Particle Creation in Charged Black Holes

Chiang-Mei Chen

Department of Physics, National Central University, Taiwan
cmchen@phy.ncu.edu.tw

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- Particle Creation in Charged Black Holes
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Overview: Spontaneous Pair Production

- **Quantum Vacuum Fluctuation:** *virtual particles*
  - Heisenberg’s uncertainty principle: \( \Delta E \Delta t \geq \frac{\hbar}{2} \)
  - creation of particle-antiparticle pairs (virtual particles)

- **Spontaneous Pair Production:** *from virtual to real*
  - Schwinger mechanism: electric field
  - Hawking radiation: causal horizon (tunneling)

  [Schwinger, 1951]
  [Parikh, Wilczek, [hep-th/9907001]]
Overview: Holographic Principle

- **Black Hole Thermodynamics**
  - Hawking temperature: surface gravity $\kappa$
    \[ T = \frac{\hbar c^3 \kappa}{kG 2\pi} \]
  - Beckenstein-Hawking entropy: area of horizon $A$
    \[ S = \frac{k c^3 A}{\hbar G 4} \]

- **Holographic Principle**

  t’ Hooft, 1993; Susskind, 1994

Gravity in Bulk $D$ dimensions $\iff$ Field Theory on Boundary $D - 1$ dimensions
Overview: Holographic Principle

- Preliminary hint: Symmetry
  - **Gravity**: anti de Sitter (AdS) space appears in near horizon geometry of extremal black holes
  - **Field Theory**: Conformal Field Theory (CFT)

- Symmetry group: \( SO(D - 1, 2) \) for both AdS\(_D\) and CFT\(_{D-1}\)

- **AdS/CFT Correspondence**: (string theory)
  
  Maldacena, 1997

\[
\text{IIB Superstring on } \text{AdS}_5 \times S^5 \quad \leftrightarrow \quad N = 4 \text{ Super Yang-Mills on Boundary of AdS}_5
\]
Overview: Holographic Principle

- **Kerr/CFT Correspondence** (extremal)

- Near horizon geometry: warped AdS$_3$ $SL(2, R) \times U(1)$
- (CFT) Central charge: $c_L = c_R = 12J/\hbar$
- (CFT) Temperature: $T_L = 1/2\pi$; $T_R = 0$
Overview: Holographic Principle

- **Reissner-Nordström (RN)/CFT Correspondence** (extremal)

- **Near horizon geometry:** $\text{AdS}_2 \times S^2$

- **Warped AdS$_3$/CFT$_2$ description**:
  - The $U(1)$ bundle of warped AdS$_3$ was recovered from the gauge field potential by uplifting the RN black hole into 5D.
  - The period of extra coordinate is $y \sim y + 2\pi \ell$
  - Gravitational constant: $G_5 = 2\pi \ell G_4$
  - (CFT) Central charge: $c_L = c_R = 6Q^3/\ell$
  - (CFT) Temperature: $T_L = \ell/2\pi Q$; $T_R = 0$
Overview: Holographic Principle

- **Kerr-Newman (KN)/CFTs Correspondence** (extremal)

- There are two different individual 2D CFTs holographically dual to the KN black hole.

- **microscopic hair theorem**: For each “hair” of black hole, except the mass, there is an associated CFT description.
Overview: Motivation

- **Purpose**: particle creation in charged black holes
  - technical simplicity: constant electric field (exactly solvable)
  - holographic description: anti de Sitter

- Reissner-Nordström (RN) Black Holes: near extremal
  - near horizon region: where the production occurs
    \[ AdS_2 \times S^2 + \text{constant electric field} \]

- extremal RN
  - vanishing HAWKING temperature: stable (thermal)
  - non-vanishing electric field: unstable (quantum)

Near horizon geometry of near extremal Reissner-Nordström Black Holes:

- Horizon radius: 
  \[ r_{\pm} = M \pm \sqrt{M^2 - Q^2} \]
- Near extremal: 
  \[ Q \to M \Rightarrow r_- \to r_+ \]
Particle Creation: Boundary Conditions

- **Pair Production**: probe massive charged scalar
  - The ratios of fluxes exhibit particle (scalar) creation with suitable boundary conditions.

  Kim, Page, [arXiv:hep-th/0005078]
  Kim, Lee, Yoon, [arXiv:0910.3363 [hep-th]]

- **Boundary condition I**: particle view point

  - **incident**: virtual particles
  - **reflected**: re-annihilated
  - **transmitted**: pair produced "particles"
Particle Creation in Charged Black Holes

Particle Creation: Boundary Conditions

- Boundary condition II: antiparticle viewpoint
  - incident: virtual particles
  - reflected: re-annihilated
  - transmitted: pair produced “antiparticles”

Equivalence:
  - particles and antiparticles should always appear in pairs
Particle Creation in Charged Black Holes

Particle Creation: Physical Quantities

- **vacuum persistence amplitude**: (probability) \( |\alpha|^2 \)
  
  (⋆ physically it should be \( 1/|\alpha|^2 \) ⋆)
  
  \[ |\alpha|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \quad \frac{1}{|\alpha|^2} \equiv \frac{D_{\text{reflected}}}{D_{\text{incident}}} \]

- **mean number of produced pairs**: \( |\beta|^2 \)
  
  \[ |\beta|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}} \]

- **absorption cross section**: (probability) \( \sigma_{\text{abs}} \)
  
  \[ \sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\beta|^2}{|\alpha|^2} \]

- **flux conservation and Bogoliubov relation**
  
  \[ |D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}| \iff |\alpha|^2 - |\beta|^2 = 1 \]
Particle Creation: Physical Quantities

- Particular values of physical quantities:
  - Normalization: \( D_{\text{incident}} = 1 \)

| Production | \( D_{\text{reflected}} \) | \( D_{\text{transmitted}} \) | \( |\alpha|^2 \) | \( |\beta|^2 \) | \( \sigma_{\text{abs}} \) |
|------------|----------------------------|-----------------|-----------------|----------------|----------------|
| None       | 1                          | 0               | 1               | 0              | 0              |
| Half       | 1/2                        | 1/2             | 1/2             | 1              | 1/2            |
| Full       | 0                          | 1               | 0               | \( \infty \)   | 1              |
Particle Creation: RN Back Holes

- Near horizon geometry of near-extremal RN
  \[ ds^2 = -\frac{\rho^2 - B^2}{Q^2}d\tau^2 + \frac{Q^2}{\rho^2 - B^2}d\rho^2 + Q^2d\Omega_2^2 \]
  \[ A_{[1]} = -\frac{\rho}{Q}d\tau; \quad F_{[2]} = \frac{1}{Q}d\tau \wedge d\rho \]

- “rescaled” deviation from extremality: \( B \propto M - Q \)
- Geometric structure: \( AdS_2 \times S^2 \) (radius \( Q \) for both)
- Electric field: constant
- Probe massive charged scalar: \( \Phi \)

\[ S_{\Phi} = \int d^4x \sqrt{-g} \left(-\frac{1}{2}D_\alpha \Phi^* D^\alpha \Phi - \frac{1}{2}m^2 \Phi^* \Phi \right) \]

- \( m \) and \( q \) are the mass and charge of \( \Phi \)
- \( D_\alpha \equiv \nabla_\alpha - iqA_\alpha \)
Particle Creation: RN Back Holes

- Field equation: Klein-Gordon (KG) equation
  \[(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2\Phi = 0\]

- Flux:
  \[D = i\sqrt{-g}g^{\rho\rho}(\Phi D_\rho \Phi^* - \Phi^* D_\rho \Phi)\]

- Ansatz:
  \[\Phi(\tau, \rho, \theta, \phi) = e^{-i\omega\tau + in\phi}R(\rho)S(\theta)\]

- Separated field equations (exactly solvable !!!)
  \[
  \partial_\rho \left[ (\rho^2 - B^2)\partial_\rho R \right] + \left[ \frac{(q\rho - \omega Q)^2 Q^2}{\rho^2 - B^2} - m^2 Q^2 - \lambda_l \right] R = 0
  \]
  \[
  \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta S) - \left( \frac{n^2}{\sin^2 \theta} - \lambda_l \right) S = 0
  \]

- \(S(\theta)\) is spherical harmonics with the eigenvalue \(\lambda_l = l(l + 1)\)
Particle Creation: RN Back Holes

- Condition for Schwinger mechanism and/or Hawking radiation
  \[(m^2 - q^2)Q^2 + (l + 1/2)^2 < 0\]
  - violation of Breitenlohner-Freedman (BF) bound in AdS$_2$
  - unstable mode

- Cosmic censorship: necessary condition
  \[q^2 > m^2\]
  - avoiding naked singularity

- Schwinger mechanism: extremal to non-extremal
Particle Creation: Results

- Bogoliubov coefficients

\[
|\alpha|^2 = \frac{\cosh(\pi a - \pi b) \cosh(\pi \tilde{a} + \pi b)}{\cosh(\pi a + \pi b) \cosh(\pi \tilde{a} - \pi b)} \\
|\beta|^2 = \frac{\sinh(2\pi b) \sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a + \pi b) \cosh(\pi \tilde{a} - \pi b)}
\]

\[
a \equiv qQ, \quad b \equiv \sqrt{(q^2 - m^2)Q^2 - (l + 1/2)^2}, \quad \tilde{a} \equiv \frac{\omega Q^2}{B}
\]

- Absorption cross section:

\[
\sigma_{\text{abs}} = \frac{\sinh(2\pi b) \sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a - \pi b) \cosh(\pi \tilde{a} + \pi b)}
\]

- Leading term of $|\beta|^2$ leads to the Schwinger formula

\[
|\beta|^2 \approx e^{-\frac{\pi m^2 Q}{q}} \approx e^{-\frac{\pi m^2 r_H^2}{qQ}}
\]
Particle Creation: Results

An interesting observation:

\[
\frac{|\beta(B = 0)|^2}{|\beta(B \neq 0)|^2} = \frac{\cosh(\pi \tilde{a} - \pi b) e^{\pi b - \pi a}}{\sinh(\pi \tilde{a} - \pi a)} = \frac{1 + e^{2\pi(b - \tilde{a})}}{1 - e^{2\pi(a - \tilde{a})}} \geq 1
\]

Production rate:

extremal (Schwinger) \geq near extremal (Schwinger+Hawking)

From the extremal to near extremal black holes

- Increasing attractive gravitational force reduces electromagnetic repulsive force for Schwinger mechanism.
- Schwinger mechanism is suppressed faster than the increasing part from the Hawking thermal radiation.

Such kind of interaction generically prohibits to distinguish the Schwinger mechanism from the Hawking radiation.
Particle Creation: Holographic Description

- CFT absorption cross section: standard formula

\[ \sigma_{abs} \sim \left( \frac{2\pi T_L}{\Gamma(2h_L)} \right)^{2h_L-1} \left( \frac{2\pi T_R}{\Gamma(2h_R)} \right)^{2h_R-1} \sinh \left( \frac{\omega_L - q_L\Omega_L}{2T_L} + \frac{\omega_R - q_R\Omega_R}{2T_R} \right) \]

\[ \times \left| \Gamma \left( h_L + i\frac{\omega_L - q_L\Omega_L}{2\pi T_L} \right) \right|^2 \left| \Gamma \left( h_R + i\frac{\omega_R - q_R\Omega_R}{2\pi T_R} \right) \right|^2 \]

- CFT dual to RN black holes

\[ c_L = c_R = \frac{6Q^3}{\ell}, \quad T_L = \frac{\ell}{2\pi Q}, \quad T_R = \frac{\ell B}{\pi Q^2} \]

- Free parameter \( \ell \) is related to measure of U(1) bundle.


- Conformal weights of dual operator: \( h_L = h_R = 1/2 \pm ib \)

Complex conformal weight \( \Rightarrow \) unstable dual operator
Particle Creation: Holographic Description

- Identification by thermodynamics:

\[
\frac{\delta M}{T_H} - \frac{\Omega_H \delta Q}{T_H} = \frac{\omega_L - q_L \Omega_L}{T_L} + \frac{\omega_R - q_R \Omega_R}{T_R}
\]

- Hawking temperature: \( T_H = \frac{B}{2\pi Q^2} \)
- Chemical potential: \( \Omega_H = A_T(B) = -B/Q \)
- \( \delta M = \omega \) and \( \delta Q = -q \)

\[
\tilde{\omega}_L \equiv \omega_L - q_L \Omega_L = -q \ell \quad \text{and} \quad \tilde{\omega}_R \equiv \omega_R - q_R \Omega_R = 2\omega \ell
\]

- The absorption cross section agrees with the CFT’s result only up to some numerical factors.
Conclusion

- We discuss the spontaneous pair production of charged scalar field in RN black holes (both extremal and near extremal limits).
- This is one of the few examples that we exactly know the Bogoliubov coefficients.
- The pair production (unstable mode) is holographically dual to an operator with complex conformal weight.
- The production rate is suppressed when the black hole temperature is turned on. (It should be enhanced in dS space.)
- The effects of the Schwinger mechanism and the Hawking radiation generically cannot be distinguished by imposing different boundary conditions.
Conclusion

- Phase Diagram:

  - Naked singularity
  - extremal $Q = M$, $T = 0$
  - stable (thermal)
  - unstable (quantum)
  - $Q_f = 0$, $M_f = 0$
    (final ?)
  - Schwarzschild
  - $Q_i = M_i$
    (initial)