THE STORY OF GRAVITATION

E. TELLER
HOOVER INSTITUTION, STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305 U.S.A.

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E. Teller

Hoover Institution, Stanford University
Stanford, CA 94305

It is a great pleasure to talk to you about some old ideas and perhaps a few new ones, some of which have to do with precise measurement of gravitational fields. However, most of what I have to say will be more about ideas than about measurements.

Just a few hundred years ago, gravitation was not understood at all. Instead, people believed that objects had natural places; heavy objects had a lower place, lighter objects a higher place. This is the more remarkable because the idea that the earth is approximately a sphere is quite old.

Two thousand years ago, people living along the River Nile, knew that when the sun came up to its highest point in the sky in the summer, it shone straight down in a certain area of the Nile valley. If one went farther south down the Nile or at an angle farther up north, the rays were not perpendicular. From that information, the size of the earth was correctly calculated.

I would love to know whether anyone in Chinese history had the idea that the earth does not stand still but rotates on its axis and describes an approximate circle around the sun.* The first person I know credited with this idea lived about 200 years before Christ, more than 2000 years ago. Aristarchos was a Greek from the island Samos, and he made this enlightened guess that it was the sun that stood still, and the earth moved around it while rotating around its axis. The Greek scientists, who called themselves philosophers, did not believe him. They did the worst thing to him that one can do to a scientist--they forgot about his proposal. However, there was at this time another very clever Greek who did not live in Greece either, but in Sicily. Archimedes, one of the greatest mathematicians, learned of Aristarchos' strange idea and found it so odd that he mentioned it in one of his books.

We don't know how Aristarchos came up with his peculiar idea. But it is possible to guess. He could make the explanation of planetary motion much simpler by saying that the planets moved on circles around the sun, and the earth while rotating around its axis also moved on a

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*Editor's note: In Chou dynasty (1122BC to 225BC) in the Chinese history, some people had the ideas that the earth is round and moving and/or rotating but none, to my knowledge, had the idea that the earth is going around the sun. Please see 成映鴻著的「古代天文學之研究」 (The Study of Ancient Astronomy) (台中長春印刷公司, 1971) for more details.

circle around the sun. If you look up at the sky, most of the stars seem to move on circles. However, the planets move in a more complicated manner. To describe their motion, the Greeks assumed that they move in circles whose centers move on circles, and whose centers move in turn on circles--epicycles. When measurements became more accurate, the final Greek word on it came from Ptolemy. He proposed epicycles of the fifth order--circles whose centers moved on circles, whose centers moved on circles, whose centers moved on circles, whose centers finally moved on circles.

Aristarchos' theory necessitated an epicycle of the third order--a circle whose center moved on a circle, whose center moved on a circle. But this simplified description was probably not his only reason for his unusual proposal. Another possibility seems more important to me.

The Greeks at that time knew the approximate distance of the moon from the earth. This had been determined by the use of parallax. Parallax results from observing an object from two points of view. If I look first through one eye and then through the other, a nearby object seems to move on the background of more distant objects. When one looks at the moon from two points on the earth, the moon appears to be covering different stars. In this way one could obtain the distance of the moon from the earth.

However, parallax could not be used to find the distance of the sun from the earth. The sun was not only so far away that the parallax would be very small, but when the sun was up one couldn't see any stars. Aristarchos devised a very clever method of making this measurement. His idea was correct, but his execution of it contained considerable error.

The phases of the moon include two times when just half the moon is illuminated. If the sun is infinitely far away, then the point when the moon is half illuminated on my right and the point when it is again half illuminated on my left will divide the moon's orbit exactly in half. But if the sun is closer, then the situation is different. The moon will be half illuminated when the sun's rays strike exactly at right angles both when it is to the right and to the left (so to speak) of the earth. However, these points will not occur exactly halfway through the moon's orbit. If the distance of the moon is known (which it was), one could estimate the distance of the sun by comparing the two arcs of the moon's orbit.

The idea is ingenious and right. The two points when the moon is just half illuminated are almost but not quite opposite. To measure the difference was very difficult, the more so because the moon's surface is not smooth which makes it difficult to find when just half of the moon's surface is illuminated. The result was that Aristarchos believed that the sun was only one-sixth as far away as it is, not 400 times as far as the moon, but maybe 65 times as far as the moon. Aristarchos' execution of his correct idea was wrong. The remarkable thing is that this mistaken calculation was believed for many centuries. Even in the 1600s Galileo believed it.

If the sun is 60 or 70 times as far away as the moon, it must be much bigger than the earth. The origin of Aristarchos' heliocentric theory may have been, "Let's not say that the tail wags the dog, let's say the dog wags the tail." That which is lighter should move, that which is heavier should remain fixed. I believe that must have been in his mind. And if so, Aristarchos had the beginning of the ideas that make up physics today. Remarkably enough, if you translate his name, the work Aristarchos means "the best beginning."

Many centuries later, Aristarchos was forgotten, and Copernicus in Poland had been asked by the Pope to correct the calendar. The sun and the moon and the stars were not behaving as they should have according to the ancient Greeks' calculations. Copernicus read Archimedes' book and noticed the reference to Aristarchos' idea. Copernicus played with the idea and came to the conclusion that it was much easier to rewrite the calendar and get a good one if you assumed that the sun was at rest and the earth and the other planets moved around it.

He hesitated. Everybody believed otherwise. The Bible said otherwise. At the very end of his life, when he was on his deathbed, he allowed his book to be published. Even then, he insisted on an introduction which said. "Dear reader, if I say that the earth moves around the sun, don't believe a word. This is not real, this is a trick to make mathematics easier. And mathematics is for the mathematicians. The rest of you, don't wprru/" But people did.

During this period, there was a great astronomer in Denmark, Tycho Brahe. He said, "If Copernicus is right, and if the earth is moving on a big circle, then I should see the distant stars from a different angle in winter and in summer. And I should notice it." But he had no telescope and only the crudest of instruments. The effect for which Tycho Brahe looked 400 years ago was observed only in 1838, because the stars are so far away that even the nearest of them will appear to move only a little bit on the background of the others. He could not see this stellar parallax, and so he did not believe that Copernicus was really right.

Still, there were other astronomers: a man whom I consider one of the greatest scientists who ever lived was among them. Johannes Kepler was only half a scientist--he was also half an astrologer. He read the fates of human events in the stars.

Yet Kepler considered the stars and the planets and the solar system as the creation of God. And he had an urge. (Our strongest urges, our strongest likes and dislikes, are difficult to explain; they are just there.) Kepler had the urge to understand how God put the solar system together. To gain this end, he stole Tycho Brahe's observations when Brahe died, for these were very accurate observations. Without computing machines, with just a goose feather and ink to put down his thoughts, Kepler worked and worked. In many a chapter he said, "In the last chapter I was a fool. I made a mistake. Now I will do it better."

He had almost explained the behavior of the most eccentric of the known planets, the most complicatedly moving planet, Mars. He almost explained it with an epicycle of the fifth order. It almost checked, but it did not fit completely.

What would you do? I am lazy. In his place I would have invented an epicycle, using not five but six circles. Then it would have been easy. But not Kepler, because he was a real scientist, who had understood that making a scientific explanation more complicated is worth nothing. If he had put in a sixth circle, then there would not have been one possible explanation but many. He could have chosen an easy way, but he would not have decided the question but only found a possibility. And he believed that God had to have had a simple way of doing it. I am not talking to you about religion. I am using the word of God only as Kepler used it -- with a deep conviction that for whatever reason the explanation must be simple.

Kepler discovered that Copernicus was right; and furthermore, the planets do not move on circules--they move in ellipses. Then he wrote a book with an introduction that was very different from Copernicus' introduction, because he said, "I have written this book, and I believe that nobody will read it for a hundred years." He was right--few if any did. But then he said, "I can wait. God Himself waited for almost 6000 years (which was at that time believed to be the age of the universe). He waited for almost 6000 years before anybody understood what He had been doing." And in a hundred years the man who really understood gravitation, Isaac Newton, did read Kepler, and did understand and explain Kepler's work.

Everybody knows that Newton explained Kepler's laws in terms of the idea that there is an attractive force between heavy bodies, a force that varies with the distance between them, as $1/r^2$, as 1 divided by the distance of the two objects squared.

There is a strange story about Newton's work that may not be familiar. Edmund Halley, a contemporary Englishman, was very much interested in comets. Comets were supposed to be bodies that came from infinity, went around the sun, and vanished again into infinity. But Halley looked up the record, and he found that some comets seemed to return. If one returned in a hundred years, and then in another hundred years, there it was again in a record. And he noticed this regularity with one particular comet which he had never seen. He predicted its periodicity. Today it is called Halley's comet. It visited when I was four years old and I didn't see it. I hope it will visit again before I die.

Newton, when he was a yound man, studied at Cambridge in England. But there was an epidemic, and he had to go home to his village, Woolsthorpe, where he had nothing to do. He did what few people do--he started to think. That is when the figurative apple fell on his head. That is when he understood gravitation. But it was a complicated subject, one which many people were discussing. And Newton did not like controversy. He did not write his ideas down. He seemed to forget about them. Then many years later, Halley came to him and said,"I don't think the comets move on parabolae which come from infinity and go to infinity. I suspect that they move on very elongated ellipses which almost look like parabolae. Tell me, can you prove Kepler's laws from your law of $1/r^2$?"

Newton said, "Yes, I proved this in Woolsthorpe some 20 years ago." Halley asked, "Could you show me the proof?" But Newton couldn't reproduce it. There seems to be another law of nature, less well-known, that the older a person gets, the more stupid he becomes. Newton was no longer as clever as he was when he was twenty. That, you can well imagine, annoyed him. So he started to work and continued for two weeks before he could find the proof that he had easily found as a yound man.

Then he said, "Now this nonsense must stop. I will write down everything so that it will never be forgotten again." And he wrote his famous book Philosophiae Naturalis Principia Mathematica, The Mathematical Principles of Natural Philosophy. I want to say only one thing about that book. The Greeks, Kepler, even Galileo, all believed that there were two kinds of laws: the laws which are valid on earth and the laws that were valid in the heavens. Apart from the mathematics he developed to a remarkable extent, Newton made one other tremendous discovery, and it was contained in this book. Newton discovered that, at least in physics, there is only one set of laws. The same laws apply on earth and in the heavens, the law which describes the falling of an apple or my dropping a piece of chalk also describes the motion of the moon and the motion of the earth and the motion of anything in the skies.

Newton had the remarkable gift of seeing things as they were, even of seeing himself as he was. At one point he said, "I don't know what I may appear to the world, but to myself I seem to have only been like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary while the great ocean of truth lay all undiscovered before me."

It is time to discuss a little mathematics. Newton had problems which he resolved in a complex manner. We know now that they can be resolved in a simpler manner. Why does the force change as 1 divided by the distance squared? Why did newton's calculations come out right when he assumed that the mass of the earth was concentrated at the center of the earth even though in fact it is spread out in a sphere? Newton solved this question by assuming the $1/r^2$ law for each part of the earth and adding all the effects, which was a really wearisome problem of integration.

Michael Faraday, a chemist, who explained most of electricity and magnetism, introduced a new idea, a mathematical tool, from which the $1/r^2$ law as applied to an extended spherical body followed more simply. He said, "If I have an electrically charged point, forces are radiating outward in all directions. I assume that the direction of the actual force will be given by the direction of the radiating arrows, but the strength will be given by the density of these lines of force. The more lines per unit surface, the stronger the force. "If instead of a charged point, I have a charged sphere, then the force has no choice but to go out radially. And since the number of the lines that originate is proportional to the total charge that is in the body, the effect is the same as though the body were concentrated in the center." The situation is the same for electricity and for gravitation. This kind of argumentation has been carried out by a theorists, Clerk Maxwell, who systematized what Faraday found and formulated in pictures.

The basic idea is that no lines of force originate (or end) in any region in which there is no electric charge. The same holds for gravitation in the absence of mass. But if mass is present, a surplus of lines of force will leave a region which is proportional to the mass contained in the region.

A further element of this formalism is the potential (electrical or gravitational). The direction of a line of force is the direction in which this potential changes most rapidly. The density of the lines of force (i.e., the strength of the field) is proportional to the rapidity of that change.

Consider a cube. Six faces. How many more lines will leave than will enter? That is found by subtracting the number of lines that leave one face from the number of lines that enter the opposite face. This must be done for all three pairs of faces. The result is one of Maxwell's equations:

$$\zeta \simeq \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

Here ζ is the density of matter or electricity. The sign of proportionality ($\simeq$) has been used instead of the sign of equality (=) to avoid a numerical discussion which for our purpose is irrelevant. The symbol $\partial^2\psi/\partial x^2$ stands for the rate with which the force $\partial\psi/\partial x$ is changing while the force itself is determined by the rate with which the potential ψ is changing. I have, of course, sneaked into this discussion a partial differential, that is a rate of change along the x direction while the two other coordinates, y and z, are fixed. The three terms on the right hand side correspond to the three pairs of faces which enclose the original cube.

All of this is merely a prelude to something remarkable which happened two hundred years after Newton--the great discoveries of Einstein. I have to start with an absurdity. Einstein said that if you and I say that two events occur simultaneously, we may have to disagree. If we move at different velocities, what is simultaneous for you is not simultaneous for me. This idea was completely new, not only to Newton, but to all of Einstein's contemporaries. It was an idea which was difficult to accept, but which was true.

What does this idea have to do with gravitation? Out of the ideas of Einstein, there grew not only special relativity but also a kind of relativity which Einstein called general relativity. It would be better called the theory of gravitation. In this theory of gravitation, Einstein did something that Newton did not dare to do. When asked why the $1/r^2$ law was so, Newton said, "Just because. I won't make hypotheses. I see that it is so and it explains the motion of the planets." However, Einstein explained Newton's law.

What I am going to tell you is in all the textbooks, but it is hidden under tons of mathematics. And yet, when you think about it carefully, Einstein's explanation of the $1/r^2$ law becomes a matter that is not really difficult. But first we need to review Einstein's ideas on special relativity.

We used to think that time passes in the exact same way for each of us. To know that this is not true is not enough. One must also ask, "What is the quantity which appears the same to all observers?" Einstein gave the answer. There is a relatively simple formula: Multiply the time difference (between two observed events) t, by the light velocity c, which is the distance that light could cover in the time t. Now square this quantity, ct, and subtract from it r^2, the square of the distance at which these two events are observed. This quantity, $(ct)^2 - r^2$, is an invariant. It remains the same for all observers.

One consequence of this statement is very important: Take two events for which this invariant is 0. Then $(ct)^2 = r^2$, and $ct = r$. Therefore, the velocity at which light can move is c. This is a more peculier statement than it seems to be. It means that the velocity of light is indeed the same for all observers.

If I want to catch up with a light beam, it will continue to move ahead of me with velocity, c. I can never catch it. This argument suggests, and indeed can be shown explicitly from, Einstein's formalism that nothing can move faster than light, and that no material substance can ever move as fast as light although it may approach light velocity. But, and this peculiarity should be repeated for emphasis, no mater how closely I approach the velocity of light, light will still appear to move ahead of me with the same old velocity, c.

Now I want to discuss one of my desires: I want to visit the Andromeda Nebula. The Andromeda Nebula is a collection of a hundred million stars just as our own Milky Way system is. It's about 2 million light years away. According to Einstein, I cannot travel faster than light. Therefore, it would take me two million years to get there. My physician tells me that having already reached 75 years, I won't live two million years longer, so I should give up all hope of ever getting to Andromeda.

But Einstein is not so pessimistic. He says, "You can do it if you just move fast enough." For you who stay at home, if I move with almost light velocity, the journey takes a little more than two million years. The quantity ct is 2 million light years or a little more because my speed was a little slower. The quantity r is two million light years. But the difference $(ct)^2 - r^2$ can be very small. If I am at the controls of the rocket at the beginning and at the end, for me r is 0. For you who stayed at home, $(ct)^2 - r^2$ is quite small. For me it must be the same. If $r=0$, then ct is small.

I can do it. I need not run out of time. If there are engineers here, they might tell you that to get a rocket to move at almost light velocity is not quite easy. So I don't really think that I can get to Andromeda. But it is the engineers who are at fault, not Einstein.

Now there is a difficult question: what will happen when I return? I take a short time to get to Andromeda; I look around, I make observations, I write them down; I come back in a short time; and I hope to be a hero in the United States and Taiwan. But of course, 4 million years have passed on earth. All people, American and Chinese, will be replaced by something very different--quite horrible! The more horrible because they will imagine that they are better, that they have evolved. I think they will be better. They will look at me as an antedeluvian animal and will put me in a zoo, I hope, with a swimming pool. And since my English is better than my Chinese, with a few volumes of Shakespeare rather than Confucius. And in time they will find out what they can find out from me because they will be quite clever.

So far I only told you a sad story. But it also seems illogical. I moved away and came back, and 4 million years passed at home, while for me hardly any time passed. From my point of view, the earth moved away and came back. Why is it not equally justified to say that I became 4 million years older, and only a short time passed on earth. What is the difference? There is an important observable difference, and this difference I want to explain to you.

You stay-at-homes have not experienced much acceleration. I did. When I started, I was accelerated, but I could look at a watch immediately, so there couldn't have been much difference in regard to the passing of time. The same thing is true when I returned. I could check on your time. But when I'm in Andromeda, no signals can reach me in less than 2 million years, and there I also was accelerated both when I arrived and when I left.

Could the accleration make a difference? Einstein said, "It can and it does." It was Galileo who first observed that on earth all matter gets accelerated by gravitation in the same way. Later this observation was verified with great accuracy.

Then, Einstein got an idea. "If I am in a falling elevator, or to be more modern, if I'm an astronaut sailing around the earth in a capsule, I don't have the sensation of acceleration. I sit there with a pen in my hand, and I drop the pen. What will happen? In the capsule, the pen will not fall. We are not affected by the earth's gravitation. The pen will just sail along on the same orbit as I. It will not appear to fall. Within the capsule it will be as if there were no gravitation.

From this it is only a step to say, "Let's seal off this room, close the door, bar the windows, forget that Taipei is outside, and then assume that we are not on earth. We are in a big rocket. The rocket engine under us accelerates us, and this acceleration is what presses us down to our seats. Then it will become obvious that everybody and everything accelerated in precisely the same way relative to the earth." This is the principle of equivalence. In any local system gravitation and acceleration are equivalent.

Now I will make a very peculiar calculation. The most peculiar quality of this calculation is that I don't know but can only guess (since he never told us) that Einstein made his calculation in the same way.

The only way to explain that only one of us aged so much during the Andromeda experience is to appeal to the acceleration that turns me around when I arrive at Andromeda. And, you will forgive me if I make this calculation in an oversimplified fashion. The acceleration that I experienced is this--I have been going away from the earth at practically light velocity, c. I want to turn around and come back toward earth with practically light velocity, c. Otherwise the trip will take too long. Therefore the velocity change is from -c to +c; the velocity change is 2c. Let us assume that this velocity change is accomplished in a short time, tau. Therefore the acceleration is $2c/\tau$.

Now assume, according to the principle of equivalence, that this is not an acceleration. It must be equivalent in every respect to a gravitation force that produces the acceleration $2c/\tau$. This accelerating force can be considered to act the whole distance between Andromeda and the earth. My rocket is stationary, and earth is moving in relationship to me.

You must be careful. You must pay attention at every step to which coordinate system applies. For the sake of simplicity, I will work in the coordinate system of the stay-at-homes. The distance is two million light years, or r. The accelerating force times the distance r is the potential difference. That is the same kind of potential ψ which I introduced when I wrote the equations about lines of force. This potential is $(2c/\tau)r$. Einstein made the assumption, that in this time, τ, time was passing at a different rate for me than for you. I feel that τ was a short time, but from your point of view, time will have passed at a different rate. The 4 million years difference which is $2r/c$ has occured in this way.

When I turn around near Andromeda, I will be accelerated toward the earth. In the language of potentials, ψ, the earth will have a high potential. This is associated with a fast passage of time on earth. The accelerating frame is equivalent to a static frame in a gravitational field. An inertial frame is equivalent to a free-falling frame. So in this analogy, earth is a free-falling frame in the gravity field.

The ratio is $(2r/c)\tau$. Since for the potential we have $\psi=(2cr)/\tau$, the ratio may be written as $(2cr/\tau)/c^2=\psi/c^2$. This actually is the famous gravitational red shift discovered by Einstein.

Now comes the surprise. People in Cambridge, Massachusetts, were interested in the small difference in potential between the top and the foot of the Harvard Tower. This is about equal to the potential difference between this room and my room in the Grand Hotel. However, the people at Harvard had very accurate nuclear clocks. They found that the difference in the passage of time was precisely the red shift predicted by Einstein.

Now I would like to derive the $1/r^2$ law. This point may be a little difficult because I want to talk about space curvature. What is space curvature? I cannot imagine curved space any more than you can. I can imagine a curved surface because I can see a surface from a third dimension. We know the earth's surface is a curved surface. If I start at the North Pole of the globe with a vector that points straight towards Taiwan and follow this vector to the equator, keeping it always pointing towards you, move it east all the way to the latitude that I came from, San Francisco, and then move back to the North Pole, the vector no longer points toward Taiwan when I have returned to the North Pole.

Having kept my vector always pointing in the same direction, always pointing south as I move from point to point, the vector surprisingly enough will still point in a different direction upon its return. By displacing vectors, being careful to keep them parallel, you find in the end a new direction. This is the sign of inherent curvature as described by Gauss.

Gauss was a famous professor at Gottingen. He had a lot to say about who else should be appointed professor at the university. A man called Riemann was to be invited to the university. He offered to talk about curved spaces in any number of dimensions. Gauss was very much interested. Riemann was invited and gave a classic paper showing how the idea of curvature can be generalized into many dimensions.

However, we don't need so many. Einstein was a simple-minded person and could only count up to four--x, y, z, and the fourth dimension, t. In four dimensions, all kinds of curvature may exist, and they are really quite complicated. In four dimensions, I can take any pair of directions, make a surface, go around it, and see how any vector changes as I move around the surface.

Therefore, curvature in four dimensions is a little complicated. I will use only a simple example of it. Even that is complicated. I can take any pair of directions, x and y, and then another vector, for instance z, and see how that vector changes in a fourth direction, in t. So the curvature is described not by one number but by a collection of numbers. This tells you how things will differ from the usual when out of four directions we have chosen four not necessarily different directions. For instance, you could ask how when moving in the xy plane, the x vector has changed in the y-direction.

According to Einstein, space is curved or warped due to the presence of mass. This curvatire is in fact what we experience as gravitation. These ideas are somewhat complex, not only because curvature in four-dimensional space is being considered but also because one of these dimension is t, which we customarily consider a little differently from dimensions in space.

Though warping is done by matter, it actually extends into empty space where no matter is present. Indeed, the $1/r^2$ law applies to empty space. We shall discuss a single example of the four-dimensional space curvature with the remarkable result that it does lead to the $1/r^2$ law.

As in previous examples, we shall take a surface, in particular a small surface in four dimensions. We shall start from a certain time ($t = 0$) and a certain point in space ($x=0$). We shall also consider (still at $t=0$) a neighboring point located at δx.

In reality the situation is further complicated by our having to perform all these operations twice, once starting from the original point ($x=0$, $t=0$) and a second time, starting from the other end of a tiny vector ($x=\delta x$, $t=0$). In each phase, the rate of the passage of time will play a role.

A detailed consideration may convince the reader that what counts is not just the red shift which itself is proportional to the rate at which a gratitational potential is changing. What really counts is the change in the red shift between x and δx. Therefore, the final result will turn out to be proportional not to gravitational force in the x direction which is $\partial\psi/\partial x$, but rather to the second derivative, $\partial^2\psi/\partial x^2$.

All of this is only the beginning. It has been pointed out that curvature in four dimensions corresponds to a complicated structure. Simple statements cannot be made about the curvature as a whole. Rather we can speak about sums of curvatures which one obtains if one repeats all the given arguments regarding x with similar arguments in regard to y and z. The result will be:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

which is the expression we met long ago, and which turned out to be zero in a vacuum.[1]

More generally, Einstein has shown that $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$ is proportional to the density of mass which corresponds precisely to the mathematical formulation of Newton's law. With mass serving as a source of the lines of force, the density of these lines around a small object will decrease as $1/r^2$ where r is the distance from the small object.

This indeed is the end of all argument, but not the end of Einstein's argument. What he called general relativity did not merely derive Newton's elementary results nor was it confined to an additional discussion of gravitational red shift. It also established further refinements in the theory of gravitation, dependent not only on the mass of the material presnet but also on its motion or momentum, and furthermore on tensions that exist in matter. This mathematically complicated theory predicts a great number of results which unfortunately are quite small and hard to observe. To the extent to which the present observations go, they have been verified in every detail.

It is a long way from the old geocentric system where gravity played the role of establishing a natural place for objects to Newton's law of universal gravitation, and from there to the abstractions and complication of space curvature discussed by Einstein. Nor it is likely that the story ends at this point. But as of 1983, no further essential part can be added.

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1 What happened to $\partial^2\psi/\partial t^2$ in four-dimensional geometry? Of the various answers that could be given, the simplest is that we are interested in the case where nothing varies with time, and we simply investigate the potential ψ around a body at rest.