IMPLICATIONS OF HUGHES-DREVER EXPERIMENTS

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in

Proceedings of the 1983 International School and Symposium on Precision Measurement and Gravity Experiment, Taipei, Republic of China, January 24 - February 2, 1983, ed. by W.-T. Ni (Published by National Tsing Hua University, Hsinchu, Taiwan, Republic of China, June, 1983)

OUTLINE

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I. Mach's Principle and Mass Anisotropy

Mach[1] holds the following point view about motions: "For me only relative motions exist.... When a body rotates relatively to the fixed stars, centrifugal forces are produced; when it rotates relatively to some different body not relative to the fixed stars, no centrifugal forces are produced. I have no objection to calling the first rotation as long as it be remembered that nothing is meant except relative rotation with respect to fixed stars." This has become known as Mach's principle. Mach's principle has been the source of imaginations of many people. It has influenced Einstein[2] in his formulation of gravitation theories. It may mean different things for different people. Mach's principle suggests that an inertial frame of reference is determined by the mass distribution in the universe, that the inertial force on a body is the gravitational interaction of distant matter on the body, and that the inertial mass of a body is determined by all the matter in the universe. In the framework of this view, one can ask whether an anisotropic distribution of matter in the universe has the consequence that inertial mass itself has a directional dependence, that is, is anisotropic[3].

If anisotropic distribution of matter in our universe can induce mass anisotropy, then since we are at the edge of our Galaxy, and our Galaxy is at the edge of Virgo Supercluster, we would see mass anisotropy at certain level.

II. Hughes-Drever Experiments

Cocconi and Salpeter[4] propose that the contribution δm to the inertial mass m by the mass δM is given by

$$\delta m \propto \frac{\delta M}{r^{\nu}} , \tag{1}$$

in which

$$0 \leq \nu \leq 1 . \tag{2}$$

As to the directional dependence, they argued that if the angular dependence is expressed as a series of Legendre polynomials, the simplest allowed anisotropic term then is $P_2(\cos\theta) = (3\cos^2\theta - 1)/2$, so that

$$\delta m \propto \frac{\delta M}{r^{\nu}} P_2(\cos\theta). \tag{3}$$

Newton's second law and the kinetic energy can then be generalized to

$$F_\alpha = \sum_{\beta=1}^{3} m_{\alpha\beta}\, a_\beta \;, \tag{4}$$

and

$$T = \sum_{\alpha,\beta=1}^{3} P_\alpha\, P_\beta \,/2m_{\alpha\beta} \;, \tag{5}$$

where $m_{\alpha\beta}$ is the mass tensor, and P_α is the momentum.

For $\nu=1$, Cocconi and Salpeter calculated that

$$\Delta m/m \sim 2 \times 10^{-5} \tag{6}$$

due to the anisotropic distribution of matter of our Galaxy, where Δm is the anisotropic part and m is the isotropic part of the mass tensor. If we use the formula

$$\frac{\delta m}{m} \sim \frac{G\delta M}{rc^2} \;, \tag{7}$$

then

$$\frac{\Delta m}{m} \sim 10^{-6} \tag{8}$$

due to the anisotropic distribution of matter of our Galaxy. (6) and (8) agree in rough orders of magnitudes.

For any bound particle, there is an additional anisotropic-mass contribution ΔE to the binding energy

$$\Delta E = (\Delta m/m)\overline{T}\, \overline{P}_2(\cos\theta)\; P_2(\cos\beta). \tag{9}$$

Here $\overline{T}$ is the average kinetic energy of the particle. θ is the angle between the direction of acceleration of the particle (For an electron in an atom, this is determined by the direction of an external magnetic field and by the magnetic quantum state.) and the direction to the galactic center. β is the angle between the magnetic field and the direction to the galactic center. For a $P_{3/2}$ electron state in an atom, the perturbed Zeeman levels are shown in Figure 1. For other atomic systems, there are corresponding shifts and splittings. There are many observations on the Zeeman levels. These give various constraints on $\Delta m/m$. Radford and Hughes[5] observed the Zeeman transitions $\Delta M_J = \pm 1, \pm 2$ in the $^3P_2$ state of atomic oxygen and gave a limit of

$$\Delta m/m < 10^{-10} \;. \tag{10}$$

In 1960, Cocconi and Salpeter[6] pointed out that because the kinetic energy for a nucleon in a nucleus is much larger than the kinetic energy

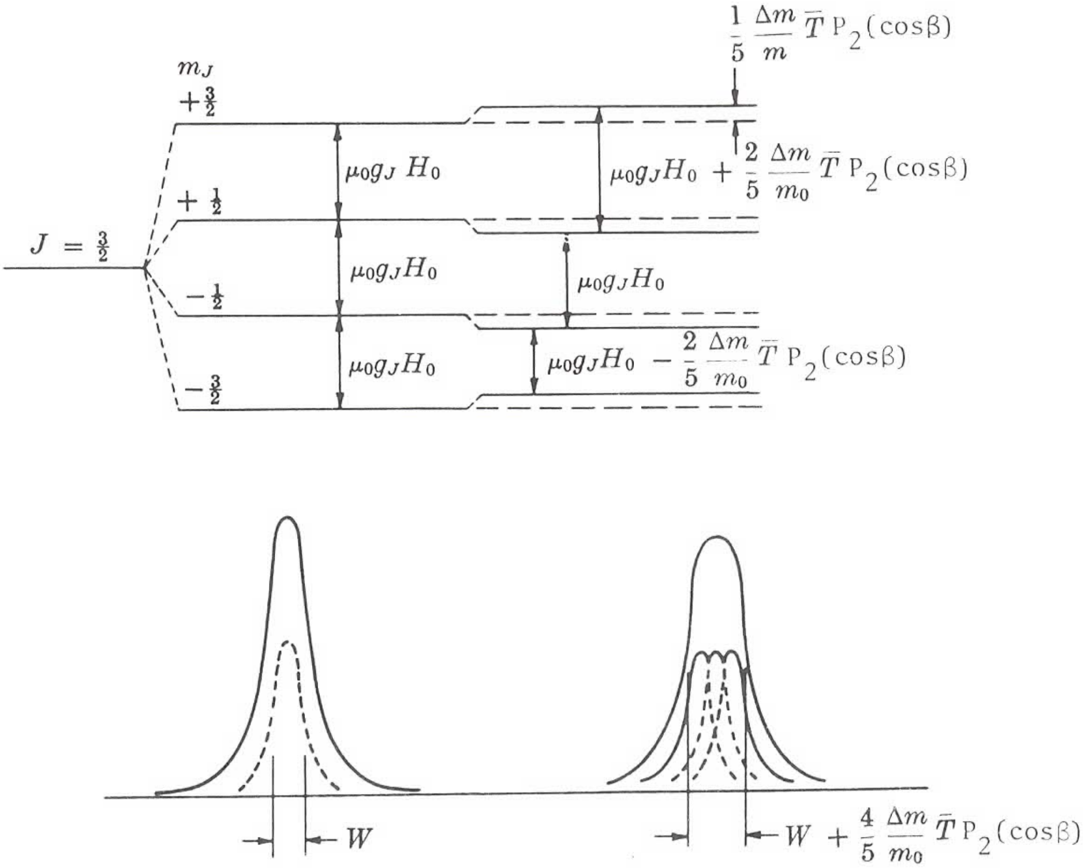

Figure 1. Zeeman energy levels and resonance lines for a $P_{3/2}$ electron as perturbed by mass anisotropy. The width of a normal Zeeman line is noted as W, and the width of the unresolved lines associated with mass anisotropy is $W + (4/5)(\Delta m/m_0)\bar{T}\ P_2(\cos\beta)$. This figure is also applicable to the nuclear energy levels and resonance lines for a nucleus with spin I = 3/2. (After 3)

for an electron in an atom, higher sensitivity in the search for mass anisotropy could be achieved by studying nuclear energy levels as compared with atomic energy levels. Sherwin et al.[7] looked for the mass anisotropic effects in the very sharp 14.4 kev Fe^{57} Mössbaner transition from an upper nuclear state with I=3/2 to the ground state with I=1/2. They concluded that

$$\Delta m/m < 5 \times 10^{-16} \quad . \tag{11}$$

In a magnetic field, the Li^7 nucleus will have four energy levels corresponding to the allowed values of the magnetic quantum number M_I. In the absence of any mass anisotropy, adjacent levels are equally spaced, and a single nuclear resonance line will be observed. If the mass anisotropy effect is present, there will be three different intervals that will lead to a triplet nuclear resonance line, if the structure is resolved, or to a single broadened line if the structure is unresolved (see Figure 1).[3] Over a 12-hour period, the width of the resonance line for Li^7 was observed by Hughes, Robinson and Beltran-Lopez[8], and by Drever[9]. To very high precision, the width does not change. From these Hughes, Robinson and

Beltran-Lopez[8] obtained the limit

$$\frac{\Delta m}{m} < 10^{-22} , \tag{12}$$

and Drever[9] obtained the limit

$$\frac{\Delta m}{m} < 5 \times 10^{-23} . \tag{13}$$

These high precision experiments are call the Hughes-Drever experiments. Comparing with (6), they rule out the model of Salpeter and Coconni entirely. (If $\nu=0$ in (1), Cocconi and Salpeter predicted $\Delta m/m \sim 3\times10^{-10}$ from the anisotropic distribution of mass in our Galaxy. This is ruled out entirely too.)

In the following, we will analyze implications of the Hughes-Drever experiments in somewhat details.

III. A Framework for Analyzing Hughes-Drever Experiments

In Cocconi-Salpeter model of Mach's principle, mass in the kinetic energy term is replaced by a mass tensor $m_{\alpha\beta}$. This mass tensor can be considered as mass times a metric tensor, $mg_{\alpha\beta}$. But the mass distribution of our Universe and gravity influence electromagnetism and electromagnetic binding energy too. If they influence the electromagnetism in the same way, by the metric $g_{\alpha\beta}$, then the anisotropic terms due to the electromagnetic binding energy may cancel the anisotropic terms in the kinetic energy and there are no observable effects. In section IV, we will explicitly demonstrate this. If the gravity-coupling to all matter and nongravitational fields (electromagnetic, weak, strong) is universal as in Einstein Equivalence Principle (EEP), then there would be no observable effects. Therefore Hughes-Drever experiments lend supports to EEP.

To put the above arguments in definite terms and to look for detailed implications of the Hughes-Drever experiments, we need a suitable framework. The framework (H-g-ψ-ϕ framework) given by equation (9) with $\chi^{ijk\ell}$ defined by (28) in reference 10, i.e.,

$$\mathcal{L}_I = -\left(\frac{1}{16\pi}\right)\chi^{ijk\ell}F_{ij}F_{k\ell} - A_k j^k(-g)^{\frac{1}{2}} - \sum_I m_I \frac{ds_I}{dt}\,\delta(\vec{x}-\vec{x}_I), \tag{14}$$

with

$$\chi^{ijk\ell} = (-H)^{\frac{1}{2}}\left(\frac{1}{2}H^{ik}H^{j\ell} - \frac{1}{2}H^{i\ell}H^{kj}\right)\psi + \phi e^{ijk\ell}, \tag{15}$$

would be a good one to start with. Here $\mathcal{L}_I$ is the Lagrangian density for an electromagnetic system in gravitational field. In according to the arguments of the preceding paragraph, we have the tensor g_{ij} (in s_I) coupled to the relalivistic mass terms and the tensor H_{ij} coupled to electromagnetism. If we want to start with a more general framework, we could use equation (14) with a general $\chi^{ijk\ell}$. But the analysis of pulsar signal propagations in reference 10 shows that to a high precision (15) is valid. Therefore, to simplify the analysis, we may almost as well use (14) with (15), i.e., H-g-ψ-ϕ framework. Two scalars (or pseudo-scalars) ψ and ϕ will not give any anisotropic effects and have little to do with our analysis of the Hughes-Drever experiments.

IV. Implications of Hughes-Drever Experiments

Even in the H-g-ψ-ϕ framework, the analysis of Hughes-Drever experiments is somewhat complicated. Therefore in this section, we demonstrate the analysis with a simplified model problem, and give qualitative arguments to derive the results of full analysis.

Consider the nonrelativistic Coulomb problem with the Hamiltonian

$$H_0 = \sum_{\alpha=1}^{3} \frac{P_\alpha P_\alpha}{2m} - \frac{Ze^2}{\left(\sum_{\alpha=1}^{3} Q^\alpha Q^\alpha\right)^{\frac{1}{2}}} . \tag{16}$$

To quantize it, canonical commutation relation

$$[Q^\alpha, P_\beta] = i\hbar\delta^\alpha{}_\beta \tag{17}$$

is imposed. In Schrödinger representation

$$P_\beta = \frac{\hbar}{i} \frac{\partial}{\partial Q^\beta} . \tag{18}$$

Suppose in a gravitational field, the Hamiltonian H becomes

$$H = \sum_{\alpha,\beta=1}^{3} \frac{g^{\alpha\beta} P_\alpha P_\beta}{2m} - \frac{Ze^2}{\left(\sum_{\alpha,\beta=1}^{3} h_{\alpha\beta} Q^\alpha Q^\beta\right)^{\frac{1}{2}}} . \tag{19}$$

Here $g^{\alpha\beta}$ is the metric coupled to the particle, and is essential equivalent to mass tensor in section II. $h_{\alpha\beta}$ is the metric coupled to the Coulomb field. To quantize it, we impose the canonical commutation relation (17) again.

Now we prove an extension of virial theorem for a quantum system with the Hamiltonian

$$H = \sum_{\alpha=1}^{3} \frac{P_\alpha P_\alpha}{2m} + V(Q^1, Q^2, Q^3) . \tag{20}$$

For a stationary state (energy eigenstate) in this system, the time derivative of the expectation value of $P_\alpha Q^\beta$ is zero:

$$\frac{d}{dt} \langle P_\alpha Q^\beta \rangle = 0 , \tag{21}$$

From the quantum dynamics, we have, therefore,

$$<[P_\alpha Q^\beta, H]> = i\hbar \frac{d}{dt} <P_\alpha Q^\beta> = 0 . \tag{22}$$

Since

$$[P_\alpha Q^\beta, H] = \frac{i\hbar}{m} P_\alpha P_\beta - i\hbar \frac{\partial V}{\partial Q^\alpha} Q^\beta , \tag{23}$$

we have

$$\frac{1}{m} <P_\alpha P_\beta> = < \frac{\partial V}{\partial Q^\alpha} Q^\beta> \tag{24}$$

for an energy eigenstate. This is an extension of the virial theorem. For Hydrogen-like atoms (16), (24) becomes

$$\frac{1}{m} <P_\alpha P_\beta> = < \frac{Ze^2 Q^\alpha Q^\beta}{r^3} > , \tag{25}$$

where

$$r = (\sum_{\alpha=1}^{3} Q^\alpha Q^\alpha)^{\frac{1}{2}} . \tag{26}$$

In a weak field, $g^{\alpha\beta}$ and $h_{\alpha\beta}$ are close to $\delta^{\alpha\beta}$. We can express them as follows:

$$g^{\alpha\beta} = \delta^{\alpha\beta} + \bar{g}^{\alpha\beta} , \tag{27}$$

$$h^{\alpha\beta} = \delta^{\alpha\beta} + \bar{h}^{\alpha\beta} , \tag{28}$$

where $h^{\alpha\beta}$ is the inverse of $h_{\alpha\beta}$ so that

$$h_{\alpha\beta} = \delta^{\alpha\beta} - \bar{h}^{\alpha\beta} + O(\bar{h}^2). \tag{29}$$

Therefore in a weak field, the Hamiltonian (19) can be expressed as

$$H = H_0 + \Delta H + O(\bar{h}^2) , \tag{30}$$

where

$$\Delta H = \sum_{\alpha,\beta=1}^{3} \frac{\bar{g}^{\alpha\beta} P_\alpha P_\beta}{2m} - \sum_{\alpha,\beta=1}^{3} \frac{Ze^2 \bar{h}^{\alpha\beta} Q^\alpha Q^\beta}{2r^3} . \tag{31}$$

The first term in ΔH is the anisotropic term in the kinetic energy and is what Cocconi and Salpeter have considered. The second term is the anisotropic term in the electromagnetic binding energy. In this system, the extended virial equation is approximately valid, i.e.,

$$\frac{1}{m}\langle P_\alpha P_\beta\rangle = \langle\frac{Ze^2Q^\alpha Q^\beta}{r^3}\rangle + O(\bar{h},\bar{g}) \ . \qquad (32)$$

Hence the change in energy ΔE for an energy eigenstate is

$$\Delta E = \sum_{\alpha,\beta=1}^{3}(\bar{g}^{\alpha\beta}-\bar{h}^{\alpha\beta})\frac{P_\alpha P_\beta}{2m} + O(\bar{h}^2,\bar{g}^2). \qquad (33)$$

Thus, if $g^{\alpha\beta} = h^{\alpha\beta}$, i.e. gravity coupled to particles and Coulomb interaction in the same way, then the anisotropic terms cancel each other in first-order approximation. Since Hughes-Drever-type experiments verify to a high precision that there are no anisotropic effects, $g^{\alpha\beta}$ equals $h_{\alpha\beta}$ to a high precision and we have universal coupling and equivalence.

If $g^{\alpha\beta} = h^{\alpha\beta}$ (or In metric theories of gravity), we transform to a local Cartesian frame (to a local Minkowskian frame in 4-dimensional spacetime), then $H=H_0$ up to curvature corrections. These curvature corrections are usually extremely small.[11] This is an easy way to see that in metric theories of gravity, there are no anisotropic effects.

In 1977, Dr. Yilmaz[12] did a calculation and claimed that general relativity violates the Hughes-Drever experiments. He used a non-inertial frame to calculate the anisotropic kinetic-energy effects. But, I believe, he missed the anisotropic potential-energy effects. In the above, I demonstrated this cancellation in a simple model. But I guess, a full calculation about Li^7 nucleus in a non-inertial frame has to be done in order to convince Dr. Yilmaz that general relativity and all other metric theories of gravity are in agreement with Hughes-Drever experiments to the present precision. Dr. Yilmaz will express his viewpoint after my talk.[13]

In (14), ds is the line element determined from the metric g_{ij}. From (15), the gravitational coupling to electromagnetism is determined by the metric H_{ij} and two scalar fields ϕ and ψ. If H_{ij} is not proportional to g_{ij}, then the hyperfine levels of the lithium atom will have additional shifts. But this is not observed to high accuracy in Hughes-Drever experiments [8,9]. Therefore H_{ij} is proportional to g_{ij} to certain accuracy. Since a change of H^{ik} to λH^{ik} does not affect $\chi^{ijk\ell}$ in (15), we can define $H_{11} = g_{11}$ to remove this scale freedom.

In Hughes-Drever experiments $\Delta m/m \le 0.5\times10^{-22}$ or $\Delta m/m_{e.m.} \lesssim 0.3\times10^{-18}$ where $m_{e.m.}$ is the electromagnetic binding energy. Using (15) in (14), we have three kinds of contributions to $\Delta m/m_{e.m.}$. These three kinds are of the order of (i) $(H_{\mu\nu}-g_{\mu\nu})$, (ii) $(H_{o\mu}-g_{o\mu})v$, and (iii) $(H_{oo}-g_{oo})v^2$ respectively. Here the Greek indices μ, ν denote space indices. Considering the motion of laboratories in our Galaxy, in the solar system and from earth rotation, we can set limits on various components of $(H_{ij}-g_{ij})$ from Hughes-Drever experiments as follows:

$$|H_{\mu\nu} - g_{\mu\nu}|/U \leq 10^{-12}$$

$$|H_{o\mu} - g_{o\mu}|/U \leq 10^{-7} - 10^{-8} \qquad (34)$$

$$|H_{oo} - g_{oo}|/U \leq 10^{-4}.$$

where U ($\sim 10^{-6}$) is the galactical gravitational potential. If we use the gravitational potential of Virgo Supercluster (we are at an edge of Virgo Supercluster), then the above limits are improved by a couple of orders.

V. Acceleration Effects

In the actual Hughes-Drever experiments, apparatuses are held stationary in the laboratory, not in a free-fall frame. Therefore, even in metric theories, there are acceleration effects. In the following, we shall obtain the order of magnitude of these acceleration effects.

The simplest way to obtain the order of magnitude of these metric acceleration effects is to add a term ΔH to the Hamiltonian H:

$$\overline{\Delta}H = mgX \ , \qquad (35)$$

where X is the coordinate operator corresponding to the height of the electron (or nucleon).

For atoms $\Delta\ell \sim 10^{-8}$cm and

$$\overline{\Delta}H/mc^2 \sim g\Delta\ell/c^2 \sim 10^{-26} \ . \qquad (36)$$

This needs a precision of 10^{-17} to 10^{-20} in infrared and visible laser spectroscopy to see it.[14] This kind or accuracy might be achievable in the future. For microwave and radiowave spectroscopy, the precision could be several orders of magnitude lower, but the achievable precisions at present are many-orders of maguitude lower than those in laser spectroscopy.

For nucleus $\Delta\ell \sim 10^{-13}$cm and

$$\overline{\Delta}H/mc^2 \sim g\Delta\ell/c^2 \sim 10^{-31} \qquad (37)$$

It is several orders of magnitude away from the precision of Hughes-Drever experiments.

VI. Concluding Remarks

Hughes-Drever experiments verify the uniqueness of metric to a high precision. This implies the universality of gravity coupling and equivalence. For such fundamental experiments, it is important to improve their precisions. It will be also interesting when the acceleration effects could be detected.

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