Particle Creation in Charged Black Holes

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Outline

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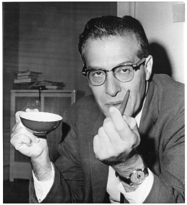
Overview: Spontaneous Pair Production

- Quantum Vacuum Fluctuation: *virtual particles*
  - Heisenberg's uncertainty principle: $\Delta E \Delta t \geq \hbar/2$
  - creation of particle-antiparticle pairs (virtual particles)
- Spontaneous Pair Production: *from virtual to real*
  - Schwinger mechanism: electric field

    Schwinger, 1951
  - Hawking radiation: causal horizon (tunneling)

    Parikh, Wilczek, [hep-th/9907001]



Overview: Holographic Principle

- Black Hole Thermodynamics
  - Hawking temperature: surface gravity κ

$$T = \frac{\hbar c^3}{kG} \frac{\kappa}{2\pi}$$

  - Beckenstein-Hawking entropy: area of horizon A

$$S = \frac{kc^3}{\hbar G} \frac{A}{4}$$

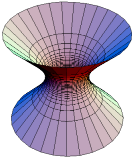
- Holographic Principle

t' Hooft, 1993; Susskind, 1994

$$\boxed{\begin{array}{c}\text{Gravity in Bulk}\\ D \text{ dimensions}\end{array}} \Longleftrightarrow \boxed{\begin{array}{c}\text{Field Theory on Boundary}\\ D-1 \text{ dimensions}\end{array}}$$

Overview: Holographic Principle

- Preliminary hint: Symmetry
  - Gravity: anti de Sitter (AdS) space appears in near horizon geometry of extremal black holes
  - Field Theory: Conformal Field Theory (CFT)



- Symmetry group: $SO(D-1,2)$
  for both AdS_D and CFT_{D-1}

- AdS/CFT Correspondence: (string theory)

Maldacena, 1997

IIB Superstring on $\text{AdS}_5 \times S^5$	$\Longleftrightarrow$	$N=4$ Super Yang-Mills on Boundary of AdS_5

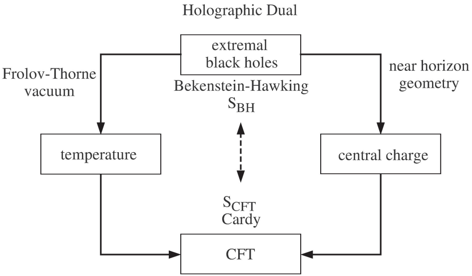
Overview: Holographic Principle

- Kerr/CFT Correspondence (extremal)

  Guica, Hartman, Song, Strominger, arXiv:0809.4266 [hep-th]

  Compere, arXiv:1203.3561 [hep-th] (review)

  - Near horizon geometry: warped AdS_3 $SL(2,R) \times U(1)$
  - (CFT) Central charge: $c_L = c_R = 12J/\hbar$
  - (CFT) Temperature: $T_L = 1/2\pi$; $T_R = 0$



Overview: Holographic Principle

- Reissner-Nordström (RN)/CFT Correspondence (extremal)

  Hartman, Murata, Nishioka, Strominger: arXiv:0811.4393 [hep-th]

  Garousi, Ghodsi: arXiv:0902.4387 [hep-th]

  CMC, Huang, Zou: arXiv:1001.2833 [hep-th]

  CMC, Sun: arXiv:1106.4407 [hep-th] (summary)

  - Near horizon geometry: $\mathrm{AdS}_2 \times S^2$

- Warped $\mathrm{AdS}_3/\mathrm{CFT}_2$ description:

  - The $U(1)$ bundle of warped AdS_3 was recovered from the gauge field potential by uplifting the RN black hole into 5D.
  - The period of extra coordinate is $y \sim y + 2\pi\ell$
  - Gravitational constant: $G_5 = 2\pi\ell G_4$
  - (CFT) Central charge: $c_L = c_R = 6Q^3/\ell$
  - (CFT) Temperature: $T_L = \ell/2\pi Q; \quad T_R = 0$

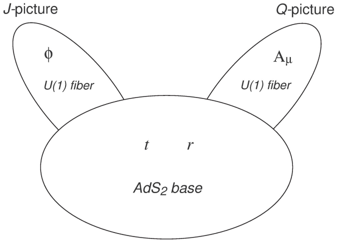
Overview: Holographic Principle

- Kerr-Newman (KN)/CFTs Correspondence (extremal)

  CMC, Huang, Sun, Wu, Zou: arXiv:arXiv:1006.4097 [hep-th]

  CMC, Sun: arXiv:1201.4040 [hep-th] (summary)

- There are two different individual 2D CFTs holographically dual to the KN black hole.



- microscopic hair theorem: For each "hair" of black hole, except the mass, there is an associated CFT discsrption.

Overview: Motivation

- Purpose: particle creation in charged black holes
  - technical simplicity: constant electric field (exactly solvable)
  - holographic description: anti de Sitter
- Reissner-Nordström (RN) Black Holes: near extremal
  - near horizon region: where the production occurs

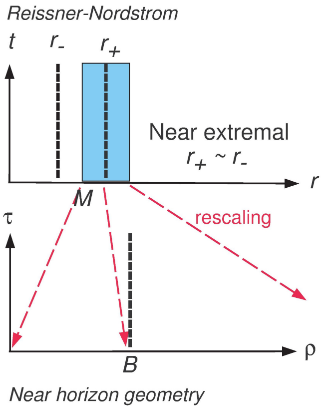
$$AdS_2 \times S^2 + \text{constant electric field}$$

- extremal RN
  - vanishing HAWKING temperature: stable (thermal)
  - non-vanishing electric field: unstable (quantum)

CMC, S.P. Kim, Y.-J. Lin, J.-R. Sun, M.-F. Wu, arXiv:1202.3224 [hep-th]

Near Horizon Geometry of RN Back Holes

- Near horizon geometry of near extremal Reissner-Nordström Black Holes:



- Horizon radius:
  $r_\pm = M \pm \sqrt{M^2 - Q^2}$
- Near extremal:
  $Q \to M \;\Rightarrow\; r_- \to r_+$

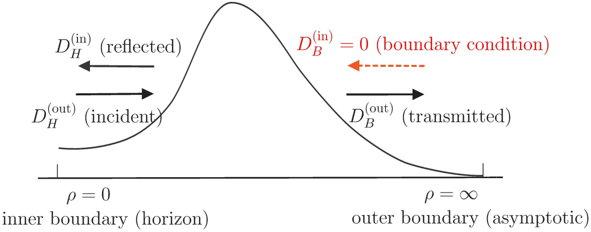
Particle Creation: Boundary Conditions

- Pair Production: probe massive charged scalar
  - The ratios of fluxes exhibit particle (scalar) creation with suitable boundary conditions.

Kim, Page, [arXiv:hep-th/0005078]

Kim, Lee, Yoon, [arXiv:0910.3363 [hep-th]]

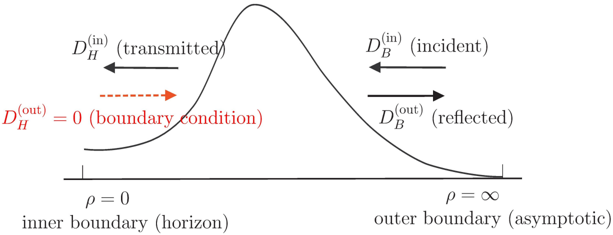
- Boundary condition I: particle view point



- incident: virtual particles
- reflected: re-annihilated
- transmitted: pair produced "particles"

Particle Creation: Boundary Conditions

- Boundary condition II: antiparticle view point



- incident: virtual particles
- reflected: re-annihilated
- transmitted: pair produced "antiparticles"

- Equivalence:
  - particles and antiparticles should always appear in pairs

Particle Creation: Physical Quantities

- vacuum persistence amplitude: (probability) $|\alpha|^2$
  ($\star$ physically it should be $1/|\alpha|^2$ $\star$)

$$|\alpha|^2 \equiv \frac{D_{\rm incident}}{D_{\rm reflected}}, \qquad \frac{1}{|\alpha|^2} \equiv \frac{D_{\rm reflected}}{D_{\rm incident}}$$

- mean number of produced pairs: $|\beta|^2$

$$|\beta|^2 \equiv \frac{D_{\rm transmitted}}{D_{\rm reflected}}$$

- absorption cross section: (probability) $\sigma_{\rm abs}$

$$\sigma_{\rm abs} \equiv \frac{D_{\rm transmitted}}{D_{\rm incident}} = \frac{|\beta|^2}{|\alpha|^2}$$

- flux conservation and Bogoliubov relation

$$|D_{\rm incident}| = |D_{\rm reflected}| + |D_{\rm transmitted}| \quad \Longleftrightarrow \quad |\alpha|^2 - |\beta|^2 = 1$$

Particle Creation: Physical Quantities

- Particular values of physical quantities:
  - Normalization: $D_{\text{incident}} = 1$

Production	$D_{\text{reflected}}$	$D_{\text{transmitted}}$	$\lvert\alpha\rvert^{-2}$	$\lvert\beta\rvert^{2}$	σ_{abs}
None	1	0	1	0	0
Half	1/2	1/2	1/2	1	1/2
Full	0	1	0	∞	1

Particle Creation: RN Back Holes

- Near horizon geometry of near-extremal RN

$$ds^2 = -\frac{\rho^2 - B^2}{Q^2} d\tau^2 + \frac{Q^2}{\rho^2 - B^2} d\rho^2 + Q^2 d\Omega_2^2$$

$$A_{[1]} = -\frac{\rho}{Q} d\tau; \qquad F_{[2]} = \frac{1}{Q} d\tau \wedge d\rho$$

  - "rescaled" deviation from extremality: $B \propto M - Q$
  - geometric structure: $\mathrm{A}dS_2 \times S^2$ (radius Q for both)
  - electric field: constant
- probe massive charged scalar: Φ

$$S_\Phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} D_\alpha \Phi^* D^\alpha \Phi - \frac{1}{2} m^2 \Phi^* \Phi \right)$$

  - m and q are the mass and charge of Φ
  - $D_\alpha \equiv \nabla_\alpha - iqA_\alpha$

Particle Creation: RN Back Holes

- Field equation: Klein-Gordon (KG) equation

$$(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi - m^2\Phi = 0$$

- Flux:

$$D = i\sqrt{-g}g^{\rho\rho}(\Phi D_\rho \Phi^* - \Phi^* D_\rho \Phi)$$

- Ansatz:

$$\Phi(\tau, \rho, \theta, \phi) = \mathrm{e}^{-i\omega\tau + in\phi} R(\rho) S(\theta)$$

- separated field equations (exactly solvable !!!)

$$\begin{aligned} \partial_\rho \left[(\rho^2 - B^2)\partial_\rho R\right] + \left[\frac{(q\rho - \omega Q)^2 Q^2}{\rho^2 - B^2} - m^2 Q^2 - \lambda_l\right] R &= 0 \\ \frac{1}{\sin\theta}\partial_\theta(\sin\theta\partial_\theta S) - \left(\frac{n^2}{\sin^2\theta} - \lambda_l\right) S &= 0 \end{aligned}$$

  - $S(\theta)$ is spherical harmonics with the eigenvalue $\lambda_l = l(l+1)$

Particle Creation: RN Back Holes

- Condition for Schwinger mechanism and/or Hawking radiation

$$(m^2 - q^2)Q^2 + (l + 1/2)^2 < 0$$

  - violation of Breitenlohner-Freedman (BF) bound in AdS_2
  - unstable mode

- Cosmic censorship: necessary condition

$$q^2 > m^2$$

  - avoiding naked singularity

- Schwinger mechanism: extremal to non-extremal

Particle Creation: Results

- Bogoliubov coefficients

$$|\alpha|^2 = \frac{\cosh(\pi a - \pi b)\cosh(\pi \tilde{a} + \pi b)}{\cosh(\pi a + \pi b)\cosh(\pi \tilde{a} - \pi b)}$$

$$|\beta|^2 = \frac{\sinh(2\pi b)\sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a + \pi b)\cosh(\pi \tilde{a} - \pi b)}$$

$$a \equiv qQ, \quad b \equiv \sqrt{(q^2 - m^2)Q^2 - (l + 1/2)^2}, \quad \tilde{a} \equiv \frac{\omega Q^2}{B}$$

- Absorption cross section:

$$\sigma_{\rm abs} = \frac{\sinh(2\pi b)\sinh(\pi \tilde{a} - \pi a)}{\cosh(\pi a - \pi b)\cosh(\pi \tilde{a} + \pi b)}$$

- Leading term of $|\beta|^2$ leads to the Schwinger formula

$$|\beta|^2 \approx e^{-\frac{\pi m^2 Q}{q}} \approx e^{-\frac{\pi m^2 r_H^2}{qQ}}$$

Particle Creation: Results

- An interesting observation:

$$\frac{|\beta(B=0)|^2}{|\beta(B\neq 0)|^2} = \frac{\cosh(\pi\tilde{a}-\pi b)}{\sinh(\pi\tilde{a}-\pi a)} e^{\pi b-\pi a} = \frac{1+e^{2\pi(b-\tilde{a})}}{1-e^{2\pi(a-\tilde{a})}} \geq 1$$

- Production rate:

extremal (Schwinger) $\geq$ near extremal (Schwinger+Hawking)

- From the extremal to near extremal black holes
  - Increasing attractive gravitational force reduces electromagnetic repulsive force for Schwinger mechanism.
  - Schwinger mechanism is suppressed faster than the increasing part from the Hawking thermal radiation.
- Such kind of interaction generically prohibits to distinguish the Schwinger mechanism from the Hawking radiation.

Particle Creation: Holographic Description

- CFT absorption cross section: standard formula

$$\sigma_{abs} \sim \frac{(2\pi T_L)^{2h_L-1}}{\Gamma(2h_L)} \frac{(2\pi T_R)^{2h_R-1}}{\Gamma(2h_R)} \sinh\left(\frac{\omega_L - q_L\Omega_L}{2T_L} + \frac{\omega_R - q_R\Omega_R}{2T_R}\right) \times \left|\Gamma\left(h_L + i\frac{\omega_L - q_L\Omega_L}{2\pi T_L}\right)\right|^2 \left|\Gamma\left(h_R + i\frac{\omega_R - q_R\Omega_R}{2\pi T_R}\right)\right|^2$$

- CFT dual to RN black holes

$$c_L = c_R = \frac{6Q^3}{\ell}, \quad T_L = \frac{\ell}{2\pi Q}, \quad T_R = \frac{\ell B}{\pi Q^2}$$

  - free parameter ℓ is related to measure of U(1) bundle.

CMC, Y.-M. Huang, S.-J. Zou, arXiv:1001.2833 [hep-th]

- Conformal weights of dual operator: $h_L = h_R = 1/2 \pm ib$

complex conformal weight $\Longrightarrow$ unstable dual operator

Particle Creation: Holographic Description

- Identification by thermodynamics:

$$\frac{\delta M}{T_H} - \frac{\Omega_H \delta Q}{T_H} = \frac{\omega_L - q_L \Omega_L}{T_L} + \frac{\omega_R - q_R \Omega_R}{T_R}$$

- Hawking temperature: $T_H = \frac{B}{2\pi Q^2}$
- chemical potential: $\Omega_H = A_\tau(B) = -B/Q$
- $\delta M = \omega$ and $\delta Q = -q$

$$\tilde{\omega}_L \equiv \omega_L - q_L \Omega_L = -q\ell \quad \text{and} \quad \tilde{\omega}_R \equiv \omega_R - q_R \Omega_R = 2\omega\ell$$

- The absorption cross section agrees with the CFT's result only up to some numerical factors.

Conclusion

- We discuss the spontaneous pair production of charged scalar field in RN black holes (both extremal and near extremal limits).
- This is one of the few examples that we exactly know the Bogoliubov coefficients.
- The pair production (unstable mode) is holographically dual to an operator with complex conformal weight.
- The production rate is suppressed when the black hole temperature is turned on. (It should be enhanced in dS space.)
- The effects of the Schwinger mechanism and the Hawking radiation generically cannot be distinguished by imposing different boundary conditions.

Conclusion

- Phase Diagram: