A PROTOTYPE MEASUREMENT OF THE NEWTONIAN GRAVITATIONAL CONSTANT USING AN ACTIVE MAGNETIC SUSPENSION TORSION FIBER

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A PROTOTYPE MEASUREMENT OF THE NEWTONIAN GRAVITATIONAL CONSTANT
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Any measurement of G ultimately faces the task of measuring a very small force. The first slide (slide 1), a quote from the Philosophical Transactions of 1821, shows one possible answer to this problem.

The next slide (slide 2) shows the frontispiece of the very fine little book containing the 1775 address by Sir John Pringle, President of the Royal Society, which was given in honor of the Reverend Nevil Maskelyne's paper entitled, "Observations made on the Mountain Schehallien for finding its attraction." The Reverend Maskelyne came up with a value for G which was good to about 20%. There were several problems with mountain measurements in which G was estimated by measuring the pull of the mountain on a pendulum hung near it. One had to measure astronomically (by observing stars) the tiny deflection of the pendulum from the vertical caused by the sideways attraction of the mountain and also estimate geologically the mass of the mountain and its "average distance" from the pendulum.

The first mountain-based big G measurement which was performed in 1735, arrived at a number that was not in good agreement with the number subsequent investigators obtained. It was later learned, however, that the mountain was an extinct volcano and this almost certainly meant that their density estimate was in error. In view of this and other similar types of measurement difficulties, the next slide (slide 3) offers a hindsight comment on Dr. Hutton's earlier pronouncement.

Before leaving Sir John Pringle's address -- noting that this is a meeting on gravitation -- I would share with you one paragraph from his text (slide 4). I leave it to you to decide just how much real progress the theory of gravitation has made in the intervening years.

Some 25 years after Maskelyne, Cavendish (and subsequently many others) measured the gravitational force between a large metal ball and a small metal ball by directly observing the attraction. Cavendish placed a pair of small metal balls on a light trapeze-like torsion beam balance which was supported by a long, silvered copper wire. He then brought large lead balls near the small ones into positions such that their attractions on the small balls pulled the bar around, twisting it until the fiber's restoring-torque balanced the effects of the tiny gravitational ball-to-ball attractions.

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A schematic of the apparatus is seen in the next slide (slide 5). Note, in particular, the fine pre-laser optical device he used to sense the position of the beam. To calculate G, Cavendish needed to know the torsion constant of his fiber as well as the masses and the distances between the small and large balls. To obtain the fiber's torsion constant he measured the pendulum's period. From that together with a calculated moment of inertia he was able to find the torsion constant and finally G. His 1% value for G has proven remarkably accurate even by today's standards -- a tribute to both his method and his experimental ingenuity. (The most recent work of Luther and Towles, Phys. Rev. Lett., Vol. 18, Number 3, 18 Jan. 1982, quotes an error of 7 parts in 10^5 while the internationally accepted value of G has an assigned uncertainty of 6 parts in 10^4.)

In the next slide (slide 6), we show another approach -- if you don't have a mountain that you trust, make one. This was von Jolly's lead sphere (45,000 kilograms). He used a pan scale weighing apparatus, performed a normal weighing operation, and then rolled the lead sphere underneath one of the pan scales and noted the increased weight of the mass on that end. From this, together with appropriate distance measurements, he could calculate G.

The next slide (slide 7) shows the exhibit in the London Science Museum of C. V. Boys' big G experiment. Note the method of casting spherical masses indicated in the foreground. Boys both invented and used a fused silica fiber to support his test masses. (You might remember Boys and his soap bubbles from my second lecture given during last week's school.) Boys' determination of G, which was carried out nearly 100 years after that of Cavendish, is considered to be a classic example of optimizing the size of an instrument. A reduction in size of the torsion balance (from that used by Cavendish) made it possible for him to greatly reduce disturbances due to temperature inequalities, and also made it convenient to increase the size of the attracting masses relative to the rest of the apparatus. His six years of effort to measure G resulted in a quoted uncertainty of 3 parts in 10^4 (and his number agrees with the accepted value to 2 parts in 10^4). His experiment is published in the Philosophical Transactions of the Royal Society for the year 1895, Volume 186. The next slide (slide 8) gives you some of the flavor of his writing. I hope some of you will want to look up this article and read it.

Common to almost all big G determinations are the problems of knowing the locations of the masses and the parameters of the torsion fiber. We started, perhaps naively, by saying, "Let's see if we can come up with a different approach that will permit us to do a little better." That was in 1970. This work was begun at Wesleyan University, but in 1972 the apparatus was moved to the Joint Institute for Laboratory Astrophysics (JILA) in Boulder, Colorado where it was completed.

The next slide (slide 9) shows a schematic of our approach: The two experimental problems that we hoped to address in our approach were:

1) the requirement of determining the mass separations with high accuracy, and 2) the makeup of the fiber itself, which we replaced with a magnetic suspension.

The masses we used were hollow cylinders -- a shape which has the useful property that it provides a maximum in force along the axis of the cylinders, and hence an insensitivity to positioning accuracy of the force experienced by a test mass located in this region. To see why the force will have a maximum, consider the following. If you were at the center of a hollow cylindrical shape, the force would be zero (because of symmetry). If you were at infinity, it would also be zero. Since you know that the force cannot vanish everywhere, this means that as you move in along the axis from infinity, somewhere (near the face of the cylinder) there is a position at which the force on a test mass is a maximum. At this point the gradient is zero and in the region surrounding it the gravitational force is nearly constant.

Another way of looking at this is to consider the LaPlace equation which the gravitational potential satisfies (slide 10). Note that the existence of a maximum in the force along the z-axis and the symmetry of the x and y directions shows the existence of a region of uniform force. An important problem not addressed in this discussion of mass geometry is that of density inhomogeneity. Implicit in the foregoing argument was an assumption of perfect density homogeneity. To illustrate the extent of this problem (imperfect density homogeneity), take the two doughnut-shaped masses used for our experiment, which were machined from adjacent pieces of the same casting of bearing bronze. Although the two shapes were dimensionally identical to 1 part in 10^4, their measured masses differed by 5 parts in 10^3! State-of-the-art metal casting can achieve a homogeneity of about 1 part in 10^3. However, the accuracy of a G measurement would be limited to about that level unless something else, such as a detailed density survey of the masses, is done to address this problem.

The other difference in our experiment was the use of a surrogate (magnetic) torsion fiber to support a mass-dumbbell torsion pendulum. The period of this pendulum is proportional to the square root of the ratio of its inertia and the, in our case, gravitationally generated angular spring constant. The optical pickup, which together with the servo maintained the vertical position of the pendulum to better than 10^{-5} cm, was quite simple. We used a sphere (as a rotationally insensitive lens) at the bottom of the pendulum to image a light-emitting diode horizontal line source onto a split photocell -- the output of which provided the information that was needed for the servo system regarding the vertical position of the pendulum.

The restoring torque acting on the pendulum comes from three things: 1) the gravitational interaction of the pendulum with the attracting masses (doughnuts), 2) the natural gravitational gradient in the laboratory, and 3) the torsion due to imperfections of our magnetic fiber. Whereas in all previous measurements of G, the restoring constant of the fiber was dominant, for our magnetic suspension, the torsional interaction due to the suspension (in principle, it should be zero) proved to be much less

than the gravitational couples. The orientation assumed by the pendulum in the absence of the attractive masses was determined (primarily) by the horizontal gravitational gradient in the laboratory. It therefore had two stable orientations, essentially 180° apart. The local gradient was due to location of the apparatus near a wall in a large basement room as well as to the presence of the Rocky Mountains directly to the west.

A block diagram of the servo electronics is shown in slide 11. The measured position noise with the pendulum suspended is shown in slide 12. The noise is due almost entirely to seismic background. The experiment was not mounted on a vibration isolating platform. Electronic noise is imperceptible on the scale shown in this slide.

To understand the principle of our experiment, let us now derive a simplified equation for G (slide 13). It will not contain all the (small) terms. (The derivation including all of the correction terms would be too tedious to present here.) It will, however, reveal all the important factors that must be taken into account to calculate G from this experiment.

The force on a sphere of mass, m, in the region of uniform force is simply given by

$$F = -GMmQ \tag{1}$$

where Q is a function of the geometry of the doughnuts only. M and m are the masses of the doughnut and sphere respectively. The equation of motion of the pendulum is thus:

$$\ddot{\theta} = \frac{2GQMmr\theta}{I'+I} - \frac{k'\theta}{I'+I} \tag{2}$$

where I is the moment of inertia due to the spheres, I' is the moment of inertia due to the rest of the pendulum, r is the radius of the pendulum and k' is the torsional restoring constant associated with all torques other than the gravitational interaction of the pendulum and the dough- nuts. This is the familiar simple harmonic oscillation equation whose solution gives:

$$\omega^2 = \frac{2GQMmr}{I'+I} + \frac{k'}{I'+I} \quad . \tag{3}$$

We may do the same experiment without the doughnuts. In this case the relevant equation is:

$$\ddot{\theta} = \frac{k'\theta}{I'+I}$$

giving

$$\omega_0^2 = \frac{k'}{I'+I} \quad . \tag{4}$$

where ω and ω_0 are the oscillation frequencies of the pendulum with and without the doughnuts. Solving Eqs. (3) and (4) to eliminate k' gives

$$G = \frac{I'+I}{2\Omega Mmr} \left(\omega^2 - \omega_0^2\right) \quad . \tag{5}$$

Further simplification is accomplished by noting that the moment of inertia due to the balls contains two terms:

$$I = 2\left(mr^2 + \frac{2}{5} mR^2\right) \tag{6}$$

where R is the radius of the spheres. Rearranging, we have:

$$I = 2mr^2 \left(1 + \frac{2R^2}{5r^2}\right) \quad ; \tag{7}$$

and letting

$$A' = \left(1 + \frac{2R^2}{5r^2}\right) \quad , \tag{8}$$

then

$$I = 2A'mr^2 \quad . \tag{9}$$

Recall also that $\omega = 2\pi/T$ where T is the period of oscillation with the doughnuts (T_0 is the period without). Inserting these into Eq. (5) we finally get

$$G = \frac{4\pi^2 A'r}{\Omega M} \left(1 + \frac{I'}{I}\right)\left(\frac{1}{T^2} - \frac{1}{T_0^2}\right) \quad .$$

It should be emphasized that this equation is only approximate. The interaction of the far doughnut, near ball, and gravitational torques on the beam, for example, have been neglected. For that reason the value obtained for G using the above equation is in error by about 15%. Nevertheless, the equation identifies the important measurements. As an example: since r appears in the leading term, the accuracy of G cannot exceed the accuracy with which r is measured. In our case, r is approximately 16.51 cm. If an accuracy of 1×10^{-4} is desired, the error in the measurement of r cannot exceed 0.00165 cm, assuming all other measurements are exact. Note also that m does not appear anywhere except in I and that as I is made much larger than I' the need to know either accurately is reduced. To the extent T_0 can be made larger by reducing the interaction of the pendulum with external masses other than the doughnuts, the accuracy of its measurement becomes less critical. However, T_0 is limited ultimately by the horizontal gradient at about 2T so that both periods need to be carefully measured.

Thus we see that the main feature of the experiment is a dumbbell-shaped object, with the test masses on its end, which is suspended horizontally in a vacuum chamber by means of a magnetic suspension. In the laboratory, it assumes an orientation dictated by the horizontal gravitational gradient. A tracking autocollimator locked onto a mirror attached to the dumbbell records its position at regular intervals as the pendulum oscillates about its preferred orientation. From this information we determined the period T_0. Next the doughnuts are introduced into the experiment in such a way that each sphere, m, is centered in the uniform gravitational force region of the nearest doughnut and the symmetry axes of the doughnuts are coincident with the preferred direction already established. Now the period T is measured. As we have seen, the two periods T_0 and T, along with the dimensions and masses of the doughnuts and the pendulum, are sufficient to compute G.

The next slide (slide 14) shows the price you pay for getting rid of C. V. Boys' 1/1000 inch diameter fused silica fiber -- you replace it with a 7 foot tall, several hundred pound, rack of electronics.

The next slide (slide 15) shows a close-up view of the vacuum chamber, magnetic coils, and external masses. We ran this at 10^{-5} mm of Hg. Pancake-shaped silicon rubber water cooling coils were made and placed between the three magnet assemblies in order to maintain the temperature stability of the entire magnet assembly. To support the 1.5 kilogram pendulum requires approximately 60 watts.

Slide 16 shows an overall view of the apparatus. Helmholtz coils were used to eliminate the horizontal component of the earth's magnetic field.

A record of our gravitational "clock" is shown in slide 17. This is the resulting sine wave as seen by a tracking autocollimator. Notice that it dies down very slowly; if you look carefully you can discover several sinful things; nevertheless it's quite a good sine wave with a reasonably high Q. The period of this local gravity-gradient-determined oscillation (for this measurement the external masses were not in place) was $\sim 3\,^1\!/_2$ hours.

Even though the magnetic suspension, as far as vertical stability was concerned, performed beyond our expectations, its torsional stability (period and zero point) was unexpectedly poor. Variations in the pendulum period (without the doughnuts in place) were about 1.5%. These variations were (ultimately) attributed to random changes of the magnetic domains in the pole faces. Both the strength of the pole face (fiber) couple and its zero position were observed to vary by nearly 100%.

Several experiments were conducted in an effort to understand this residual torsional couple better. First of all, if the core of the supporting magnet is not exactly vertical, its upper pole interacts with the bottom pole of the pendulum causing it to tip slightly. If the pendulum is unbalanced it will rotate so the heaviest part is lowest, and it will proceed to oscillate in that position with a tip-dependent zero and

period. This behavior was observed. Two things were done to eliminate
that effect. 1) The pendulum was very carefully balanced by mounting
the ferrite rod horizontally in air bearings; balance screws were then
adjusted until its period became very long (slide 18). 2) The upper core
was set vertical to within 0.2 arc seconds. It was monitored continuous-
ly and found to remain within one arc second of the vertical over many
days. With these precautions implemented, the period and zero were still
found to vary; however the size of the changes could not be accounted for
by any possible remaining tilt effects.

In the process of carrying out various experiments we noticed that
opening the laboratory door caused a small change in the error signal.
This was determined to be an atmospheric pressure effect -- the lab was
at a slightly different pressure than the adjacent hallway. Thereafter
the pressure change bent the vacuum chamber slightly and changed the gap
between the cores. The pressure was continuously monitored with an
electronic pressure transducer. A cross correlation of barometric and
period data however showed no effect.

Experimenting with a pendulum which has a period of $3\frac{1}{2}$ hours is
very tedious. To shorten the period and at the same time isolate any
(possible) pole face interaction, the dumbbell was replaced with a short
tungsten rod of the same mass. The suspension thus saw the same weight
but the moment of inertia and quadrupole interaction with the room were
greatly reduced. The results of this set of measurements showed the con-
tribution of the pole faces to the restoring force would set the period
(with the dumbbell in place) at about 20 hours. That is a spring con-
stant of 1.15×10^{-10} Nt-m/rad. However, this spring constant fluctuated
nearly 100% and the zero shifted, randomly, many degrees. This observa-
tion is consistent with the uncertainty limit of the final result, and
our contention that the observed (and experimentally damaging) period
fluctuations were due -- at least to a considerable degree -- to changes
in the magnetic interactions of the pole faces.

An attempt was made to use another material for the cores in the
hope that the ends would be more stable magnetically. Armco Electromag-
net Iron was selected and cores were made. These cores had a steeper
point on the ends, made possible by the higher saturation flux density.
However, another difficulty arose. Eddy currents introduced a lag in the
servo (it was impossible to change the pole strength quickly enough) re-
sulting in an interaction of the servo with the first structural reso-
nance of the pendulum at 300 Hz (the "wing" flapping mode). As a result
the system could not be made to work stably and quietly without substan-
tial and very time-consuming modifications. A possible, but again diffi-
cult to implement, solution to the pole face problem that occurred to us
was to spin the upper rod so as to average out the mgnetic character of
the upper pole face.

A particularly illuminating observation was made when there was an
electrical power outage. The laboratory lights flickered and we felt
sure something dreadful would happen to the apparatus (which was run-
ning). What we found was that the oscillation period of the pendulum had

suddenly changed from $3\frac{1}{2}$ hours to 30 minutes. We took the apparatus
apart, expecting to find a chip out of one side of the top and the bottom
pieces of ferrite. (If you introduce little magnetic hills or antihills
then you'd expect to find a preferred orientation between the pole faces
and therefore a torque generated.)

However, at least to the eye, both were still perfect. I called the
company that had made the ferrite and asked, "Is there any way you can
test for micro cracks, etc.?" After I explained what had happened, the
company's representative said, "You have to remember from freshman physics
that if you take a bar of iron and beat it with a hammer in a magnetic
field, you're going to magnetically align that bar." That's exactly what
happened when the two pole pieces hit. They were in a magnetic field, for
after all the system uses a magnetic field: they hit, and it obviously
changed them magnetically. We now believe that the same process is the
source of the observed slow variation in the magnetic character of the
pole faces; as the ground moves up and down, the servo changes the applied
magnetic field to keep the vertical position stable so both rods are con-
stantly being hit with a little "magnetic hammer."

The next slide (slide 19), while summarizing the restoring torques
acting on the fiber, also compares their relative strengths. As we have
mentioned earlier, the restoring torque acting on the pendulum can be
divided into three parts: 1) the gravitational interaction of the pendu-
lum with the attracting masses (doughnuts), 2) the natural gravitational
gradient in the laboratory, and 3) the torsion due to the suspension
fiber. In all previous measurements of G, the restoring constant of the
fiber was the dominant factor, and while in our case the fiber constant
amounted to only 1% of the total, the fact that it (and its zero) varied
by virtually 100% was the fundamental limitation in the accuracy of our
result.

At this point further progress would have required extensive changes
as well as at least a year's work in order to incorporate them into the
apparatus. Since this work had successfully demonstrated a new and still
promising method for measuring G, in which all errors except those from
the period were held below the level required to increase the accuracy to
which G was known at that time, a thesis was written,[*] and approved and
the work was suspended at that time.

The final result of our attempts to measure G using a (ferrite) mag-
netic suspension is

$$G = 6.575 \pm 0.17 \times 10^{-11} \ \text{ntm}^2/\text{kg}^2 \quad .$$

The main contributor to the uncertainty comes from the suspension which
we have mentioned and which contributes an uncertainty 100 times larger
than the next largest effect -- which was the uncertainty in our mass
density.

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[*]W. A. Koldewyn, Ph.D. Thesis, Wesleyan University, 1976.

Though, in principle, one could easily have justified a second Ph.D. thesis (given a suitable student) to continue this effort and solve the problems which always arise in this kind of work even though we felt we had identified a solution to many of them, one of us (JEF) was somewhat reluctant to further this work using a magnetic suspension. We now recognize (a realization that did not come at t = 0) that dia- and paramagnetic effects associated with the materials from which the test masses were fabricated would also need to be dealt with. And this problem is only compounded by the fact that we are using them in close proximity to a magnetic suspension which (in the case of our geometry) results in a magnetic field gradient in the "interaction" region almost two orders of magnitude worse than what was there without the use of a magnetic suspension. To be sure, one could remove the magnetic portions of the fiber from their close proximity to the experiment (at the price of a long intervening rod) -- but again, it would present one more problem yet to be addressed and solved. Further, by this time, we suspected that a fluid-based fiber (see paper by Faller in PMGE School, "The Fluid-Fiber Based Torsion Pendulum: An Alternative to Simply Getting a Bigger Hammer") might in the long run prove to be the better surrogate with which to proceed on this very challenging and demanding experiment. Thank you for your kind attention.

"From the closest and most scrupulous attention I can employ on this question, the preference, in point of <u>accuracy</u>, appears to be decidedly in favour of the large mountain experiment, over that of small balls."

Dr Charles Hutton
Phil. Trans. (1821)

(1)

Not So !

~~Faller~~

(1983)

(Precision versus Accuracy)

(3)

A
D I S C O U R S E
ON THE
Attraction of Mountains,
DELIVERED AT THE
Anniverfary Meeting of the ROYAL SOCIETY,
November 30, 1775.

By Sir JOHN PRINGLE, Baronet,
PRESIDENT.

PUBLISHED BY THEIR ORDER.

LONDON:
Printed for the ROYAL SOCIETY.

M.DCC.LXXV.

(2)

I fhall not confider the fubject of attraction at large, nor touch upon any fpecies of it, excepting what in latter times, by the effects, has been diftinguifhed by the name of *gravity* or *gravitation*; a property of bodies perceptible to the vulgar when things fall to the ground, but long acknowledged by this Society, to be a quality impreffed by the Creator on all matter, whether of the earth or of the heavens, whether at reft or in motion: *He commanded, and it was created.*

(4)

(5)

(6)

(7)

'I had hoped to have made a greater number of experiments under more widely differing conditions, but the strain which they entail is too severe, for not only have I had to give up holidays for the last three years, but to leave London on Saturdays and occasionally to sit up all Saturday and Sunday nights at the end of a week's work. The conditions, therefore, are too difficult for such an extended series as I should like to make to be possible, and I must after one more effort, leave the problem to others who have leisure, and what is of far greater consequence, a quiet country place undisturbed by road and railway traffic, and who possess the knowledge and manipulative skill which the experiment requires...."

Professor C. V. Boys[1]

[1] Professor C. V. Boys, "On The Newtonian Constant of Gravitation Philosophical Transactions of the Royal Society of London for Year MDCCCXCV. Volume 186.

(8)
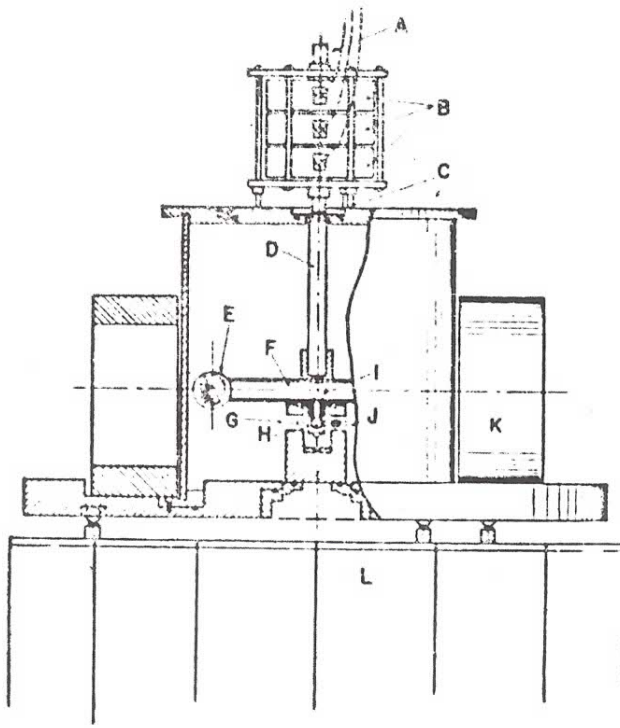
A	FERRITE CORE	G	LIGHT EMITTING DIODE
B	ALUMINUM FOIL COILS	H	SAPPHIRE SPHERE
C	FERRITE MAGNETIC "WINDOW"	I	MIRROR
D	FERRITE RODE	J	SPLIT PHOTODIODE
E	ALUMINUM BRONZE BALLS	K	BRASS DOUGHNUT MASSES
F	QUARTZ TUBE	L	ALUMINUM SUPPORT TUBES

(9)

$$\nabla^2 \phi = 0$$

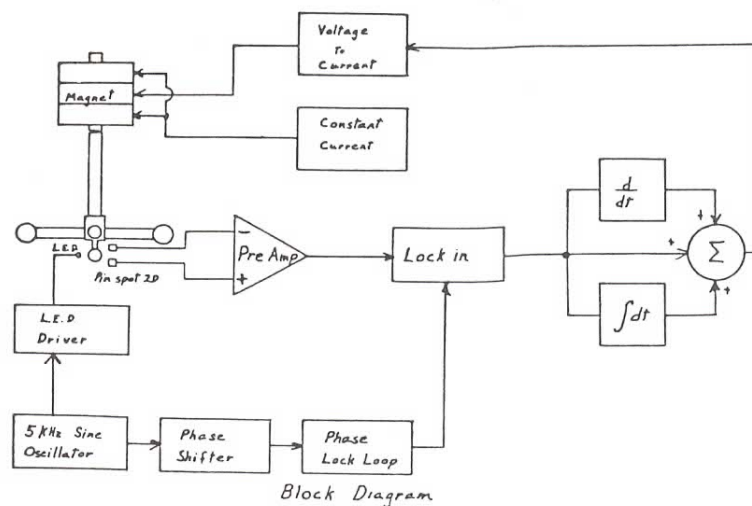$$\therefore \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0$$

@ Max Force $\frac{\partial F_z}{\partial z} = 0$

by symmetry $\frac{\partial F_y}{\partial y} = \frac{\partial F_x}{\partial x}$

$$\therefore \frac{\partial F_y}{\partial y} = \frac{\partial F_x}{\partial x} = 0$$
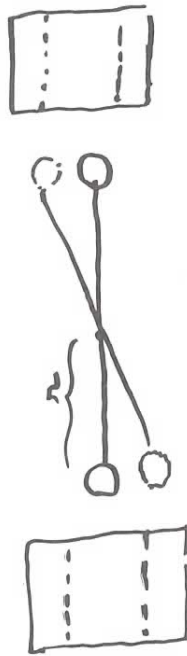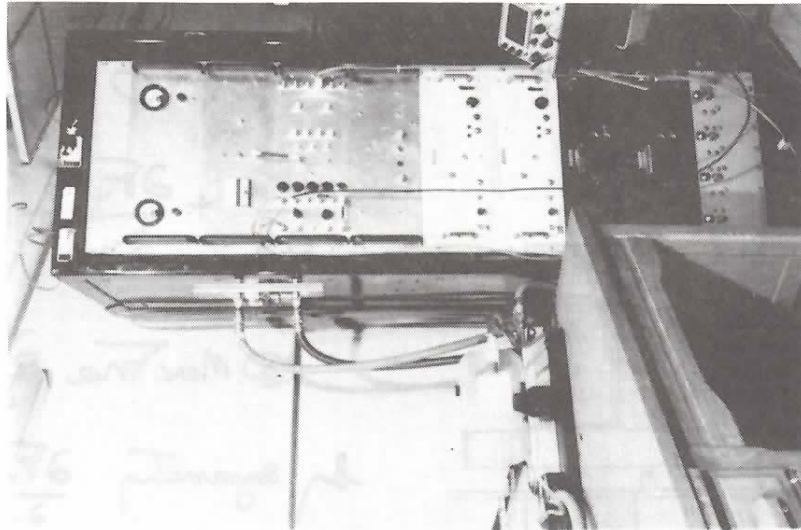
$\Longrightarrow$ Region of uniform force

(10)

Block Diagram

(11)

(14)

(12)

5×10^{-5} in

.004 sec

seen from above

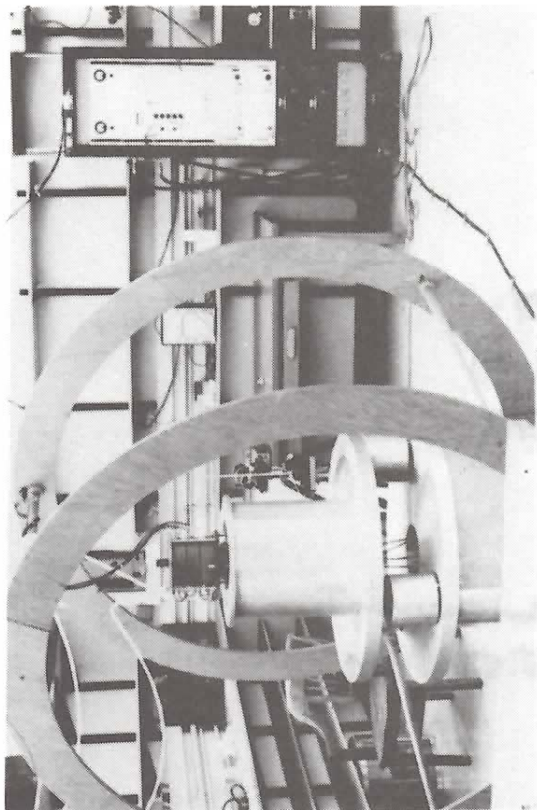$$\ddot{\theta} = \frac{2GQ\,Mm\,r\,\delta}{I'+I} - \frac{K'\theta}{I+I'}$$

algebra

$$G \sim \frac{4\pi^2 A' r}{Qm}\left(1 + \frac{I'}{I}\right)\left(\frac{1}{T_i} - \frac{1}{T_0}\right)$$
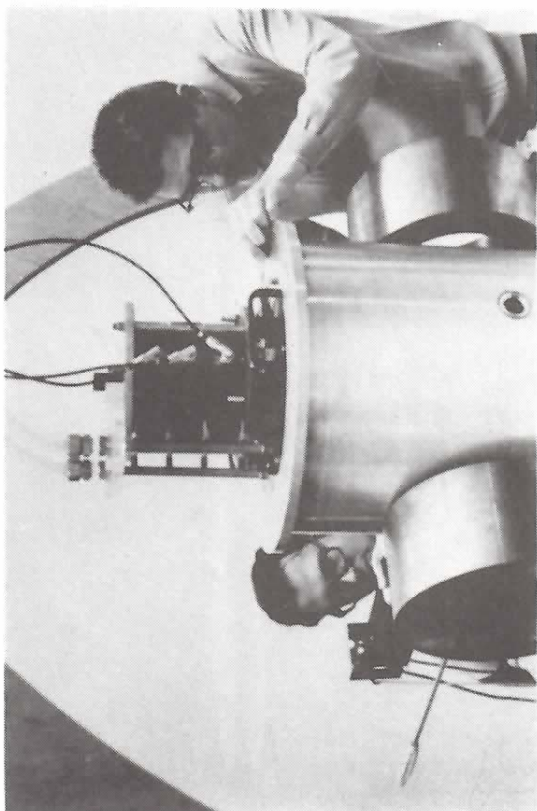
(13)
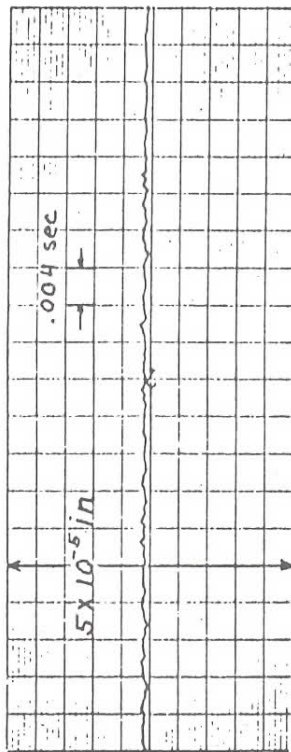
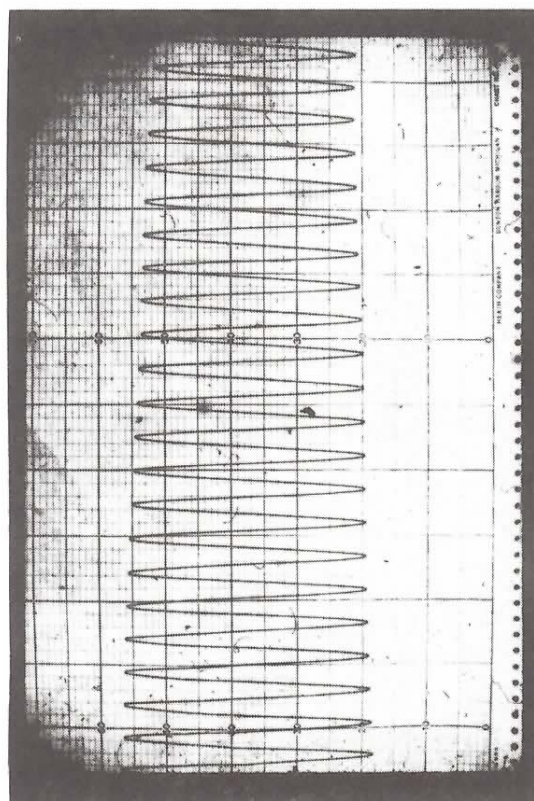(16)

(18)

(15)

(17)

Restoring torques

1) Grav. interaction with attracting masses (doughnuts)

2) Grav. gradient in laboratory

3) Suspension fiber

In all previous torsion pendulum measurements of G, the restoring torsional const. of the fiber has dominated

"K"

1) $T \sim 90$ min $\equiv 1$

2) $T \sim 4$ hours $\sim \frac{1}{7}$ $10\% g(1)$

3) $T \sim 20$ hours $\sim \frac{1}{170}$ $1\% g(1)$

(19)