D-branes in Double Field Theory

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Outline

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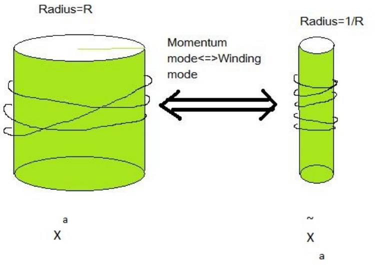
T-duality

Geometrically, T-duality arises from compactifying a theory on a circle with radius R, and such a theory describes the same physics as a theory compactified on a circle with radius $1/R$ with the winding mode and momentum mode exchanged.



Motivation

Idea of doubled geometry:

$$
\begin{array}{ccc}
 & \text{A doubled torus with coordinates}(x, \tilde{x}) & \\
\swarrow & & \searrow \\
\text{Physical space} & & \text{T dual}
\end{array}
$$

Idea of double field theory:

To write down the complete field theory in coordinate space, one must include both x^a and its dual $\tilde{x}_a$:

$$S = \int dx^a d\tilde{x}^a dx^\mu \mathcal{L}(x^a, \tilde{x}_a, x^\mu),$$

i.e. all fields are doubled and we call it a double field theory.

Developments

[1990-1991] Tsytlin proposed a T-duality symmetric formulation by adding the dual coordinates to the standard worldsheet theory. In this theory, the world sheet action takes the form of a FJ-type action originally proposed by Floreanini and Jackiw (FJ formulation, 89'), however the action is not manifestly Lorentz covariant.

[2009-2011] Hull et. al. reformulated the double framework and the resulting double field theory for the massless sector of closed string is manifestly Lorentz covariant. In this formalism, one can eliminate half of the degrees of freedom by imposing a self-duality constraint.

[2011] Albertsson, Dai, Lin and K, double field theory formulation for doubled D-branes.

[2011] Supersymmetric double field theory–Hohm and Kwak, Jeon, Lee and Park.

[2011] Double field formation of Yang-mills theory–Jeon, Lee and Park.

And more...

Outline

Doubled geometry setup

Let $X^i = (X^a, X^\mu)$, where $X^a \backsim X^a + 2\pi$, $i = 1, \cdots, n$. With metric and B-field defined by

$$G_{ij} = \begin{pmatrix} g_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} b_{ab} & 0 \\ 0 & 0 \end{pmatrix},$$

where η is the flat worldsheet metric.
G and B can be combined into a string background field E defined by

$$E_{ij} = G_{ij} + B_{ij} = \begin{pmatrix} E_{ab} & 0 \\ 0 & \eta_{\mu\nu} \end{pmatrix}, \quad E_{ab} = g_{ab} + b_{ab}.$$

$O(n,n)$ T-duality group

Let h be an element of $O(n,n;\mathbb{Z})$ defined by

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

Then h preserves the form

$$\eta = \begin{pmatrix} 0 & \mathbb{I}_{n\times n} \\ \mathbb{I}_{n\times n} & 0 \end{pmatrix},$$

i.e. $h^t \eta h = \eta$. T-duality group acts on a string background $E_{ab} = g_{ab} + b_{ab}$ $(a, b = 1, \ldots, n)$ on the n-torus via

$$\tilde{E} \equiv h(E) = (a(E) + b)(c(E) + d)^{-1}.$$

In doubled geometry, the key component is a $2n \times 2n$-matrix $\mathbb{H}$ called a generalized metric which transform as an $O(n,n)$-tensor:

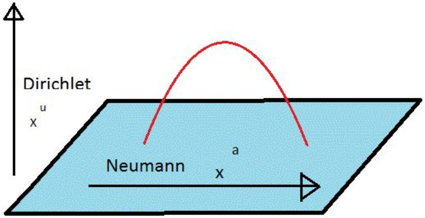
$$\mathbb{H} = \begin{pmatrix} G - BG^{-1}B & BG^{-}1 \\ -G^{-1}B & G^{-1} \end{pmatrix},$$

and also an $O(n,n)$-invariant constant matrix $\mathbb{L}$ conveniently chosen as

$$\mathbb{L} = \begin{pmatrix} 0 & \mathbb{I}_{n\times n} \\ \mathbb{I}_{n\times n} & 0 \end{pmatrix}.$$

Doubled coordinate is defined by $\mathbb{X}^I = (X^i, \tilde{X}_i)$ where $\tilde{X}_i = (\tilde{X}_a, \tilde{X}_\nu)$.

D-branes



Neumann boundary condition:

$$\partial_1 X^a|_{\partial\Sigma} = 0.$$

Dirichlet boundary condition:

$$\delta X^\mu|_{\partial\Sigma} = 0.$$

T-duality exchanges Dirichlet and Neumann boundary conditions.

Projectors for a doubled D-brane

On doubled space, we can define the corresponding projectors $(\Pi_D, \Pi_N \in GL(2n))$:

- Dirichlet projector: Π_D,
  Neumann projectors: Π_N,
- Projectors by definition: ${\Pi_{N,I}}^J \equiv (\mathbb{I} - \Pi_D^t)_I{}^J$,
- The projectors need to satisfy the following conditions
  1. Normal condition: $\Pi_D^2 = \Pi_D$, and $\Pi_N^2 = \Pi_N$.
  2. Null condition: $\Pi_D^t \mathbb{L} \Pi_D = 0$, $\Pi_N \mathbb{L} \Pi_N^t = 0$.
  3. Orthoganality condition: $\Pi_N \mathbb{H} \Pi_D = 0$.
  4. Integrability condition: ${\Pi_{N,I}}^K {\Pi_{N,J}}^L \partial_{[K} \Pi^t_{N,L]}{}^{.M} = 0$.

Boundary conditions

The Dirichlet projector is used to express the Dirichlet boundary condition in a covariant way:

$$\Pi^{I}_{D,J}\partial_0\mathbb{X}|_{\partial_\Sigma} = 0,$$

this is equivalent to the Neumann boundary condition:

$$\Pi_{N,I}{}^{J}\mathbb{H}_{JK}\partial_1\mathbb{X}^K|_{\partial_\Sigma} = 0.$$

T-duality transformation

Let $h \in O(n, n; Z)$.
The doubled coordinate, generalized metric and Dirichlet/Neumann projectors transform under T-duality via

$$\begin{aligned} \mathbb{H}_{IJ} \mapsto & \quad \tilde{\mathbb{H}}_{IJ} = (h^{-1}\mathbb{H}h)_{IJ}, \\ \mathbb{X}^I \mapsto & \quad \tilde{\mathbb{X}}^I = (h^{-1})^I{}_J \mathbb{X}^J, \\ \Pi_D \mapsto & \quad \tilde{\Pi}_D = h^{-1}\Pi_D h, \\ \Pi_N \mapsto & \quad \tilde{\Pi}_N = h^{-1}\Pi_N h. \end{aligned}$$

Besides, a self-duality condition is required to reduce the degrees of freedom to half:

$$\partial_\alpha \mathbb{X}^I = \epsilon_\alpha^\beta \mathbb{L}^{IJ} \mathbb{H}_{JK} \partial_\beta \mathbb{X}^K.$$

2-Dimensional Example

Consider a 2-dimensional model with

$$\mathbb{H} = \begin{pmatrix} R^2 & 0 \\ 0 & R^{-2} \end{pmatrix}, \quad \mathbb{L} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and double coordinates $X = (x, \tilde{x})^t$.
The possible allowed Dirichlet projectors are

$$\Pi_D^{(1)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \Pi_D^{(2)} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

since by definition $\Pi_D = (\mathbb{I} - \Pi_N^t)$, the corresponding Neumann projectors are

$$\Pi_N^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Pi_N^{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

The self-duality condition $\partial_\alpha \mathbb{X}^I = \epsilon_\alpha^\beta \mathbb{L}^{IJ} \mathbb{H}_{JK} \partial_\beta \mathbb{X}^K$ gives

$$\partial_0 x = R^{-2} \partial_1 \tilde{x}, \quad \partial_0 \tilde{x} = R^2 \partial_1 x.$$

Case I: Dirichlet projector $\Pi_D^{(1)}$

Neumann boundary condition ${\Pi_{N,I}}^J \mathbb{H}_{JK}\partial_1 \mathbb{X}^K|_{\partial_\Sigma} = 0$ gives:

$$R^{-2}\partial_1 \tilde{x} = 0.$$

Substitute the above relation into the self-duality condition gives

$$\partial_0 x = 0, \quad \partial_1 \tilde{x} = 0,$$

i.e. $\tilde{x}$ is a Neumann direction and resulting a $D1$-brane on the T-dual:

$$\begin{tikzcd} & (x, \tilde{x}) \arrow[dl] \arrow[dr] & \\ \mathrm{D0-brane} & & \mathrm{D1-brane} \end{tikzcd}$$

Case II: Dirichlet projector $\Pi_D^{(2)}$

Neumann boundary condition ${\Pi_{N,I}}^J \mathbb{H}_{JK}\partial_1 \mathbb{X}^K|_{\partial_\Sigma} = 0$ gives:

$$R^2 \partial_1 x = 0.$$

Substitute the above relation into the self-duality condition gives

$$\partial_0 \tilde{x} = 0,$$

i.e. x is a Neumann direction and resulting a $D1$-brane on the physical space.

$$\begin{tikzcd} & (x, \tilde{x}) \arrow[dl] \arrow[dr] & \\ \mathrm{D1-brane} & & \mathrm{D0-brane} \end{tikzcd}$$

T-duality generated by

$$h = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

exchanges the role of x and $\tilde{x}$.
T-duality generate by h acts on the projectors via

$$\tilde{\Pi}_D^{(1)} = h^{-1}\Pi_D^{(0)}h = \Pi_D^{(2)},$$

i.e.

$$\Pi_D^{(1)} \longleftrightarrow^{h} \Pi_{(D)}^{(2)},$$

which has the intepretation of

$$\mathrm{D0-brane} \longleftrightarrow^{\mathrm{T-duality}} \mathrm{D1-brane}$$

4-dimensional example

Consider a four dimensional doubled flat space with metric $G_{ij} = \delta_{ij}$ and constant B field $B_{ij} = \epsilon_{ij} b$. The doubled coordinates split as $\mathbb{X} = (x, y, \tilde{x}, \tilde{y})^t$. Thus the generalized metric becomes

$$\mathbb{H} = \begin{pmatrix} 1+b^2 & 0 & 0 & b \\ 0 & 1+b^2 & -b & 0 \\ 0 & -b & 1 & 0 \\ b & 0 & 0 & 1 \end{pmatrix} .$$

The self-duality conditions read

$$\begin{aligned} \partial_0 \tilde{x} &= \partial_1 x + b \partial_0 \tilde{y}, \\ \partial_0 \tilde{y} &= \partial_1 y - b \partial_0 \tilde{x}, \\ \partial_1 \tilde{x} &= \partial_0 x + b \partial_1 \tilde{y}, \\ \partial_1 \tilde{y} &= \partial_0 y - b \partial_1 \tilde{x}, \end{aligned}$$

If we consider the following Dirichlet projector

$$\Pi_D(D0) = \begin{pmatrix} \mathbb{I}_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & 0_{2\times 2} \end{pmatrix},$$

we obtain the boundary conditions for a $D0$-brane in the $\{X, Y\}$-space, i.e.,

$$\partial_0 X|_{\partial\Sigma} = 0, \quad \partial_0 Y|_{\partial\Sigma} = 0.$$

Similarly, if we consider the following Dirichlet projector,

$$\Pi_D(D2) = \begin{pmatrix} 0_{2\times 2} & 0_{2\times 2} \\ 0_{2\times 2} & \mathbb{I}_{2\times 2} \end{pmatrix},$$

we obtain a $D2$-brane in a constant B-field in the $\{X, Y\}$-space, i.e.

$$(\partial_1 X - B\partial_0 Y)|_{\partial\Sigma} = 0, \quad (\partial_1 Y + B\partial_0 X)|_{\partial\Sigma} = 0.$$

Outline

FJ-action

Consider the Floreanini-Jackiw (FJ)-type action as the bulk action

$$S_{FJ} = \frac{1}{2}\int_{\Sigma} d^2\sigma \left(-\mathbb{H}_{IJ}\partial_1\mathbb{X}^I\partial_1\mathbb{X}^J + \mathbb{L}_{IJ}\partial_1\mathbb{X}^I\partial_0\mathbb{X}^J\right).$$

The FJ-action is Lorentz invariant as a worldsheet action, thus a consistent doubled formalism for quantization.

We propose the boundary source term for the gauge fields

$$S_b = -\int_{\partial\Sigma} d\tau(\mathbb{A}_I\partial_0\mathbb{X}^J).$$

Inserting $1 = \Pi_D + \Pi_N^t$ in the boundary term

$$S_b = -\int_{\partial\Sigma} d\tau \left(\mathbb{A}_I(\Pi_D + \Pi_N^t)\partial_0\mathbb{X}^I\right) = -\int_{\partial\Sigma} d\tau \left(\mathbb{A}_I\Pi_N^t\partial_0\mathbb{X}^I\right)$$

$$\equiv -\int_{\partial\Sigma} d\tau \left(\mathcal{A}_p\partial_0\mathcal{X}^p\right).$$

where $\mathcal{A}$ and $\mathcal{X}$ are $\mathbb{A}$ and $\mathbb{X}$ restricted to the Neumann subspace.

For example, consider the previous $2D$-example where
$\Pi_N^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ corresponds to a $D1$-brane extends in the $\tilde{x}$-direction, and it's T-dual $\Pi_N^{(2)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ corresponds to a $D1$-brane extends in the x-direction.
In this case the doubled gauge field is defined by $\mathbb{A} = (A, \tilde{A})^t$.
Then the gauge field boundary terms are
$\mathbb{A}_I ((\Pi_N)^t)^I{}_J \partial_0 \mathbb{X}^J = \tilde{A} \partial_0 \tilde{x}$
and
$\mathbb{A}_I ((\Pi_N)^t)^I{}_J \partial_0 \mathbb{X}^J = A \partial_0 x$, respectively.

The boundary action can be rewritten as

$$S_b = -\int_{\partial\Sigma} d\tau \mathcal{A}_a \partial_0 \mathcal{X}^a + \text{(irrelevant Dirichlet part)}.$$

Let $\mathbb{F}$ be the full doubled space field strength and we define $\mathcal{F} = d\mathcal{A} = \Pi_N \mathbb{F} \Pi_N^t$ as the pull back of $\mathbb{F}$ onto the Neumann subspace.
Similarly, we can pull back $\mathbb{H}$ to the Neumann subspace and denote it as $\mathcal{H}$.

Next, we introduce quantum fluctuation ξ^I over the classical background $\mathbb{X}^I$ as

$$\mathbb{X}^I \to \mathbb{X}^I + \xi^I,$$

where ξ^I has doubled degrees of freedom.
We assume a flat target space and a constant B-field.
The bulk action in (Euclidean worldsheet signature) expands to

$$S_E = \frac{1}{2}\int_\Sigma d^2\sigma(\mathbb{H}_{IJ}\partial_1\mathbb{X}^I\partial_1\mathbb{X}^J - i\mathbb{L}_{IJ}\partial_1\mathbb{X}^I\mathbb{X}^J$$
$$+(2\mathbb{H}_{IJ}\partial_1\xi^I\partial_1\mathbb{X}^J - i\mathbb{L}_{IJ}\partial_1\xi^I\partial_0\mathbb{X}^J - i\mathbb{L}_{IJ}\partial_1\mathbb{X}^I\partial_0\xi^J)$$
$$+(\mathbb{H}_{IJ}\partial_1\xi^I\partial_1\xi^J - i\mathbb{L}_{IJ}\partial_1\xi^I\partial_0\xi^J) + \mathcal{O}(\xi^3)).$$

Similarly for the boundary term

$$S_{E_b} = i\int_{\partial\Sigma} d\tau(\mathcal{A}_a\partial_0\mathcal{X}^a + \xi^a\mathcal{F}_{ab}\partial_0\mathcal{X}^b + \frac{1}{2}(\xi^c\xi^a\nabla_c\mathcal{F}_{ab}\partial_0\mathcal{X}^b$$
$$+\xi^a\partial_0\xi^b\mathcal{F}_{ab}) + \frac{1}{3}(\frac{1}{2}\xi^c\xi^d\xi^a\nabla_c\nabla_d\mathcal{F}_{ab}\partial_0\mathcal{X}^b$$
$$+\xi^c\xi^a\partial_0\xi^b\nabla_c\mathcal{F}_{ab}) + \mathcal{O}(\xi^4) + \cdots).$$

The bulk equations of motion for the background fields $\mathbb{X}^I$ are

$$\partial_1(\mathbb{H}_{IJ}\partial_1\mathbb{X}^J) - i\mathbb{L}_{IJ}\partial_0\partial_1\mathbb{X}^J = 0,$$

while the Neumann boundary conditions read

$$\mathcal{H}_{pq}\partial_1\mathcal{X}^q + \mathrm{i}\,\mathcal{F}_{pa}\partial_0\mathcal{X}^a\,|_{\partial\Sigma} = 0,$$

and the Dirichlet boundary conditions are

$$\Pi_D\,\mathbb{X}\,|_{\partial\Sigma} = 0.$$

The equations of motion and boundary conditions for ξ are very similar.
Using these equations of motion and boundary conditions, we have the doubled space Green's function G^{AB} which satisfies

$$(\mathbb{H}_{IJ}\partial_1^2 - i\mathbb{L}_{IJ}\partial_0\partial_1)G^{JK}(\sigma,\sigma') = -\delta_I{}^K 2\pi\delta(\sigma-\sigma'),$$

while the Neumann boundary conditions are given by

$$\mathcal{H}_{ab}\partial_1 G^{bc}(\tau,\tau') + i\mathcal{F}_{ab}\partial_0 G^{bc}(\tau,\tau')|_{\sigma=0} = 0.$$

Chiral frame

To quantize the worldsheet doubled formalism, the first difficulty we encounter is how to incorporate the self-duality condition in a Lorentz invariant way.
It turns out that the self-duality condition is equivalent to the chiral conditions.
For instance, in 4-dimensions if we choose chiral scalars

$$Z_{1\pm} \equiv \tilde{x} \pm x - by, \quad Z_{2\pm} \equiv \tilde{y} \pm y + bx,$$

the self-duality conditions are equivalent to the chiral scalar conditions:

$$\partial_\pm Z_{1\mp} = \partial_\pm Z_{2\mp} = 0,$$

here $\partial_\pm = \partial_0 \pm \partial_1$.

In the chiral frame, $\mathbb{H}$ and $\mathbb{L}$ are diagonal,

$$\mathbb{H} = \begin{pmatrix} \mathbb{I}_{n\times n} & 0 \\ 0 & \mathbb{I}_{n\times n} \end{pmatrix}, \quad \mathbb{L} = \begin{pmatrix} \mathbb{I}_{n\times n} & 0 \\ 0 & -\mathbb{I}_{n\times n} \end{pmatrix}.$$

We can always change back to the original light-like frame using the vielbeins $\mathcal{V}_{\bar{I}}{}^{J} \in GL(2n)$, i.e. the coordinates and the generalized metric transform as

$$\mathbb{X}^{\bar{I}} = \mathcal{V}^{\bar{I}}{}_{I}\mathbb{X}^{I}, \quad \mathbb{H}_{\bar{I}\bar{J}} = \mathcal{V}_{\bar{I}}{}^{I}\mathcal{V}_{\bar{J}}{}^{J}\mathbb{H}_{IJ}.$$

Quantizing the worldsheet doubled theory is then equivalent to quantizing the action of chiral scalars.

Master equation

We solve the Green's function in the chiral frame and then transform it back to the original frame, and use the doubled Green's function to evaluate the one-loop counter term for the boundary coupling, which take the form

$$\frac{i}{2}\int_{\partial\Sigma} d\tau\ \Gamma_a \partial_0 \mathcal{X}^a.$$

The relevant leading contribution to the one-loop counter term read from the expanded boundary action is

$$S_{int} = \frac{1}{2}\int_{\partial\Sigma} d\tau\ \xi^c \xi^a \nabla_c \mathcal{F}_{ab} \partial_0 \mathcal{X}^b.$$

Thus the counter term reads

$$\Gamma_a \equiv \lim_{\epsilon\to 0} G^{bc}(\epsilon \equiv \tau - \tau')\nabla_c \mathcal{F}_{ab}.$$

where ϵ is the short-distance UV cutoff.

The beta-function for the boundary gauge coupling can be obtained as

$$\beta_a \equiv -2\pi\epsilon \frac{\partial \Gamma_a}{\partial \epsilon} = 0.$$

We find the resulting equation of motion of the effective double field theory for double *D*-branes is:

$$\beta_a \equiv \left\{ \left(\mathcal{H} - \mathcal{F}^2 \right)^{-1} \right\}^{bc} \nabla_b \mathcal{F}_{ca} = 0.$$

Theorem (Abouelsaoodas et al.)

The equations of motion of the form

$$\left((g - F^2)^{-1} \right)^{ab} \nabla_b F_{ac} = 0,$$

can be obtained from the Lagrangian

$$\mathcal{L} = \sqrt{-\det(g + F)}.$$

Therefore we propose the following action for the double D-brane

$$S_{eff} = \int d^{n+1}(\mathcal{X})\sqrt{-\det(\mathcal{H}+\mathcal{F})}.$$

This master action describes all doubled D-brane configurations related by T-duality transformations. Choosing a set of specific projectors would reduce the theory on a pair of T-dual D-branes.

Issues:

However, the doubled DBI-action does not reduce to the conventional DBI action for a single D-brane.
For the case of $D2$-brane, the DBI-determinant reads

$$det(\mathcal{H} + \mathcal{F}) = -1 + B^2 + F_{XT}^2 + F_{YT}^2 - 2\ B\ F_{XY} - F_{XY}^2.$$

When $B = 0$,

$$det(\mathcal{H} + \mathcal{F}) = -1 + F_{XT}^2 + F_{YT}^2 - F_{XY}^2,$$

which is identical to the conventional DBI determinant for $B = 0$.
Possible reasons:

- Gauge fields depend on the doubled coordinates and upon reduction do not reduce to functions live on the Neumann subspace.
- We do not double the time coordinate, so the lack of T-dual covariance associate with the time component may cause a lack of compensation between the world-sheet and the boundary.

Summary

We considered the doubled formalism for open strings on a *D*-brane in flat space with a constant *B*-field, and we derived the master equation for a doubled *D*-brane.

A double *D*-brane in this formalism is described by a set of projectors that encode the boundary conditions. The Neumann projectors are then used to formulate the boundary action for open strings on double *D*-branes.

Finally, we applied the background field method to the open string doubled formalism and derived the effective double field theory for double *D*-branes. The resulting effective action takes a DBI-like form.