SPIN, TORSION AND POLARIZED TEST-BODY EXPERIMENTS

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I. Spin and Torsion

In 1921, Eddington[1] mentioned the notion of an asymmetric affine connection in discussing possible extensions of general relativity. In 1922, Cartan[2] introduced torsion as the anti-symmetric part of an asymmetric affine connection and laid the foundation of this generalized geometry. Cartan[3] proposed that the torsion of spacetime might be connected with the intrinsic angular momentum of matter. In 1921-22, Stern and Gerlach[4] discovered the space quantization of atomic magnetic moments. In 1925-26, Goudsmit and Uhlenbeck[5] introduced our present concept of electron spin as the culmination of a series of studies of doublet and triplet structures in spectra. Following the idea of Cartan, Sciama[6,7] and Kibble[8] developed a theory of gravitation which is commonly called the Einstein-Cartan-Sciama-Kibble (ECSK) theory of gravity. In this and the subsequent development, spin and torsion are closely connected.

Let us consider the Dirac equation of a spin 1/2 particle in special relativity:

$$i\hbar\gamma^i \frac{\partial}{\partial x^i} \psi(x) - mc\psi(x) = 0 \ . \tag{1}$$

To write the Dirac equation (1) in a general coordinate system, the usual comma-goes-to-semicolon rule is not enough since in a general coordinate system, the Dirac γ-matrices are not yet defined. The natural way to define γ-matrices in a general coordinate system is to use tetrad.

At each point, a basis of four orthonormal vectors, i.e., an orthonormal tetrad (vierbein) can be introduced as follows:

$$\underline{e}_a(x) = e^i{}_a \partial_i \tag{2}$$

with

$$\underline{e}_a \cdot \underline{e}_b = \eta_{ab} \equiv \mathrm{diag.}(+---). \tag{3}$$

Here the ∂_i are the tangent vectors of the coordinate lines. They represent the coordinate basis; the $e^i{}_a$ are the tetrad components with respect to this coordinate basis and η_{ab} is the Minkowski metric. The anholonomic (tetrad or Lorentz) indices a,b...=0,1,2,3 number the tetrads. Furthermore, we define the reciprocal reference frame (vierbein)

$$\underline{e}^a = e_i{}^a dx^i , \tag{4}$$

with

$$e_i{}^a e^i{}_b = \delta^a{}_b . \tag{5}$$

The metric g_{ij} and the tetrad connection $\Gamma^a{}_{bc}$ are

$$g_{ij} = e_i{}^a e_j{}^b \eta_{ab} , \tag{6}$$

and

$$\Gamma^a{}_{bc} = e^a{}_i e^j{}_b e^k{}_c \Gamma^i{}_{jk} + e^a{}_i e^i{}_{b,c} . \tag{7}$$

Thus we see that in a tetrad frame, because of the anholonomic term $e^a{}_i e^i{}_{b,c}$, the resulting tetrad connection is in general, not symmetric in b and c; there is an antisymmetric part $\Gamma^a{}_{[bc]}$.

Now the Dirac equation (1) in general coordinates can easily be written down as

$$i\hbar \gamma^a D_a \psi(x) - mc\psi(x) = 0 , \tag{8}$$

where D_a is the covariant differentiation in the tetrad frame. γ^a (a=0, 1,2,3) are the usual Dirac matrices.

After the works of Utiyama[9], Sciama[6,7] and Kibble[8], interest and activities in gauge-type and torsion-type theories of gravity have continuously increased. Various different theories postulate somewhat different interaction of matter with gravitational field(s). In ECSK theory and in some other torsion theories, there is a torison gravitational field besides the usual metric field.[10] As we saw above, in special relativity, if we use a nonholonomic tetrad frame, there is an antisymmetric part of the affine connection. Therefore many people working on torsion theory take the equivalence principle to mean something different from EEP so that torsion can be included. This is mostly clearly stated in P. von der Heyde's article "The Equivalence Principle in the U_4 Theory of Gravitation"[11]: Locally the properties of special relativistic matter in a noninertial frame of reference cannot be distinguished from the properties of the same matter in a corresponding gravitational field. This modified equivalence principle (MEEP) allows for formal inertial effects in a nonholonomic tetrad frame and hence allows torsion.

To be specific, in general relativity, we have the gravitational field g_{ij}. If we use tetrad frame, both $e_i{}^a$ and $\Gamma^{ab}{}_c$ ($\equiv \eta^{bd} \Gamma^a{}_{dc}$) are natural candidates for gravitational fields. Therefore in a torsion theory, we have both $e_i{}^a$ and $\Gamma^{ab}{}_c$ as gravitational fields. If the metric g_{ij} ($=e_i{}^a e_j{}^b \eta_{ab}$) and the connection $\Gamma^{ab}{}_c$ are compatible, i.e.,

$$D_a g_{ij} = 0 , \tag{9}$$

then $e_i{}^a$ and $\Gamma^{ab}{}_c$ are equivalent to g_{ij} and $\Omega^i{}_{jk}$ where $\Omega^i{}_{jk}$ is the torsion defined to be the antisymmetric part of $\Gamma^i{}_{jk}$, i.e.

$$\Omega^i{}_{jk} = \frac{1}{2}(\Gamma^i{}_{jk} - \Gamma^i{}_{kj}). \tag{10}$$

Since the properties (wavelengths, frequencies, etc.) of matter we observe do not depend on the past history in gravitational field, the metric compatibility condition (9) seems to be justified. If (9) is valid, the underlying geometry is called Riemann-Cartan spacetime U_4. In gauge formulations of gravity, the connection Γ is usually considered as guage potential. It would be interesting if we can derive the metric from the connection and dynamical equations. Therefore we looked for necessary and sufficient conditions for the existence of a compatible metric given a connection Γ.[12-14] Although we have found local necessary and sufficient conditions, we have not been successful in giving these conditions a dynamical meaning.

II. Empirical Foundations of Equivalence Principles

In this section we will discuss those parts of empirical foundations of equivalence principles which are relevant to the present context.

The gravity experiments relevant to spin 1/2 particles include neutron free-fall experiments and Colella-Overhauser-Werner (COW) experiment. For a brief review of these experiments, please see reference 15. If around the earth, there is a significant torsion field and if spin coupled to torsion, then particles with different polarization would behave differently. In the neutron free-fall experiment of Dabbs, Harvey, Paya, Horstmann[16], no splitting greater then a few percent of g is found for two vertical neutron-spin projections $\pm\frac{1}{2}$. To the accuracy involved, this is a confirmation of EEP for polarized neutrons. Because of limited precisions, there are still large untested gaps between various torsion theories and experiments.

As to the torsion coupling to the electromagnetism, we first look at the empirical constraints to the equivalence principles on electromagnetic systems. From Table II in reference 15, we see that in the general χ-g framework, there is only one degree of freedom (out of 21) of χ that is largely unconstrained, i.e.,

$$\chi^{ijk\ell} = (-g)^{\frac{1}{2}}(\frac{1}{2}g^{ik}g^{j\ell} - \frac{1}{2}g^{i\ell}g^{kj}) + \phi e^{ijk\ell}. \tag{11}$$

This is the nonmetric theory we will study in details in the next section. We will show that it is equivalent to a torsion theory.

III. A Nonmetric Theory of Gravity - A Torsion Theory

In this section, we study the nonmetric theory (11) in some details.[17,18]

From (9) in reference 15 and (11) above, the interaction Lagrangian is

$$\mathcal{L}_I = \mathcal{L}_I^{(M)} + \mathcal{L}_I^{(NM)} \tag{12}$$

where $\mathcal{L}_I^{(M)}$ is the usual metric Lagrangian and

$$\mathcal{L}_I^{(NM)} = (-\frac{1}{16\pi})\phi F_{ij}F_{k\ell}\, e^{ijk\ell} = (-\frac{1}{4\pi})\ \phi_{,i}\ A_j\ A_{k,\ell}\ e^{ijk\ell}\ \text{(mod. div.)}. \tag{13}$$

The Maxwell equations are

$$F^{ik}{}_{;k} + \varepsilon^{ikm\ell}\ F_{km}\ \phi_{,\ell} = -4\pi j^i . \tag{14}$$

The Lorentz force law is the same as in metric theories of gravity. Gauge invariance and charge conservation are guaranteed. The Maxwell equations (14) are also conformally invariant.

This theory can be put into the form of a torsion theory. Define a metric compatible affine connection as

$$\Gamma^i{}_{jk} = \{^{\,i}_{jk}\} + \Omega^i{}_{jk} \tag{15}$$

where $\{^{\,i}_{jk}\}$ is the Christoffel symbol obtained from g_{ij} and

$$\Omega^i{}_{jk} = 2\phi_{,\ell}\,\varepsilon^{\ell i}{}_{jk}\ , \tag{16}$$

is the torsion field. The Maxwell equations (14) can be written as

$$\overline{F}^{ik}{}_{|k} = -4\pi j_i \tag{17}$$

where $|$ denotes covariant differentiation with respect to $\Gamma^i{}_{jk}$ (e.g., $V^i{}_{|j} = V^i{}_{,j} + \Gamma^i{}_{jm}V^m$ etc.) and

$$\overline{F}_{ik} \equiv A_{k|i} - A_{i|k} = A_{k,i} - A_{i,k} + 2\Omega^\ell{}_{ik}\ A_\ell\ . \tag{18}$$

The nonmetric part of the Lagrangian can be written in the form

$$\mathcal{L}_I^{(NM)} = 2A_j\ A_{k,\ell}\ \Omega^{jk\ell}\sqrt{-g}\ \ . \tag{19}$$

To complete this theory as a gravitational theory, we have to add a gravitational Lagrangian to it. For example, the gravitational

Lagrangian $\mathcal{L}_G$ could be

$$\mathcal{L}_G = \frac{\sqrt{-g}}{16\pi} R(\Gamma^i{}_{jk}), \tag{20}$$

$$\mathcal{L}_G = \frac{\sqrt{-g}}{16} [R(\Gamma^i{}_{jk}) + \eta\phi_{,i}\ \phi^{,i}], \tag{21}$$

or

$$\mathcal{L}_G = \frac{\sqrt{-g}}{16\pi} [\phi R(\{^i{}_{jk}\}) - \frac{1}{\phi}\ \omega(\phi)\ \phi_{,i}\ \phi^{,i}], \tag{22}$$

where η is a parameter and $\omega(\phi)$ is a function of ϕ.

Defining the electromagnetic stress-energy as

$$T_{ij}^{(em)} = \frac{1}{4\pi} (-F_{i\ell}F_j{}^{\ell} + \frac{1}{4}F_{\ell m}F^{\ell m}g_{ij}), \tag{23}$$

we have the following matter response equation:

$$T_i{}^k{}_{;k} = -\frac{1}{4\pi}\ \varepsilon^{jkm\ell}\ F_{ij}\ F_{km}\ \phi_{,\ell}\ . \tag{24}$$

From Theorem I in reference 15, we know that test bodies follow geodesics in this theory. Choose geodesic frames such that $g_{ij,k} = 0$ along the geodesic of the test body considered. Define the angular momentum tensor as

$$J_{ik} = \int (x_i\ T_{ko} - x_k\ T_{io})dV\ . \tag{25}$$

The rate of change of the three angular momentum for the test body is

$$\frac{dJ_{\mu\nu}}{dt} = I_\mu\phi_{,\nu} - I_\nu\phi_{,\mu} \quad \text{or}\ \underset{\sim}{J} = \underset{\sim}{I} \times \underset{\sim}{\nabla}\phi \tag{26}$$

where

$$I_\mu = \frac{1}{16\pi} \int \varepsilon^{njk\ell}\ F_{nj}\ F_{k\ell}\ x_\mu\ dV \tag{27}$$

is the gravitational polarization vector. Equation (26)is similar to the one for a magnet in a magnetic field. Thus, the motion would be oscillation about the $\underset{\sim}{\nabla}\phi$ axis. The order of magnitude of the torque would be $E_{e.m.}$ x fraction of polarization x d x $\underset{\sim}{\nabla}\phi$ where d is the dimension of the polarization region. To constrain the remaining ϕ freedom of χ and to test this nonmetric theory, experiments on polarized test bodies with I_μ large are suggested. (See next section)

In this theory, the rate of change of angular momentum is depending on the degree of polarization and the strength of torsion field. Since the motion of a macroscopic test body is determined not only by its trajectory but also by its rotation state, we have proposed[19,20] the following stronger weak equivalence statement (WEP[II]) to be tested by experiments, which states that in a gravitational field, the motion of a test body with a given initial motion state is independent of its internal structure and composition (universality of free fall motions).

From Table II of reference 15, the nonmetric coupling (13) of gravity to electromagnetism is the only one that is largely allowed in the χ-g framework by experiments. In 1978, Hojman, Rosenbaum, Ryan and Sheply[21] proposed a propagating torsion theory. The coupling bewteen torsion and electromagnetic field is contained in the χ-g framework and different from that given by (13). Soon this theory was found to violate the Eötvös-Dicke-Braginsky experiments.[22] In 1980, Sabbata and Gasperini[23] found an indirect coupling between torsion and electromagnetism. This coupling is the same as that given by (13), and is, therefore empirically viable.

Hehl and some people hold the point of view that torsion does not couple to the electromagnetic field, only coupled to leptons and quarks. My point of view on this is: we should look for more empirical or phenomenological connections first.

IV. Polarized Test-Body Experiments

Within χ-g framework, the significance of polarized test-body experiments is as follows: The acceleration measurements on polarized test bodies would be tests of WEP[I]. The importance of these experiments as tests of WEP[I] has been emphasized by Morgan and Peres[24]. From Theorem I of reference 15, WEP[I] constrains 20 degree of freedom of χ. Eötvös-Dicke-Braginsky experiments constrain 2 degrees of freedom of χ. The acceleration measurements on polarized test bodies would constrain the other 18 degrees of freedom. This would constitute a check on the constraints obtained by other experiments in Table II of reference 15. According to Theorem II of reference 15, the acceleration and torque measurements of both polarized the unpolarized test bodies would be a complete set of tests for EEP.

Polarized test body experiments can also probe the role of spin in gravity. This has to be treated in an extended phenomenological framework. But experimentalists should not be bound by the restrictedness of theoretical analysis. As we have seen in Figure 1 of reference 15, there is a vast range of regions in gravity that is unprobed empirically.

Theoretically, it would be the best if we could do particle-particle gravity experiments. But this is not feasible at the present. Therefore we have to do either particle-in-an-external-gravitational-field experiments (like neutron free-fall experiments or COW experiment) or test-body experiments. There are two kinds of test-body experiments: (i) test-body-in-an-external-gravitational-field experiments; (ii) test-body-test-body gravity experiments.

To prepare a polarized test-body with net total spin but without net total magnetic moment, we use two different magnetic materials to make two halves of a ring and magnetize it. (Fig. 1) Most of the magnetic flux is contained in the ring. The leakage mostly comes from the contacting boundary and is quite small. Wrapping by μ-metals makes the leakage even smaller. To do the experiment, we could either use a torsion balance comparing polarized body with unpolarized best body, or float them in a

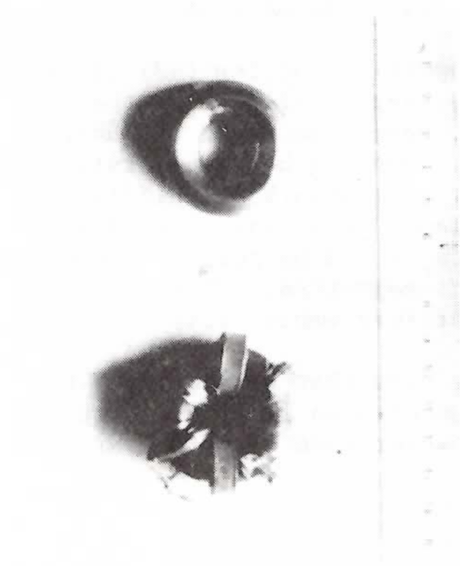

Figure 1. The two halves of the upper ring are made of nickel. The two halves of the lower ring are made of iron. To prepare a polarized body, we use one half upper ring and one half lower ring and magnetize them as shown in the lower part of the figure.

"fluid fiber" developed by Faller, Keiser and Keyser.[25] These experiments would improve the current limits on polarization effects by several orders of magnitude.

Ultimately, we have to use low temperature techniques.[26] We could either replace the quartz gyro in the stanford gyroscope experiment[27] by a polarized solid He^3 gyro, or wait until Dr. Ritter and his group at University of Virginia have developed their ultra stable low-temperature rotors and persuade them to use polarized rotors too.

To end this talk, I will just mention some possible significances of the polarized test-body experiments in the following and leave the others to the imaginations of the audience:

(i) testing the nonmetric theory in this talk,
(ii) testing long range spin interactions,
(iii) testing the spontaneous-symmetry breaking piece $\chi^{(0)ijk\ell}$ of $\chi^{ijk\ell}$
(iv) possible tests of cosmology.

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