Non-perturbative study of the Higgs-Yukawa model using lattice field theory

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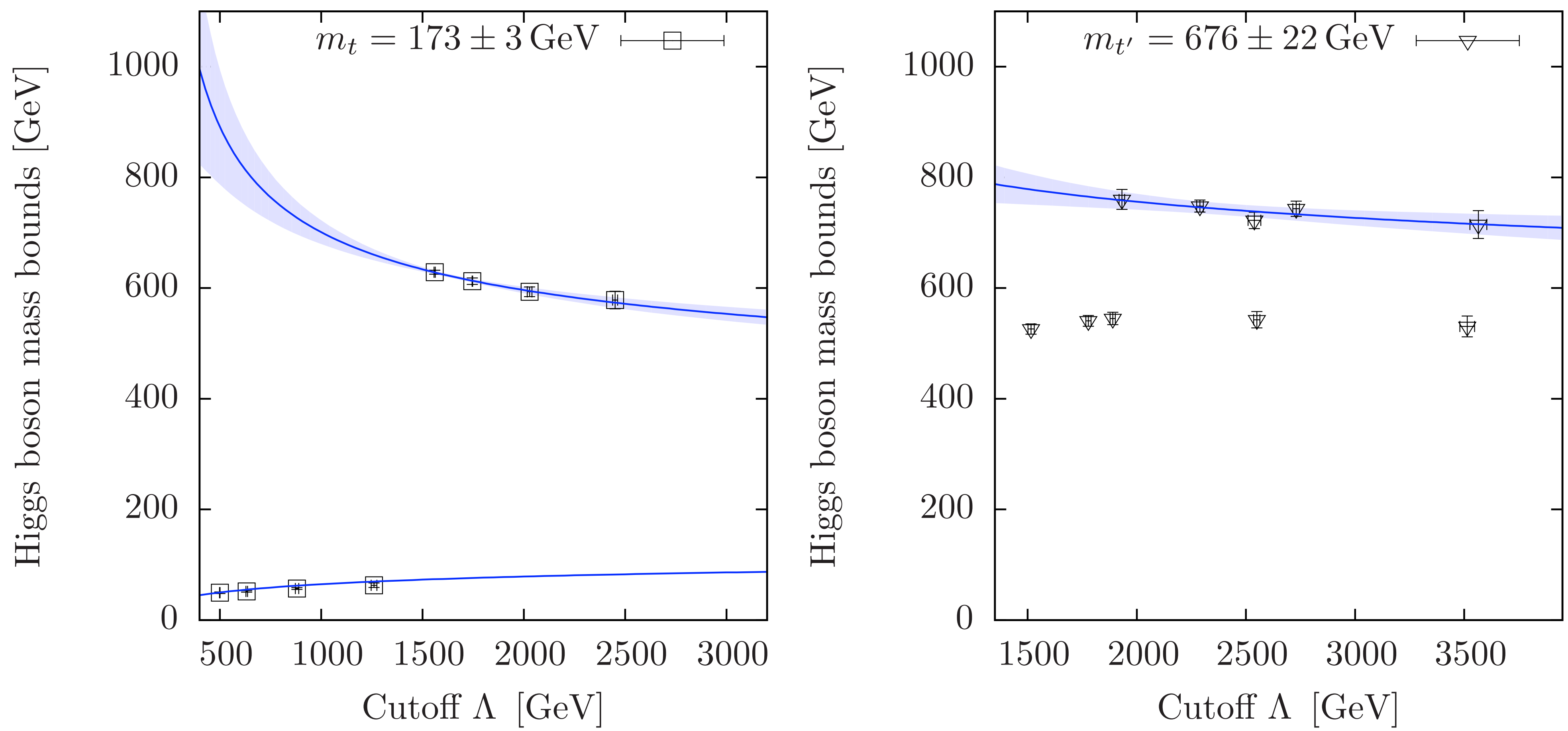
Outline

- Motivation.
- Do Higgs and Yukawa live close to a critical point?
  --- ideas and strategy (non-chiral example).
- Preliminary results (chiral theory) from our on-going study.
- Outlook.

Motivation



P. Gerhold and K. Jansen, 2011

* Constraints on the masses of extra-generation fermions from the 125 GeV scalar.

The 125 GeV scalar

- It may be a dilaton in a strongly-coupled theory:

  ➡ Does it have to be walking technicolour?
  ➡ HY model exhibits quasi scale inavariance?

  P.Q. Hung and C. Xiong, 2009

- It may be the Standard Model Higgs:

  ➡ Evade the hierarchy problem w/o SUSY?

- Both require non-perturbative studies:

  ➡ Second-order non-thermal phase transitions.

Hierarchy and triviality problem: perturbation theory (misleading)

- Scalar mass operator is of dimension 2 and is not protected by chiral symmetry.
- The one-loop beta-functions for the scalar and Yukawa coupling are positive.
- Perturbation theory over-simplifies the problem and may lead to misleading statements.

The scalar field theory as a spin model

- Scalar theory on the lattice (a=1),

$$S_\varphi \;=\; -\sum_{x,\mu} \varphi_x^\alpha \varphi_{x+\hat{\mu}}^\alpha + \sum_x \left[\frac{1}{2}(2d + m_0^2) \varphi_x^\alpha \varphi_x^\alpha + \frac{1}{4} \lambda_0 (\varphi_x^\alpha \varphi_x^\alpha)^2 \right]$$

- Perform the change of variables,

$$\Phi^\alpha = \sqrt{2\kappa}\phi^\alpha, \quad \lambda_0 = \frac{\hat{\lambda}}{\kappa^2}, \quad \bar{m}_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa} \quad .$$

- Bulk phase structure of the resulting spin model,

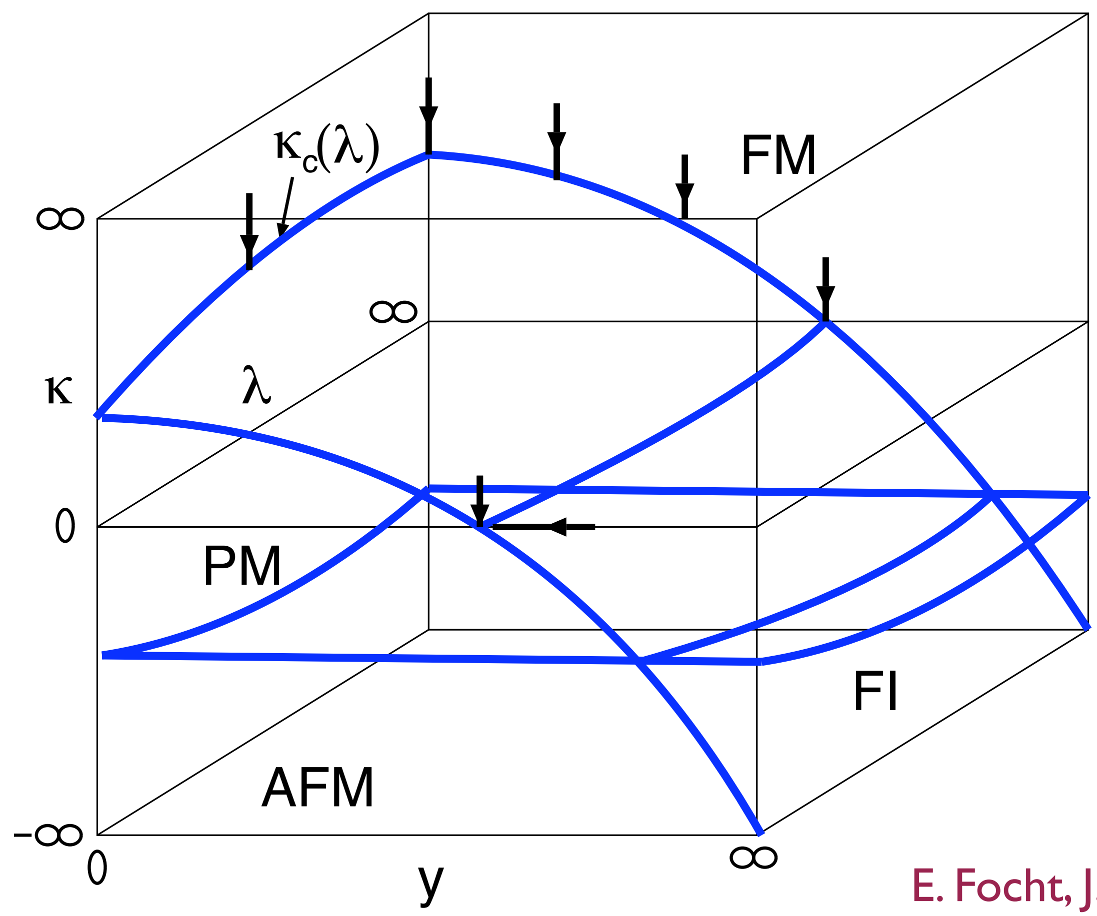
$$Z_\phi = \int \prod_{x,\alpha} d\phi_x^\alpha \; \exp(-S_\phi) = \int \prod_{x,\alpha} d\mu(\phi_x^\alpha) \; \exp\left(2\kappa \sum_{x,\mu} \phi_x^\alpha \phi_{x+\hat{\mu}}^\alpha \right),$$

$$d\mu(\phi_x^\alpha) = d\phi_x^\alpha \; \exp\left[-\phi_x^\alpha \phi_x^\alpha - \hat{\lambda}(\phi_x^\alpha \phi_x^\alpha - 1)^2 \right].$$

Fermions

- The overlap fermion (exact chiral symmetry).
- The lattice Yukawa operator takes the same form as its continuum counterpart.
- Extremely computationally demanding.

What is it like with the Yukawa coupling

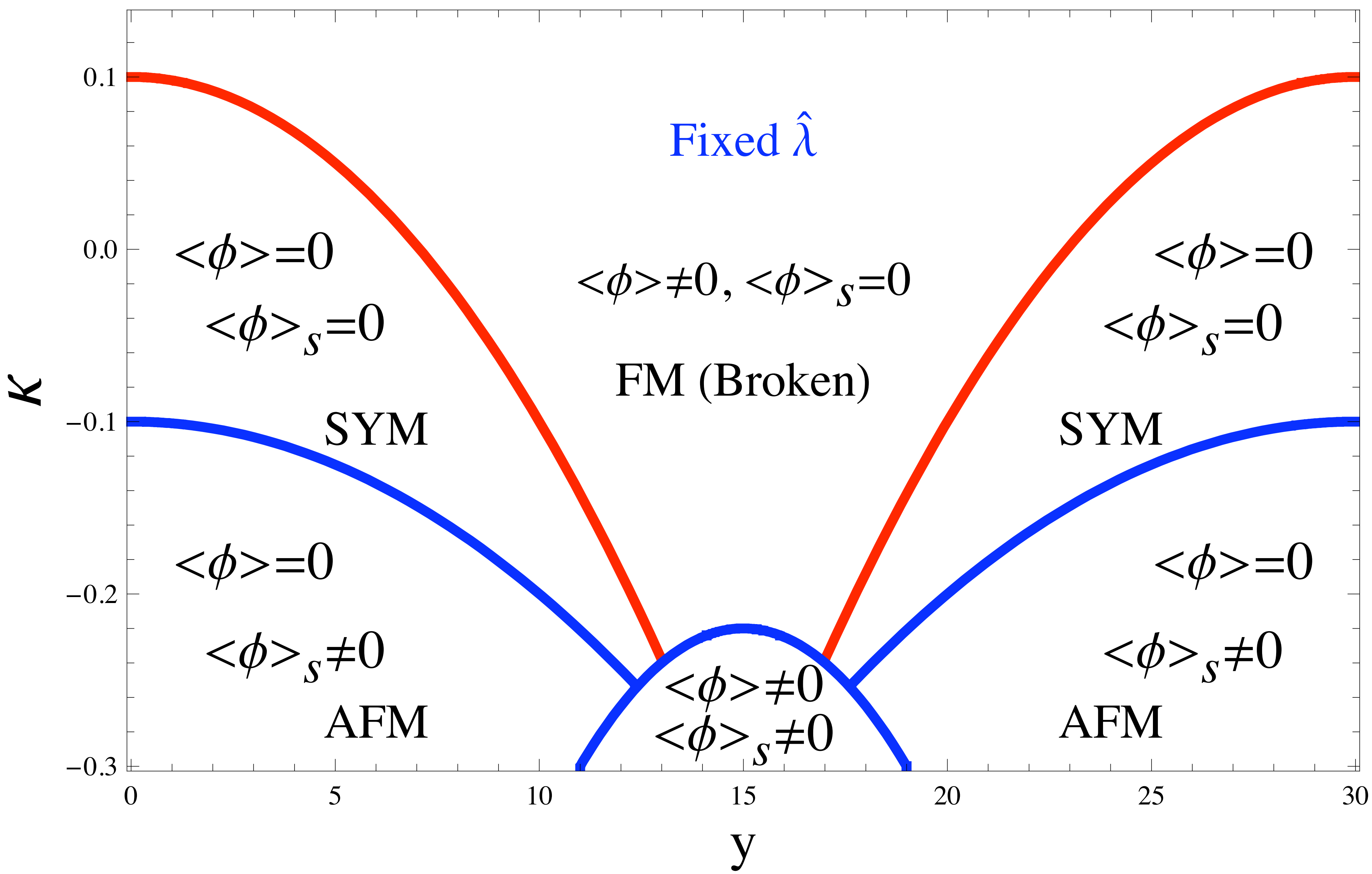


E. Focht, J. Jersak, J. Paul, 1995

Second-order phase transitions ⟶ Natural scale separation (continuum limit)

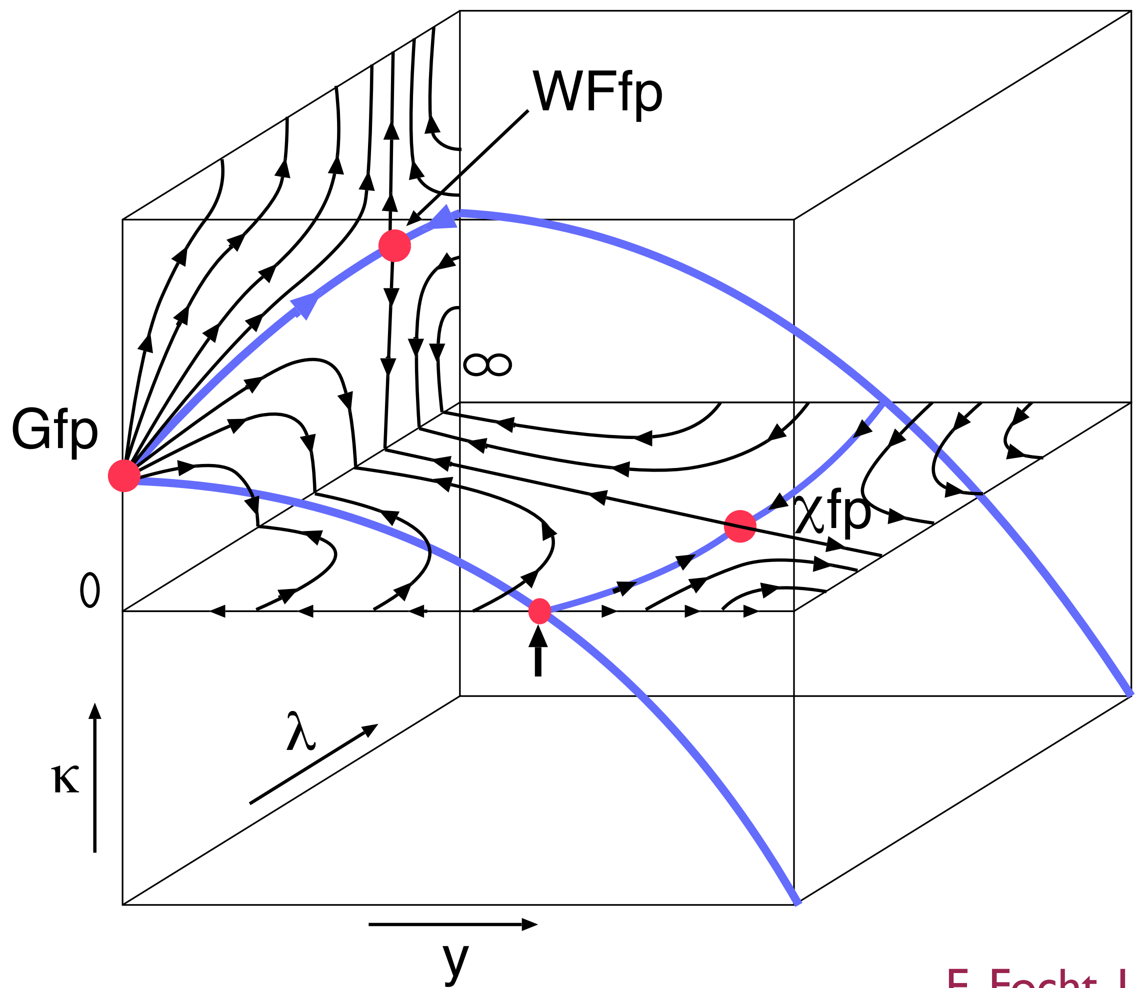
* Question: Is the theory non-trivial in 4D?

At stronger bare Yukawa coupling



Wednesday, May 15, 13

The bulk phase structure (3D)

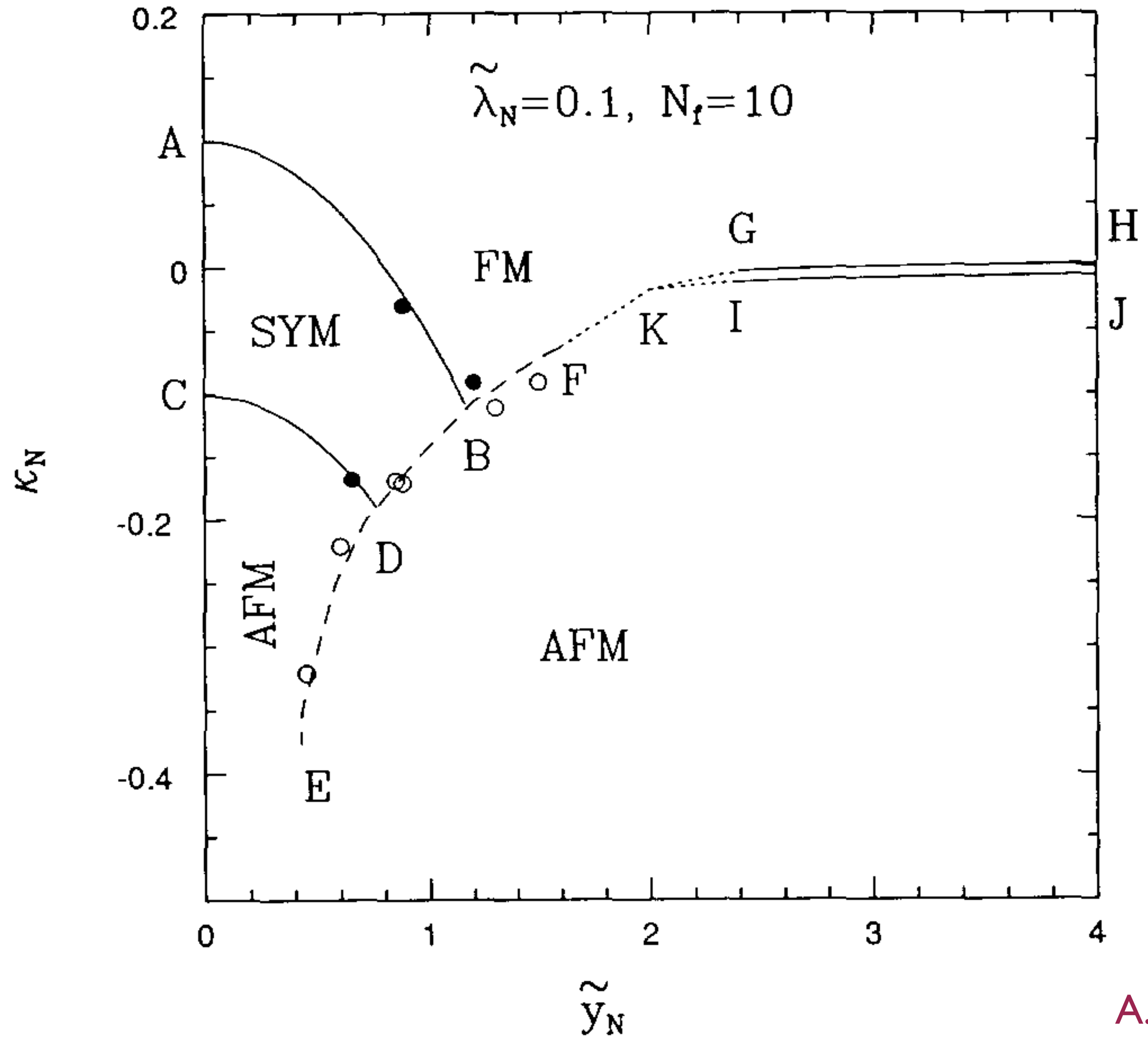


E. Focht, J. Jersak, J. Paul, 1995

Only the Gfp remains in 4D scalar sector....

*The hierarchy problem is a consequence of triviality in 4D

The 4D bulk phase structure



A.Hasenfratz *et al.*, 1993

Evidence for a tri-critical point?

If so, is the Yukawa coupling non-trivial there?

Our target

- Study the chiral theory.
- Investigate the phase structure in detail.
- Make contact with phenomenology.

Finite-size scaling (*a'la* M. Fisher)

- Renormalisation Group near fixed points.
- Central statement: "Universal" function

$$\frac{P_L(t)}{P_\infty(t)} = f\left(\frac{L}{\xi_\infty(t)}\right)$$, with observable P.

- Magnetic susceptibility and Binder's cumulant:

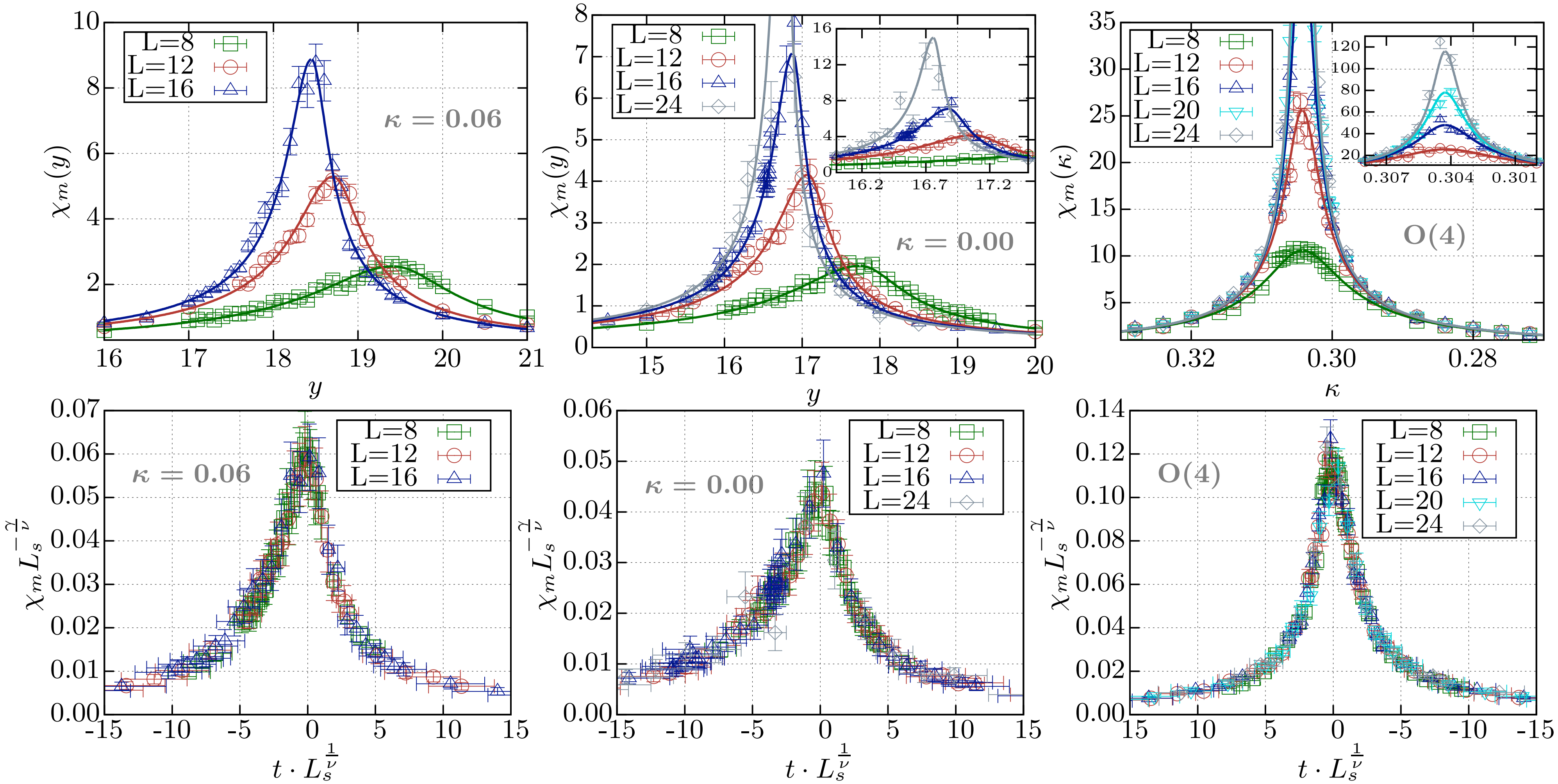
$$\chi_m(t,L) \cdot L_s^{-\gamma/\nu} = g\left(\hat{t} L_s^{1/\nu}\right), \text{ with } \hat{t} = \left[T/\left(T_c^{(L=\infty)} - C \cdot L_s^{-b}\right) - 1\right]$$

$$Q_L = g_{Q_L}\left(t L^{1/\nu}\right)$$

- γ and ν are the critical exponents.

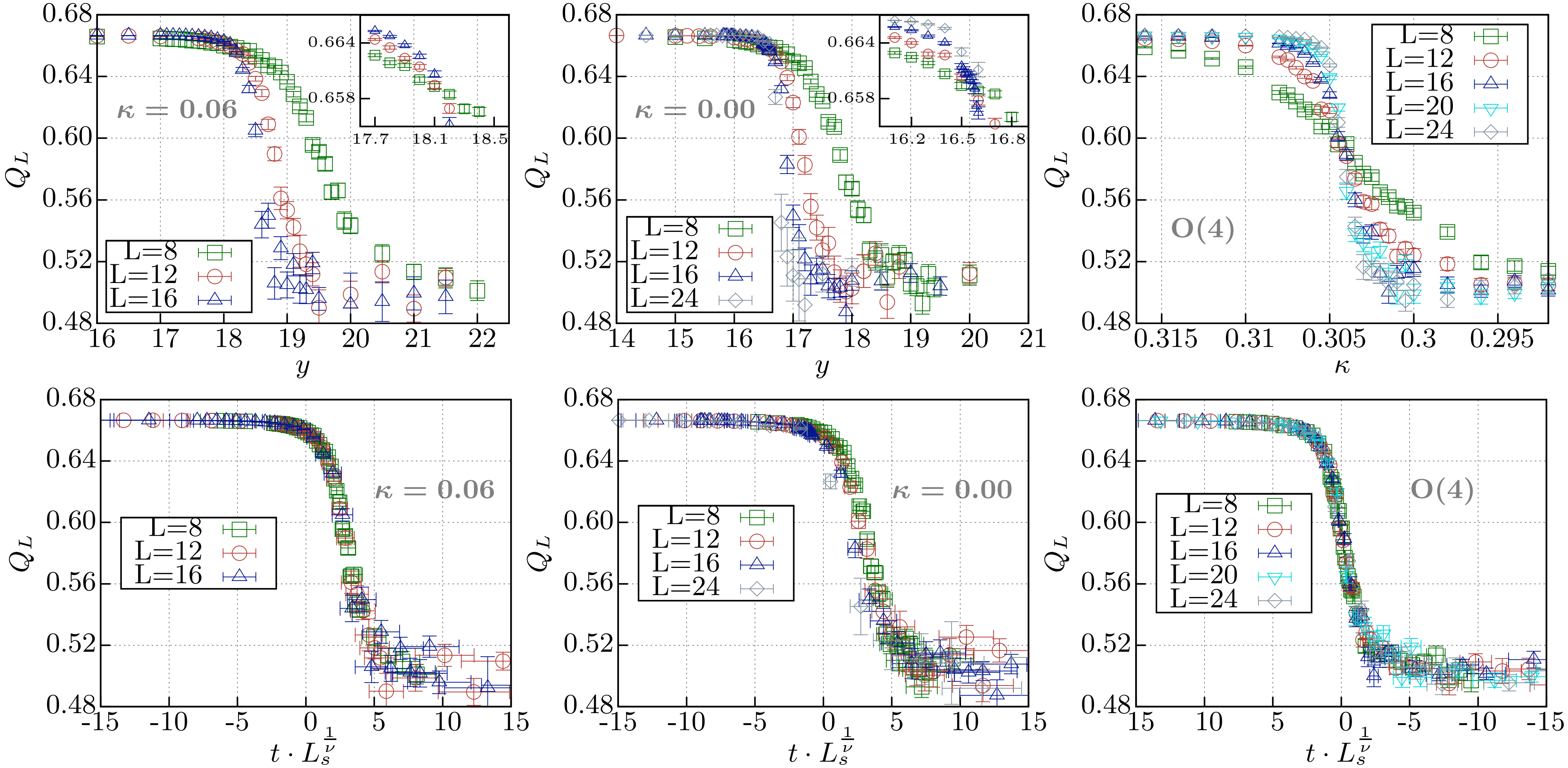
➡ How different are they from the mean-field values?

4D scaling test, susceptibility



	$T_c^{(L=\infty)}$	ν	γ	C	b	fit interval
$\kappa = 0.06$	18.119(67)	0.576(28)	1.038(30)	4.7(1.6)	1.95(18)	17.5, 20.0
$\kappa = 0.00$	16.676(15)	0.541(22)	0.996(15)	10(2)	2.42(10)	15.0, 19.0
O(4)	0.304268(27)	0.499(12)	1.086(19)	N/A	N/A	0.300, 0.308

4D scaling test, Binder's cumulant



	$T_c^{(L=\infty)}$	ν	interval
$\kappa = 0.06$	18.147(24)	0.550(1)	17.4, 18.8
$\kappa = 0.00$	16.667(27)	0.525(6)	16.0, 17.2
O(4)	0.3005(34)	0.50000(3)	0.294, 0.314

Concluding remarks and outlook

- Evidence for novel FP in the HY model.
- Complication in 4d (work in progress)

  ➨ Gaussian FP in the scalar sector.
  ➨ Does it remain in the HY model?
  ➨ Logarithmic corrections to FSS.

- Spectrum calculation (on-going work).