AN EXAMPLE OF FEEDBACK CONTROL:
A PRECISION VISCOMETER

CARL H. LEYH

DEPARTMENT OF PHYSICS, UNIVERSITY OF VIRGINIA
CHARLOTTESVILLE, VIRGINIA 22901 U.S.A.

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Carl H. Leyh
Department of Physics, University of Virginia
Charlottesville, VA 22901

I. Introduction

This presentation is given in response to Dr. W.K.S. Cheung's request for examples of feedback control to illustrate the points he has been discussing in his talks entitled "Feedback Control in Physical Science: A Layman's Approach".[1] In Dr. Cheung's lectures he has used numerous examples of how feedback control can be used to make an unstable open loop system into a stable system by closing a feedback loop. This is a very important use of feedback control, but it is not the only use for feedback control. Even if a measurement system is open loop stable, many times a closed loop system using feedback control can be used to increase the precision of the open loop system. Not only is precision improved, but frequency response can also be improved. Data can be obtained with greater ease and automation using a feedback control system without added instrumentation by using the feedback parameters of the servo loop. There are many advantages to a closed loop system that an open loop system, although stable, can not provide.

The purpose of this talk is to provide an example of an open loop stable system that was improved by incorporating a feedback loop into the design. The system is the viscodensimeter[2-4] developed by Dr. J. W. Beams at the University of Virginia. This device was capable of measuring the density and viscosity of liquids simultaneously. Only the viscosity measuring system is pertinent to this talk.

II. The "Open Loop" Viscometer

The principle of the viscosity measuring system is quite simple. First a cylindrical object is magnetically suspended while submerged in a liquid. The magnetic suspension provides a frictionless bearing for the cylinder. Two pairs of coils which are positioned orthogonally to each other are located outside the liquid, but with the suspended cylinder located at the center of the four coil configuration. A 10 kHz sinusoidal current passes through one coil pair while another sinusoidal current, 90° out of phase with the first, passes through the other coil pair. This creates a rotating magnetic field at the cylinder's position and causes the cylinder to rotate. This magnetic torque will accelerate the cylinder until the viscous drag of the liquid on the cylinder establishes an equilibrium angular velocity.

Since the viscous drag (torque) is proportional to the velocity of the cylinder, the equation of motion can be written as,

$$I\dot{\omega} + b\omega = \Gamma, \tag{1}$$

where I is the moment of inertia of the cylinder, ω is the angular velocity of the cylinder, Γ is the magnetic torque on the cylinder, and b is the viscous damping coefficient. By definition $b\omega$ is the viscous drag. For Γ being a constant, Eq. (1) can be solved and shown to be,

$$\omega = \frac{\Gamma}{b}\,(1 - e^{-b/I\ t}). \qquad (2)$$

At equilibrium, $t \to \infty$,

$$\omega = \Gamma/b, \qquad (3)$$

or

$$T = b/\Gamma, \qquad (4)$$

where T is the period of rotation of the cylinder. Remember, $b = b(\eta)$, i.e. the damping coefficient is a function of the viscosity, η. Therefore, if Γ is well known and constant, η can be measured by measuring T, and this can be done with a stopwatch for a slowly rotating cylinder.

Figure 1 shows a schematic diagram of the "open loop" system.

Let me emphasize, that this system works. It is stable and can provide some precise viscosity measurements. However, it has poor frequency response. Let me explain this in more detail.

In order to obtain precise period measurements, a number of revolutions are averaged. If the viscosity of the liquid is changing during the measurement, due to temperature changes or chemical reactions, the change would not be detected. Similarly if the equilibrium period, T, is long, it is hard to determine if equilibrium has been reached, or more importantly if equilibrium is being maintained. Even if the period measurement system was automated these problems would still exist along with the additional burden of the extra instrumentation due to the automated data acquisition.

III. The "Closed Loop" Viscometer

A new viscometer was designed to overcome these difficulties. It worked on the same principles as the old system but with the following important changes. First, instead of rotating the suspended cylinder, the liquid is rotated instead. This of course produces a viscous drag on the cylinder which makes the cylinder begin to rotate. Secondly, the magnetic torque operates in the direction opposite to that of the cylinder's motion. In this way the magnetic torque cancells the viscous drag. When these two torques are equal the cylinder will not rotate. A feedback control system is used to detect any rotation of the cylinder and then regulates the magnetic torque in such a way that the viscous drag is always equal to the magnetic torque. This system is shown schematically in Fig. 2.

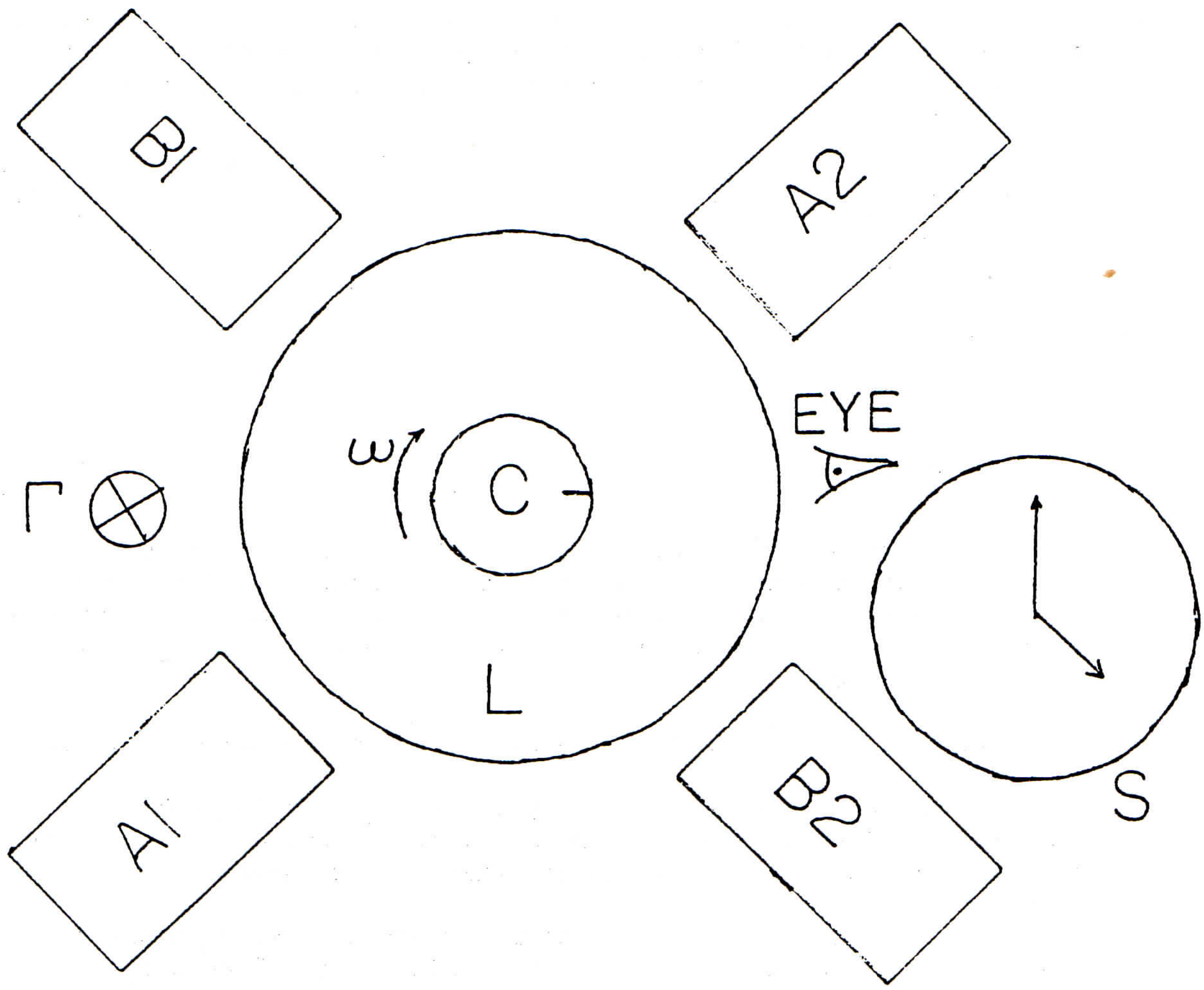

Figure 1 Open Loop System - A cylinder, C, is magnetically suspended in a liquid L. This shows a top view of the set up. Drive coil pairs A1-A2 and B1-B2 create a rotating magnetic field which produces a torque, Γ, on the cylinder and causes it to rotate. The angular velocity of the cylinder, ω, is recorded by eye and a stopwatch, S. Γ is into the page.

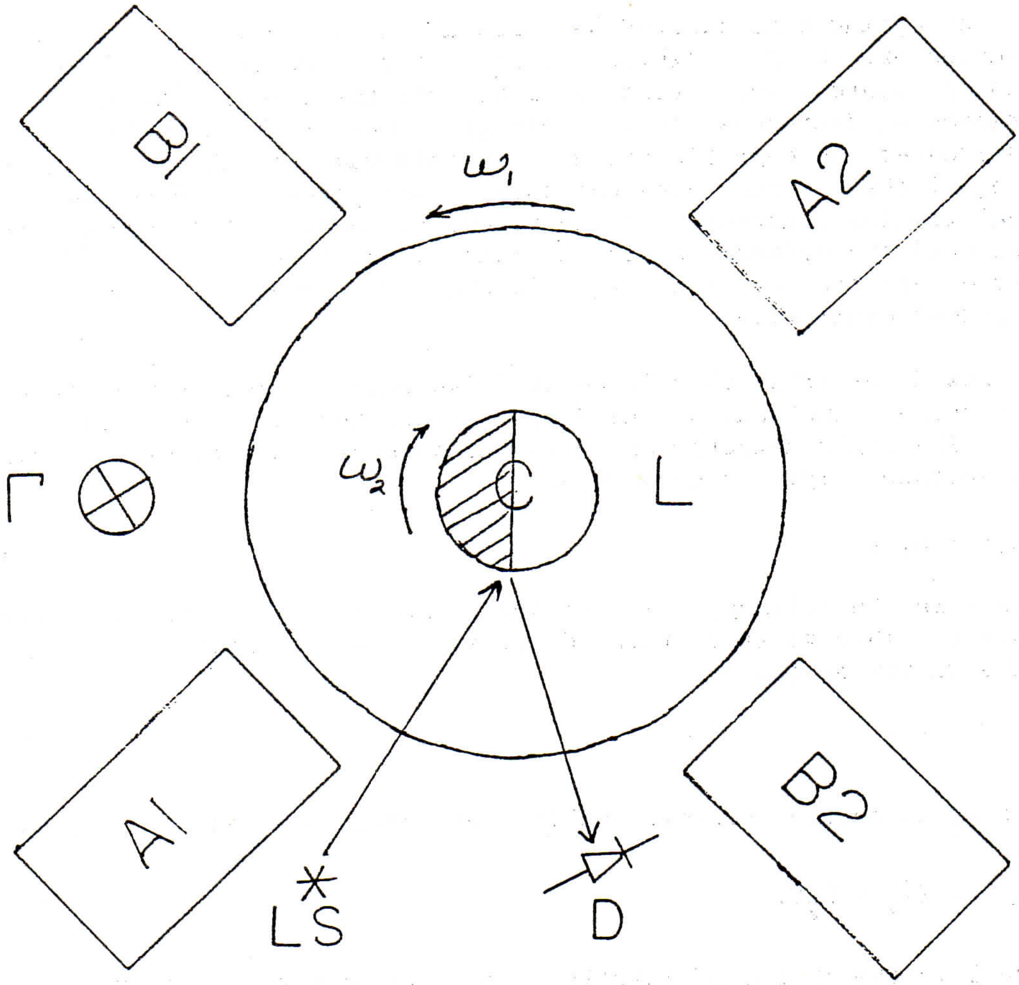

Figure 2 Closed Loop System - The same top view as in Figure 1, except that now the liquid, L, is rotated with an angular velocity, ω_1. The half white, half black cylinder C, tries to rotate with velocity ω_2, due to the magnetic torque, Γ, produced by the two pairs of drive coils, A1-A2 and B1-B2. However, light from the light source, LS, reflects off the white/black interface into the photodetector, D, and provides a voltage signal that is fedback to regulate Γ. Hence, the feedback control acts to make $\omega_2 = 0$, i.e. Γ equals the viscous drag of L.

The cylinder is painted half black and half white. Light is reflected off the black/white interface into a detector. If the cylinder rotates, the reflected light into the detector either increases or decreases, so the voltage signal of the detector increases or decreases the magnetic torque. In this way when the two torques are equal the cylinder does not rotate, and hence, the detector signal remains constant. If the viscosity changes, the torques become unbalanced the cylinder begins to rotate, but this motion is "immediately" detected and the magnetic torque is changed in order to re-establish the torque equilibrium

How do we write this in control language? First, you write down the transfer functions of the different components of the control loop. The light detector converts the angular position of the cylinder to a voltage. This is written as,

$$V = K_1\theta, \tag{5}$$

where V is the voltage, θ is the angle, and K_1 is a proportionality constant. This voltage, then, determines the new magnetic torque by the relationship,

$$\Gamma = K_2V, \tag{6}$$

where K_2 is another proportionality constant. However, Eq. (6) is also,

$$\Gamma = I\mathring{\omega}_2 = K_2V, \tag{7}$$

where I is the moment of inertia of the cylinder and $\mathring{\omega}_2$ is the angular acceleration. This acceleration produces a new angular position as,

$$\theta = \iint \mathring{\omega}_2 dt^2. \tag{8}$$

So that,

$$\theta = \frac{\Gamma}{I}\iint \mathring{\omega}_2 dt^2, \tag{9}$$

or, using Laplace transforms,

$$\theta(s) = \frac{\Gamma(s)}{Is^2}\ . \tag{10}$$

Now the feedback control loop can be described with a block diagram, shown in Fig. 3, by using Eqs. (5), (6), and (10), after their appropriate Laplace transformations. Although the block diagram can be written many ways, I've written it so that V is the output which is being fedback. This is because V contains the viscosity information since the viscous drag must equal the magnetic torque. Eq. (6) can be written as,

$$\Gamma = K_2V = b\omega_1, \tag{11}$$

and hence

$$b(\eta) = K_2 \frac{V}{\omega_1} . \tag{12}$$

Therefore, if K_2 and ω_1 are well known, measuring V gives you the viscosity signal. Note that this voltage is a lot easier to measure than the cylinder's angular velocity of the open loop method. And since this is part of the feedback loop there is no extra instrumentation other than a voltmeter.

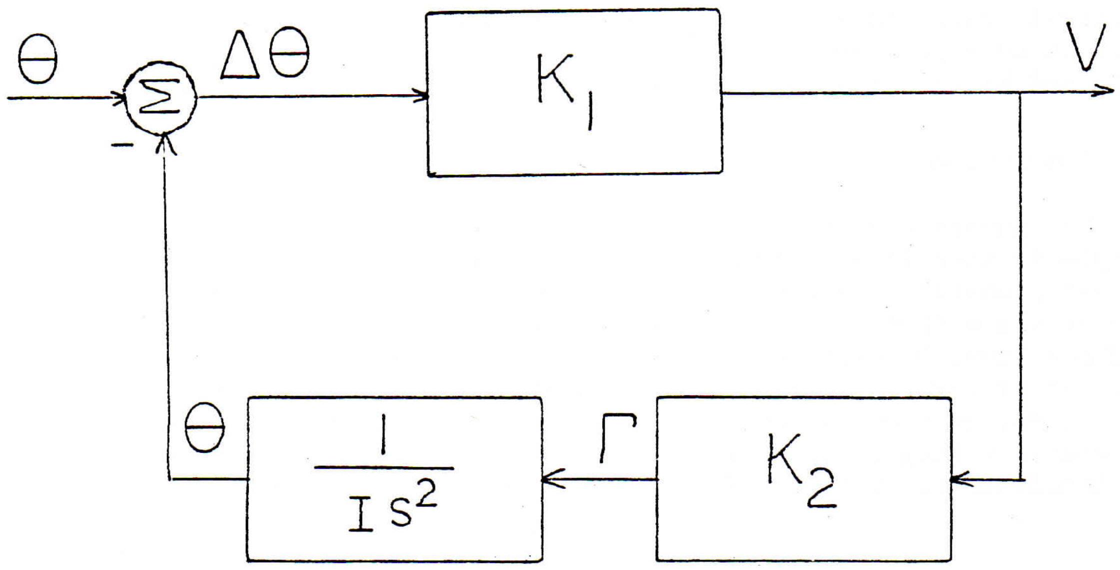

Figure 3 Block Diagram of the Feedback Loop - Using the Laplace transforms of Eqs. (5), (6), and (9) where K_1 and K_2 are coupling constants, and I is the moment of inertia of the cylinder. The output V is a voltage signal which is fedback to produce a torque, Γ, and hence a new cylinder position θ, which when combined (negative feedback) with the original θ, produces the control signal, Δθ. Negative feedback drives the control signal to zero, hence θ does not change when the system is in equilibrium.

Another important point that is hidden in this example is that although the viscous drag causes the cylinder to rotate, this is not the measured quantity. In the open loop case, this is exactly what is measured. But in the closed loop case the angular velocity is driven to zero. This is another important property of feedback control. Instead of measuring a parameter, that parameter is driven to zero and the parameter that does the driving is the actual measured quantity. In this way the feedback loop "measures" the unknown for you.

Notice from Fig. 3 that $\Delta\theta$ is the control signal and the loop is trying to make $\Delta\theta$ equal zero. Zero can be accurately determined and the system does it for you. By working this way the control loop gives you the proper amount of voltage, V, continuously. Feedback is ideally suited for this type of "nulling" experiment. It "nulls" the signal of interest to zero and allows you to easily determine how much voltage, V was needed to arrive at $\Delta\theta$ equal to zero. Not only does the feedback control work continuously but relatively instantaneously, i.e. good frequency response. Obviously there is an upper limit to the response of the loop, but the values of K_1 and K_2 can be adjusted to optimize the frequency response.

IV. Conclusion

To summarize what I have said. The most important point of this example is that it demonstrates that feedback control is not restricted to making unstable systems stable, but can also be used to make open loop stable systems into better closed loop systems. Secondly, the feedback loop, by driving the control signal to zero, makes the measurement for you. All you do is monitor one of the feedback parameters, as shown in Eq. (12). And this parameter is an easily measurable voltage. Thirdly, the feedback control works continuously, and immediately, thus providing improved frequency response.

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