

EQUIVALENCE PRINCIPLES, THEIR EMPIRICAL FOUNDATIONS,
AND THE ROLE OF PRECISION EXPERIMENTS TO TEST THEM

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I. Introduction — Equivalence Principles and the Precision Measurement Frontiers

Equivalence principles play very important roles both in the Newtonian theory of gravity and relativistic theories of gravity. The ranges of validity of these equivalence principles or their possible violations give clues and/or constraints to the microscopic origins of gravity. They will be even more important when the precisions of the tests become higher.

To pursue further tests of EEP, we have to look into precise experiments and observations in our laboratory, in the solar system and in diverse astrophysical situations. All of these depend on the progress in the field of precision measurement, and demands more precise standards.

On the other hand, in my talk about metrology¹, we have noticed that all basic standards except the prototype mass standard are based on physical laws, their fundamental constants and the microscopic properties of matter. The Einstein Equivalence Principle (EEP) says, in essence, local physics is the same everywhere. Therefore, to the precision of its empirical tests, EEP warrants the universality of these standards and their implementations.¹

In this paper, we will give a detailed discussion of various different equivalence principles, their empirical foundations and the role of precision experiments to test them. In the next section, we will discuss the history and meaning of various equivalence principles. In section III, we describe a general phenomenological framework for analyzing and testing equivalence principles. Section IV gives theorems and relations among various equivalence principles. Section V analyzes pulsar signal propagations as precision tests of EEP. Section VI quotes constraints from the Hughes-Drever experiments on the uniqueness of the metric. Section VII describes the relevancy of test-body experiments. Section VIII analyzes redshift experiments. In section IX and X, we describe the constraints on the variability of fundamental constants and microscopic particle experiments respectively. In the last section, we discuss prospects.

II. History and Meaning of Various Equivalence Principles

Our current understanding and formulation of gravity can be simply described in the following picture: Matter produces gravitational field

and gravitational field influences matter. In Newton's theory of gravity, the Galileo's weak equivalence principle² (WEP[I]) determines how matter behaves in a gravitational field, and Newton's inverse square law determines how matter produces gravitational field. In a relativistic theory of gravity such as a metric theory, the Einstein's equivalence principle (EEP) determines how matter behaves in a gravitational field, and the field equations determine how matter produces gravitational field(s). In Einstein's general relativity, with a suitable choice of the stress-energy tensor, the Einstein equation can imply the Einstein equivalence principle. In non-metric theories of gravity, other versions of equivalence principles may be used. The above situations can be summarized in the following table together with electromagnetism.

Table I. Gravity and Electromagnetism

$\text{Matter} \xrightarrow{\text{produces}} \text{Gravitational Field(s)} \xrightarrow{\text{influence(s)}} \text{Matter}$		
Newtonian Gravity	Inverse Square Law	WEP[I]
Relativistic Gravity	Field Equation(s) e.g., Einstein equation	EEP or substitute
$\text{Charges} \xrightarrow{\text{produce}} \text{Electromagnetic Field} \xrightarrow{\text{influences}} \text{Charges}$		
Electromagnetism	Maxwell Equations	Lorentz Force Law

From Table I, we see the crucial role played by equivalence principles in the formulation of gravity. In the following, we will discuss the history and meaning of various equivalence principles.

A. Ancient concepts of inequivalence

From the observations that heavy bodies fall faster than light ones in the air, ancient people, both in the orient and in the west, believe that objects with different constituents behave differently in a gravitational field. We now know that this is due to the inequivalent responses to different buoyancy forces and air resistances.

B. Macroscopic equivalence principles

(i) Galileo equivalence principle² (WEP[I])

Using an inclined plane, Galileo (1564-1642) showed that the distance a falling body travels from rest varies as the square of the time. Therefore, the motion is one of constant acceleration. Moreover, Galileo

demonstrated that "the variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of 100 cubits [about 46 meters] a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this, I came to the conclusion that in a medium totally void of resistance all bodies would fall with the same speed [together]"². The last conjecture is the famous Galileo equivalence principle and serves as the beginning of our understanding of gravity. More precisely, Galileo equivalence principle states that in a gravitational field, the trajectory of a test body with a given initial velocity is independent of its internal structure and composition (universality of free fall trajectories).

From Galileo's observations, we can arrive at the following two conclusions.

(a) The gravitational force (weight) at the top of the inclined plane and that at a middle point of the inclined plane can be regarded the same to the experimental limits in those days. Hence a falling body experiences a constant force (its weight). The motion of a falling body is one of constant acceleration. Therefore a constant force f induces a motion of constant acceleration a . Hence force and acceleration (not velocity) are closely related. If one changes the inclinations of the plane to get different "dilutions" of gravity, one finds

$$f \propto a \quad (1)$$

for a falling body. From Galileo's observation of the universality of free fall trajectories, we know that a is the same for different bodies. But f (weight) is proportional to mass m . Hence for different bodies,

$$\frac{f}{m} \propto a. \quad (2)$$

If one chooses appropriate units, one arrives at

$$f = ma \quad (3)$$

for falling bodies. If one further assumes that all kind of forces are equivalent in their ability to accelerate and notices the vector nature of forces and accelerations, one would arrive at Newton's second law,

$$\underline{f} = m \underline{a}. \quad (4)$$

(b) From Galileo equivalence principle, the gravitational field can be described by the acceleration of gravity \underline{g} . Newton's second law for N particles in external gravitational field \underline{g} is

$$m_I \frac{d^2 \underline{x}_I}{dt^2} = m_I \underline{g}(\underline{x}_I) + \sum_{J=1}^N \underline{F}_{IJ}(\underline{x}_I - \underline{x}_J), \quad (I=1, \dots, N) \quad (5)$$

where \underline{F}_{IJ} is the force acting on particle I by particle J . At a point \underline{x}_0 , expand $\underline{g}(\underline{x})$ as follows

$$\underline{g}(\underline{x}_I) = \underline{g}_0 + \underline{\lambda} \cdot (\underline{x}_I - \underline{x}_0). \quad (6)$$

Choosing x_0 as origin and applying the following non-Galilean spacetime coordinate transformation

$$\underline{x}' = \underline{x} - \frac{1}{2} g_0 t^2, \quad t' = t, \quad (7)$$

(5) is transformed to

$$m_I \frac{d^2 \underline{x}'_I}{dt'^2} = \sum_{J=1}^N F_{IJ}(\underline{x}'_I - \underline{x}'_J) + O(\underline{x}'_J) \quad (8)$$

Thus we see that locally the effect of external gravitational field can be transformed away. Thus we arrive at a strong equivalence principle. Therefore in Newtonian mechanics,

Galileo Weak Equivalence Principle \Leftrightarrow Strong Equivalence Principle.

In the days of Galileo and Newton, the nature of light and radiation was controversial and had to wait for further development to clarify it.

(ii) The second weak equivalence statement (WEP[II])

Since the motion of a macroscopic test body is determined not only by its trajectory but also by its rotation state. From previous studies,^{3,4} we have proposed the following stronger weak equivalence statement (WEP[II]) to be tested by experiments, which states that in a gravitational field, the motion of a test body with a given initial motion state is independent of its internal structure and composition (universality of free fall motions). By a test body, we mean an uncharged macroscopic body whose size is small compared to the length scale of the inhomogeneities of the gravitational field. More will be said about WEP[II], in my talk "Spin, Torsion and Polarized Test-Body Experiments" in the Symposium.⁵

C. Microscopic equivalence principles

The development of physics in the nineteenth century brought to improved understanding of light and radiations and the development of special relativity. In 1905, Einstein obtained the equivalence of mass and energy and derived the famous Einstein formula $E=mc^2$. A natural question came in at this point: How light and radiations behave in a gravitational field? This led to the formulation of microscopic equivalence principles.

(i) Einstein equivalence principle (EEP)

Two years after he proposed special relativity and the formula $E=mc^2$, Einstein⁶, in the last part (Principle of Relativity and Gravitation) of his comprehensive 1907 essay on relativity, proposed the complete physical equivalence of a homogeneous gravitational field to a uniformly accelerated reference system: "We consider two systems of motion, Σ_1 and Σ_2 . Suppose Σ_1 is accelerated in the direction of its X axis, and γ is the magnitude (constant in time) of this acceleration. Suppose Σ_2 is at rest, but situated in a homogeneous gravitational field, which imparts to all objects an acceleration $-\gamma$ in the direction of the X axis. As far as we know,

the physical laws with respect to Σ_1 do not differ from those with respect to Σ_2 , this derives from the fact that all bodies are accelerated alike in the gravitational field. We have therefore no reason to suppose in the present state of our experience that the systems Σ_1 and Σ_2 differ in any way, and will therefore assume in what follows the complete physical equivalence of the gravitational field and the corresponding acceleration of the reference system." From this equivalence, Einstein derived clock and energy redshifts in a gravitational field. When applied to a spacetime region where inhomogeneities of the gravitational field can be neglected, this equivalence dictates the behavior of matter in gravitational field. The postulate of this equivalence is called the Einstein Equivalence Principle (EEP). EEP is the cornerstone of the gravitational coupling of matter and non-gravitational fields in general relativity and in metric theories of gravity.

EEP is a microscopic principle and may mean slightly different things for different people. To most people, EEP is equivalent to the comma-goes-to-semicolon rule for matter (not including gravitational energy) in gravitational field. Therefore, EEP means that in any and every local Lorentz frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special-relativistic forms.⁷ In other words, EEP says that the outcome of any local, nongravitational test experiment is independent of the velocity of the apparatus. For example, the fine structure constant $\alpha = \frac{e^2}{\hbar c}$ must be independent of location, time, and velocity.

(ii) Modified Einstein equivalence principle (MEEP)

In 1921, Eddington⁸ mentioned the notion of an asymmetric affine connection in discussing possible extensions of general relativity. In 1922, Cartan⁹ introduced torsion as the anti-symmetric part of an asymmetric affine connection and laid the foundation of this generalized geometry. Cartan¹⁰ proposed that the torsion of spacetime might be connected with the intrinsic angular momentum of matter. In 1921-22, Stern and Gerlach¹¹ discovered the space quantization of atomic magnetic moments. In 1925-26, Goudsmit and Uhlenbeck¹² introduced our present concept of electron spin as the culmination of a series of studies of doublet and triplet structures in spectra. Following the idea of Cartan, Sciama^{13,14} and Kibble¹⁵ developed a theory of gravitation which is commonly called the Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity.

After the works of Utiyama¹⁶, Sciama^{13,14} and Kibble¹⁵, interest and activities in gauge-type and torsion-type theories of gravity have continuously increased. Various different theories postulate somewhat different interaction of matter with gravitational field(s). In ECSK theory and in some other torsion theories, there is a torsion gravitational field besides the usual metric field.¹⁷ In special relativity, if we use a nonholonomic tetrad frame, there is an antisymmetric part of the affine connection. Therefore many people working on torsion theory take the equivalence principle to mean something different from EEP so that torsion can be included. This is mostly clearly stated in P. von der Heyde's article "The Equivalence Principle in the U_4 Theory of Gravitation"¹⁸: Locally the properties of special relativistic matter in a noninertial frame of reference cannot be distinguished from the properties

of the same matter in a corresponding gravitational field. This modified equivalence principle (MEEP) allows for formal inertial effects in a non-holonomic tetrad frame and hence allows torsion. There are two ways to treat the level of coupling of torsion: one can consider torsion on the same level as symmetric affine connection (MEEP[I]) or one can consider torsion on the same level as curvature tensors (MEEP[II]). Hehl, and von der Heyde¹⁸ hold the second point of view. For a test body, curvature effects are neglected; so MEEP[II] is essentially equivalent to EEP for test bodies. Test bodies with nonvanishing total intrinsic spin feel torques from the torsion field. Hence MEEP[I] does not imply WEP[II]. Moreover MEEP[I] does not imply WEP[I] either.⁵ Therefore we have

$$\begin{array}{ccc} \text{EEP} & \Rightarrow & \text{MEEP[I]} \\ \searrow & \nearrow & \searrow \\ & \text{WEP[II]} & \Rightarrow \text{WEP[I]} \end{array}$$

D. Equivalence principles including gravity

How does gravitational energy behaves in a gravitational field? Is local gravity experiment depending on where and when in the universe it is performed? These involve nonlinear gravity effects.

(i) WEP[I] for massive bodies

This weak equivalence principle says that in a gravitational field, the trajectory of a massive test body with a given initial velocity is independent of the amount of gravitational self-energy inside the massive body. In Brans-Dicke theory and many other theories, there are violations of this equivalence principle. The violations are called Nordtvedt effects.^{19,20} General relativity obeys WEP[I] for massive bodies in the post-Newtonian limit and for black holes. In the Symposium, Professor J.E. Faller will talk about the lunar laser ranging experiment. The precise nature of this experiment verifies WEP[I] for massive bodies to about 2% in the gravitational self-energy.^{21,22}

(ii) Dicke's²³ strong equivalence principle (SEP)

This is a microscopic equivalence principle. It says that the outcome of any local test experiment—gravitational or nongravitational—is independent of where and when in the universe it is performed, and independent of the velocity of the apparatus. If this equivalence principle is valid, G should be a true constant. Brans-Dicke theory with its variable "gravitational constant" as measured by Cavendish experiments, satisfies EEP but violates SEP.

The violations of SEP seem to be linked with the violations of WEP[I] for massive bodies in many cases. It is interesting to know how SEP and WEP[I] for massive bodies are connected. The violations of SEP may also be connected to the violations of WEP[I] at some level in some cases.

We note in passing that there are other versions of equivalence principles which we are not able to list them here one-by-one.

III. A General Phenomenological Framework for Analysing and Testing Equivalence Principles

The renaissance of general relativity in the last two decades together with the fundamental discoveries and developments in particle physics leads to renewed interests in the microscopic origin of gravity. The discovery of parity violation^{24,25} and CP violation²⁶ in the weak interaction puts us into a symmetry broken world. The success of Weinberg-Salam^{27,28} theory of unified electroweak interaction makes spontaneous symmetry breaking a promising way to generate "fundamental" constants and to unify interactions. The incorporation of spontaneous symmetry breaking in gravity to generate Newton's gravitational constant G has been considered by various authors²⁹. Considering the role of spins and different gauge groups, various torsion theories and gauge-type theories of gravity have been proposed. Many of these theories violate EEP in one way or another at certain level.

With our present knowledge, a unification scale can be drawn as in figure 1. Electromagnetism and weak interaction are unified as electroweak interaction at W^\pm and Z^0 energies (~ 100 GeV).^{30,31} According to the grand unification schemes³², the electroweak and strong interactions would be unified at an energy of 10^{15} GeV. At present accelerator energy ($\sqrt{s} \sim 540$ GeV), Quantum Chromodynamics (QCD) looks promising in explaining the strong interaction. From this energy to the grand unification energy, there is a 12-order of magnitude gap. The important question is whether in this wide gap, there are more structures besides electroweak unification and QCD. Recently, theoretical developments of supersymmetry and technicolor point to this direction. In gravitation, equivalence principle and Einstein equation are only verified empirically at lower energies and larger distances. Using dimensional arguments, quantum phenomena should be important in gravity at Planck-mass energy ($(\hbar c^5/G)^{1/2} = 1.223 \times 10^{19}$ GeV or Planck distance $(\hbar G/c^3)^{1/2} = 1.616 \times 10^{-33}$ cm. Above (or About) this energy, it is also possible that the grand unified interaction and quantum gravity can further be unified. Here the important question is whether the gravitational coupling of matter still obeys the Einstein equivalence principle. To answer this question, ever precise measurements at ever diverse situations are desired. This is a paramount challenge to the field of precision measurement.

As an illustration of why the naive point of view of no more structures might not be right, we quote the following historical example. Einstein's efforts of unifying electromagnetism and gravity were not successful. Now we understand why. Before electromagnetism and gravity can be unified, one has to take into account weak interaction and, possibly, strong interaction. Similar things could happen again and there would be more structures.

Both the fundamental role of EEP in general relativity and its possible violation at certain level demand a close scrutiny of its empirical foundations. For this purpose, we need a general framework to study the empirical foundations of EEP, to analyze the theoretical significance of various experiments and observations, and to propose new experiments.

Since here we are only concerned with the behavior of matter in a gravitational field, we treat gravitational field as external field. Most experiments concern electromagnetism. Therefore to be more specific, we consider electromagnetically interacting system in this article. Generalizations to strong and weak interactions will be presented elsewhere. In previous work we have used a general framework — the χ -g framework^{3,4} to study Schiff's conjecture and theoretical relations of various equivalence principles. The χ -g framework is rather comprehensive and we use it here to study the empirical formations of EEP.

This framework can be summarized in the following interaction Lagrangian density

$$\mathcal{L}_I = -\left(\frac{1}{16\pi}\right)\chi^{ijkl} F_{ij}F_{kl} - A_k j^k (-g)^{\frac{1}{2}} - \sum_I m_I \frac{ds_I}{dt} \delta(\underline{x} - \underline{x}_I), \quad (9)$$

where $\chi^{ijkl} = \chi^{klij} = -\chi^{klji}$ is a tensor density of the gravitational fields (e.g., g_{ij} , ϕ , etc.), and j^k , $F_{ij} \equiv A_{j,i} - A_{i,j}$ have the usual meaning. The gravitational constitutive tensor density χ^{ijkl} dictates the behavior of electromagnetism in a gravitational field and has 21 independent components in general. For a metric theory (when EEP holds), χ^{ijkl} is determined completely by the metric g^{ij} and equals $(-g)^{\frac{1}{2}}(\frac{1}{2}g^{ik}g^{jl} - \frac{1}{2}g^{il}g^{kj})$. (9) is the most general interaction Lagrangian density with the conditions: (i) uncharged particles following geodesics of a Riemannian metric, (ii) electric charge being conserved, (iii) only gravitational fields (potentials), not their gradients, being involved in \mathcal{L}_I , (iv) quadratic in the gradient of the electromagnetic potential and no mass-like terms involved, in $\mathcal{L}_I^{(EM)}$.

IV. Theorems and Relations Among Various Equivalence Principles

A. Schiff's conjecture

In 1960, Leonard Schiff³³ argued as follows: "The Eötvös experiments show with considerable accuracy that the gravitational and inertial masses of normal matter are equal. This means that the ground state eigenvalue of the Hamiltonian for this matter appears equally in the inertial mass and in the interaction of this mass with a gravitational field. It would be quite remarkable if this could occur without the entire Hamiltonian being involved in the same way, in which case a clock composed of atoms whose motions are determined by this Hamiltonian would have its rate affected in the expected manner by a gravitational field." He suggested that EEP and, hence, the metric gravitational redshift are consequences of WEP[I]. In short, Schiff believes that

$$\text{WEP[I]} \iff \text{EEP}.$$

This conjecture is known as Schiff's conjecture. The scope of validity of Schiff's conjecture has great importance to the analysis of the empirical foundations of EEP.

B. Two theorems^{3,4}Stress-energy tensor density, 4-momentum and center-of-mass —

In conformity with the definitions for the standard Lagrangian formulation and for dielectric materials, we define the electromagnetic stress-energy tensor density as

$$\begin{aligned} \mathcal{T}_i^{k(EM)} &= A_{\ell,i} (\partial \mathcal{L}^{(EM)} / \partial A_{\ell,k}) - \delta_i^k \mathcal{L}^{(EM)} \\ &= (1/4\pi) (-\chi^{k\ell mn} A_{\ell,i} F_{mn} + 1/4 \chi^{j\ell mn} F_{j\ell} F_{mn} \delta_i^k). \end{aligned} \quad (10)$$

The total stress-energy tensor density is $\mathcal{T}_i^k = \mathcal{T}_i^{k(EM)} + \mathcal{T}_i^{k(P)}$ where $\mathcal{T}_i^{k(P)}$ is the usual stress-energy tensor density of particles. The 4-momentum vector of a test body is $P_i = \int \mathcal{T}_i^0 d^3x$. Defining the center of mass as $\dot{X}^i = (\int x^i \mathcal{T}_0^0 d^3x / P_0)$, then one can readily show that

$$\dot{X}^i = P^i / P^0 \quad (11)$$

for a test body.

Matter-response equation — From the Euler-Lagrange equations, we derive the matter-response equation

$$\mathcal{T}_i^k{}_{,k} = - \frac{\partial \mathcal{L}}{\partial x^i} \quad (12)$$

From Eq.(12), one can show that

$$\dot{P}_m = \frac{1}{16\pi} \chi^{ijk\ell}{}_{,m} \int F_{ij} F_{k\ell} d^3x + \frac{1}{2} g_{k\ell,m} \int \mathcal{T}^{k\ell(P)} d^3x. \quad (13)$$

We now impose the condition of WEP[I]. Since a nonelectromagnetically interacting test body follows a geodesic in the metric g^{ij} , any other test body will follow such a geodesic too. Choose a Fermi-normal coordinate system such that the test body is at rest in the system and the Christoffel symbols vanish along the geodesic. We then have $\dot{P}_m = d(P^0 \dot{X}_m) / dt = 0$. Compare this with Eq.(13), we conclude that

$$\chi^{ijk\ell}{}_{,m} \int F_{ij} F_{k\ell} d^3x = 0. \quad (14)$$

Lemma. In a fixed coordinate system, Eq.(14) holds for every test body if and only if $\chi^{ijk\ell}{}_{,m} = \phi_{,m} e^{ijk\ell}$, where

$$e^{ijk\ell} = \begin{cases} 1 & \text{if } (ijk\ell) \text{ is an even permutation of } (0123) \\ -1 & \text{if } (ijk\ell) \text{ is an odd permutation of } (0123) \\ 0 & \text{otherwise.} \end{cases}$$

— Expanding F_{ij} in powers of $\chi^{ijk\ell}$ and $\chi^{ijk\ell}{}_{,m}$ and substituting in

Eq.(14), every order in the expansion of (14) must vanish. From the vanishing of the first-order expansion,

$$\chi^{ijkl}_{,m} \int F_{ij}^{(0)} F_{kl}^{(0)} d^3x = 0, \quad (15)$$

where $F_{ij}^{(0)}$ is the solution of Maxwell's equations in special relativity. Using Eq.(15) and considering test bodies consisting of a parallel-plate capacitor and a solenoidal coil of current, one can derive (i) $\chi^{0123}_{,m} = \chi^{0231}_{,m} = \chi^{0312}_{,m}$ and (ii) all other components of $\chi^{ijkl}_{,m}$ not related to the components in (i) by the symmetry property vanish. Therefore $\chi^{ijkl}_{,m} = \phi_{,m} e^{ijkl}$ where $\phi \equiv \chi^{0123}$ and we prove the only if part of the Lemma. If $\chi^{ijkl}_{,m} = \phi_{,m} e^{ijkl}$, we have

$$\int \chi^{ijkl}_{,m} F_{ij} F_{kl} d^3x = 4 \int \phi_{,mj} e^{ijkl} A_i A_{k,\ell} d^3x = 0 \quad (16)$$

because the second derivatives of gravitational fields can be neglected for test bodies. In the derivation of first equality in Eq.(16), an average over a dynamical timescale for the body has been performed in order to make surface terms vanish. This is the standard virial theorem technique used in treating a macroscopic body. Q.E.D..

Theorem I. For a system whose Lagrangian density is given by (1), WEP[I] holds if and only if

$$\chi^{ijkl} = (-g)^{1/2} \left(\frac{1}{2} g^{ik} g^{jl} - \frac{1}{2} g^{il} g^{kj} + \phi \epsilon^{ijkl} \right) \quad (17)$$

where ϕ is a scalar function of the gravitational fields and $\epsilon^{ijkl} = (1/\sqrt{-g}) e^{ijkl}$. — In a Fermi-normal coordinate system of a test body geodesic, by the Lemma, WEP[I] holds if and only if

$$\chi^{ijkl}_{,m} = \phi_{,m} e^{ijkl} \quad (18)$$

In a region R where gravitational effects are negligible, $\chi^{ijkl} = \frac{1}{2} \eta^{ik} \eta^{jl} - \frac{1}{2} \eta^{il} \eta^{kj}$ (η^{ij} is the Minkowski metric). Starting from this region, integrating Eq.(18) along the geodesic, and transforming to an arbitrary coordinate system, we get Eq.(17). Assuming there exists a geodesic network connecting every point in spacetime to such a region R, then Eq.(17) holds for every spacetime point. Now it is easy to see that ϕ is a scalar function of the gravitational fields. Moreover, if Eq.(17) holds, then Eq.(14) holds in the Fermi-normal frame, and hence WEP[I] holds. Q.E.D..

If $\phi \neq 0$ in (17), the gravitational coupling to electromagnetism is not minimal and EEP is violated. Hence WEP[I] does not imply EEP. But WEP[I] does constrain the 21 degrees of freedom of χ to only one degree of freedom (ϕ). In the actual empirical situation, since Eötvös-Dicke-Braginsky experiments are performed on unpolarized bodies, they constrain only 2 degrees of freedom of χ (cf. §VII). Only when these experiments are performed on polarized bodies with various different electromagnetic energy configurations, can they constrain the other 18 degrees of freedom.

From this theorem, one can easily prove the results of Lightman and Lee³⁴:

Corollary. For an electromagnetically interacting system in a static spherical-symmetric gravitational field whose Lagrangian density is given by

$$\mathcal{L}_I = (1/8\pi)(\epsilon E^2 - B^2/\mu) - A_k j^k (-g)^{1/2} - \sum_I m_I (ds_I/dt) \delta(\underline{x} - \underline{x}_I)$$

where ϵ and μ are functions of the gravitational fields, WEP[I] implies EEP. — For the above Lagrangian density $\chi^{1234} = 0$. Hence $\phi = 0$. Q.E.D.

In my Symposium talk on "Spin, Torsion and Polarized Test-Body Experiments"⁵, I will demonstrate that in the theory with χ^{ijkl} given by (17), there are anomalous torques on electromagnetic-energy polarized test bodies unless $\phi = 0$. For $\phi = 0$ in (17), the theory reduces to metric theory and EEP holds. Therefore we arrive at the following theorem:

Theorem II. For the Lagrangian (9), WEP[II] implies EEP.

Therefore, in the χ -g framework, we have the following relations among equivalence principles:

$$\text{WEP[I]} \Leftrightarrow \text{WEP[II]} \Leftrightarrow \text{EEP}.$$

Eötvös-type experiments on polarized test bodies — To test WEP[II], it is crucial to perform Eötvös-type experiments on polarized bodies. In view of Theorem II, this is also an excellent test for EEP.

C. Remarks

As we have seen in subsection A of this section, Schiff argued in 1960 that EEP should be a consequence of WEP[I], hence the metric gravitational redshift should also be a consequence of WEP[I]. However Dicke³⁵ held a different point of view and believed that the redshift experiment has independent theoretical significance. In the eikonal approximations of the χ -g framework, I have shown that the first-order gravitational redshifts are metric.³

In November 1970, the interests in the issue of the validity of Schiff's conjecture were rekindled during a vigorous argument between L. Schiff and K.S. Thorne at the Caltech-JPL Conference on Experimental Tests of Gravitation Theories.

In 1973, Thorne, Lee, and Lightman³⁶ analyzed the fundamental concepts and terms involved in detail and gave a plausibility argument supporting Schiff's conjecture. Lightman and Lee³⁴ proved Schiff's conjecture for electromagnetically interacting systems in a static, spherically symmetric gravitational field using a particular mathematical formalism known as the $\text{Th}\epsilon\mu$ formalism. $\text{Th}\epsilon\mu$ formalism is a special case in the χ -g framework. Lightman-Lee theorem is proved as corollary to Theorem I in subsection B of this section.

I started to work on Schiff's conjecture in September 1972. David L. Lee and Alan P. Lightman kept me informed about their progress. I tried to look for a counterexample to Schiff's conjecture. Early December, I found the candidate expressed by (9) with (17). I used Lightman-Lee³⁴ method to calculate the test body trajectories in this candidate counterexample and found that it agrees with Eötvös-Dicke experiments. So I suspected that this is a real counterexample. When I told Lee and Lightman about my findings in the December 1972 Texas Symposium, they have already completed their theorem.

Since it became more and more difficult to do higher-order calculations in the candidate counterexample using Lightman-Lee method, I looked for a different approach. Using this new approach (similar to the one in proving Theorem I), I proved that the candidate counterexample is indeed a real one. In May 1973, I presented this result in a seminar in the department of physics of the University of British Columbia. Later, when this result is written and typed, I found that there could be anomalous torque on a polarized test body. So I have only sent pre-prints³⁷ out.

I kept looking for counterexamples to Schiff's conjecture without anomalous torques, but I failed. So I guessed that Schiff's conjecture should be largely valid. Therefore I designed the χ -g framework and used it to prove Theorem I and II in the subsection B. These theorems together with the counterexample were presented in the June, 1974 Salt Lake City APS Meeting.³

Subsequently, I found that the χ -g framework is not only good for analyzing equivalence principles, but also comprehensive enough to study the empirical foundations of EEP. These studies will be presented in the following sections.

Recently, Coley^{38,39,40} have used a general formalism with seven nonmetric degrees of freedom to study the validity of Schiff's conjecture in a spherically symmetric and static (SSS) gravitational field. His results actually show that WEP \rightarrow EEP in SSS gravitational field in his formalism and, therefore, extend Lightman-Lee Theorem in SSS gravitational field.

V. Pulsar Signal Propagations as Tests of EEP

If EEP is observed, photons with different polarizations as test particles shall follow identical trajectories in a gravitational field. In pulsar observations, the pulses and micropulses with different polarizations are correlated in general structure and timing⁴¹. Due to precise timing and rich polarization data, pulsar signal propagations in galactic gravitational field constitute high precision tests of EEP.

A. Equation for electromagnetic wave propagation in a weak gravitational field

Since our galactic Newtonian potential U is of the order of 10^{-6} , we use weak field approximation in the χ -g framework. The vacuum Maxwell equation, derived from the Lagrangian (9), is

$$(\chi^{ijk\ell} A_{k,\ell})_{,j} = 0. \quad (19)$$

Neglecting $\chi^{ijk\ell}_{,p}$ in slowly varying field, (19) becomes

$$\chi^{ijk\ell} A_{k,\ell j} = 0. \quad (20)$$

For weak field, we assume

$$\chi^{ijk\ell} = \chi^{(0)ijk\ell} + \chi^{(1)ijk\ell}, \quad (21)$$

where

$$\chi^{(0)ijk\ell} = \frac{1}{2} \eta^{ik} \eta^{j\ell} - \frac{1}{2} \eta^{i\ell} \eta^{kj} \quad (22)$$

with η^{ij} the Minkowski metric and $|\chi^{(1)s}| \ll 1$.

B. Conditions for gravitational nonbirefringence — Photons propagate along a metric H_{ik}

Using eikonal approximation, we look for plane-wave solution propagating in the z -direction. Imposing radiation condition in the zeroth order and solving the dispersion relation for ω , we obtain

$$\omega_{\pm} = k \left\{ 1 + \frac{1}{4} [(K_1 + K_2) \pm \sqrt{(K_1 - K_2)^2 + 4K^2}] \right\} \quad (23)$$

where

$$\begin{aligned} K_1 &= \chi^{(1)1010} - 2\chi^{(1)1013} + \chi^{(1)1313}, \\ K_2 &= \chi^{(1)1202} - 2\chi^{(1)1203} + \chi^{(1)1323}, \\ K &= \chi^{(1)1020} - \chi^{(1)1023} - \chi^{(1)1320} + \chi^{(1)1323}. \end{aligned} \quad (24)$$

Photons with two different polarizations propagate with different speed $v_{\pm} = \frac{\omega_{\pm}}{k}$ and would split in 4-dimensional spacetime. The conditions for no splitting (no retardation) is $\omega_+ = \omega_-$, i.e.

$$K_1 = K_2, \quad K = 0. \quad (25)$$

(25) gives two constraints on $\chi^{(1)s}$.

The conditions for no splitting (no retardation) of electromagnetic waves propagating in every direction give the following ten constraints on $\chi^{(1)s}$:

$$\begin{aligned}
\chi^{(1)1220} &= \chi^{(1)1330} , \\
\chi^{(1)2330} &= \chi^{(1)2110} , \\
\chi^{(1)3110} &= \chi^{(1)3220} , \\
\chi^{(1)1020} &= -\chi^{(1)1323} , \\
\chi^{(1)2030} &= -\chi^{(1)2131} , \\
\chi^{(1)3010} &= -\chi^{(1)3212} , \\
\chi^{(1)1320} &= -\chi^{(1)1230} , \\
\chi^{(1)1320} &= -\chi^{(1)2310} , \\
\chi^{(1)1010} + \chi^{(1)1313} &= \chi^{(1)2020} + \chi^{(1)2323} , \\
\chi^{(1)1010} + \chi^{(1)1212} &= \chi^{(1)3030} + \chi^{(1)3232} .
\end{aligned} \tag{26}$$

Now define $H^{(1)ij}$, ψ and ϕ as

$$\begin{aligned}
H^{(1)10} &\equiv H^{(1)01} \equiv -2\chi^{(1)1220} , \\
H^{(1)20} &\equiv H^{(1)02} \equiv -2\chi^{(1)2330} , \\
H^{(1)30} &\equiv H^{(1)03} \equiv -2\chi^{(1)3110} , \\
H^{(1)12} &\equiv H^{(1)21} \equiv -2\chi^{(1)1020} , \\
H^{(1)23} &\equiv H^{(1)32} \equiv -2\chi^{(1)2030} , \\
H^{(1)31} &\equiv H^{(1)13} \equiv -2\chi^{(1)3010} , \\
H^{(1)11} &\equiv 2\chi^{(1)2020} + 2\chi^{(1)2121} - H^{(1)00} , \\
H^{(1)22} &\equiv 2\chi^{(1)3030} + 2\chi^{(1)3232} - H^{(1)00} , \\
H^{(1)33} &\equiv 2\chi^{(1)1010} + 2\chi^{(1)1313} - H^{(1)00} , \\
\psi &\equiv 1 + 2\chi^{(1)1212} + \frac{1}{2}\eta_{00}(H^{(1)00} - H^{(1)11} - H^{(1)22} \\
&\quad - H^{(1)33}) - H^{(1)11} - H^{(1)22} , \\
\phi &\equiv \chi^{(1)0123} .
\end{aligned} \tag{27}$$

Note that in these definitions $H^{(1)00}$ is not defined and free. It is straightforward to show that if the ten constraints (26) are satisfied then χ can be written to first-order in $\chi^{(1)}$'s in the form

$$\chi^{ijkl} = (-H)^{\frac{1}{2}} \left(\frac{1}{2} H^{ik} H^{jl} - \frac{1}{2} H^{il} H^{kj} \right) \psi + \phi e^{ijkl}, \quad (28)$$

where

$$H^{ij} = \eta^{ij} + H^{(1)ij},$$

$$H = \det(H_{ij}), \quad (29)$$

$$H_{ij} H^{jk} = \delta_i^k,$$

and

$$e^{ijkl} = \begin{cases} 1, & \text{if } (ijkl) \text{ is an even permutation of } (0123), \\ -1, & \text{if } (ijkl) \text{ is an odd permutation of } (0123), \\ 0, & \text{otherwise.} \end{cases} \quad (30)$$

C. Observational constraints from pulsars.

In actual observations, the pulses and micropulses with different polarizations are correlated in general structure and no retardation with respect to polarizations are observed. This means that conditions similar to (25) are satisfied to observational accuracy. For Crab pulsar, the micropulses with different polarizations are correlated in timing to within 10^{-4} sec, the distance of the Crab pulsar is 2200 pc, therefore to within 10^{-4} sec / (2200 x 3.26 light yr.) = 5×10^{-16} accuracy two conditions similar to (25) are satisfied. Over 300 pulsars in different directions are observed. Many of them have polarization data. Combining all of them, (26) is satisfied to an accuracy of $10^{-14} - 10^{-16}$. Since $U \sim 10^{-6}$, $\chi^{(1)}/U$ (or χ/U) agrees with that given by (28) to an accuracy of $10^{-8} - 10^{-10}$. Detailed analysis will reveal better results.

Thus, to high accuracy, photons are propagating in the metric field H^{ik} and two additional scalar fields ϕ and ψ . A change of H^{ik} to λH^{ik} does not affect χ^{ijkl} in (28) — this corresponds to the freedom of $H^{(1)00}$ in the definition (27) of $H^{(1)ij}$. Thus we have eleven degrees of freedom in (28).

Recently, McCulloch, Hamilton, Ables and Hunt⁴² have observed a radio pulsar in the large Magellanic Cloud. Backer, Kulkarni, Heiles, Davis and Goss⁴³ have discovered a millisecond pulsar which rotates 20 times faster than the Crab pulsar. The progress of these observations would potentially give better constraints on some of the conditions (26) due to larger distance or fast period involved.

Analysis of optical and X-ray polarization data from various astrophysical sources will give better accuracy to some of the ten constraints in (26). Results of this analysis will be presented in the future.

VI. Hughes-Drever Experiments and the Uniqueness of the Metric

Since (28) is verified empirically to high accuracy from pulsar observations, in the following we start from (28) as a base to analyze other experiments.

In (9), ds is the line element determined from the metric g_{ij} . From (28), the gravitational coupling to electromagnetism is determined by the metric H_{ij} and two scalar fields ϕ and ψ . If H_{ij} is not proportional to g_{ij} , then the hyperfine levels of the lithium atom will have additional shifts. But this is not observed to high accuracy in Hughes-Drever experiments^{44,45}. Therefore H_{ij} is proportional to g_{ij} to certain accuracy. Since a change of H_{ik} to λH_{ik} does not affect χ_{ijkl} in (28), we can define $H_{11} = g_{11}$ to remove this scale freedom.

In my Symposium talk on "Implications of Hughes-Drever Experiments"⁴⁶, I will discuss Hughes-Drever experiments in more details, and will show that from these experiments:

$$\begin{aligned} |H_{\mu\nu} - g_{\mu\nu}|/U &\leq 10^{-12} \\ |H_{0\mu} - g_{0\mu}|/U &\leq 10^{-7} - 10^{-8}, \\ |H_{00} - g_{00}|/U &\leq 10^{-4}. \end{aligned} \quad (31)$$

where U ($\sim 10^{-6}$) is the galactical gravitational potential.

VII. Test-Body Experiments

A. Eötvös-Dicke-Braginsky experiments

Eötvös-Dicke-Braginsky^{47,48,49} experiments are performed on unpolarized test bodies. In essence, it says that the unpolarized electric and magnetic energies follow the same trajectories as other forms of energy to certain accuracy. The constraints on (28) are

$$|1 - \psi|/U < 10^{-9} \quad (32)$$

and

$$|H_{00} - g_{00}|/U < 10^{-5} \quad (33)$$

where U is the solar gravitational potential at earth.

B. Polarized test-body experiments⁵⁰

These experiments will give informations on the other 18 degrees of freedom which can be constrained by WEP[I], and one more degree of freedom (by measuring torque) that can be constrained by WEP[II].

More details can be found in the Symposium article "Spin, Torsion and Polarized Test-Body Experiments".⁵

VIII. Redshift Experiments

Einstein, in his comprehensive 1907 essay on relativity⁶, derived the gravitational redshift from his equivalence principle (EEP). About 1920, the gravitational redshift of light in the white dwarf Sirius B was measured. This confirmed the existence of gravitational redshift. At that time, the state of matter in white dwarf had not been understood. The high redshift in Sirius B served as an independent clue to the small size and high density of white dwarfs. More recent measurement of the Sirius B gravitational redshift gives the value $\Delta\lambda/\lambda = \Delta U/c^2 \equiv v/c = 89 \pm 16$ (km/sec)/c. Together with stellar model, this is a confirmation of Einstein's prediction to 20% accuracy.⁵¹

Earlier observations of the gravitational redshift 0.6(km/sec)/c from the surface of the Sun is ambiguous and controversial. The presence of nongravitational effects on the solar surface such as Doppler shifts in the high-temperature gas, possible high electric fields due to gas ionization, vertical currents, etc, make the measurement difficult. Using direct electronic techniques instead of photographic methods, Brault and Dicke^{52,23} measured the displacement of the center of the D₁ line of sodium as a function of the radial distance across the solar disk. After corrections for the small line asymmetry, they found that the shift was constant and agreed with Einstein's prediction to within 5%.

In 1958, Mössbauer⁵³ discovered highly monochromatic nuclear transitions in solids. This discovery makes possible a laboratory determination of the gravitational redshift. Pound and Rebka⁵⁴ (1959) worked on the resonant absorption of the 14.4KeV γ -ray from 0.10 μ -sec Fe⁵⁷ for an experimental determination of the gravitational redshift. They found that this line is monochromatic to 1 part in 10^{12} . By placing the emitter and absorber of the gamma rays at the bottom and top of a tower at Harvard University separated by a height $h=22.5$ meters, Pound and Rebka⁵⁵ (1960) measured redshift in agreement with Einstein's prediction to 10% accuracy. Later, Pound and Snider⁵⁶ (1965) improved the agreement to 1 % accuracy.

Gravitational redshift corrections are employed for clock synchronization stations with different heights throughout the world. The consistency of this procedure serves as a routine check on Einstein's prediction.

In 1976, Vessot et al^{57,58} used an atomic hydrogen maser clock in a space probe to test and confirm the metric gravitational redshift to an accuracy of 1.4×10^{-4} .⁵⁸ The space probe attained an altitude of 10,000km above the earth's surface. With (32), the constraint on (28) is

$$|H_{00} - g_{00}|/U \leq 1.4 \times 10^{-4} \quad (34)$$

Thus, we see that for the constraint on $|H_{00} - g_{00}|/U$, Hughes-Drever experiments, Eötvös-Dicke-Braginsky experiments⁵ and Vessot-Levine experiment compete among themselves in accuracies.

The empirical constraints from last three sections and this section can be summarized in the following table.

Table II. Empirical Foundations of the Einstein Equivalence Principle.

Experiments	Constraints	Accuracy
Pulsar Signal Propagation	$\chi^{ijkl} \rightarrow (-H)^{\frac{1}{2}} (\frac{1}{2} H^{ik} H^{jl} - \frac{1}{2} H^{il} H^{kj}) \psi + \phi e^{ijkl}$	$10^{-8} - 10^{-10}$
Hughes-Drever Experiments	$H_{\mu\nu} \rightarrow g_{\mu\nu}$	10^{-12}
	$H_{0\mu} \rightarrow g_{0\mu}$	$10^{-7} - 10^{-8}$
	$H_{00} \rightarrow g_{00}$	10^{-4}
Eötvös-Dicke-Braginsky Experiments	$\psi \rightarrow 1$	10^{-9}
	$H_{00} \rightarrow g_{00}$	10^{-5}
Vessot-Levine Redshift Experiment	$H_{00} \rightarrow g_{00}$	10^{-4}

* With the above constraints, $\chi^{ijkl} = (-g)^{\frac{1}{2}} (\frac{1}{2} g^{ik} g^{jl} - \frac{1}{2} g^{il} g^{kj}) + \phi e^{ijkl}$ to various degrees of accuracy, i.e. EEP is verified to various degrees of accuracy except for the freedom in ϕ .

If EEP is not valid, the relative rates of different types of clocks could vary with potential. In 1974, C.M. Will⁵⁹ analyzed this situation employing the THE μ framework. Recently Turneaure, Will, Farrell, Mattison and Vessot⁶⁰ performed such a comparison of clocks: "The experiment compared the rates of a pair of hydrogen maser clocks with those of a set of three superconducting cavity stabilized oscillator clocks as a function of the solar gravitational potential. During the experiment, the solar potential in the laboratory varied approximately linearly at 3 parts in 10^{12} per day because of the Earth's orbital motion, and diurnally with an amplitude of 3 parts in 10^{13} because of the Earth's rotation. An upper limit on the relative frequency variation of 1.7 parts in 10^2 of the external potential was set. The accuracy was limited by the frequency stability of the clocks and by unmodeled environmental effects. The result is consistent with the EEP at the two percent level."

IX. Variability of Fundamental Constants

A. $e^2/\hbar c$ and other elementary particle constants

EEP implies that the fine structure constant $\alpha = e^2/\hbar c$ and other dimensionless elementary particle constants are independent of time and location of measurement. Various laboratory, geological, and astrophysical observations give constraints on the temporal and spatial variations of these constants. At present, the best constraints on the temporal variations of the fundamental interaction constants come from Shlyakhter's⁶¹ analysis of the natural Oklo reactor over geological time scale:

- (i) fine structure constant $\alpha = e^2/\hbar c$: $|\dot{\alpha}/\alpha| < 10^{-17}/\text{yr}$
- (ii) weak interaction constant $\beta = g_f m^2 c/\hbar^3$: $|\dot{\beta}/\beta| < 10^{-12}/\text{yr}$
- (iii) strong interaction constant g_s^2 : $|\dot{g}_s/g_s| < 10^{-18}/\text{yr}$.

The recent clock comparison experiment of Turneaure et al.⁶⁰ sets a limit on the spatial variation of the logarithm of the fine-structure constant of 0.007 of the variation of the local gravitational potential.

B. G

SEP implies that the Newtonian gravitational constant G is independent of time and location of measurement. But Dirac's Large Number Hypothesis and certain views of Mach's Principle suggest that G should vary over cosmological time. These suggests $\dot{G}/G \sim 10^{-10}$ to $10^{-11}/\text{yr}$. From radio metric observations of the planets, Anderson et al.⁶² give a positive upper bound of $1.4 \times 10^{-10} \text{ yr}^{-1}$ for $|\dot{G}/G|$, while Reasonberg and Shapiro⁶³ give the upper bound $|\dot{G}/G| < 2.4 \times 10^{-10} \text{ yr}^{-1}$.

Ranging data from the Viking lander on Mars have been accumulating since 1976. It now appears possible to determine \dot{G}/G to 10^{-11} yr^{-1} accuracy. This is a very interesting moment. Theories of gravity⁶⁴⁻⁶⁶ with varying G together with a phenomenological framework⁶⁷ have been proposed.

X. Microscopic Experiments

EEP implies that elementary particles fall with the same acceleration as test bodies and photons. Moreover EEP implies that in the free-fall frame, elementary particles interact in the same way as those in Minkowski frame in special relativity. In the following, we describe the results of two experiments to test them.

A. Neutron free-fall experiments

In 1951, McReynolds⁶⁸ measured the free-fall acceleration of neutrons to be $935 \pm 70 \text{ cm/sec}^2$. In 1965, Dabbs, Harvey, Paya and Horstmann⁶⁹ improved the accuracy by one order of magnitude and obtained the values $975.4 \pm 3 \text{ cm/sec}^2$ and $973.1 \pm 7 \text{ cm/sec}^2$ for neutrons in agreement with the local value $g = 979.74 \text{ cm/sec}^2$ to 0.5%. In 1976, Koester⁷⁰ compared neutron scattering lengths measured dependent on and independent of gravity and obtained the value $\gamma = 1.00016 \pm 0.00025$ for the ratio of gravitational to inertial mass for the neutron. This verified the equivalence for the neutron with an accuracy (250 ppm) comparable to the Vessot-Levine photon redshift experiment.

B. Colella-Overhauser-Werner (COW) experiment

In 1975, Colella, Overhauser and Werner^{71,72} used a neutron interferometer to observe the quantum-mechanical phase shift of neutrons caused by their interaction with Earth's gravitational field. This experiment probe the simultaneous effects of gravity and quantum mechanics on the motion of the neutron. The outcome depends on both Planck's constant and the gravitational constant. In 1980, Staudenmann, Werner, Colella and Overhauser⁷³ refined the experiments significantly. Their results show 0.2% agreement with the predictions of EEP. COW experiment is of foremost importance in verifying the validity of equivalence principles in quantum mechanics.

Besides the above laboratory experiments, there are tests from astrophysical observations, please see Beall⁷⁴ for details.

XI. Outlook

Physics extends its frontiers to progress. There are two ways to extend the frontiers: One is to look for extreme or diverse situations; the other is to look for minute effects. Therefore, astrophysical observations to look for extreme and diverse conditions and precision experiments to look for minute effects become more and more important for the progress of physics. In particle physics, people look for cosmological observations and inferences about early universe. Experimenters put great efforts in doing precision experiments such as neutron electric dipole moment experiment, (g_e-2) experiment, double beta decay experiment and proton decay experiment.

Astrophysical observations and precision experiments are even more crucial for the progress of gravitational physics. As we can see from Figure 1, more extreme conditions and/or better precisions are needed to look for the microscopic origins of gravity. We have a long way to go. But we have to start somewhere. To look for possible violations of EEP at some level or to look for an alternate equivalence principle, ever precise experiments are needed. From the empirical foundations of EEP in Table II, polarized test body experiment seems to be a good one to begin with. With the advance of space technology, precision space experiments provide important opportunities too.

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