Quantum Field Theory

Ling-Fong Li

11			• • •		- `
11	n	c†	ıt.	+	e 1
		36		սւ	\sim

æ

・ロト ・聞ト ・ ほト ・ ほト

QCD

Quark Model

Isospin symmetry

To a good approximation, nuclear force is independent of the electromagnetic charge carried by the nucleons — charge independence. To implement this, we say strong interaction has an SU(2) symmetry $n \leftrightarrow p$. These SU(2) generators T_1, T_2, T_3 satisfy commutation,

$$[T_i, T_j] = i\epsilon_{ijk}T_k$$

Acting on n or p

$$T_3|p\rangle = \frac{1}{2}|p\rangle, \quad T_3|n\rangle = -\frac{1}{2}|n\rangle, \quad T_+|n\rangle = |p\rangle, \quad T_-|p\rangle = |n\rangle \cdots$$

This means n and p form a doublet under isospin transformation. Isospin invariance simply means that

$$[T_i, H_s] = 0$$

where H_s is strong interation Hamiltonian.

We can extend isospin assignments to other hadrons . For example we get the following isospin multiplets,

$$(\pi^+, \pi^0, \pi^-)$$
 $I = 1$, (K^+, K^0) , $(\bar{K^0}, K^-)$ $I = \frac{1}{2}$, η , $I = 0$

$$(\Sigma^+, \Sigma^0, \Sigma^-)$$
 $I = 1$, (Ξ^0, Ξ^-) , $I = \frac{1}{2}$, Λ , $I = 0$
 (ρ^+, ρ^0, ρ^-) $I = 1$, (K^{+*}, K^{0*}) , $(K^{\overline{0}*}, K^{*-})$ $I = \frac{1}{2}$...

If isospin symmetry were exact, then all particles in multiplets have same masses, which is not the case in nature. But the mass difference within the isospin multiplets seems to be quite small.

$$\frac{m_n - m_p}{m_n + m_p} \sim 0.7 \times 10^{-3}, \qquad \frac{m_{\pi +} - m_{\pi 0}}{m_{\pi +} + m_{\pi 0}} \sim 1.7 \times 10^{-2} \qquad \cdots$$

Thus isospin symmetry as approximate one and maybe it is good to few %.

3

イロト イポト イヨト イヨト

SU(3) symmetry and Quark Model

When Λ and k particles were discovered, they were produced in pair (associated production) with longer life time. It was postulated that these new particles possessed a new additive quantum number, **strangeness** S, conserved by strong interaction but violated in decays,

$$S(\Lambda^0) = -1$$
, $S(K^0) = 1$...

Extension to other hadrons, we can get a general relation,

$$Q=T_3+\frac{Y}{2}$$

where Y = B + S is called hyperchargee, and B is the baryon number. This is known as Gell-Mann-Nishijima relation.

Eight-fold way : Gell-Mann, Neeman

Group togather mesons or baryons with same spin and parity,

A B F A B F







(c)



These are the same as irreducible representations of SU(3) group. The spectra of hadrons show (Institute) Quark Model 5 / 14

Quark Model

One peculiar feature of eight fold way is that octet and decuplet are not the fundamental representation of SU(3) group. In 1964, Gell-mann and Zweig independently propose the quark model: all hadrons are built out of spin $\frac{1}{2}$ quarks which transform as the fundamental representation of SU(3),

$$q_i = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

with the quantum numbers

	Q	Т	T_3	Y	S	В
и	2/3	1/2	+1/2	1/3	0	1/3
d	-1/3	1/2	-1/2	1/3	0	1/3
s	-1/3	0	0	-2/3	$^{-1}$	1/3

In this scheme, mesons are $q \bar{q}$ bound states. For examples,

$$\begin{array}{lll} \pi^+ & \sim & \bar{d}u & \pi^0 \sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d). & \pi^- \sim \bar{u}d \\ \\ K^+ & \sim & \bar{s}u & K^0 \sim \bar{s}d \end{array}, \quad K^- \sim \bar{u}s. \quad \eta^0 \sim \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \end{array}$$

3

and baryons are qqq bound states,

$$\begin{array}{lll} p & \sim & uud \ , \ n \sim ddu \\ \Sigma^+ & \sim & suu \ , \ \Sigma^0 \sim s \left(\frac{ud + du}{\sqrt{2}} \right) \ , \ \Sigma^- \sim sdd \\ \Xi^0 & \sim & ssu \ , \ \Xi^- \sim ssd \ , \ \Lambda^0 \sim \frac{s(ud - du)}{\sqrt{2}}. \end{array}$$

Quantum numbers of the hadrons are all carried by the quarks. But we do not know the dynamics which bound the quarks into hadrons. Since quarks are the fundamental constituent of hadrons it is important to find these particles. But over the years none have been found. **Paradoxes of simple quark model**

- Quarks have fractional charges while all observed particles have integer charges. At least one of the quarks is stable. None has been found.
- Hadrons are exclusively made out qq, qqq bound states. In other word, qq, qqqq states are absent.
- **3** The quark content of the baryon N^{*++} is *uuu*. If we chose the spin state to be $\left|\frac{3}{2}, \frac{3}{2}\right\rangle$ then all quarks are in spin-up state $\alpha_1 \alpha_2 \alpha_3$ is totally symmetric. If we assume that the ground state has I = 0, then spatical wave function is also symmetric. This will leads to violation of Pauli exclusion principle.

イロト イヨト イヨト

Color degree of freedom

One way to get out of these problems, is to introduce color degrees of freedom for each quark and postulates that only color singlets are physical observables. 3 colors are needed to get antisymmetric wave function for N^{*++} and remains a color singlet state. In other words each quark comes in 3 colors,

$$u_{\alpha}=(u_1,u_2,u_3)$$
 , $d_{\alpha}=(d_1,d_2,d_3)\cdots$

All hadrons form singlets under $SU(3)_{color}$ symmetry, e.g.

$$N^{*++} \sim u_{\alpha}(x_1) \alpha_{\beta}(x_2) u_{\gamma}(x_3) \varepsilon^{\alpha \beta \gamma}$$

Futhermore, color singlets can not be formed from the combination qq, qqqq and they are absent from the observed specrum. Needless to say that a single quark is not observable.

Gell-Mann Okubo mass formula

Since SU(3) is not an exact symmetry, we want to see whether we can understand the pattern of the SU(3) breaking. Experimentally, SU(2) seems to be a good symmetry, we will assume isospin symmetry to set $m_u = m_d$. We will assume that we can write the hadron masses as linear combinations of quark masses.

0 <u>o</u> mesons

$$\begin{split} m_{\pi}^{2} &= (m_{o}+2m_{u}) \\ m_{k}^{2} &= m_{o}+m_{u}+m_{s} \\ m_{\eta}^{2} &= m_{o}+\frac{2}{3}(m_{u}+2m_{s}) \end{split}$$

Eliminate the quark masses we get

$$4m_k^2 = m_\pi^2 + 3m_\eta^2$$

This known as the Gell-Mann Okubo mass formula. Experimentally, we hav $LHS = 4m_k^2 \simeq 0.98(Gev)^2$ while $RHS = m_\pi^2 + 3m_\eta^2 \simeq 0.92(Gev)^2$ This seems to show that this formula works guite well.

• • = • • = •



We get the mass relation

$$\frac{m_{\Sigma}+3m_{\Lambda}}{2}=m_{N}+m_{\Xi}$$

Expermentally,
$$\frac{m_{\Sigma} + 3m_{\Lambda}}{2} \simeq 2.23 \text{ Gev}$$
, and $m_N + m_{\Xi} \simeq 2.25 \text{ Gev}$.
 $\frac{3}{2}^+ \text{ baryon}$
 $m_{\Omega} - m_{\Xi^*} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Sigma^*} - m_{N^*}$

This is referred to as equal spacing rule. In fact when this relation is derived the particle Ω has not yet been found and this relation is used to predicted the mass of Ω and subsequent discovery gives a very strong support to SU(3) symmetry.

nstitute)	Quark Model	

2

・ロト ・ 日 ・ ・ ヨ ト ・ ヨ ト ・

 $\omega - \phi$ mixing

For 1^- mesons, the situation seems to be different. If we make an analogy with o^- mesons, we would get Gell-Mann Okubo mass relation as,

$$3m_{\omega}^2 = 4m_{K^*}^2 - m_{\rho}^2$$

Using $m_{K^*} = 890 \text{ Mev}$, $m_{\rho} = 770 \text{ Mev}$ we get $m_{\omega} = 926.5 \text{ Mev}$ from this. But experimentally, $m_{\omega} = 783 \text{Mev}$ which is quite far away. On the other hand there is a ϕ meson with mass $m_{\phi} = 1020 \text{ Mev}$ and has same SU(2) quantum number as ω . In principle, when SU(3) symmetry is broken, $\omega - \phi$ mixing is possible. Suppose for some reason there is a significant $\omega - \phi$ mixing we want to see whether this can save the mass relation. Denote the SU(3) octet state by V_8 and singlet state by V_1

$$V_8 = rac{1}{\sqrt{6}} (ar{u} u + ar{d} d - 2ar{s}s)$$
 , $V_1 = rac{1}{\sqrt{3}} (ar{u} u + ar{d} d + ar{s}s)$

Write the mass matrix as

$$M = \left(egin{array}{cc} m_{88}^2 & m_{18}^2 \ m_{18}^2 & m_{11}^2 \end{array}
ight)$$

Assume that the octet mass is that predicted by Gell-Mann Okubo mass relation, i.e.

$$3m^2_{88} = 4m^2_{K^*} - m^2_{\rho}$$

After diagonalizing the mass matrix M, we get

$$R^+MR = M_d = \begin{pmatrix} m_{\omega}^2 & 0\\ 0 & m_{\phi}^2 \end{pmatrix}$$
, with $R = \begin{pmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{pmatrix}$

Thus

$$\omega = \cos \theta V_8 - \sin \theta V_1$$

$$\phi = \sin \theta V_8 + \cos \theta V_1$$

and

$$\sin heta=\sqrt{rac{(m^2_{88}-m^2_\omega)}{(m^2_\phi-m^2_\omega)}}$$

Using $m_{88} = 926.5$ Mev from Gell-Mann Okubo mass formula, we get

 $\sin \theta = 0.76$

This is very close to the ideal mixing sin $\theta = \sqrt{\frac{2}{3}} = 0.81$ where mass eigenstates have a simple form,

$$\omega = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$\phi = \bar{s}s$$

This means that the physical ϕ meson is mostly made out of s quarks in this scheme. **Zweig rule**

(Institute)

Since ω and ϕ have same quantum numbers under SU(2), one expects they have similar decay widths. Experimentally, $\omega \to 3\pi$ mostly, but $\phi \to 3\pi$ is very suppressed relative to $\phi \to KK$ channel even though $\phi \to KK$ has very small phase space since $m_{\phi} = 1020 \text{ Mev}$ and $m_k \approx 494 \text{ Mev}$.

$$B(\phi
ightarrow KK) pprox 85\%$$
 , $B(\phi
ightarrow \pi\pi\pi) \sim 28\%$

Quark diagrams

In term quarks contents, the decays of ϕ meson proceed as following diagrams



FIG. 4.10. ϕ decays: (a) disallowed; (b) allowed by the Zweig rule.

Zweig rule postulates that processes involving quark-antiquark annihilation are highly suppressed for some reason. This explains why ϕ has a width $\Gamma_{\phi} \approx 4.26$ Mev smaaler than $\Gamma_{\omega} \approx 8.5$ Mev.

 J/ψ and charm quark

In 1974 the $\psi/J(3100)$ particle was discovered with unusually narrow width, $\Gamma \sim 70 \text{ kev}$, compared to $\Gamma_{\rho} \sim 150 \text{ Mev}, \Gamma_{\omega} \sim 10 \text{ Mev}$.

Simple explanation, $\psi/J \sim \bar{c}c$ and is below the threshold of decaying into 2 mesons containing charm quark. It can only decay by $c\bar{c}$ annihilation in the initial state. By Zweig rule, these decays are highly suppressed and have very narrow width.

< 3 > < 3 >