國立清華大學

Inter-satellite Laser Interferometry for Space Science Missions

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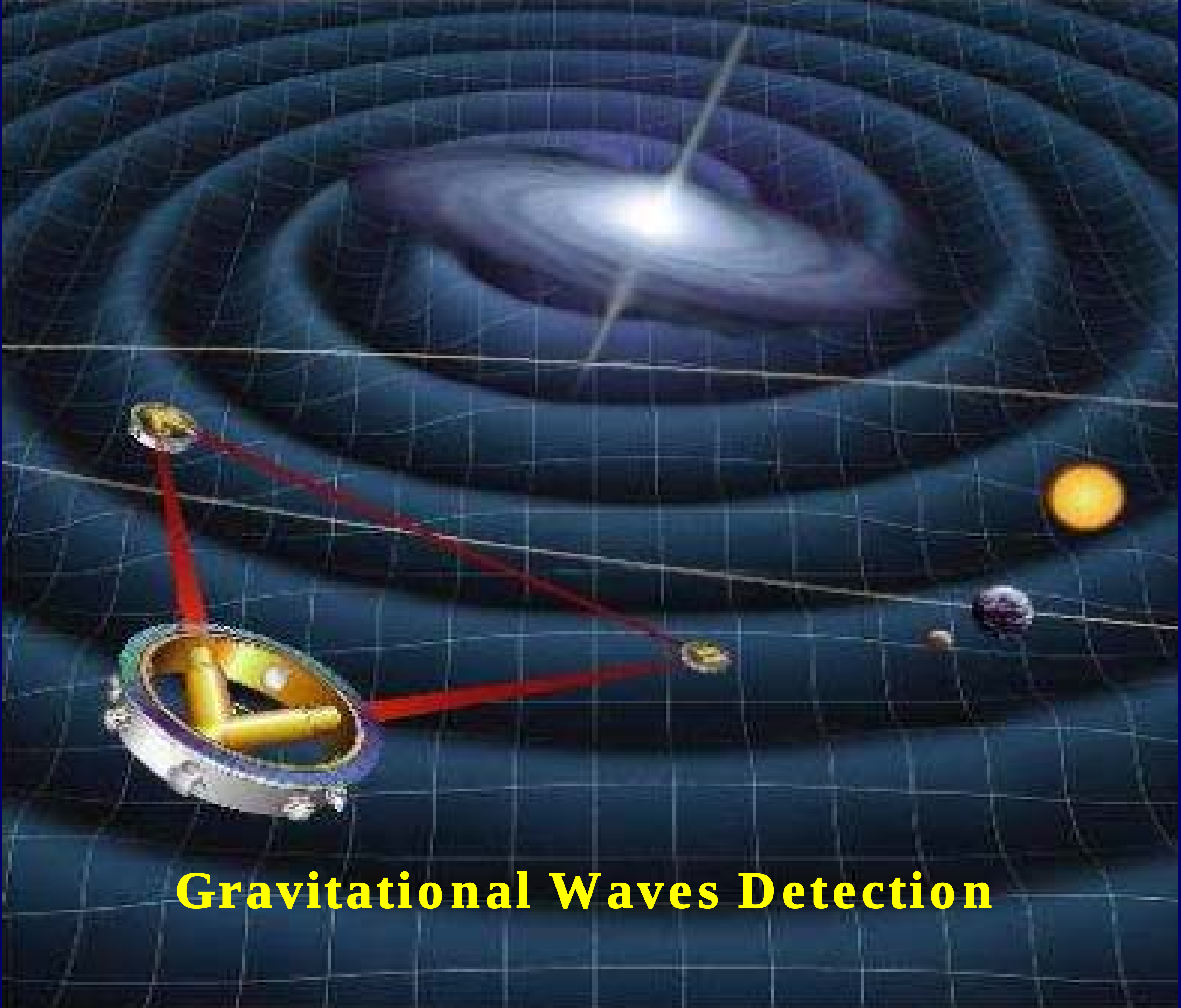
Outline

1. Gravitational waves detection
2. Earth gravity recovery
3. Development roadmap
4. Conclusion

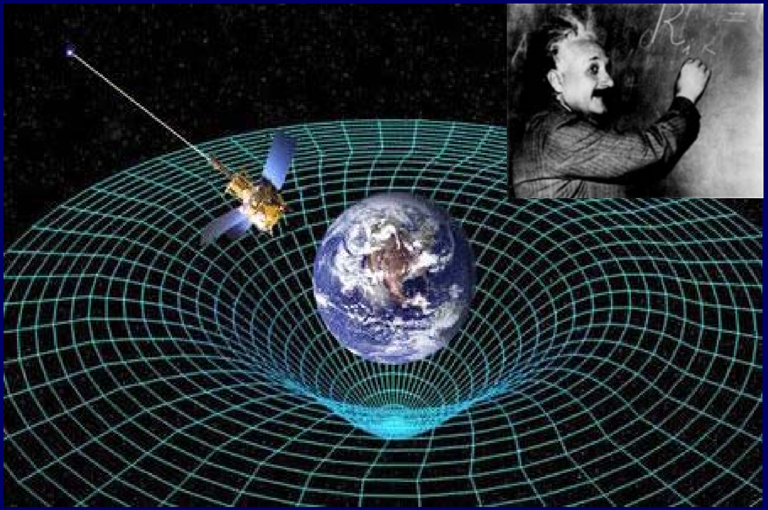


Gravitational Waves Detection

Basic Ideas of General Relativity

Matter tells space how to curve;
space tells matter how to move.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu}$$

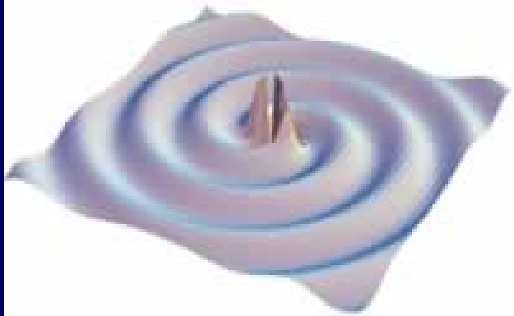


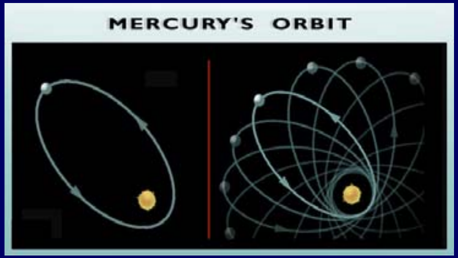
What we feel as gravity arises from the curvature of spacetime. Mass causes spacetime to curve, and the curvature determines the paths of freely moving masses.

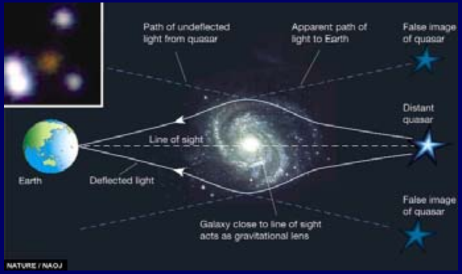
Predicted Effects of GR:

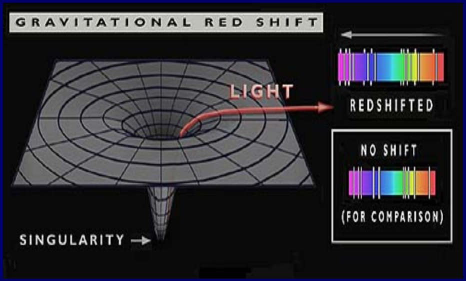
- Orbit precession in the perihelion of planets
- Deflection of light by solar gravity
- Redshift of spectral lines
- Frame dragging
- Gravitational radiation

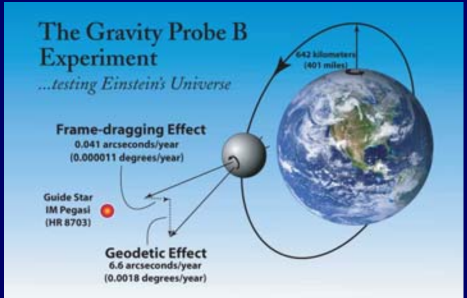
  （gravitational waves）





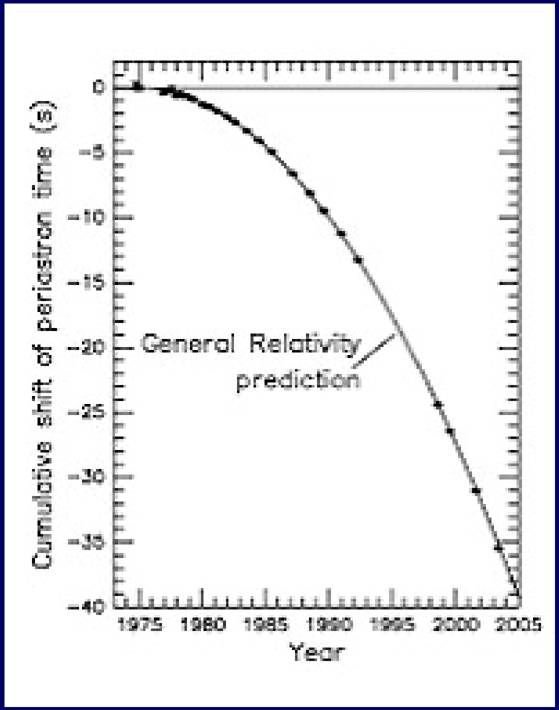






Indirect Evidence of GW

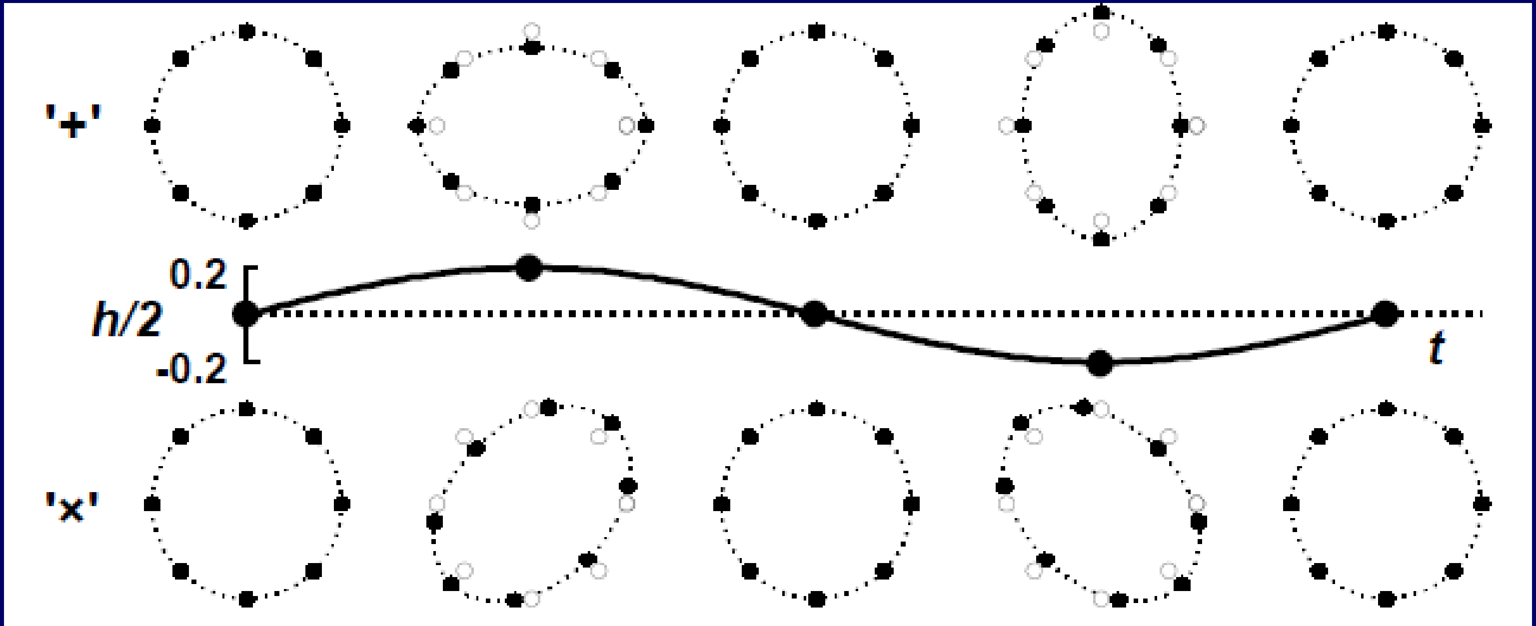
In 1974, PSR B1913+16 was discovered by R. A. Hulse and J. H. Taylor, Jr., of Princeton University

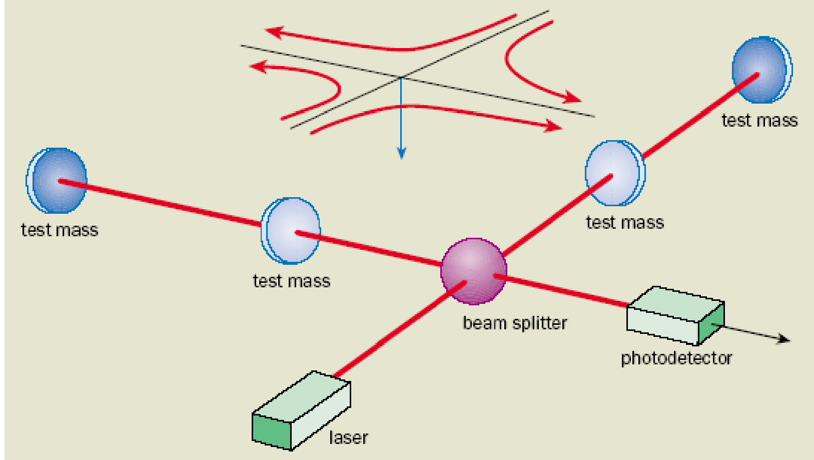


PSR B1913+16 is a pulsar in a binary star system, in orbit with another neutron star around a common center of mass.

1993 Nobel Prize in Physics.

Direct GW Detection using Laser Interferometers





Gravitational waves will change the light travel time of both arms (i.e. spatial strain) asymmetrically. Hence a phase shift between two beams could be measured.

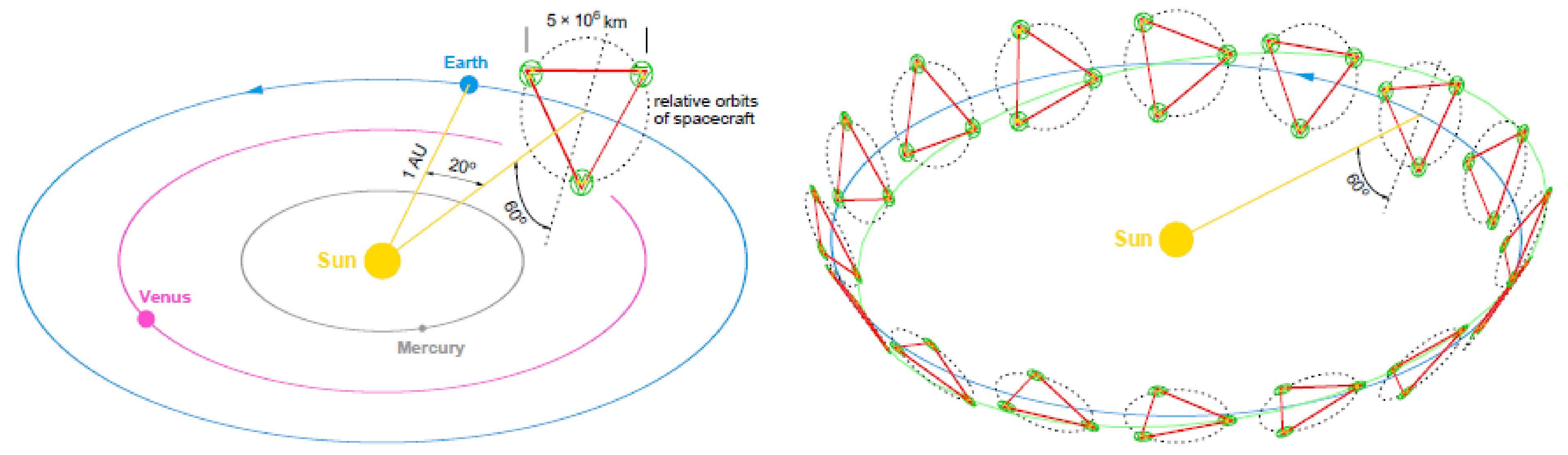
LIGO – Laser Interferometer Gravitational-waves Observatory

LIGO Hanford Observatory

LIGO Livingston Observatory

Baseline: 4 km
Strain sensitivity: $10^{-23}/\mathrm{Hz}^{1/2}$
Sensing frequency: 40 ~ 1000Hz

LISA: Laser interferometer Space Antenna



Detection of GW with laser interferometry & drag-free control

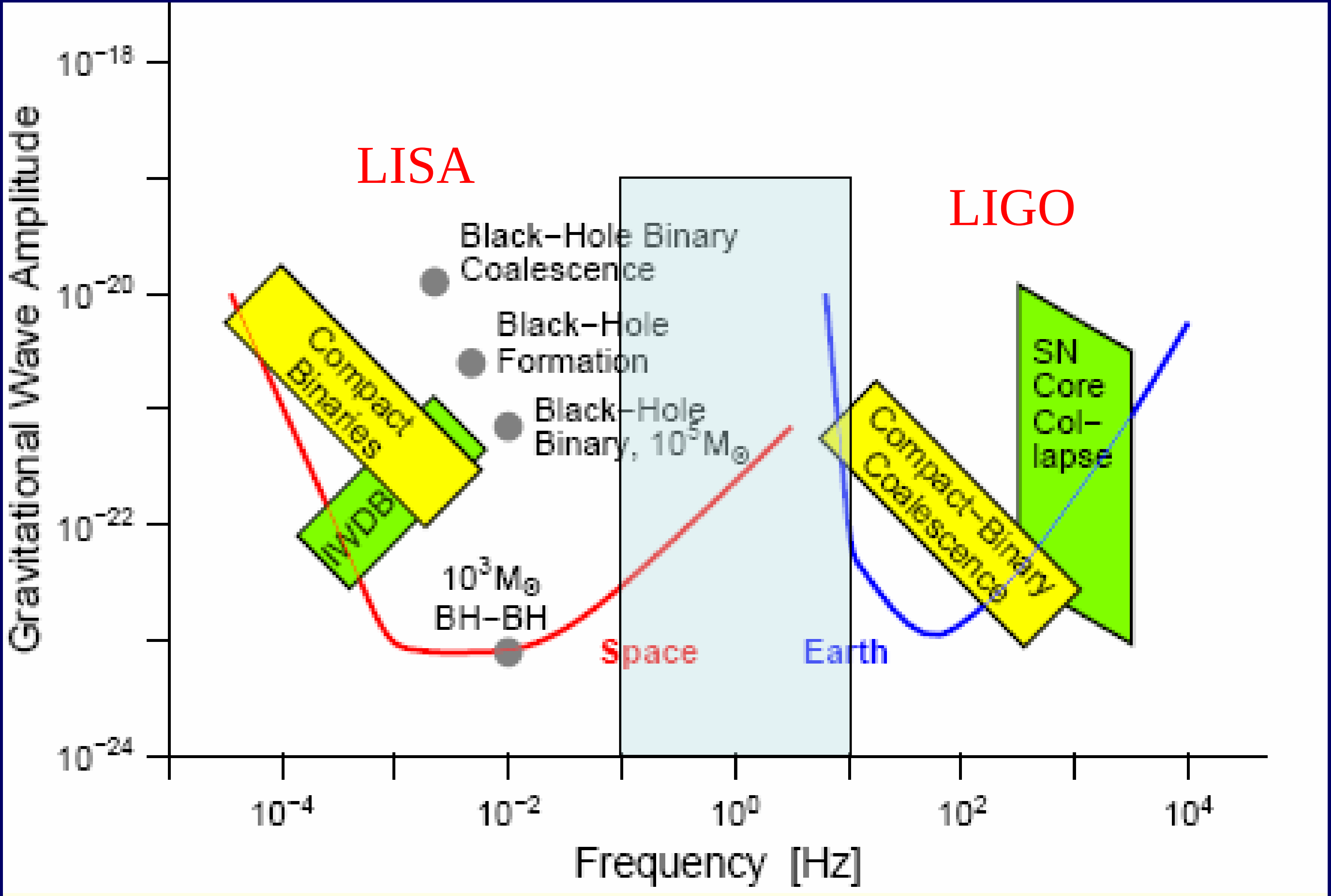
Sources:

- White dwarf binaries
- Binaries of super massive black holes
- Extreme-mass-ratio spirals

Sensitivity $h < 4\times10^{-21}/\mathrm{Hz}^{1/2}$

Freq. GW f: 10^{-4}~0.1Hz

Scientific Goals of Space-based & Ground-based GW Detections



Gravitational Waves Detection

around 0.1 Hz

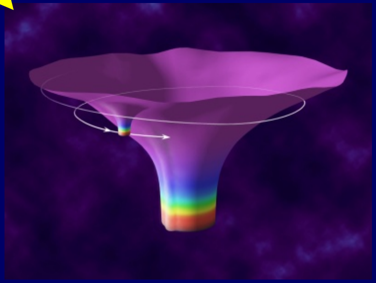
Gravitational Wave Sources
in the Deci-Hertz Window

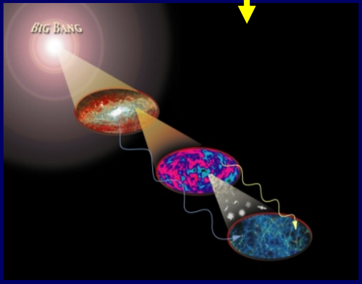
Astrophysical:

- Intermediate-mass black holes (IMBH) binary coalescence
- NS-NS binaries, non-axisymmetric neutron stars
- Supernovae, black hole-white dwarf, black hole-neutron star binaries in spiraling.
- Burst events – parabolic or hyperbolic encounter of a black hole with a star.

Cosmological:

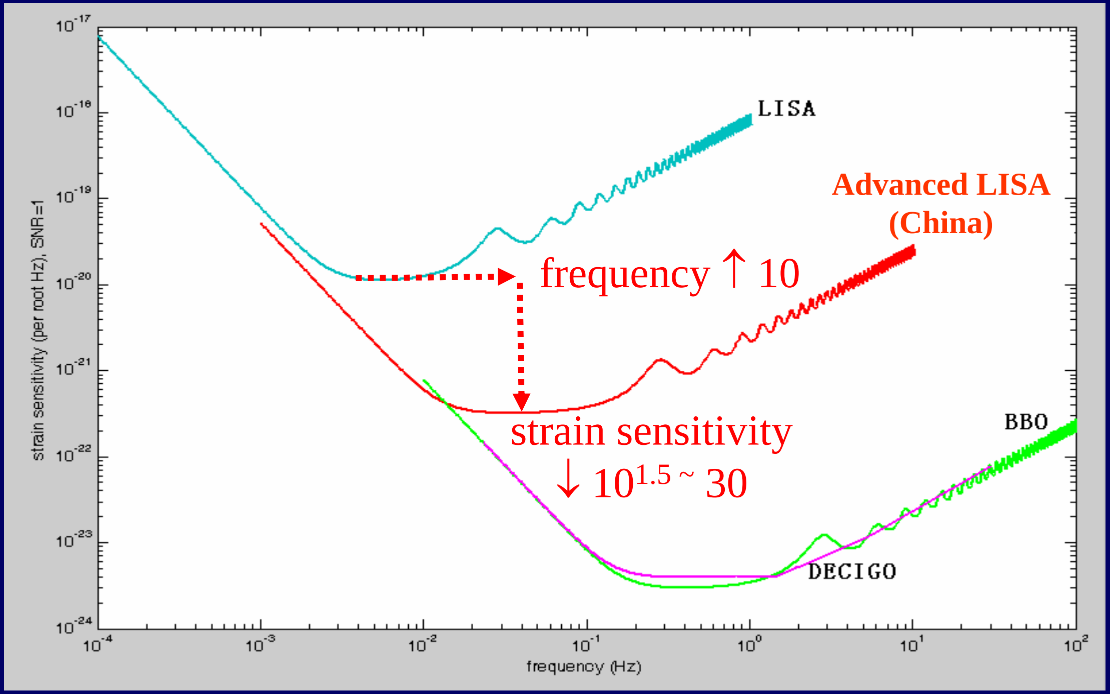
- Primordial Gravitational Wave Background
- Bursts from hypothetical cosmological structures like cosmic string and other topological defects in the early Universe





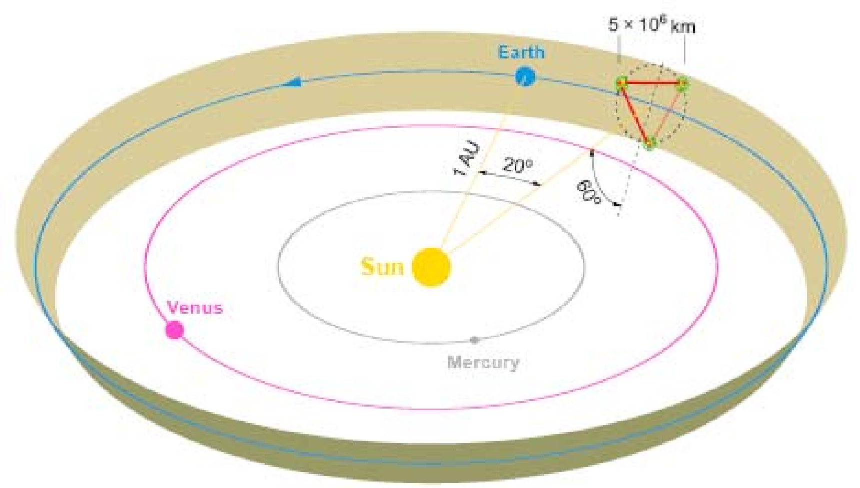
Missions under consideration around 0.1 Hz

- BBO: most sensitive at 1Hz, resolution: 10^{-26}
- DECIGO:
  most sensitive at 0.1Hz



	Frequency range (Hz)	Arm length	Displacement noise (pm/$Hz^{1/2}$)	Acceleration noise ($ms^{-2}/Hz^{1/2}$)
LISA	$10^{-4} \sim 10^{-1}$	5×10^{9}m	40	3×10^{-15}
Adv. LISA	$10^{-3} \sim 1$	5×10^{8}m	0.13	5×10^{-14}

Orbit



HCO: heliocentric orbit

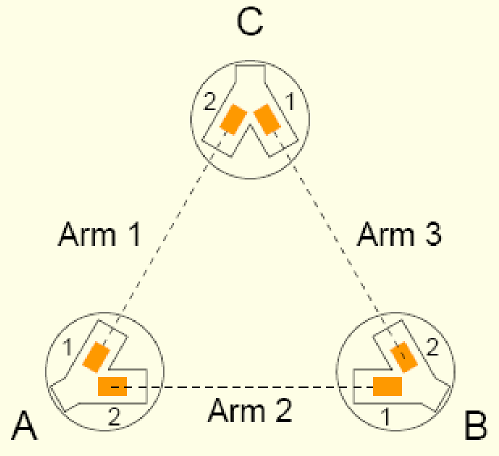
L: 5 million km (5×10^8 m)

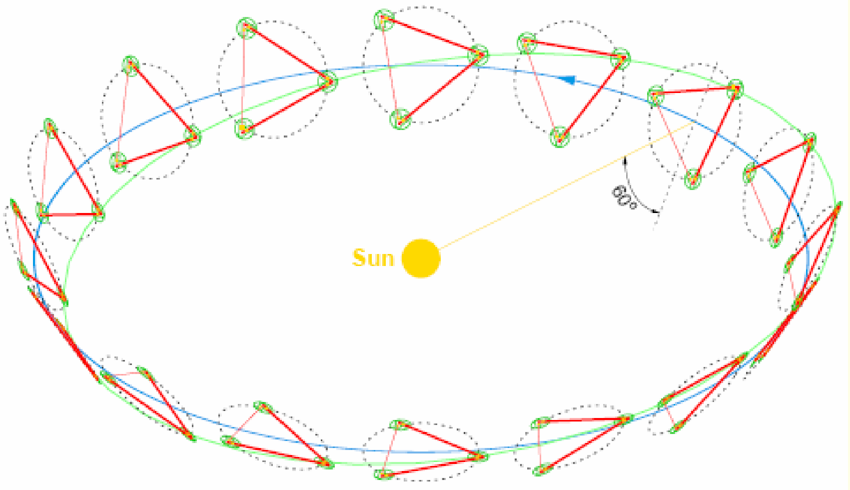
L variance: 5×10^5 m

Configuration: Equilateral Triangle

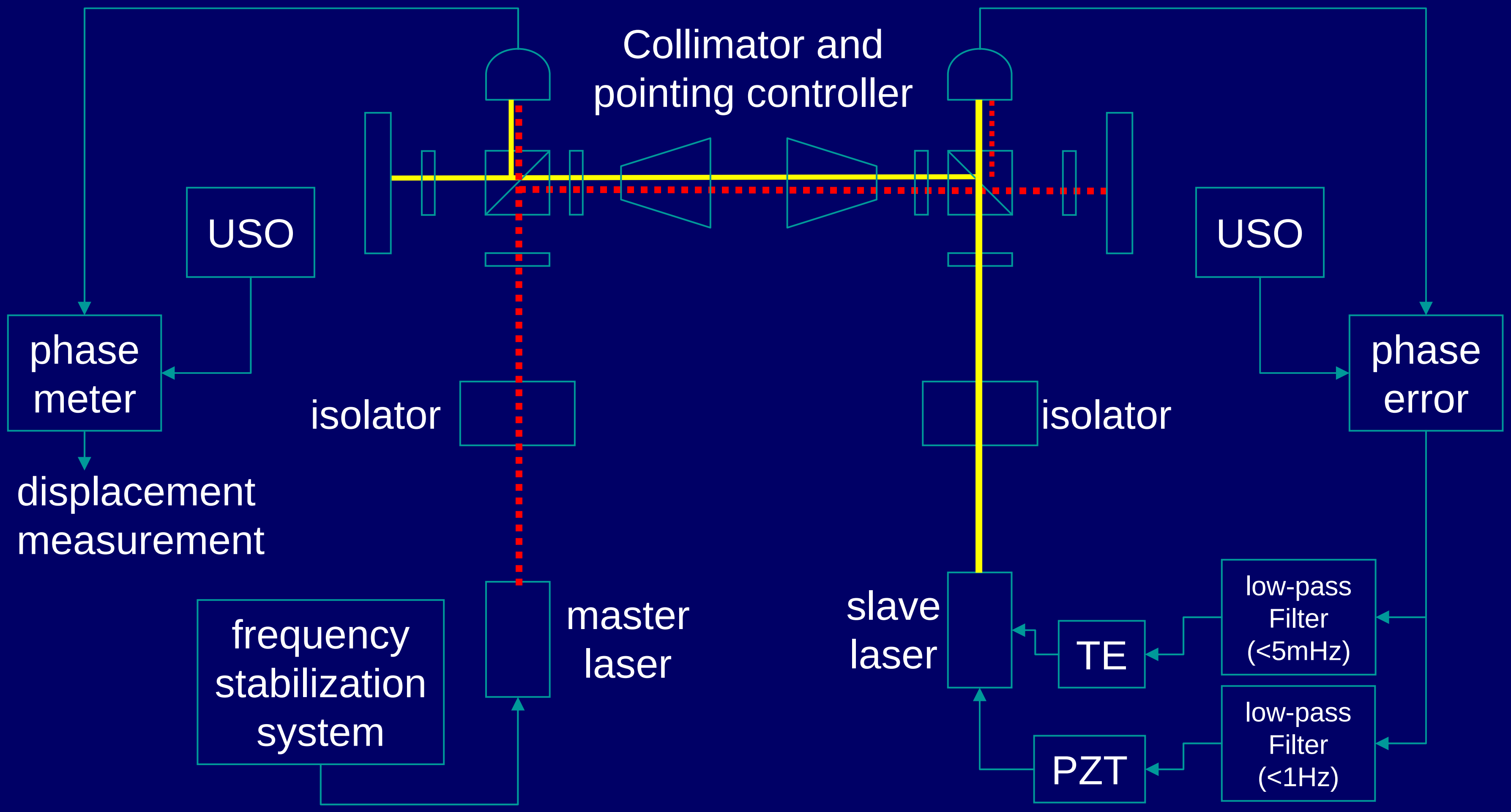
Configuration inclined against ecliptic by 60º (thermal stabilization)

Trailing the Earth by 20º (50 million km)

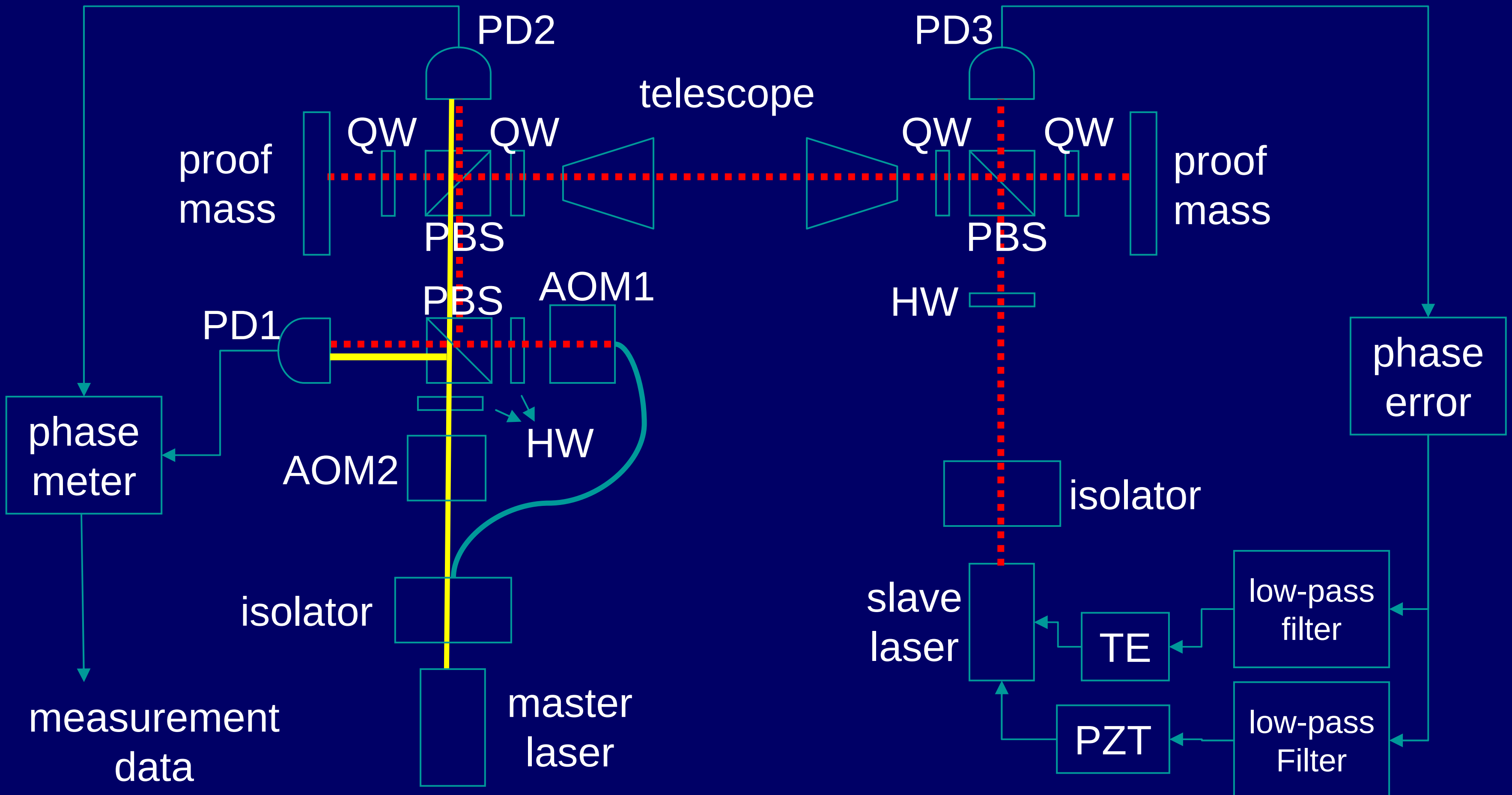




Heterodyne Laser Ranging System
(transponder using heterodyne OPLL)



Heterodyne Laser Ranging System
(transponder using homodyne OPLL)



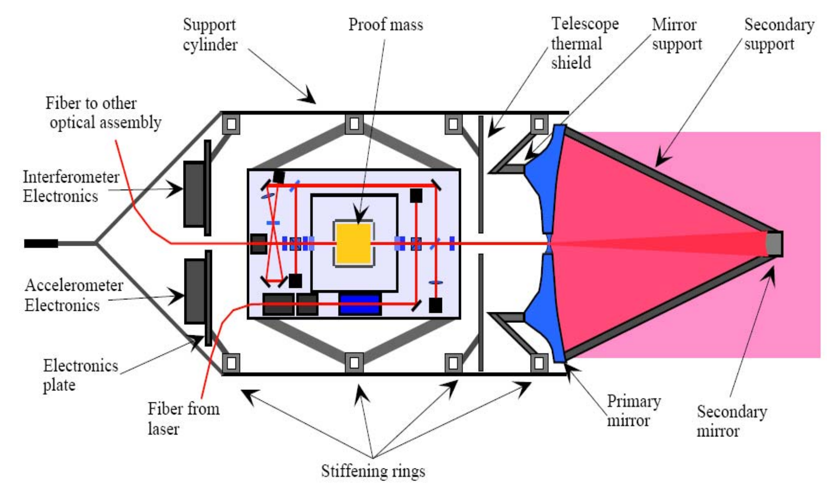
Shot Noise of Phase Measurement

Telescope diameter D: 60cm

Laser power P_0: 12.5W

Laser wavelength λ: 1064nm

Optical efficiency ε: 0.3



$$P_r = \varepsilon^2 P_0 \left(\frac{D^2}{L\lambda} \right)^2 = 5 \times 10^{-7} W$$

(power attenuation ~ 4×10^{-8})

$$\Delta x_{shot} = \left(\frac{1}{2\pi} \right) (hc)^{1/2} \left(\frac{\lambda}{P_r} \right)^{1/2} = 1 \times 10^{-13} m / \sqrt{Hz}$$

Error Sources of Optical Bench

Cyclic error: < 2 nm

Thermal drift:

n (glass) = 1.5

α_1(thermal expansion coefficient) = 10^{-6}

α_2(thermal coefficient of refractive index) = 10^{-6}

d (OPL in glass) ~ 10 cm

$\Delta T = 1\ \mu\mathrm{K}/\sqrt{\mathrm{Hz}}$

$$\Delta x = (n-1)\times\alpha_1\times d\times\Delta T + n\times\alpha_2\times\Delta T\times d \approx 0.2\, pm/\sqrt{Hz}$$

It’s better to use ultra-low expansion materials.

Ultra-Stable Optical Bench

Key technologies：

- Optical layout
- Fiber coupling
- ULE material machining

  ULE，Zerodur

  （$\alpha=10^{-7}\sim10^{-8}/K$）
- Silicate bonding

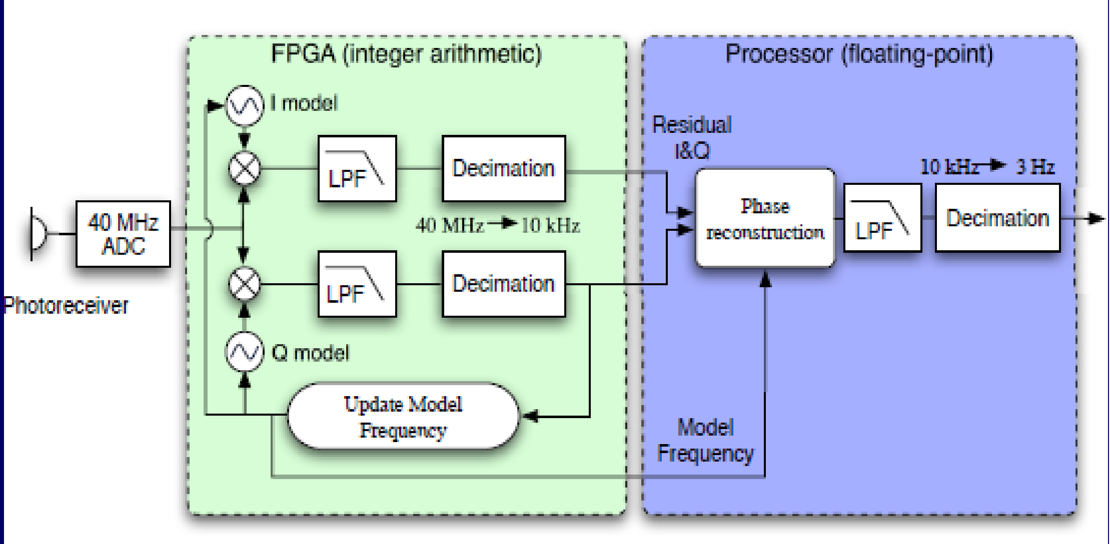
  stress：10kPa

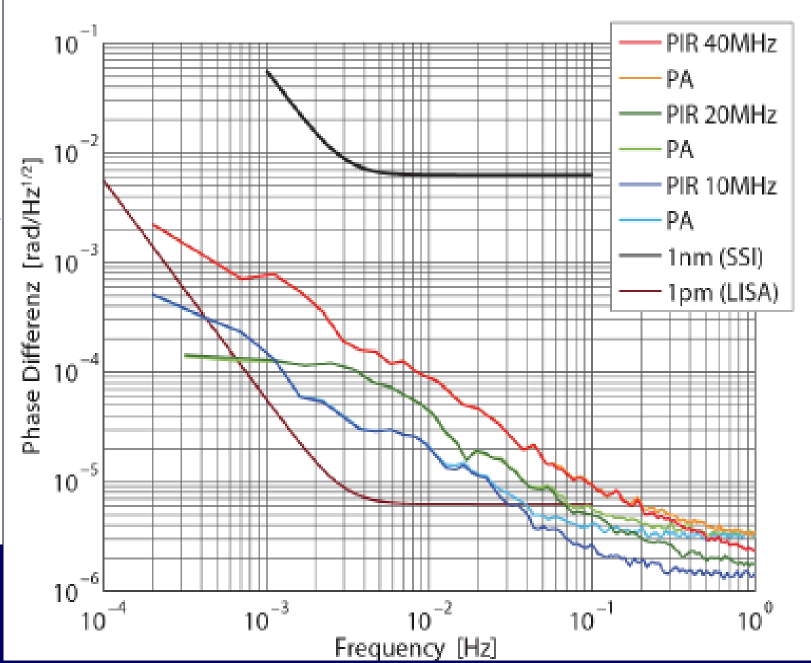
  tilt angle：10μrad

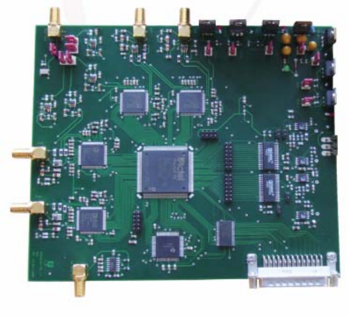
  position error：10μm

Resolution of Phase-meter

$$\Delta x = 2.4\times 10^{-14}\, m/\sqrt{Hz} \boxed{\Rightarrow \Delta\phi = 1.4\times 10^{-7}\, rad/\sqrt{Hz}}$$







FPGA-based digital phasemeter

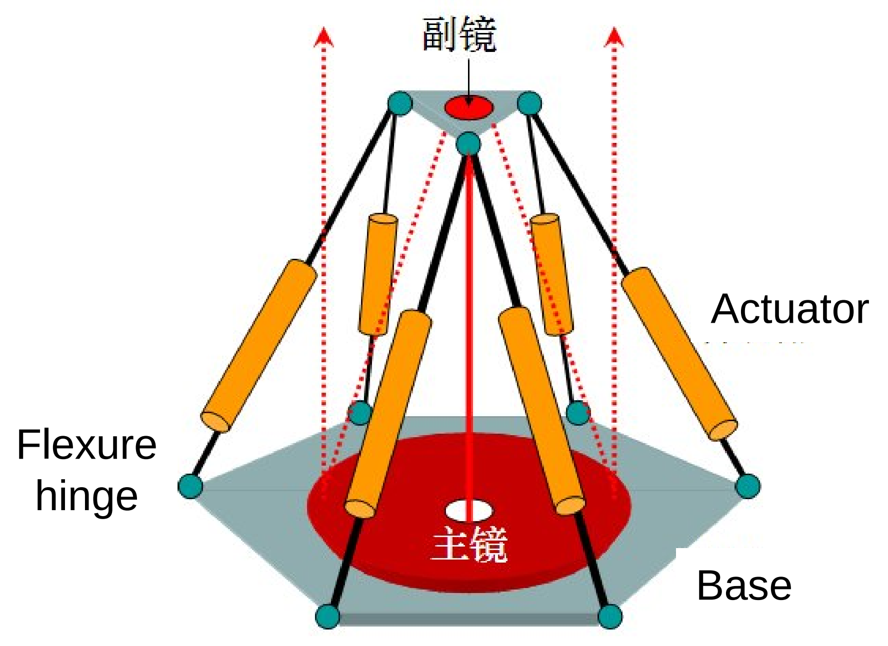
Pointing Offset and Jitter Errors

$$\Delta x_{\text{pointing}} \approx \delta \frac{\theta_{dc}\theta_{jitter}}{\theta_{div}^{2}} = \frac{\lambda}{10} \times \frac{\theta_{dc}\theta_{jitter}}{(\lambda / D)^{2}}$$

$$\theta_{dc}\theta_{jitter} \approx 10 \times \frac{\lambda \Delta x}{D^{2}} = 6 \times 10^{-19} rad^{2} / \sqrt{Hz}$$

$$\theta_{dc} \sim 3 \times 10^{-9} rad$$

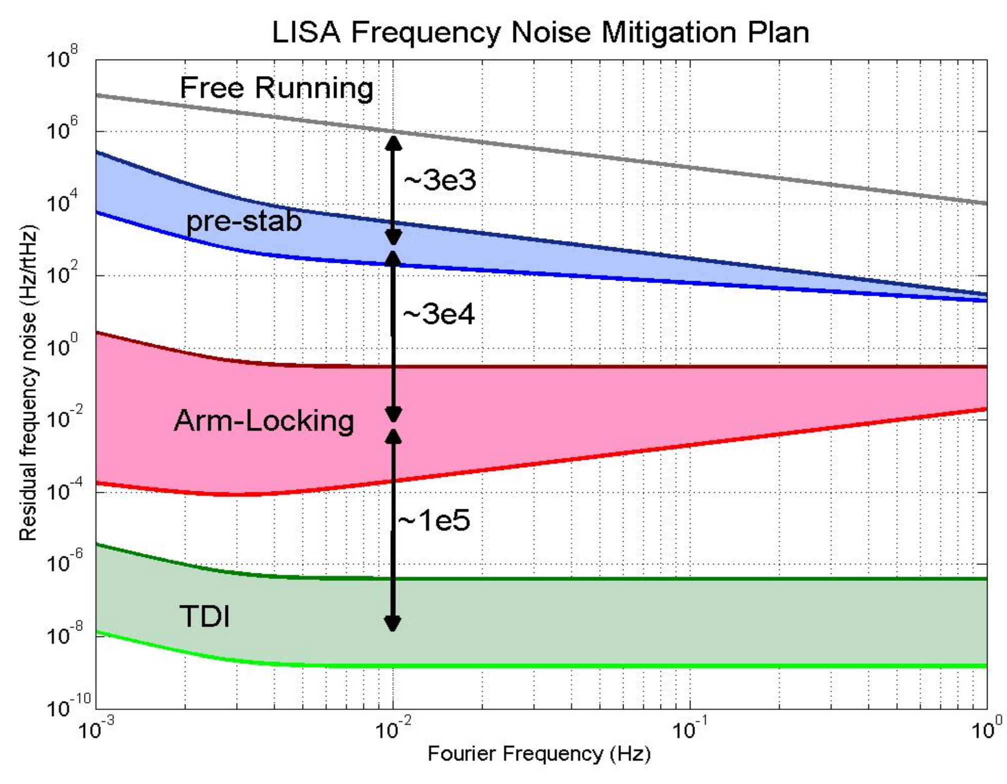
$$\theta_{jitter} \sim 2 \times 10^{-10} rad / \sqrt{Hz}$$



- Z-axis range： 20μm
- Positioning precision： 0.1nm
- Pointing-angle range： 10^{-5}rad
- Pointing-angle precision： 10^{-10}rad

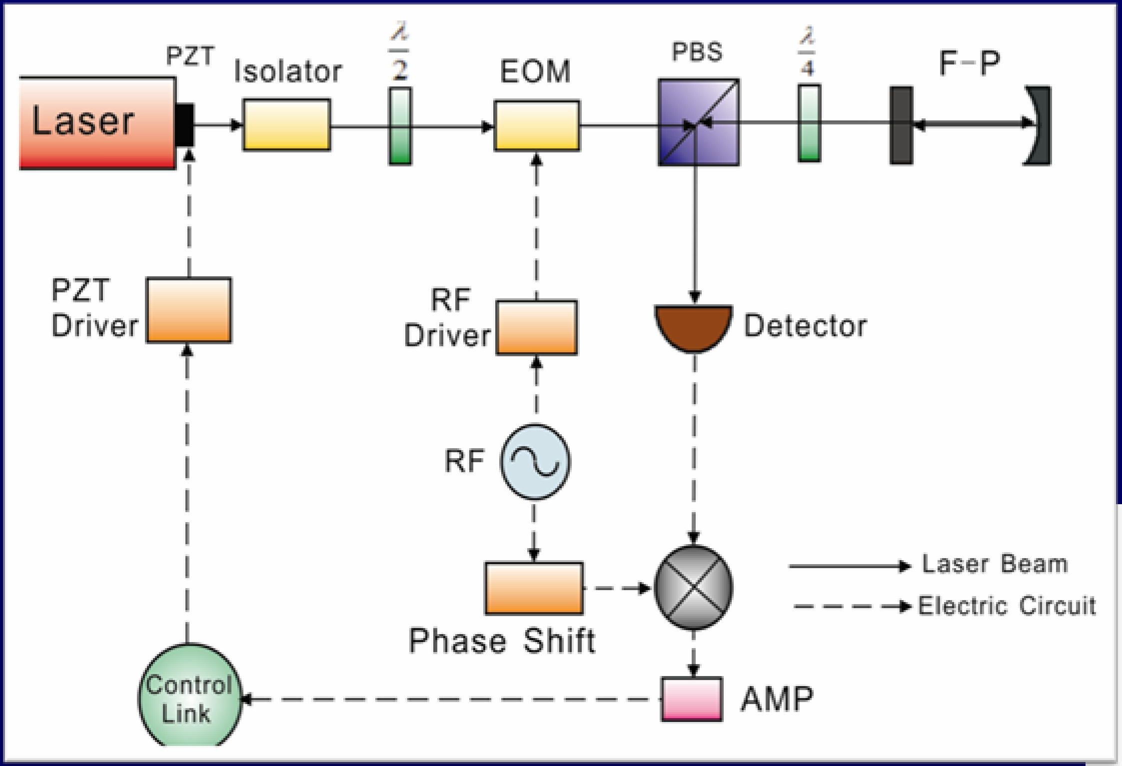
Laser Frequency Instability

- Three-stage system (two active one passive) to achieve overall suppression of ~10^{13}
- Running pre-stabilization and arm-locking in series reduces gain (bandwidth) requirements on arm-locking.

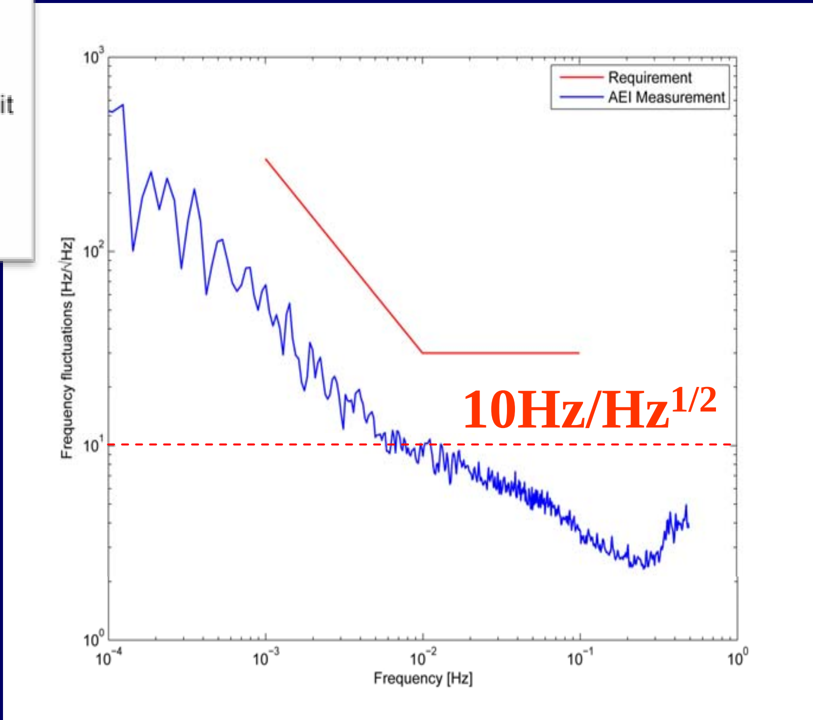


$$\delta x = \delta x_1 - \delta x_2 = L_1 \frac{\delta \nu}{\nu} - L_2 \frac{\delta \nu}{\nu} = \Delta x \frac{\delta \nu}{\nu} \qquad \delta x \sim 2\ \text{pm/Hz}^{1/2}$$

Laser Frequency Stabilization: PDH Scheme



AEI experimental result



$10Hz/Hz^{1/2}$

Key factors：

- EOM modulation depth and modulation frequency
- Mode matching of F-P cavity
- Stability of F-P cavity
- Environmental control

Comparison between LISA and Adv. LISA

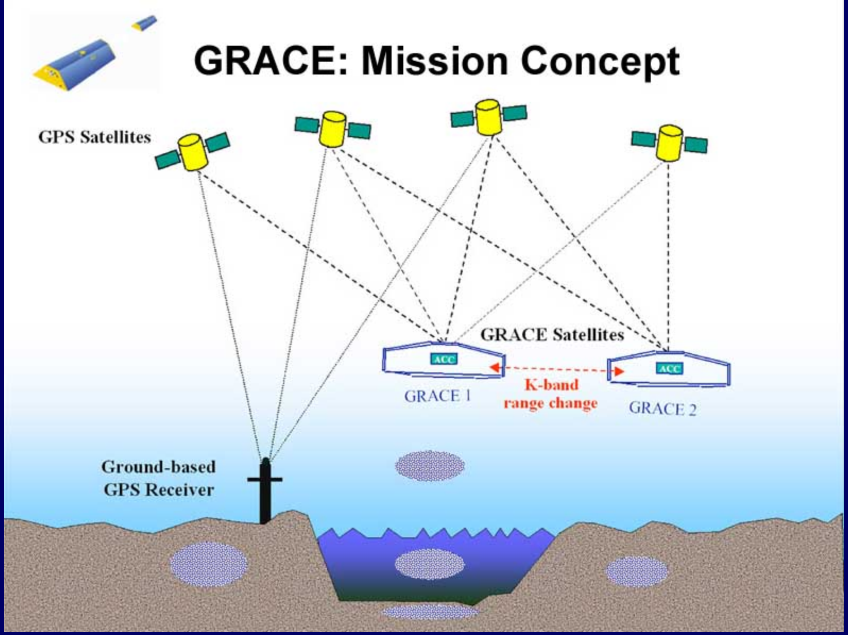
	LISA	Advanced LISA
Laser power	1 W	12.5 W
Diameter of telescope	40 cm	60 cm
Unequal arm length $\Delta L/L$	1%	~ 0.1%
Phase measurement	3×10^{-5} rad/Hz$^{1/2}$	1.4×10^{-7} rad/Hz$^{1/2}$
Static offset error in pointing	15 nrad	3 nrad
Pointing jitter	6 nrad/Hz$^{1/2}$	0.2 nrad/Hz$^{1/2}$
Laser frequency stability	6×10^{-7} Hz/Hz$^{1/2}$	1×10^{-6} Hz/Hz$^{1/2}$

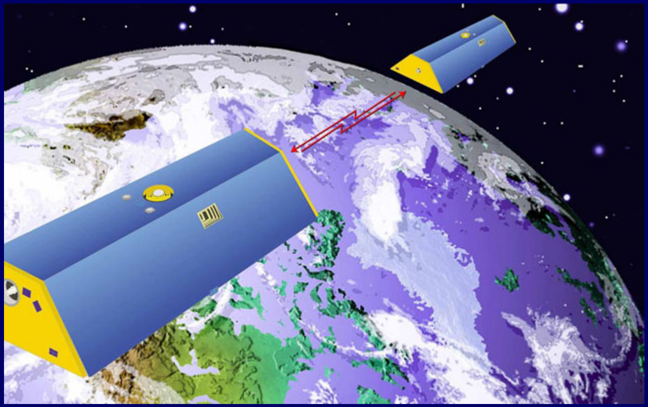
Earth's Gravity Recovery Mission
for
Earth Science and Geophysics

GRACE: Overview

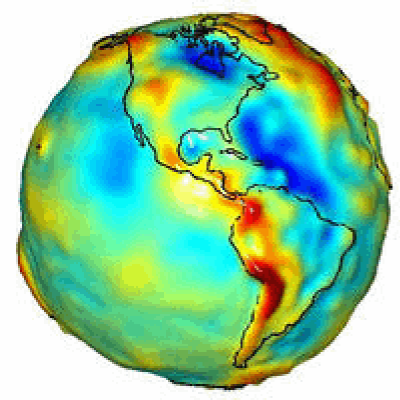
Satellite to satellite tracking (SST):

- Separation: 220 km
- Altitude: ~500 km (decline orbit)
- Measurement:
  microwaves in K-band (10μm)
  GPS (1mm)

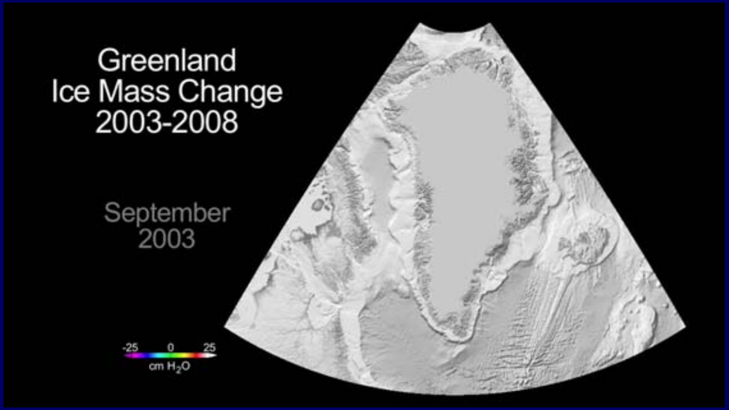




GRACE, twin satellites launched in March 17, 2002. [2]

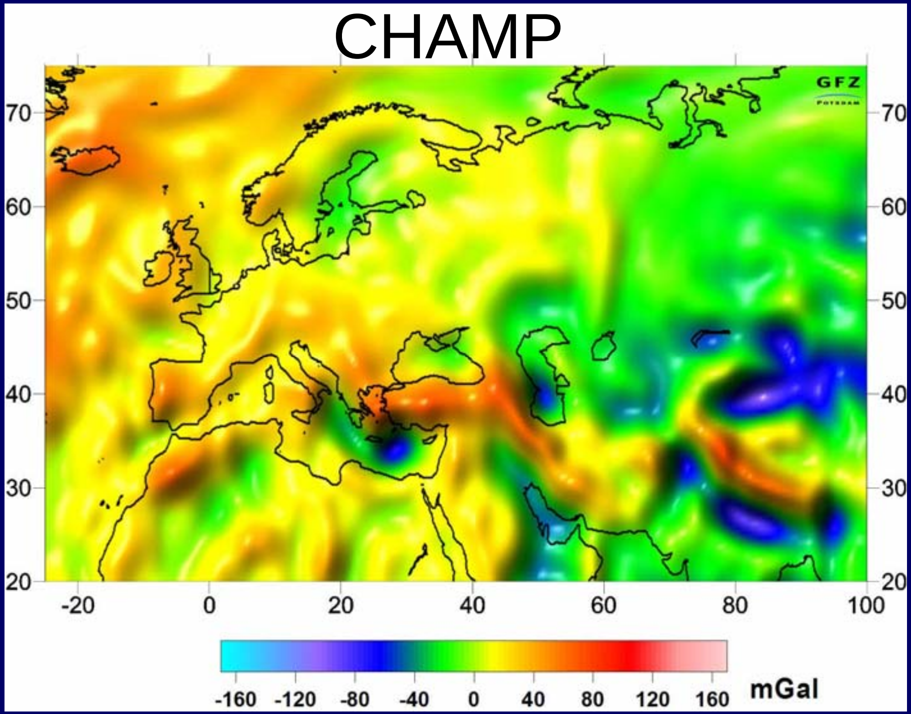


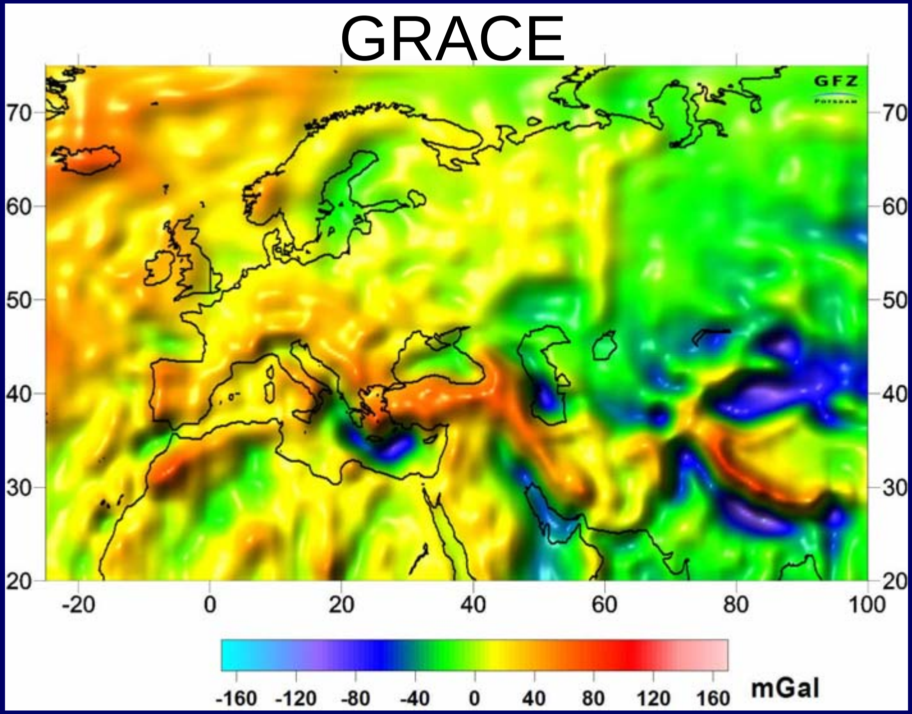
GRACE Global Gravity Animation [2]

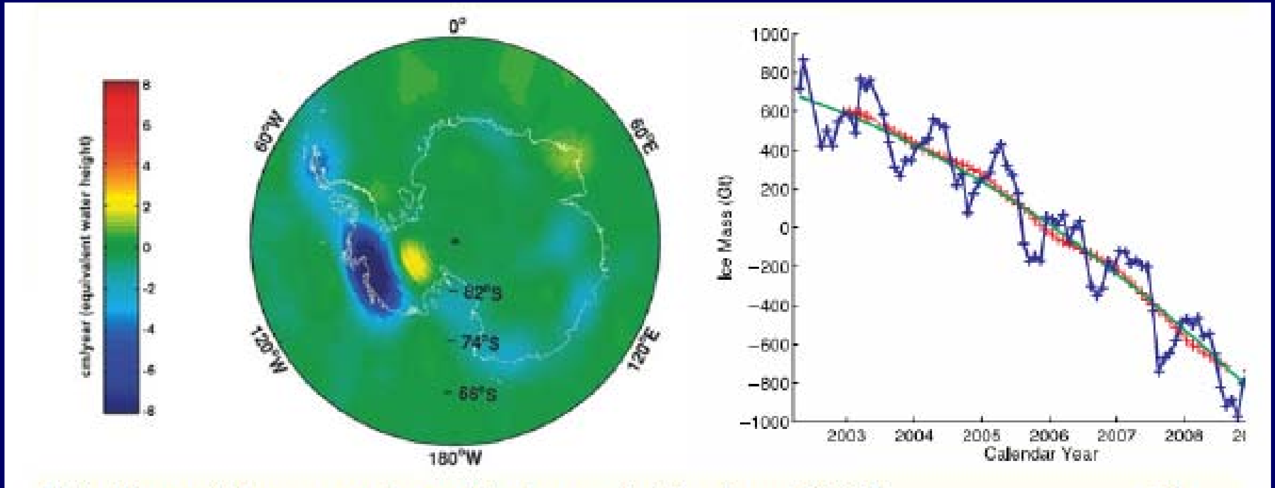


Greenland Ice Mass Change 2003-2008 [2]

Resolution of Gravity Anomalies

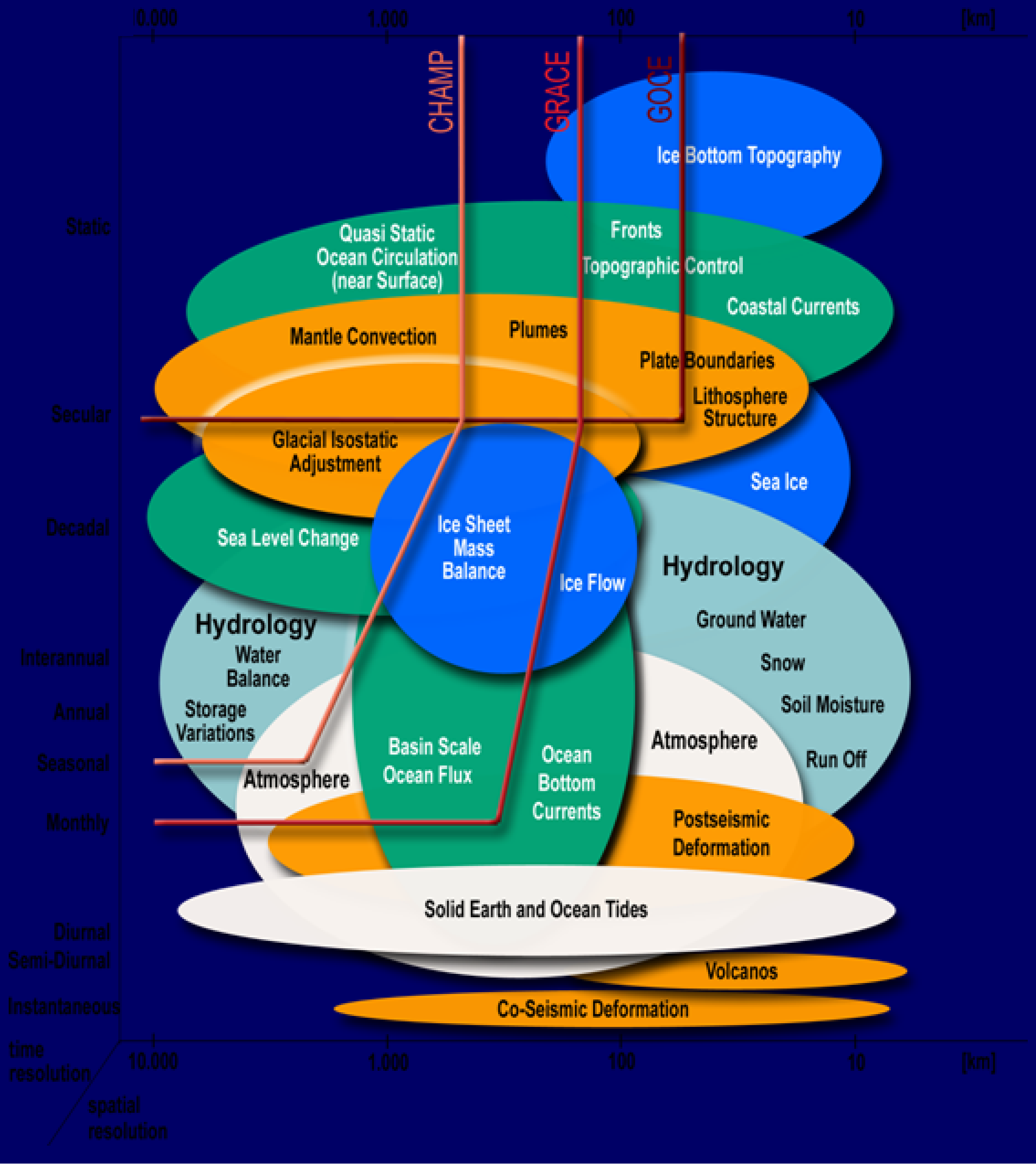

CHAMP


GRACE



[http://www.jpl.nasa.gov/news/features.cfm?feature=2378] [1]

Challenge of Future Satellite Missions



Doppler Frequency Shift and Range-Rate Measurement

Homodyne OPLL: $\omega_S(t) = \omega_M(t-\tau_1)\times\left[1-u(t-\tau_1)/c\right]$

$$\omega_b(t) = \omega_M(t) - \omega_S(t-\tau_2)\cdot[1-u(t-\tau_2)/c]$$

$$= [\omega_M(t) - \omega_M(t-\tau_2-\tau_1)] + \omega_M(t-\tau_2-\tau_1)\cdot[\frac{u(t-\tau_2-\tau_1)}{c} + \frac{u(t-\tau_2)}{c}]$$

$$\tau_1 \approx \tau_2 \approx 0.33\,\text{ms}$$

$$\omega_\text{M}(t) \approx \omega_\text{M}(t-\tau_2-\tau_1)$$

$$\frac{u(t)}{c} \approx \frac{u(t-\tau_2)}{c} \approx \frac{u(t-\tau_2-\tau_1)}{c} < 3.3\times10^{-8}$$

$$\Rightarrow \quad \boxed{\omega_b(t) \approx 2\omega_M(t)\cdot\frac{u(t)}{c}}$$

Phase Shift and Range Measurement

Slave satellite

$$\phi_S(t) \approx \omega_M(t'-\tau_1)\cdot[1-\frac{u(t'-\tau_1)}{c}]\cdot\tau_1+\phi_M(t-\tau_1)+\phi_{OPLL}$$

Master satellite

$$\phi_D(t)=\omega_S(t-\tau_2)\cdot[1-\frac{u(t-\tau_2)}{c}]\cdot\tau_2+\phi_S(t-\tau_2)$$

$$\begin{aligned}\phi_D(t)&=\omega_M(t-\tau_2-\tau_1)\cdot(\tau_2+\tau_1)+\phi_M(t-\tau_2-\tau_1)+\phi_{OPLL}\\&-\omega_M(t-\tau_2-\tau_1)\cdot\frac{u(t-\tau_2-\tau_1)}{c}\cdot(\tau_1+\tau_2)-\omega_M(t-\tau_2-\tau_1)\cdot\frac{u(t-\tau_2)}{c}\cdot\tau_2\end{aligned}$$

$$\boxed{\Delta L=\left(\frac{c}{2\omega_M}\right)\left[\Delta\phi_D(t)-2L\frac{\Delta\omega_M}{c}-\Delta\phi_{OPLL}\right]}$$

Error Sources of Laser Ranging System

Laser	Laser frequency noise Shot noise
Optical bench	Optical cyclic error Thermal drift of optical bench Photodetector noise
Phase measurement	Phase-meter resolution OPLL residual noise
Pointing control	Laser beam pointing and jitter errors
Environment	Refractive index fluctuation Ionosphere effect

Experimental Conditions for Earth's Gravity Mapping

Inter-satellite distance L = 100 km

Relative speed between satellites v = 10 m/s

Laser beam divergence angle $\theta_{\mathrm{div}} = 10^{-3}$ rad

Laser beam pointing error $\theta_{\mathrm{dc}} = 10^{-4}$ rad

Laser power P_0 = 1.0 W

Laser wavelength λ = 1064 nm ($f = 2.8\times10^{14}$ Hz)

Temperature variation $\Delta T = 0.01\ \mathrm{K}/\sqrt{\mathrm{Hz}}$

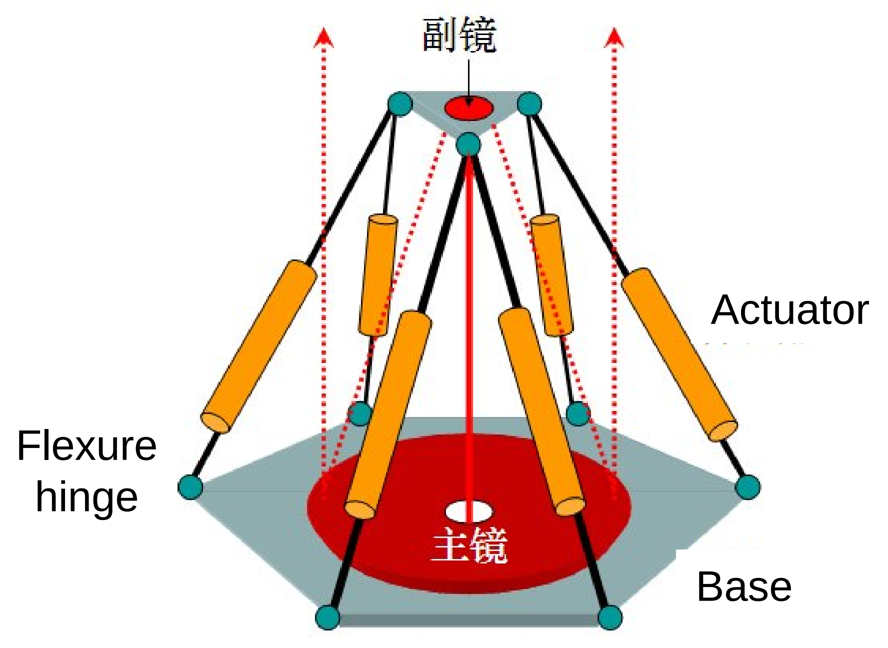
Error Budget

Major Error Source	Estimated error
Pre-stabilized laser: $\Delta f < 20$ Hz/Hz$^{1/2}$	7 nm/Hz$^{1/2}$
Stable optical bench: (1) thermal control: $\Delta T = 0.001$K (2) optical non-linearity (cyclic error)	 2 nm/Hz$^{1/2}$ 2 nm
Divergence angle of laser beam: $\theta_{div} \sim 10^{-3}\ rad$ **Pointing control:** $\theta_{dc} \sim 5\times10^{-4}\ rad,\ \theta_{jit} \sim 10^{-4}\ rad/Hz^{1/2}$	6.5 nm/Hz$^{1/2}$
Phasemeter resolution: 6×10^{-3}rad/Hz$^{1/2}$ @ 0.1Hz	1 nm/Hz$^{1/2}$
Shot noise: $P_0 = 1$W, $L = 100$ km	<1 pm/Hz$^{1/2}$
(RSS) Total	**10 nm/Hz$^{1/2}$**

Beam Pointing and Jitter Errors

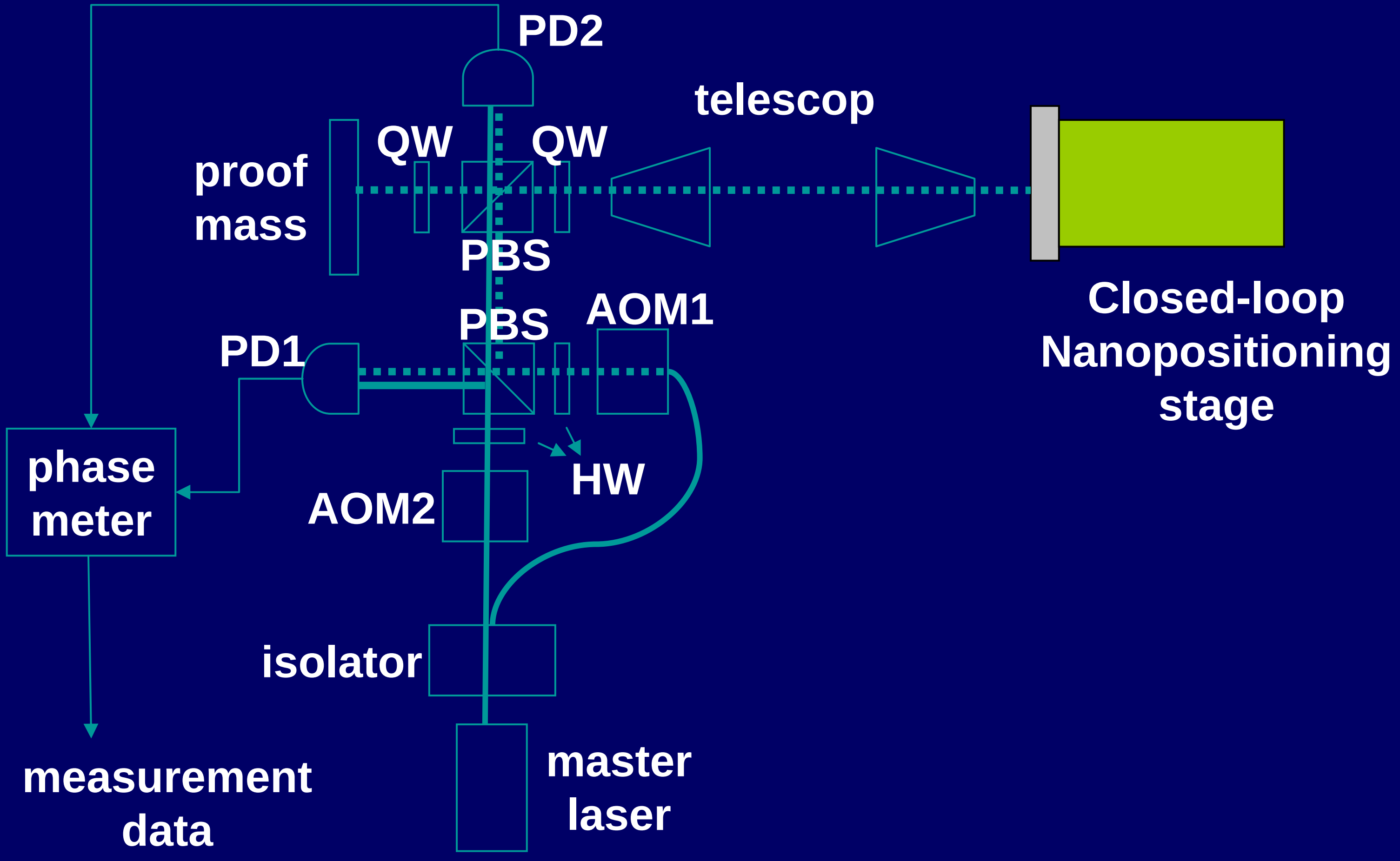
$$\Delta x = \frac{4\pi^2}{32}\delta\frac{\theta_{dc}\theta_{jitter}}{\theta_{div}^2}$$

$$= \frac{4\pi^2}{32}\times\frac{\lambda}{10}\times\frac{5\times10^{-4}\,rad\times10^{-4}\,rad/\sqrt{Hz}}{(10^{-3}\,rad)^2} = 6.5nm$$



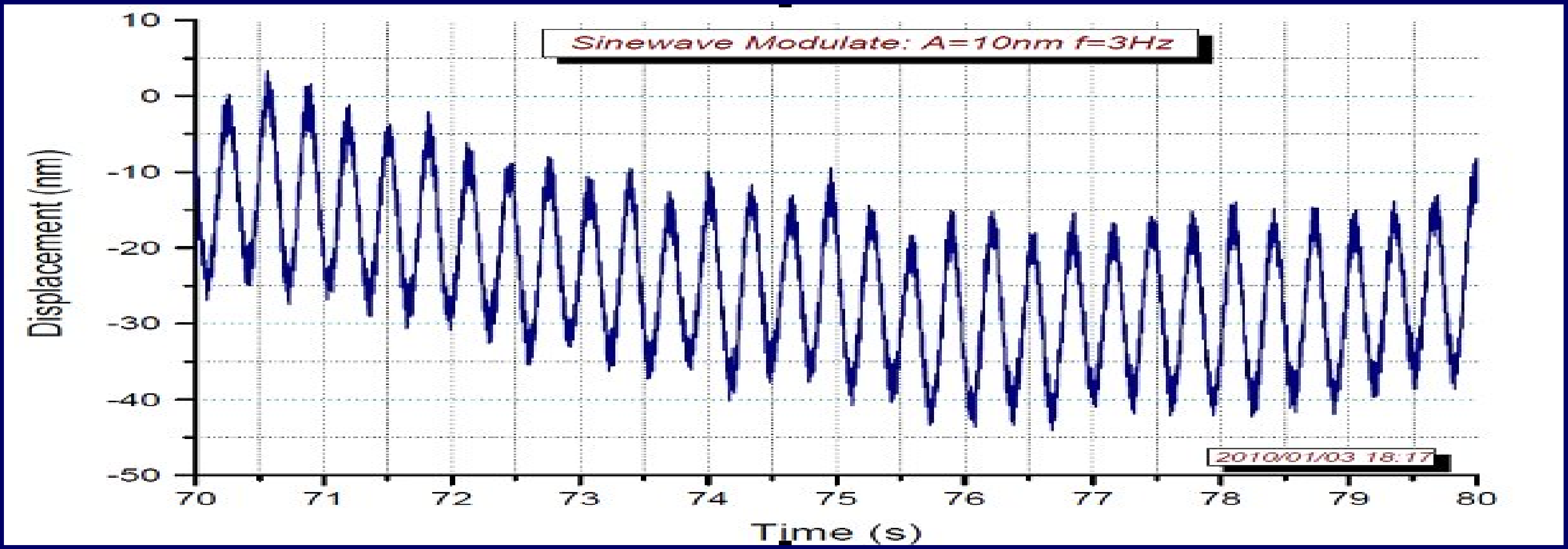
- Z-axis range：220μm
- Positioning precision：0.2μm
- Pointing-angle range：10^{-3}rad
- Pointing-angle precision：10^{-5}rad

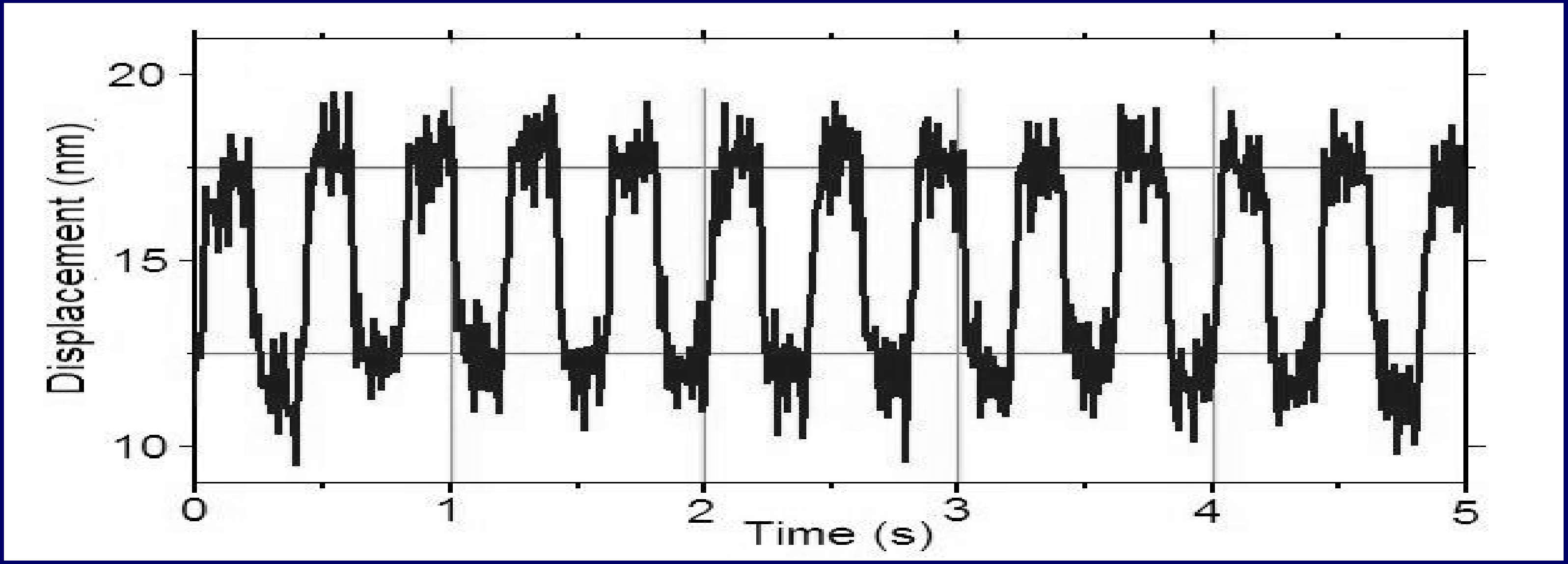
Heterodyne Laser Ranging System
(transponder using homodyne OPLL)



Prototype of Laser Ranging System Installed at HUST

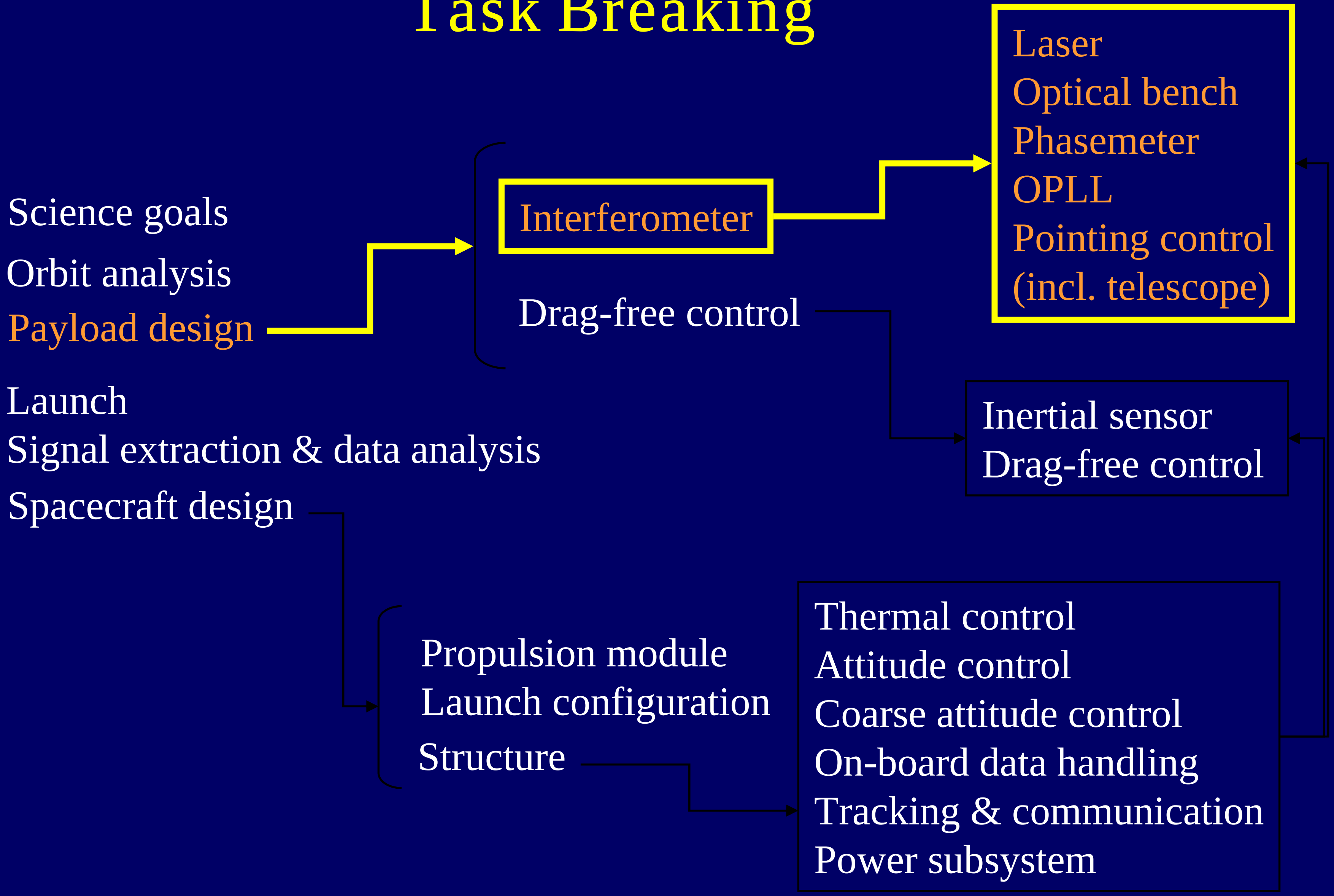
Preliminary Results
of the Prototype of Laser Ranging System





Scope and Roadmap

Task Breaking



Science goals

Orbit analysis

Payload design

Launch

Signal extraction & data analysis

Spacecraft design

Interferometer

Drag-free control

Laser
Optical bench
Phasemeter
OPLL
Pointing control
(incl. telescope)

Inertial sensor
Drag-free control

Propulsion module
Launch configuration
Structure

Thermal control
Attitude control
Coarse attitude control
On-board data handling
Tracking & communication
Power subsystem

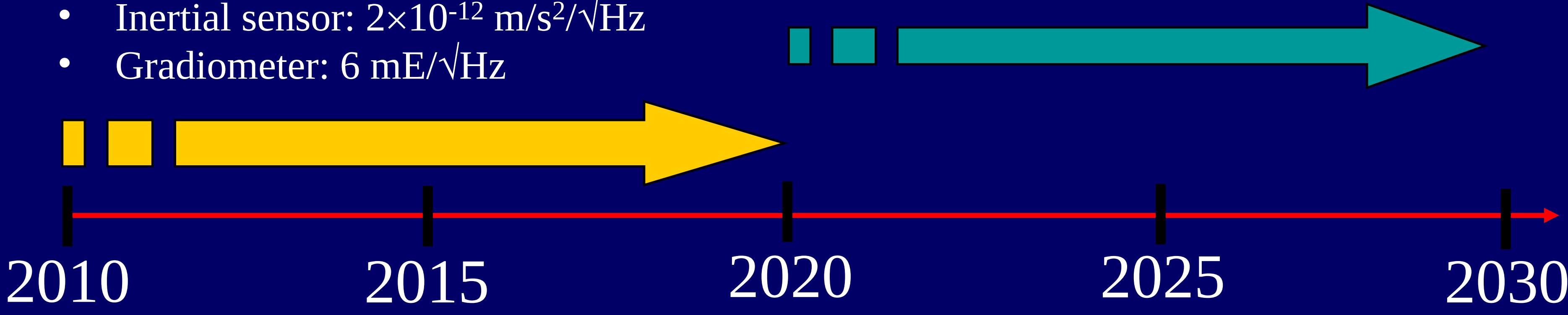
Inter-Satellite Laser Interferometry

- Transponder-type heterodyne interferometer
  - Freq./Inten.-stabilized laser
    - (1) PDH pre-stabilization
    - (2) Arm-locking
    - (3) TDI interferometry
  - phasemeter
    - (1) FPGA-based
    - (2) Pure analog
    - (3) Low noise photodetector
  - Transponder
    - (1) Homodyne OPLL
    - (2) Heterodyne OPLL
  - Pointing control
    - (1) Nanopositioning control
    - (2) Telescope
  - Optical bench
    - (1) Interferometer scheme
    - (2) Beam quality
    - (3) Optical loss
    - (4) Mounting & solidification
    - (5) Thermal drift

Roadmap

- Laser: $\Delta f \sim 1\text{Hz}/\sqrt{\text{Hz}}$@1mHz (space-qualified Nd:YAG)
- Phasemeter: $\phi_{res} \sim 1$ pm (6×10^{-6} rad)
- Optical phase locking: $\phi_{err} < 10^{-5}$ rad/$\sqrt{\text{Hz}}$
- Optical bench:
  - nonlinearity < 1 pm
  - thermal drift < 10 nm/K
- Pointing control
  - $\theta_{dc} \sim 10^{-6}$ rad
  - $\theta_{jitter} < 10^{-7}$ rad/$\sqrt{\text{Hz}}$
- Inertial sensor: 2×10^{-12} m/s^2/$\sqrt{\text{Hz}}$
- Gradiometer: 6 mE/$\sqrt{\text{Hz}}$

- Laser: $\Delta f \sim 1\text{Hz}/\sqrt{\text{Hz}}$@1mHz
- Phasemeter: $\phi_{res} \sim 10^{-7}$ rad
- Optical phase locking: $\phi_{err} < 10^{-7}$ rad/$\sqrt{\text{Hz}}$
- Pointing control
  - $\theta_{dc} \sim 3$ nrad
  - $\theta_{jitter} < 0.2$ nrad/$\sqrt{\text{Hz}}$
- Inertial sensor: 5×10^{-14} m/s^2/$\sqrt{\text{Hz}}$
- Thermal control: 5×10^{-7} K/$\sqrt{\text{Hz}}$
- …
- Engineering model of Adv. LISA



Conclusions

- GW detection (long-term goal), and Earths gravity recovery (short-term goal): We development all key technologies step by step.

- GW detection is a challenging mission that provides a strong driving force to develop the state-of-the-arts technologies. Many scientists and engineers will be well-trained during the whole developing stage.

Thank you
for your attentions!