Anomaly in pion-induced **Drell-Yan processes** Hsiang-nan Li (李湘楠) Academia Sinica Presented at NTHU Mar. 14, 2013

# Outlines

- Introduction
- Violation of Lam-Tung relation
- Resolutions in the literature
- More relevant observations
- Pion-induced Drell-Yan processes
- Summary

# Introduction

- Drell-Yan is one of most intensively studied processes in QCD
- Data have been used to extract various PDF, TMD (unpolarized and polarized)
- But anomaly still exists since 80's:
- Lam-Tung relation, supposed to be obeyed by lepton angular distributions, is violated in pion-induced Drell-Yan

#### Violation of Lam-Tung relation

# Lepton pair angular distribution



# Lam-Tung Relation

 $\frac{d\sigma}{d\Omega} \propto [W_T (1 + \cos^2 \theta) + W_L (1 - \cos^2 \theta) + W_\Delta \sin 2\theta \cos \phi + W_{\Delta\Delta} \sin^2 \theta \cos 2\phi]$ 

 $\frac{d\sigma}{d\Omega} \propto (1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi)$ 

*qq* annihilation parton model:  $O(\alpha_s^0) \lambda = 1, \mu = \nu = 0; W_T = 1, W_L = 0$ pQCD:  $O(\alpha_s^1), W_L = 2W_{\Delta\Delta}; 1 - \lambda - 2\nu = 0$ 

Lam and Tung (PRD 18, 2447, 1978)

## LT relation holds in pp, pd DY



E866 (PRL 99, 082301, 2007; PRL 102, 182001, 2009)

#### Pion-induced DY



Fig. 3a-c. Parameters  $\lambda$ ,  $\mu$ , and  $\nu$  as a function of  $P_T$  in the CS frame. a 140 GeV/c; b 194 GeV/c; c 286 GeV/c. The error bars correspond to the statistical uncertainties only. The horizontal bars give the size of each interval. The dashed curves are the predictions of perturbative QCD [3]

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NA10 (Z. Phys. C 37, 545, 1988)
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### LT violation in pion-induced DY

140 GeV/c



NA10 (Z. Phys. C 37, 545, 1988)

#### Resolutions in the literature

Vacuum effect; Boer-Mulders function D. Boer

#### Angular asymmetries in Drell-Yan in theory



Dashed lines:  $\mathcal{O}(\alpha_s)$ ; Solid lines:  $\mathcal{O}(\alpha_s^2)$ ; Q = 8 GeVBrandenburg, Nachtmann & Mirkes, ZPC 60 (1993) 697

# need nonperturbative dynamics

Miniworkshop on Dihadron Fragmentation Functions (DiFF), Pavia, Sept 7, 2011

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#### **Collinear factorization**

Collinear quarks ( $p_{quark} = xP_{hadron}$ ) inside unpolarized hadrons are unpolarized too

$$ho^{(q,\bar{q})} = rac{1}{4} \{ {f 1} \otimes {f 1} \}$$



#### transversely polarized photon, structure WT only

Miniworkshop on Dihadron Fragmentation Functions (DiFF), Pavia, Sept 7, 2011

D. Boer

#### Angular asymmetry requires helicity flip

The  $\cos 2\phi$  asymmetry arises from an interference between +1 and -1 photon helicities  $\nu \neq 0$ 



This requires transversely polarized quark-antiquark annihilation

then parton transverse d.o.f comes in to play sine term appears, which breaks LT relation

D. Boer

#### Explanation as a QCD vacuum effect

The QCD vacuum can induce a spin correlation between an annihilating  $q\,\bar{q}$ 

Chromo-magnetic Sokolov-Ternov effect: spin-flip gluon synchrotron emission leading to a correlated polarization of q and qbar.

The spin density matrix becomes:



$$\rho^{(q,\bar{q})} = \frac{1}{4} \{ \mathbf{1} \otimes \mathbf{1} + F_j \, \boldsymbol{\sigma}_j \otimes \mathbf{1} + G_j \, \mathbf{1} \otimes \boldsymbol{\sigma}_j + H_{ij} \, \boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j \}$$

Lam-Tung relation could be violated

$$1 - \lambda - 2\nu = -4\kappa = -4\frac{H_{22} - H_{11}}{1 + H_{33}}$$

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## **Resolution via vacuum Effect**

#### parameterize the effect and fit to data

$$\kappa = \kappa_0 \frac{|q_T|^4}{|q_T|^4 + m_T^4},$$
  

$$\kappa_0 = 0.17, m_T = 1.5 GeV$$

Brandenburg, et. al (Z. Phy. C60,697, 1993)



#### **Resolution via BM Effect**



•  $h_1^{\perp}$  represents a correlation between quark's  $k_T$  and hadron mass transverse spin in an unpolarized hadron



# Remarks

- Vacuum effect is flavor blind, how to differentiate pion-proton and proton-proton DY? The pp DY obeys LT relation.
- BM can explain pp DY by arguing that sea quark is involved, and sea quark BM function is small

#### More relevant observations

# Clue 1: $k_{T}$ factorization

#### k<sub>T</sub> factorization

- Collinear and k<sub>T</sub> factorizations are fundamental tools in PQCD
- $k_T$  factorization applies to small x (high energy), to final-state spectra at low  $q_T$ , to exclusive process with end-point  $xP^+ \approx k_T$ , singularity (heavy-quark decays)  $q_T \approx k_T$
- Keep parton  $k_T$  in hard kernel, and  $k_T$  is not integrated out in PDF-> TMD
- $k_T$  factorization appropriate for studying low  $q_T$  spectra in Drell-Yan processes

## Clue 2: kT factorization breakdown

Collins, Qiu 07 Vogelsang, Yuan 07 Collins 0708.4410

#### Hadron hadroproduction

- $k_{T}$  factorization holds for simple processes like DIS, but breaks down for complicated ones like hadron production  $H_1(p_1) + H_2(p_2) \rightarrow H_3(p_3) + H_4(p_4) + X$
- Can lower TMD be factorized from the full process? for illustration,

consider scalar particles from collinear

eikonal line approximation



#### **NLO** factorization

 The first two diagrams do not contribute to lower TMD, k<sub>T</sub> factorization holds at NLO



Canceled by gluons on RHS. Cross section must be real.



Define lower PDF

anomalous

• Two gluons on the same side



#### Glauber gluons in Drell-Yan



for finite  $q_T$ 



 k<sub>T</sub> factorization broken by Glauber gluons in processes involving at least 3 hadrons

# Clue 3: Glauber phase at low $q_T$

# **Glauber-gluon** factorization

- Eikonal approximation holds at low q<sub>T</sub>
- Glauber gluons factorize



- TMD maintains its universality Chang, Li 2011
- Generalized k<sub>T</sub> factorization applies to Drell-Yan at low  $q_{T}$

## Clue 4: Other pion-involved puzzles

#### Puzzles in B decays

- $B(\pi^0\pi^0), B(\pi^0\rho^0)$  much larger than predictions
  - $$\begin{split} B(\pi^0\pi^0) &= 1.55 \pm 0.19, \quad [(0.29^{+0.50}_{-0.20})] & \text{data [theory]} \\ B(\pi^0\rho^0) &= 2.0 \pm 0.5, \quad [\approx 0.7] \quad ~1.91 \text{ X 10E-6} \\ B(\rho^0\rho^0) &= 0.74^{+0.30}_{-0.27}, \quad [(0.92^{+1.10}_{-0.56})], \end{split}$$
- $A_{CP}(\pi^0 K^{\pm})$  much different from  $A_{CP}(\pi^{\pm} K^{\pm})$
- New physics or QCD effect?
- If new physics, how about  $B(\pi^0\pi^0), B(\pi^0\rho^0)$
- If QCD, but  $B(\rho^0 \rho^0)$  is normal

#### Puzzles in D decays

	Mode	Representation	quark amplitudes	$\mathcal{B}_{\mathrm{exp}}$	$\mathcal{B}_{ ext{theory}}$		
			· ·	$(\times 10^{-3})$	$(\times 10^{-3})$		
$D^0$	$\pi^+\pi^-$	$\lambda_p[(T+E)\delta_{pd} + $	$P^p + PE + PA$ ]	$1.400\pm0.026$	$2.24 \pm 0.10$		
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}}\lambda_p[(-C+E)\delta_p]$	$_{pd} + P^p + PE + PA)]$	$0.80\pm0.05$	$1.35 \pm 0.05$		
	$\pi^0\eta$	$\lambda_p [-E\delta_{pd}\cos\phi -$	$\frac{1}{\sqrt{2}}C\delta_{ps}\sin\phi + (P^p + PE)\cos\phi]$	$0.68\pm0.07$	$0.75\pm0.02$		
	$\pi^0 \eta'$	$\lambda_p [-E\delta_{pd}\sin\phi +$	$\frac{1}{\sqrt{2}}C\delta_{ps}\cos\phi + (P^p + PE)\sin\phi$ ]	$0.89 \pm 0.14$	$0.74\pm0.02$		
	$\eta\eta$	$\frac{1}{\sqrt{2}}\lambda_p\{[(C+E)\delta_p$	$d_d + P^p + PE + PA ] \cos^2 \phi$	$1.67\pm0.20$	$1.44\pm0.08$		
		$+(-\frac{1}{\sqrt{2}}C\sin)$	$2\phi + 2E\sin^2\phi)\delta_{ps}\}$				
	$\eta\eta'$	$\lambda_p \{ \frac{1}{2} [ (C+E) \delta_{pd} \}$	$+ P^p + PE + PA ] \sin 2\phi$	$1.05\pm0.26$	$1.19\pm0.07$		
		$+(\frac{1}{\sqrt{2}}C\cos 2$	$\phi - E\sin 2\phi)\delta_{ps}\}$				
Ν	Note discrepancy in pi pi						

Even including symmetry breaking, pi pi puzzle persists.

See C.D. Lu's talk Cheng, Chiang 2012

# Pion hadroproduction

Bylinkin, Rostovtsev 1112.5734

- Charged particle production in hadron collisions at RHIC and LHC
- Spectrum shape parametrization

 $\frac{d\sigma}{P_T dP_T} = A_e \exp\left(-E_{Tkin}/T_e\right) + \frac{A}{(1 + \frac{P_T^2}{T^2 \cdot n})^n}$ 

$$\bar{E_{Tkin}} = \sqrt{P_T^2 + M^2} - M$$

#### M: hadron mass

 $A_e, A, T_e, T$  and "n" are free parameters

#### Fit to kaon and proton data

#### Negligible exponential term



#### Fit to pion data-sizable exp term



UA1, Phys. Lett. B366, 434 (1996)

FIG. 1. Pion spectrum [9] fitted with a modified Tsallis function (1): the red (dashed) line shows the exponential term and the green (solid) one - the power law.

# In terms of ratio R



but not these data fewer hadrons involved

ratio of power-law contribution with  $p_T$  integrated over

$$R = \frac{AnT}{AnT + A_e(2MT_e + 2T_e^2)(n-1)}$$

# **Common features**

- All anomalous processes demand  $k_T$  factorization (end-point singularity in heavy-flavor decays, and  $p_T$  spectra)
- All anomalous processes involve at least three hadrons
- The above are necessary conditions for Glauber divergences to appear
- All anomalous processes involve pions
- Do we really understand pion?

### Clue 5: Nambu-Goldstone boson

# Unique role of pion

- Due to confinement, ordinary hadron as quark bound state must have finite mass
- No contradiction
- If massless Nambu-Goldstone boson is elementary, no contradiction
- Pion is Nambu-Goldstone boson
- But pion is also  $q\overline{q}$  bound state with confinement, which must have finite mass
- How could it be possible?

# **Reconciliation?**

- Use different Fock states to meet different roles of pion
- Leading Fock state  $q\overline{q}$  is tight to lower confinement potential, higher Fock state gives soft cloud (Lepage, Brodsky 79; Nussinov, Shrock 08; Duraisamy, Kagan 08)
- But parton model is under approximation of neglect confinement...?
- Anyway, pion is unique.
- Strong Glauber effect from this soft cloud? Are those puzzles due to Glauber phase?

# Clue 6: Resolutions of puzzles with Glauber phase of pion

# Spectator diagrams involve 3 hadrons



#### pi K puzzle in B decays

- Treat Glauber phase  $exp(iS_e)$  as a constant
- pi K puzzle resolved for Se ~ -0.5



# D decay BRs with Se=-0.5 Li, Lu, Yu, 2011

Modes	Br(FSI)	Br(diagram)	Br(pole)	Br(exp)	Br(this work)
$D^0 \to \pi^+ \pi^-$	1.59	$2.24 {\pm} 0.10$	$2.2\pm0.5$	$1.45 {\pm} 0.05$	1.43 📛
$D^0 \to K^+ K^-$	4.56	$1.92{\pm}0.08$	$3.0 \pm 0.8$	$4.07 {\pm} 0.10$	4.19
$D^0 \to K^0 \overline{K}^0$	0.93	0	$0.3 \pm 0.1$	$0.320 {\pm} 0.038$	0.36
$D^0 \to \pi^0 \pi^0$	1.16	$1.35 {\pm} 0.05$	$0.8\pm0.2$	$0.81 {\pm} 0.05$	0.57
$D^0 \to \pi^0 \eta$	0.58	$0.75 {\pm} 0.02$	$1.1\pm0.3$	$0.68 {\pm} 0.07$	0.94
$D^0 \to \pi^0 \eta'$	1.7	$0.74 {\pm} 0.02$	$0.6\pm0.2$	$0.91 {\pm} 0.13$	0.65
$D^0 \to \eta \eta$	1.0	$1.44 {\pm} 0.08$	$1.3\pm0.4$	$1.67 {\pm} 0.18$	1.48
$D^0 \to \eta \eta'$	2.2	$1.19 {\pm} 0.07$	$1.1\pm0.1$	$1.05 {\pm} 0.26$	1.54
$D^+ \to \pi^+ \pi^0$	1.7	$0.88 {\pm} 0.10$	$1.0\pm0.5$	$1.18 {\pm} 0.07$	0.89
$D^+ \to K^+ \overline{K}^0$	8.6	$5.46 {\pm} 0.53$	$8.4\pm1.6$	$6.12 \pm 0.22$	5.95
$D^+ \to \pi^+ \eta$	3.6	$1.48 {\pm} 0.26$	$1.6\pm1.0$	$3.54{\pm}0.21$	3.39 📛
$D^+ \to \pi^+ \eta'$	7.9	$3.70 {\pm} 0.37$	$5.5 \pm 0.8$	$4.68 {\pm} 0.29$	4.58
$D_S^+ \to \pi^0 K^+$	1.6	$0.86 {\pm} 0.09$	$0.5 \pm 0.2$	$0.62 {\pm} 0.23$	0.67
$D_S^+ \to \pi^+ K^0$	4.3	$2.73 {\pm} 0.26$	$2.8\pm0.6$	$2.52 \pm 0.27$	2.21
$D_S^+ \to K^+ \eta$	2.7	$0.78 {\pm} 0.09$	$0.8\pm0.5$	$1.76 {\pm} 0.36$	1.00
$D_S^+ \to K^+ \eta'$	5.2	$1.07 {\pm} 0.17$	$1.4\pm0.4$	$1.8 \pm 0.5$	1.92

# Pion-induced Drell-Yan processes

Put all clues together...

# **Collins-Soper frame**



- $q_T$ : lepton-pair transverse momentum
- Q: lepton-pair invariant mass
- θ<sub>1</sub>: related to boost of Collins-Soper frame

## LO diagrams for finite $q_T$







# Glauber gluons

 Sum over Glauber-gluon attachments to ladder diagrams





one more collinear gluon is needed next-to-leading logarithm

## Glauber phase

 Assign exp(iSe) and exp(-iSe) to first two diagrams, coefficient of each angular term

$$\begin{split} \hat{\sigma}_{0} &= \left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right) \left(1 + \frac{1}{2}s_{1}^{2}\right) + \underline{(c_{e} - 1)} & c_{e} \equiv \cos S_{e} \\ &\times \left\{2\left[\frac{E_{1}E_{2}}{k^{2}}c_{1}^{2} + \frac{k}{E_{1}} + \frac{k}{E_{2}} - \frac{1}{2}\left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right) - 2\right] \\ &- \left[\frac{E_{1}E_{2}}{k^{2}}c_{1}^{2} - \frac{1}{2}\left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right)\right]s_{1}^{2}\right\}, \quad (3) \\ \hat{\sigma}_{1} &= \left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right) \left(c_{1}^{2} - \frac{1}{2}s_{1}^{2}\right) + (c_{e} - 1) \\ &\times \left\{\left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}} - 2\right)c_{1}^{2} + \frac{E_{2}}{E_{1}}\right)\right]s_{1}^{2}\right\} & \text{E1: hadron 1 energy} \\ &+ \left[\frac{E_{1}E_{2}}{k^{2}}c_{1}^{2} - \frac{1}{2}\left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right)\right]s_{1}^{2}\right\} \end{split}$$

#### More formulas

$$\begin{aligned} \hat{\sigma}_{2} &= \left(\frac{E_{1}}{E_{2}} - \frac{E_{2}}{E_{1}}\right) c_{1}s_{1} + (c_{e} - 1) \\ &\times \left(\frac{E_{2} - E_{1}}{k} + \frac{E_{1}}{E_{2}} - \frac{E_{2}}{E_{1}}\right) c_{1}s_{1} \\ \hat{\sigma}_{3} &= \left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right) s_{1}^{2} - (c_{e} - 1) \\ &\times 2 \left[\frac{E_{1}E_{2}}{k^{2}}c_{1}^{2} - \frac{1}{2}\left(\frac{E_{1}}{E_{2}} + \frac{E_{2}}{E_{1}}\right)\right] s_{1}^{2}, \end{aligned}$$

$$\sin^2 \theta_1 = \frac{q_T^2}{Q^2 + q_T^2}$$

corresponding to boost in Collins-Soper frame

$$\lambda \sim \hat{\sigma}_1 / \hat{\sigma}_0 \quad \mu \sim \hat{\sigma}_2 / \hat{\sigma}_0 \quad \nu \sim \hat{\sigma}_3 / \hat{\sigma}_0$$

• As Se->0  $\lambda = \frac{c_1^2 - \frac{1}{2}s_1^2}{1 + \frac{1}{2}s_1^2}, \ \mu = \frac{\frac{E_1}{E_2} - \frac{E_2}{E_1}}{\frac{E_1}{E_2} + \frac{E_2}{E_1}} \frac{c_1s_1}{1 + \frac{1}{2}s_1^2}, \ \nu = \frac{s_1^2}{1 + \frac{1}{2}s_1^2}$ 

obey Lam-Tung relation

#### Numerical results



#### $q_T$ dependence



# Comparison

- Like vacuum effect, Glauber gluon causes fact. breakdown (but from soft cloud)
- Vacuum effect is flavor blind, but Glauber effect is strong only from pion
- Compared to BM function, parton transverse d.o.f introduced by small  $q_T$
- Glauber gluon: Nambu-Goldstone nature
- BM or Nambu-Goldstone nature? Discriminated by measuring  $\overline{p}p$  Drell-Yan (valence anti-quark from  $\overline{p}$ )

## CDF $\overline{p}p$ data

- Lam-Tung relation is respected:  $A_0 = A_2$
- at large Q=mZ, consistent with LT
- p<sub>T</sub>/Q not small, support Glauber gluons



# Summary

- Violation of Lam-Tung relation is one of pion-involved anomalous data
- Glauber-gluon effect leads to phase factor in k<sub>T</sub> factorization, which could resolve several pion-involved puzzles
- Propose to measure anti-proton-proton
   Drell-Yan process to discriminate the BM or Nambu-Goldstone nature
- Next target: can pion hadroproduction be resolved by Glauber gluons?

#### Back-up slides

#### Total NNLO

#### Two gluons on opposite sides

$$\begin{split} I_1(k_T) &= \frac{\lambda^2 g_1^2 g_2(g_2 + g_1)}{(2\pi)^{12}} x p^+ \int dk^- \, d^4 l_1 \, d^4 l_2 \frac{[2(p^+ - k^+) + l_1^+] \, [2(p^+ - k^+) + l_2^+]}{(l_1^2 - m_g^2) \, (l_2^2 - m_g^2) \, [(k - l_1) - m_q^2 + i\epsilon] \, [(k - l_2) - m_q^2 + i\epsilon]} \\ &\times \frac{(2\pi)^2 \delta(l_1^+) \delta(l_2^+) \, 2\pi \delta\left((p - k)^2 - m_q^2\right)}{[(p - k + l_2) - m_q^2 + i\epsilon]} \\ &= \frac{\lambda^2 g_1^2 g_2(g_2 + g_1) x(1 - x)}{256\pi^7} \int d^2 l_{1T} \, d^2 l_{2T} \prod_{j=1,2} \frac{1}{(l_{jT}^2 + m_g^2) \, [(k_T - l_{jT})^2 + m_q^2]}. \end{split}$$

Two gluons on the same sides

$$I_2(k_T) = \frac{-\lambda^2 g_1^2 g_2(g_2 + g_1) x(1 - x)}{256\pi^7} \int d^2 l_{1T} \, d^2 l_{2T} \, \frac{1}{(l_{1T}^2 + m_g^2) \left(l_{2T}^2 + m_g^2\right) \left[(k_T - l_{1T} - l_{2T})^2 + m_q^2\right] \left(k_T^2 + m_q^2\right)}$$

 No cancellation. Lower TMD is not universal. k<sub>T</sub> fact. fails. Integrated over k<sub>T</sub>, they cancel. Collinear factorization holds.

#### Glauber region

- Collinear region (I<sup>+</sup>, I<sup>-</sup>, I<sub>T</sub>) ~ (E, m<sup>2</sup>/E, m)
- Soft region ~ (m,m,m)
- Breakdown appears in Glauber region (0, m<sup>2</sup>/E, m) due to δ(I+), I<sup>2</sup> = - I<sub>T</sub><sup>2</sup>
- Consider spectator propagator  $(p_1-k_1+l)^2=2(p_1-k_1)+l^2+(k_{1T}+l_T)^2$
- Two terms are of the same order of m<sup>2</sup>
  - $\Rightarrow$  (k<sub>1T</sub> + I<sub>T</sub>)<sup>2</sup> is not negligible
  - $\Rightarrow$  Eikonal approximation does not hold
- IR gluons usually not factorized in Glauber region. Universality of TMD is lost



- I- is of order m<sup>2</sup>/E and m, respectively
- For latter,  $2(p_1 k_1)^+ l^- >> (k_{1T} + l_T)^2$

# **Reconciliation?**

- As r decreases, interaction decreases, m decreases, but quarks not allowed to move freely
- To reconcile pion's role, need to create state "no interaction, no free motion"
- Namely, short distance, low mass
- As r decreases,  $k_T$  must be frozen
- Pion has large soft cloud in this sense,  $k_T <<1/r$
- Violation of uncertainty principle?

# Large soft cloud in pion

• Model of Brodsky et al. (1980)

$$\phi_p(x,k_T) = \frac{4\pi m_p^2}{[m_p^2 + k_T^2/x + k_T^2/(1-x)]^2}$$

- m<sub>p</sub>: meson mass carried by quark
- Distribution in b space carried by anti-quark



# Clue 6: Glauber phase from pion

#### Color-suppressed tree

•  $B(\pi^0\pi^0), B(\rho^0\rho^0)$  both depend on colorsuppressed tree amplitude C



 C is an important but least understood quantity in B decays

# Glauber phase factor

- Glauber factor is factorized and universal. How is it different between  $\pi$  ,  $\rho$  ?
- Though Glauber gluons are factorized, loop momentum  $I_{\rm T}$  flows through mesons
- If mesons have different intrinsic  $k_T$  dependence, Glauber effects are different
- Consider Glauber factor

$$G_2(l_T) = \int d^2b' \exp(il_T \cdot b') \exp[iS(b')]$$

parametrization  $S(b) = \alpha b^2$ 

# Convolution in $k_T$ space

• Consider intrinsic  $k_T$  dependence

.

$$\int \frac{d^2 k_T}{(2\pi)^2} \frac{d^2 k_{1T}}{(2\pi)^2} \frac{d^2 k_{2T}}{(2\pi)^2} \int \frac{d^2 l_T}{(2\pi)^2} \phi_B(k_T) \phi_1(k_{1T})$$



#### Glauber effect

 Resum Glauber divergence to all orders, like summing collinear divergence into meson wave function





# Numerical results

depend on T

#### • With $\alpha = -0.42$ for all following

Mode	Data [1]	NLO $[4]$	NLO (this work)
$B^0 \to \pi^+ \pi^-$	$5.11\pm0.22$	$6.5^{+}_{-3.8(-1.8)}$	$6.58^{+2.21}_{-1.62}(\omega_B)^{+0.24}_{-0.19}(a_2^{\pi})$
$B^+ \to \pi^+ \pi^0$	$5.48^{+0.35}_{-0.34}$	$4.0^{+}_{-1.9(-1.2)}$	$5.60^{+0.00}_{-1.90}(\omega_B)^{+0.00}_{-2.39}(a_2^{\pi})$
$B^0 \to \pi^0 \pi^0$	$1.91\substack{+0.22\\-0.23}$	$0.29^{+0.50(+0.13)}_{-0.20(-0.08)}$	$1.10^{+0.00}_{-0.88}(\omega_B)^{+0.00}_{-0.65}(a_2^{\pi}) \Leftarrow$
$B^0  o  ho^0  ho^0$	$0.73\substack{+0.27 \\ -0.28}$	$0.92^{+1.10(+0.64)}_{-0.56(-0.40)}$	$0.61^{+1.02}_{-0.00}(\omega_B)^{+0.15}_{-0.06}(a_2^{\rho})$

Li, Liu, Xiao, to appear

 Pion indeed shows stronger Glauber effect than rho

Pion multiplicity in e+e- anni.

Hadron productions at Z pole





#### BM, vacuum effect, and Glauber gluons

	$h_1^{\perp} \neq 0$	QCD vacuum effect	Blauber gluon
$ ho^{(q,ar q)}$	$ ho^{(q)}\otimes ho^{(ar q)}$	possibly entangled pos	sibly entangled
Q dependence	$\kappa \sim 1/Q$	?	1/Q <sup>2</sup>
large $Q_T$ limit	$\kappa \to 0$	need not disappear ( $\kappa  ightarrow \kappa_0$	) should disappear
flavor dependence	yes	flavor blind	yes
x dependence	yes	if yes, then not hadron bline	<sub>d</sub> yes

D.B., Brandenburg, Nachtmann & Utermann, EPJC 40 (2005) 55

Different experiments ( $\pi^{\pm}, p, \bar{p}, \ldots$  beams) are needed in different kinematical regimes

CDF data at Q=mZ, consistent with LT, may not discriminate BM or Glauber gluons (1103.5699)