^c_{\ da\dot{b}} P^d$

with $\zeta_a^{\ A}$ are the spin frame (dyad).

Dirac equation of curved space-time, in terms of spin coefficients:

$$(D+\varepsilon-\rho)F_1+(\bar{\delta}+\pi-\alpha)F_2=imG_1,$$
$$(\Delta+\mu-\gamma)F_2+(\delta-\beta-\tau)F_1=imG_2,$$
$$(\delta+\bar{\pi}-\bar{\alpha})G_1-(D+\bar{\varepsilon}-\bar{\rho})G_2=-imF_2,$$
$$(\Delta+\bar{\mu}-\bar{\gamma})G_1-(\bar{\delta}+\bar{\beta}-\bar{\tau})G_2=imF_1.$$

Proposed by Penrose *et al*. (1962)

ТЕОРЕТИЧЕСКАЯ
И МАТЕМАТИЧЕСКАЯ
ФИЗИКА
Том 70, № 2
февраль, 1987

О КРИТЕРИЯХ РАЗДЕЛИМОСТИ ПЕРЕМЕННЫХ В УРАВНЕНИИ ДИРАКА В ГРАВИТАЦИОННЫХ ПОЛЯХ

Андрушкевич И. Е., Шишкин Г. В.

Исследуются условия разделимости переменных в уравнении Дирака в гравитационных полях. Доказаны строгие теоремы, указывающие необходимые и достаточные условия разделимости переменных при диагональной калибровке тетрады. Получены операторы, определяющие зависимость волновой функции от разделившихся переменных.

1. Введение. К важнейшим проблемам сегодняшнего дня следует отнести построение теории, объединяющей квантовую теорию и теорию гравитации. Квантовая физика вышла в настоящее время на тот рубеж, где становится насущной проблемой необходимость учета не только релятивистских эффектов, но и влияния гравитации, в частности искривления пространства-времени. С другой стороны, физика гравитации не может обходиться уже без учета квантовых закономерностей, например в исследованиях по релятивистской астрофизике; здесь уместно упомянуть широко известный эффект Хокинга, теоретическое открытие которого коренным образом изменило наши представления о черных дырах.

Последовательное построение объединения квантовой теории и теории гравитации невозможно без тщательного изучения одночастичных состояний, т. е. без исследований точных решений ковариантных обобщений релятивистских волновых уравнений. В этом отношении наибольшие трудности, как известно, возникают при изучении ковариантного обобщения уравнения Дирака (КОУД).

Ввиду сложности уравнения Дирака (это система четырех уравнений в частных производных) уже в специальной теории относительности (СТО) число известных точных решений долгое время оставалось весьма ограниченным. Из них к важнейшим можно отнести следующие: свободное движение [1], частица в кулоновском поле [2–6], частица в однородном магнитном поле [7–9], частица в поле плоской электромагнитной волны [10]. Несомненный прогресс в поисках точных решений уравнения Дирака в СТО наблюдается в последние годы в связи с разработкой новых методов (см., например, [11, 12]).

Возможности выявления точных решений уравнения Дирака оказываются непосредственно связанными с возможностями разделения переменных и выделения соответствующих физически интересных операторов. Можно указать два типа трудностей, возникающих на этом пути в СТО: первый обусловлен самой природой той или иной конкретной задачи,

2. Постановка задачи. Обозначения. Принимаем метрику пространства-времени в виде

(2.1) $ds^2=a_{11}(dx^1)^2+a_{22}(dx^2)^2+a_{33}(dx^3)^2-a_{44}(dx^4)^2,$

где a_{mm} — произвольные положительно определенные функции переменных x^1, x^2, x^3, x^4, и используем диагональную калибровку тетрады

(2.2) $h_\mu{}^i=\mathrm{diag}(a_{11}^{1/2}, a_{22}^{1/2}, a_{33}^{1/2}, a_{44}^{1/2}),$

здесь греческие индексы отнесены к РП, латинские — касательному пространству Минковского (ПМ); те и другие пробегают значения от 1 до 4.

Связь γ-матриц Дирака РП и ПМ ($\bar\gamma$) устанавливается соотношениями

(2.3) $\gamma_\mu=h_\mu{}^i\bar\gamma_i,\quad \gamma^\mu=h^\mu{}_i\bar\gamma^i.$

При этом

(2.4) $[\gamma^\mu, \gamma^\nu]_+=2g^{\mu\nu}I,\qquad g^{\mu\nu}=\mathrm{diag}(a_{11}^{-1}, a_{22}^{-1}, a_{33}^{-1}, -a_{44}^{-1}),$

$[\bar\gamma^m, \bar\gamma^n]_+=2\eta^{mn}I,\quad \eta^{mn}=\mathrm{diag}(1, 1, 1, -1),\quad I=\mathrm{diag}(1, 1, 1, 1).$

Обращаясь к общепринятому определению спинорной связности [23, 24]

(2.5) $4\Gamma_\lambda=g_{\mu\alpha}\{\partial_\lambda h_\nu{}^k\cdot h^\alpha{}_k-\Gamma_{\nu\lambda}{}^\alpha\}s^{\mu\nu},\quad 2s^{\mu\nu}=\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu,$

$\partial_\lambda=\frac{\partial}{\partial x^\lambda},\quad 2\Gamma^\alpha_{\nu\lambda}=g^{\alpha\beta}(\partial_\lambda g_{\beta\nu}+\partial_\nu g_{\lambda\beta}-\partial_\beta g_{\nu\lambda}),$

в нашем случае получаем

(2.6) $4\Gamma_\nu=-(\partial_\lambda g_{\nu\nu}\cdot\gamma^\nu\gamma^\lambda-\partial_\nu g_{\nu\nu}\cdot g^{\nu\nu}),$

где по ν нет суммирования. Переходя к КОУД

(2.7) $\{\gamma^\nu(\partial_\nu-\Gamma_\nu)+m_0\}\Psi=0,$

имеем

(2.8) $\left\{\frac{\bar\gamma^1}{\sqrt{a_{11}}}\left(\partial_1-\frac{\partial_1 a_{11}}{4a_{11}}\right)+\frac{\bar\gamma^2}{\sqrt{a_{22}}}\left(\partial_2-\frac{\partial_2 a_{22}}{4a_{22}}\right)+\right.$

$\left.+\frac{\bar\gamma^3}{\sqrt{a_{33}}}\left(\partial_3-\frac{\partial_3 a_{33}}{4a_{33}}\right)+\frac{\bar\gamma^4}{\sqrt{a_{44}}}\left(\partial_4-\frac{\partial_4 a_{44}}{4a_{44}}\right)+m_0\right\}\Phi=0,$

(2.9) $\Psi=(a_{11}a_{22}a_{33}a_{44})^{-1/8}\Phi.$

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I.E. Andrushkevich, and G.V. Shishkin

Dirac equation in an arbitary curvilinear coordiante system,

described by the metric tensor $(g_{uv}) = \mathrm{diag}(f_1^2(g), f_2^2(g), f_3^2(g))$ can be expressed as

$$\left\{\frac{1}{f_1}\gamma^1\left(\partial_1 - \frac{\partial_1 f_1^2}{4 f_1^2}\right) + \frac{1}{f_2}\gamma^2\left(\partial_2 - \frac{\partial_2 f_2^2}{4 f_2^2}\right)\right.$$
$$\left.\frac{1}{f_3}\gamma^3\left(\partial_3 - \frac{\partial_3 f_3^2}{4 f_3^2}\right) - \gamma^3 E + M_0\right\}\frac{1}{\sqrt{f_1 f_2 f_3}}\Psi = 0.$$

That Dirac equation has the equivalent representation

$$\left\{ \sum_{u=1}^{3} \gamma^{\mu} \frac{1}{E_{uu}} \left(\partial_u - A_u^{\text{eff}} \right) - \gamma^4 + \frac{M_0}{E} \right\} \Psi = 0.$$

in which the vector field

$$\begin{aligned} \mathbf{A}^{\text{eff}} &= \frac{xg'(r)}{2r} \frac{d}{dg} \left(\ln f_1 f_2 f_3 + \ln f_1 \right) \hat{e}_1 \\ &+ \frac{yg'(r)}{2r} \frac{d}{dg} \left(\ln f_1 f_2 f_3 + \ln f_2 \right) \hat{e}_2 \\ &+ \frac{zg'(r)}{2r} \frac{d}{dg} \left(\ln f_1 f_2 f_3 + \ln f_3 \right) \hat{e}_3, \end{aligned}$$

and $(E_{uu}) = \text{diag}\left(Ef_1, Ef_2, Ef_3\right)$, $(M_{uu}) = \text{diag}\left(Mf_1, Mf_2, Mf_3\right)$, the quantity $(M/E) \equiv \left[tr(M_{uu}) / tr(E_{uu}) \right]$ is invariant under arbitrary coordinate transformations.

With $\Psi=(\Phi_1,\Phi_2,\Phi_3,\Phi_4)^T$, the Dirac equation has the explicit coupling expression

$$-i\frac{1}{f_1}(\partial_x - i\partial_y)\Phi_4 + i(x-iy)\left(\frac{g'(r)}{2rf_1}\right)\frac{d}{dg}(3\ln f_1 + \ln f_3)\Phi_4 - i\frac{1}{f_3}\partial_z\Phi_3$$
$$-i\left(\frac{zg'(r)}{2rf_3}\right)\frac{d}{dg}(2\ln f_1 + 2\ln f_3)\Phi_3 - (E-M_0)\Phi_1 = 0,$$

$$-i\frac{1}{f_1}(\partial_x + i\partial_y)\Phi_3 + i(x+iy)\left(\frac{g'(r)}{2rf_1}\right)\frac{d}{dg}(3\ln f_1 + \ln f_3)\Phi_3 + i\frac{1}{f_3}\partial_z\Phi_4$$
$$-i\left(\frac{zg'(r)}{2rf_3}\right)\frac{d}{dg}(2\ln f_1 + 2\ln f_3)\Phi_4 - (E-M_0)\Phi_2 = 0,$$

$$i\frac{1}{f_1}(\partial_x - i\partial_y)\Phi_2 - i(x-iy)\left(\frac{g'(r)}{2rf_1}\right)\frac{d}{dg}(3\ln f_1 + \ln f_3)\Phi_2 - i\frac{1}{f_3}\partial_z\Phi_1$$
$$-i\left(\frac{zg'(r)}{2rf_3}\right)\frac{d}{dg}(2\ln f_1 + 2\ln f_3)\Phi_2 + (E+M_0)\Phi_3 = 0,$$

$$i\frac{1}{f_1}(\partial_x + i\partial_y)\Phi_1 - i(x+iy)\left(\frac{g'(r)}{2rf_1}\right)\frac{d}{dg}(3\ln f_1 + \ln f_3)\Phi_1 - i\frac{1}{f_3}\partial_z\Phi_2$$
$$+i\left(\frac{zg'(r)}{2rf_3}\right)\frac{d}{dg}(2\ln f_1 + 2\ln f_3)\Phi_2 + (E+M_0)\Phi_4 = 0.$$

Conjucture the solution:

$$j = l + \frac{1}{2}: \begin{cases} \Phi_1 = \sqrt{\frac{l+m+1/2}{2l+1}} f(g(r)) Y_{l,m-1/2} \\ \Phi_2 = -\sqrt{\frac{l-m+1/2}{2l+1}} f(g(r)) Y_{l,m+1/2} \\ \Phi_3 = -i\sqrt{\frac{l-m+3/2}{2l+3}} \tilde{f}(g(r)) Y_{l+1,m-1/2} \\ \Phi_4 = -i\sqrt{\frac{l+m+3/2}{2l+3}} \tilde{f}(g(r)) Y_{l+1,m+1/2} \end{cases}, l = 0, 1, 2, \cdots,$$

$$j = l - \frac{1}{2}: \begin{cases} \Phi_1 = \sqrt{\frac{l-m+1/2}{2l+1}} f(g(r)) Y_{l,m-1/2} \\ \Phi_2 = \sqrt{\frac{l+m+1/2}{2l+1}} f(g(r)) Y_{l,m+1/2} \\ \Phi_3 = -i\sqrt{\frac{l+m-1/2}{2l-1}} \tilde{f}(g(r)) Y_{l-1,m-1/2} \\ \Phi_4 = i\sqrt{\frac{l-m-1/2}{2l-1}} \tilde{f}(g(r)) Y_{l-1,m+1/2} \end{cases}, l = 1, 2, 3, \cdots.$$

$$\frac{d\tilde{f}}{dg}+\left[\frac{g}{2}\frac{d}{dg}(3\ln f_1+\ln f_3)+\frac{g}{2rg'}\right]\frac{1}{g}\tilde{f}$$

$$+\left(\frac{f_1}{f_3}\frac{g}{rg'}\right)\frac{l+3/2}{g}\tilde{f}+\left(\frac{f_1}{g'}\right)(E-M_0)f=0,$$

$$\frac{df}{dg}+\left[\frac{g}{2}\frac{d}{dg}(3\ln f_1+\ln f_3)+\frac{g}{2rg'}\right]\frac{1}{g}f$$

$$-\left(\frac{f_1}{f_3}\frac{g}{rg'}\right)\frac{l+1/2}{g}f-\left(\frac{f_1}{g'}\right)(E+M_0)\tilde{f}=0.$$

The conditions of invisibility cloaking of the spinor fields are

$$\frac{df}{dg}+\left[g\frac{d}{dg}(3\ln f_1+\ln f_3)+\frac{g}{rg'}\right]=1,\ \left(\frac{f_1}{f_3}\frac{g}{rg'}\right)=1,\ \text{and}\ \left(\frac{f_1}{g'}\right)=1.$$

6. Cloaking the Spinor Fields

The conditions of invisibility cloaking of the spinor fields are

$$\frac{df}{dg}+\left[g\frac{d}{dg}(3\ln f_1+\ln f_3)+\frac{g}{rg'}\right]=1,\quad \left(\frac{f_1}{f_3}\frac{g}{rg'}\right)=1,\ \text{and}\ \left(\frac{f_1}{g'}\right)=1.$$

They are solved exactly by

$$f_1(g)=g'(r),\ f_3(g)=g(r)/r,\ \text{and}\ g''=0.$$

$$\frac{df}{dg}+\frac{1+\kappa}{g}f-(E+M)\tilde{f}=0,$$

$$\frac{df}{dg}+\frac{1-\kappa}{g}f+(E-M)\tilde{f}=0.$$

Here $\kappa=\pm(j+1/2)=\pm 1,\pm 2,\pm 3,\cdots.$

The spinor field in the cloaking shell is given by

$$\Psi = \sum_{j=1/2}\sum_{m=-j}^{j}\begin{pmatrix} \sqrt{\frac{k_0(E+M)}{\pi}}\, j_\kappa(k_0 g)\phi^A_{jm}(\theta,\varphi) \\ -i\sqrt{\frac{k_0(E-M)}{\pi}}\, j_{\kappa-1}(k_0 g)\phi^B_{jm}(\theta,\varphi) \end{pmatrix} + \sum_{j=1/2}\sum_{m=-j}^{j}\begin{pmatrix} -\sqrt{\frac{k_0(E+M)}{\pi}}\, j_{|\kappa|-1}(k_0 g)\phi^B_{jm}(\theta,\varphi) \\ -i\sqrt{\frac{k_0(E-M)}{\pi}}\, j_{|\kappa|}(k_0 g)\phi^A_{jm}(\theta,\varphi) \end{pmatrix},$$

where the spherical spinors

$$\phi^A_{jm}(\theta,\varphi) = \begin{pmatrix} \sqrt{\frac{j-m+1}{2j+2}}\, Y_{j+1/2,m-1/2} \\ \sqrt{\frac{j+m+1}{2j+2}}\, Y_{j+1/2,m+1/2} \end{pmatrix}, \quad \phi^B_{jm}(\theta,\varphi) = \begin{pmatrix} \sqrt{\frac{j+m}{2j}}\, Y_{j-1/2,m-1/2} \\ -\sqrt{\frac{j-m}{2j}}\, Y_{j-1/2,m+1/2} \end{pmatrix}.$$

The unti-spinor field in the cloaking shell is given by

$$\Psi = \sum_{j=1/2} \sum_{m=-j}^{j} \begin{pmatrix} \sqrt{\dfrac{k_0(||E|-M)}{\pi}}\, j_\kappa(k_0 g)\phi_{jm}^{A}(\theta,\varphi) \\ i\sqrt{\dfrac{k_0(|E|+M)}{\pi}}\, j_{\kappa-1}(k_0 g)\phi_{jm}^{B}(\theta,\varphi) \end{pmatrix} + \sum_{j=1/2} \sum_{m=-j}^{j} \begin{pmatrix} \sqrt{\dfrac{k_0(|E|-M)}{\pi}}\, j_{|\kappa|-1}(k_0 g)\phi_{jm}^{B}(\theta,\varphi) \\ -i\sqrt{\dfrac{k_0(|E|+M)}{\pi}}\, j_{|\kappa|}(k_0 g)\phi_{jm}^{A}(\theta,\varphi) \end{pmatrix}.$$

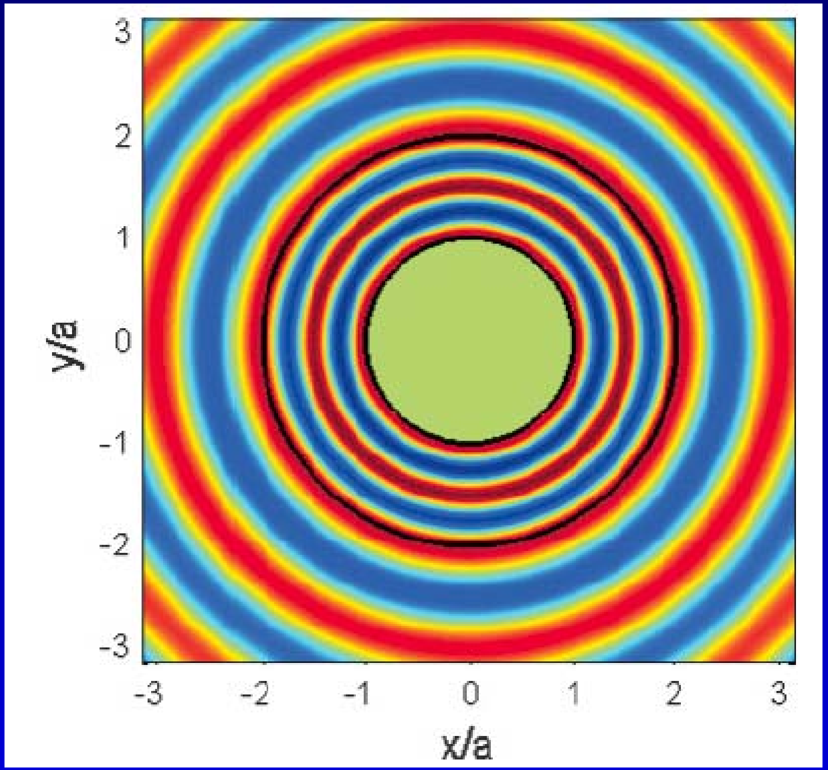
Using the the linear scaling function $g(r)=(r-a)/\lambda$ for a spherical cloaking shell with inner (outer) radius a (b), the energy and mass tensors become $(E_{uv})=\mathrm{diag}(E_{\theta\theta},E_{\varphi\varphi},E_{rr})=\mathrm{diag}(E/\lambda,E/\lambda,E(r-a)/\lambda)$ and $(M_{uv})=\mathrm{diag}(M_{\theta\theta},M_{\varphi\varphi},M_{rr})=\mathrm{diag}(M/\lambda,M/\lambda,M(r-a)/\lambda)$.

The effective vector potential and the corresponding magetic field $\mathbf{B}^{\mathrm{eff}}=\nabla\times\mathbf{A}^{\mathrm{eff}}$ are given by

$$\mathbf{A}^{\mathrm{eff}}=\frac{1}{2}\left(\frac{g'}{g}-\frac{1}{r}\right)\left(\hat{e}_r+\cos\theta\hat{e}_z\right),$$

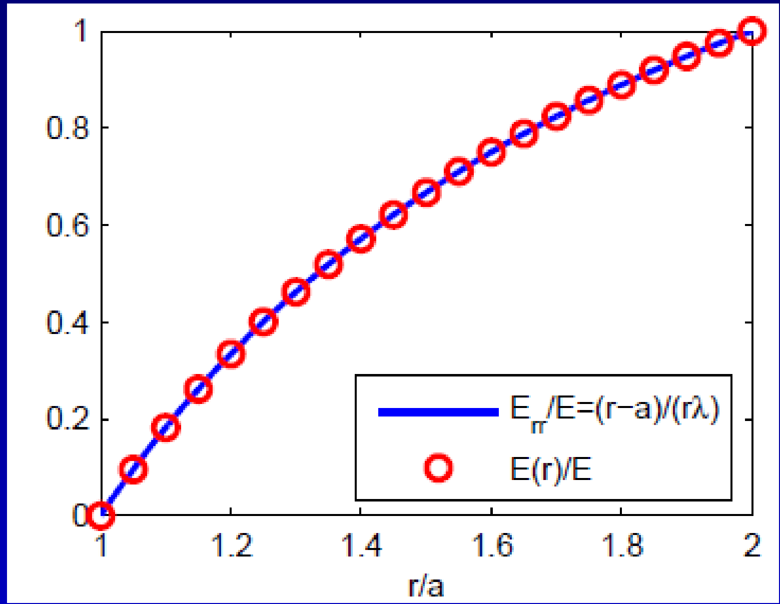
and

$$\mathbf{B}^{\mathrm{eff}}=\frac{a\sin 2\theta}{4r^2(r-a)^2}(3r-2a)\hat{e}_\varphi.$$



The quantum cloaking of the large component of relativistic spin-1/2 particles. The particles are incident along the z axis and the figure is plotted at the plane of the equator,i.e., $\theta=\pi/2$. The figure exhibits the large component as perfectly cloaked and guided by the quantum cloak.

Outside the cloaking shell, the outgoing spherical waves completely coincide with the pattern of the free space.



The modulation of the radial energy of relativistic spin-1/2 particles in the cloaking shell. The solid curve corresponds to the modulated radial energy $E_{rr} = E(r-a)/\lambda r$ with $\lambda = (b-a)/b$, where E is the incident energy of spin particles and we have taken $a=1$ and $b=2$. When the spin-1/2 particles are controlled moving on a 2D plane, such as the fermions on the graphene, the modulation can be achieved by an aggregation of ring-shaped magnets that produces a gradually alterable magnetic field to change the radial energy of spin particles according to $E(r) = \alpha\sqrt{B(r)}$ as shown by small circles, where $\alpha = \sqrt{2e\hbar v_F^2 \,|\, n \,|}$ is a constant, with v_F^2 the Fermi velocity and $|n|$ the Landau levels index.

Using magnets to cloak matter waves

By Chris Lee | Last updated 3 months ago

Cloaking started out as a cool way to demonstrate an insight that Pendry had: you can run the mathematics for electromagnetism in any coordinate system, and the differences between the arbitrary coordinate system and real space can be interpreted as physical properties of the medium through which real light waves propagated. (We'll explain this idea more in a minute.)

This is a fantastic insight, and cloaking is only one of many ideas to spring forth from that fount.

Of course, there are many physics situations that are similar to electromagnetism in terms of their mathematics. Acoustic cloaking joined the bandwagon early. Cloaking matter waves joined the club a couple of years ago. The matter waves considered were rather special, but now matter waves and structures that you can test in the real world have turned up in a *Physical Review A* publication.

Designing the light flow you need

Where to start? Perhaps by returning to the start of the story. The movement of light through a medium is described by a type of equation called the wave equation. It doesn't really matter what the medium is, or what wavelength the light is; this equation will let you predict what the light will do.

But this is just an equation, and the *form* of an equation depends on the coordinate system we use (e.g., what sort of grid we use to divide up space). This is actually quite helpful. For instance, if you want to look at light propagating inside a sphere, you can use a spherical coordinate system that makes the math easy to solve. This makes life simpler. But what if you don't like the easy life and wanted to make things more difficult?

Then you might do what Pendry did and use a coordinate system that is "irregular." Imagine a grid where the squares change size and shape depending where you are. This makes the wave equation a proper nightmare to solve—at least as far as I'm concerned; others probably dream of the challenge—but Pendry had good reason to do this.

You see, if you choose this coordinate system correctly, you can create regions of space where the light waves simply will not go. Instead, they travel around the region and return to their original path as if it had not deviated at all—the perfect optical cloak.

At this stage, such ideas are just so much mental wankery because, although we can imagine stretched and deformed space, we have difficulty *making* them. Pendry's genius was to note that stretched and deformed space in a vacuum looks identical to physical structures with particular optical properties as far as the light is concerned.

In other words, he had just created a tool that lets you design the light flow you want and then calculate backwards to figure out exactly the structure and optical properties that would allow you to create such a light flow. This work has been vindicated with experiments showing that cloaking does actually work.

No invisibility cloak yet

Now the story repeats itself, with De Hone-Lin of National Sun Yat-Sen Univeristy of Taiwan figuring out how to cloak matter waves.

Instead of the wave equation for light, we use the quantum mechanical wave equation for particles that have spin half—electrons and protons are spin half particles and fit the bill here—and are moving at a fair clip.

Spin—a type of angular momentum similar in properties to, but in no way physically the same as, the angular momentum associated with a spinning top or the Earth's rotation—is important to the story only insofar as it makes the math more difficult, but also far closer to something physically realizable.

Hone-Lin showed that to cloak an object from a stream of electrons, for instance, one needed to vary the electrons' effective mass and energy *and* have some kind of vector field present. Now, if one were to think of something like neutrons, this would be pretty difficult to achieve, but for electrons this appears to be feasible.

Hone-Lin points out that the effective mass and energy of an electron in graphene (a single layer of graphite) can be varied by a magnetic field, which just also happens to be a vector field. He concludes that a cloaked region of graphene could be created by placing concentric rings of magnets with different strengths around the region.

So, what are the applications? On this point Hone-Lin has some interesting suggestions. One of the interesting questions in materials science is often how strongly particles with spin interact; being able to cloak and uncloak spins at will could make these measurements easier. It could also provide new tests for nonlocality and entanglement in quantum systems. This should herald more good times for the graphene community.

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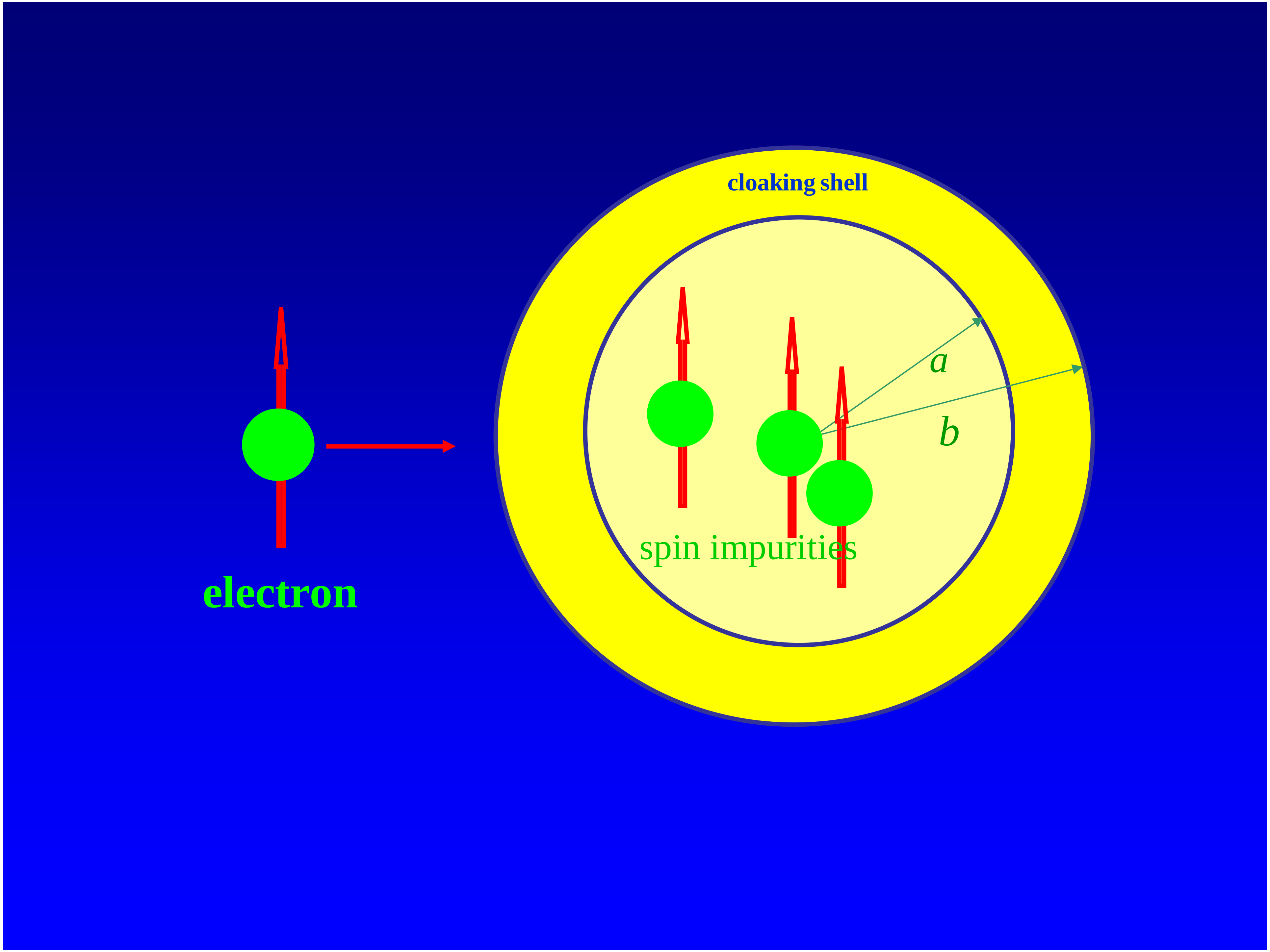
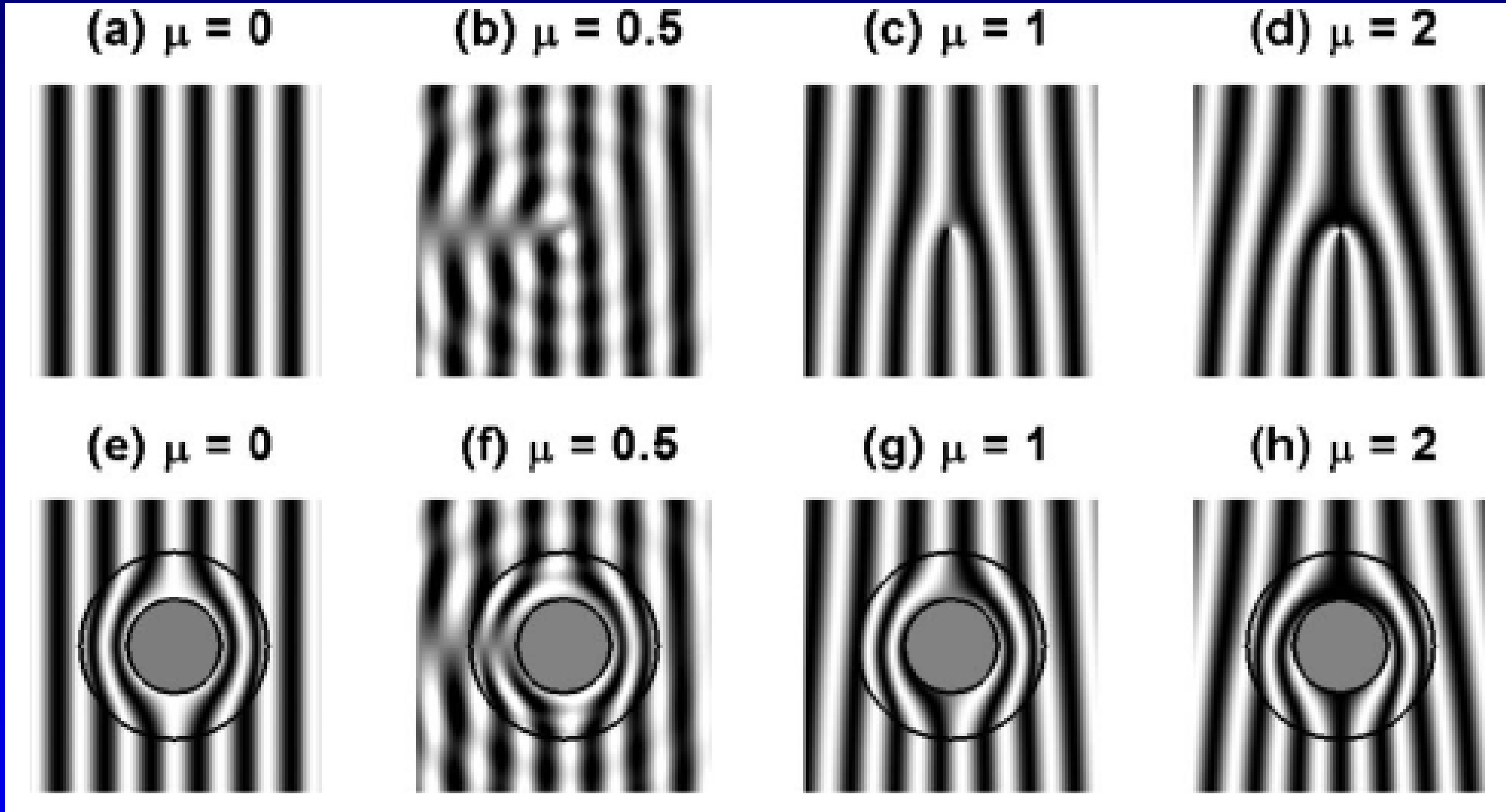
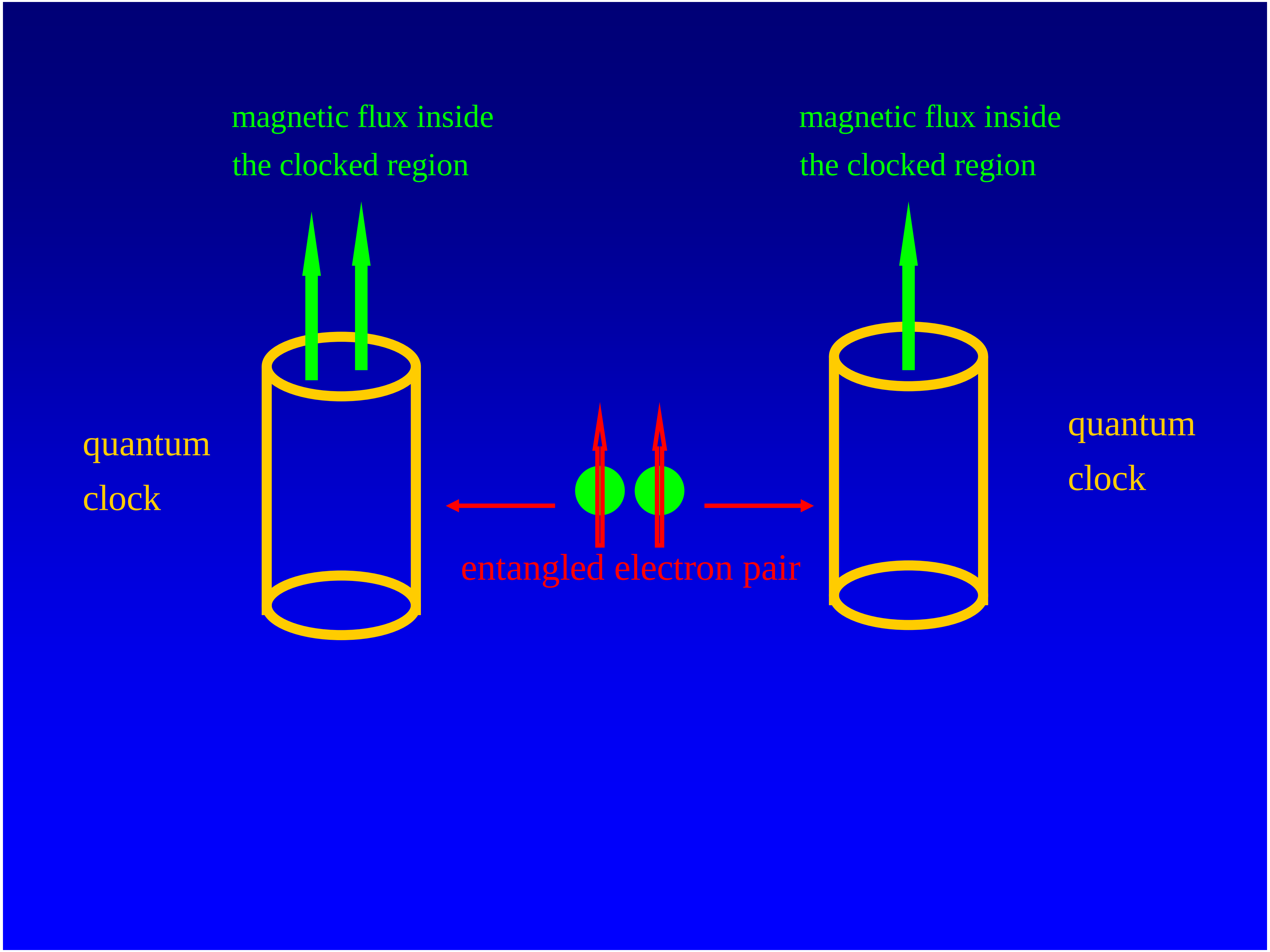
7. Applications

(1) Spin cloak provides a novel method to guide the spin-1/2 matter waves. As shown in the foregoing sections, the quantum cloak can perfectly guide the spin-1/2 matter waves by controlling theirenergy and mass moving in an effective vector field. One of the benefits of the method is that there is no leakage of the guided spin waves in the specific cloaked region. Thus, the quantum cloak actually guarantees the decuple of the spin waves within the cloaked region. This perfect property may be very useful for accurately revealing and measuring some spin-spin interactions by turning the cloak on and off such as the interaction between an electron and a spin impurity, or even among an electron and some spin impurities concealed in the cloaked region.


cloaking shell
a
b
spin impurities
electron

(2) The quantum cloak of spin waves provides an ideal apparatus for testing the nonlocal interferences of matter waves from the global effects of gauge fields. The nonlocal physical phenomena such as from the Aharonov-Bohm (AB) effect, hybrid AB and Einstein-Podolsdy-Rosen (AB-EPR) experiment, and so forth all exhibit nonlocal interferences of matter waves from a remote magnetic field. In these experiments, the allowed propagating paths of the matter waves must circumscribe a region of space within which the magnetic field is confined and from which the matter waves are excluded. Since the quantum cloak is able to perfectly guide the matter waves through the cloaking shell without any leakage, a quantum cloak that conceals a magnetic flux inside the cloaked region provides an ideal setup for testing these nonlocal physical phenomena.

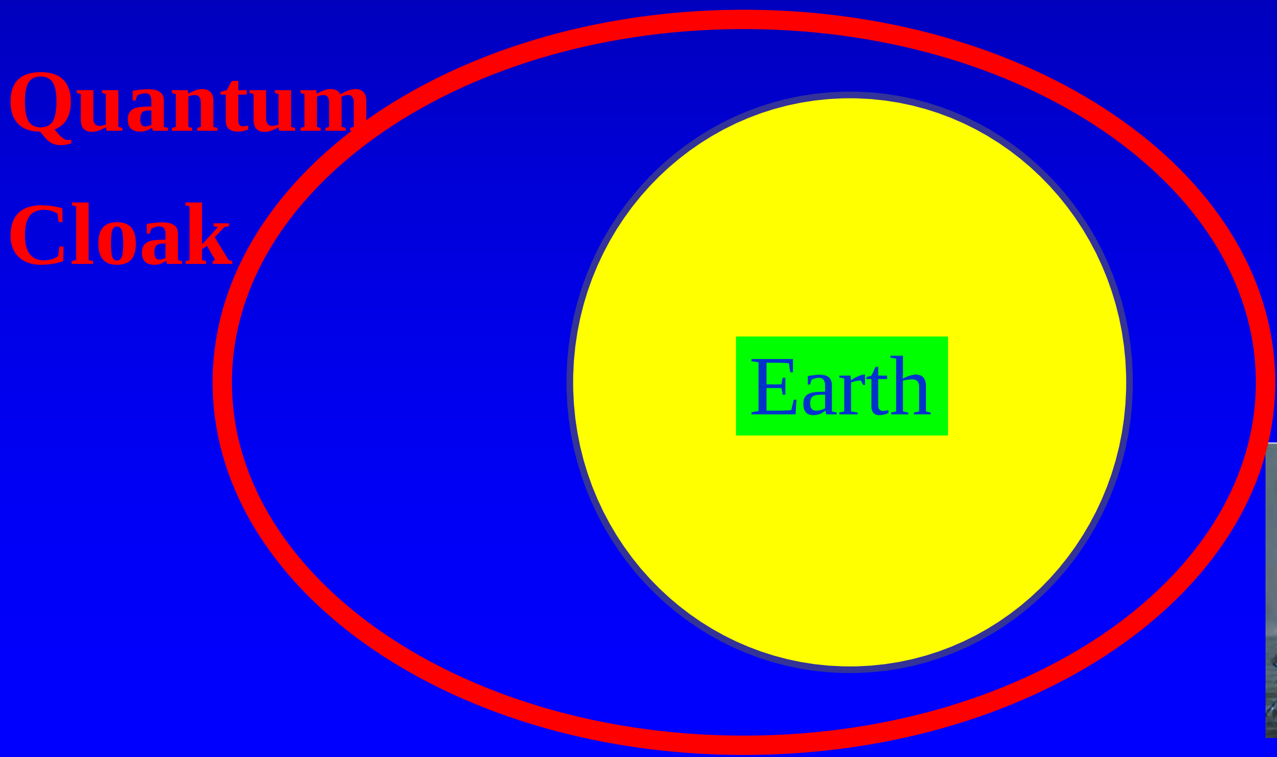

(a) μ = 0
(b) μ = 0.5
(c) μ = 1
(d) μ = 2
(e) μ = 0
(f) μ = 0.5
(g) μ = 1
(h) μ = 2


magnetic flux inside
the clocked region
magnetic flux inside
the clocked region
quantum
clock
quantum
clock
entangled electron pair

(3) The present cloaking construct provide a prototype to controlling a spin-$\frac{1}{2}$ matter wave at will. Since there is no restriction on the coordinate transformation for the Dirac equation, it is then possible, through the desired coordinate transformation, to control the spin wave porgagating along the specified paths. For example, a transformation that lead to two propagating channels of a spin wave with different lengths, would be an ideal setup to exhibit the interesting self-interference phenomenon of the spin wave.

(4) protect human being from bombing of high energy cosmic rays. Cosmic rays include different kinds of high energy particles in which charged ones can be deflected by the magnetic field of the Earth while the uncharged particles such as neutron would go through the aerosphere and seriously injure the human body. A spin cloak with wide enough band could be used to guide the matter waves and protect the human being. Moreover, if the outer shell of buildings such as the international space station is built with the cloaking matter, the cloak would protect the space station from their injuries.

(5) protect the Earth from the bombing of asteroids. All matter are composed of spin-1/2 particles. If we could make a large enough quantum cloak with frequencies sufficient to cover the matter of asteroids and put it on the orbit of the Earth, it would guide the asteroids bypassing the goals of bombing, and the Earth can be saved from the disaster.

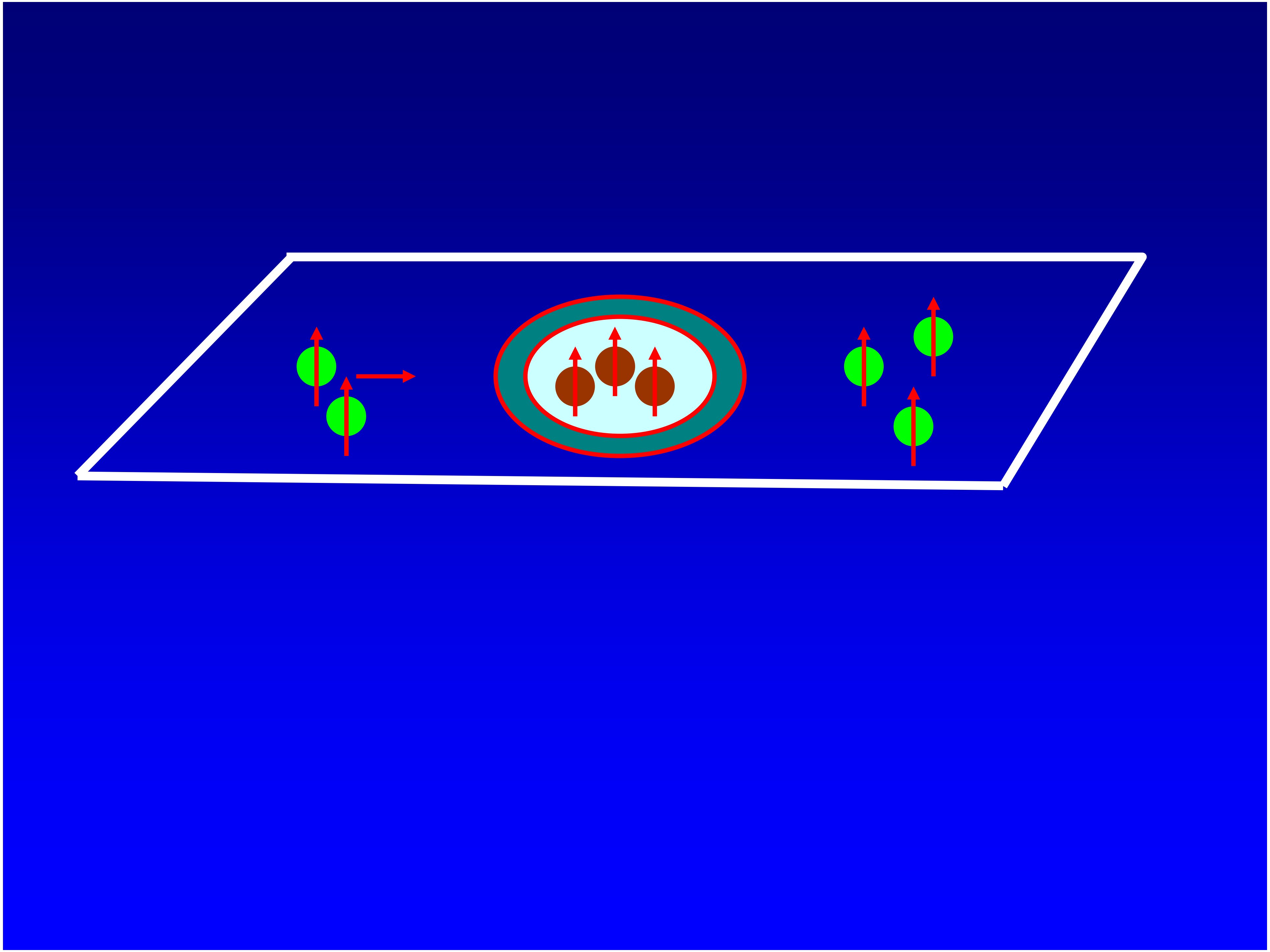


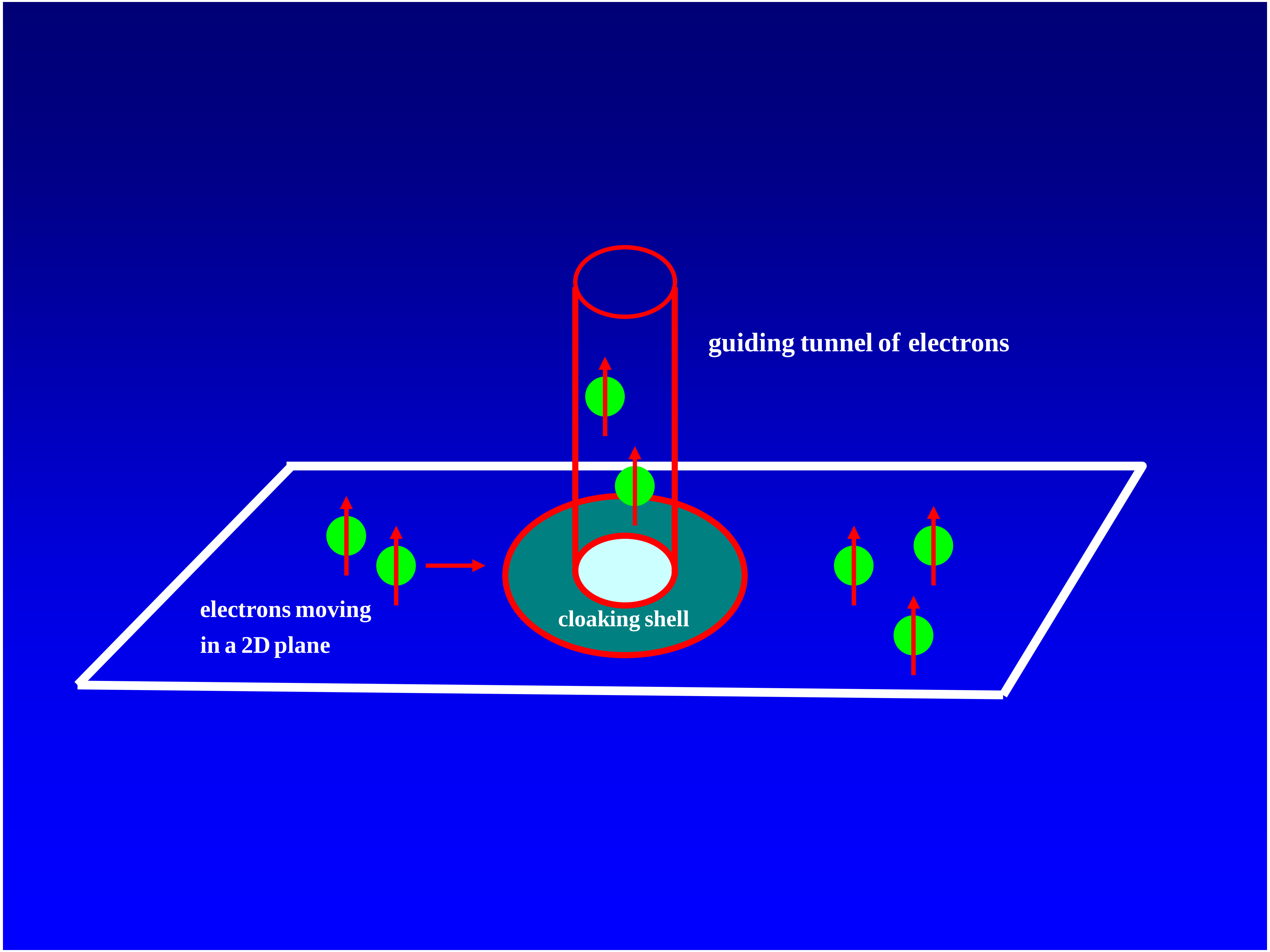
(6) make a wall-through clothes.

Since matter are constructed from spin-1/2 particles, a quantum cloak that can cloak all frequencies of spin-1/2 matter waves is equivalent to a sci-fi "clothes". When one wears the sci-fi clothes and go through a wall, the matter of wall would perfectly flow through the cloaking shell without leaving any marks. Thus, such a cloak is a sci-fi wall-through clothes.

I would like to thank Pi-gang Luan for introducing me to this interesting area.

Thank you!




guiding tunnel of electrons
electrons moving
in a 2D plane
cloaking shell